On Measuring Aggregate Price Changes with Product Turnover

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Abstract

When product turnover occurs, the Jevons index also experiences significant chain drift. Additionally, the GEKS index does not have economic interpretations. This paper proposes a **transitive** and economically meaningful index formula for situations involving product turnover. Specifically, it assumes a CES-type utility function and demonstrates that the corresponding cost-of-living index can vary significantly depending on whether the turnover is driven by demand or supply factors. It then proposes a method for **identifying demand and supply shocks** and applies it to various types of data.

Product Turnover and Chain Drifts

- Scanner data tend to exhibit a high rate of product turnovers, which makes it difficult to use direct indices for long run price comparisons.
- High rate of product turnover often causes serious chain drifts.

Ordinal COLI

- Balk (1989) introduces an Ordinal COLI, accounting for heterogeneous preferences:

$$COLI^O(s,t,q_f) = \frac{E_t(p_t, U_t(q_f))}{E_s(p_s, U_s(q_f))}.$$

- Here, $q_f = (q_{1f}, q_{2f}, ..., q_{Nf})$ represents an exogenous reference quantity vector.
- The Ordinal COLI remains invariant to monotonic transformations of utility functions. We do not need a normalization condition because it relies on ordinal utility.
- The COLI is transitive, free from chain drifts.

COLI with CES Preferences

When the utility function is CES, the ordinal COLI can be written as

$$COLI\left(s,t,q_f\right)\_B = \frac{\left(\sum_{i=1}^{N_t}\left(\frac{p_{it}}{\varphi_{it}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \times \left(\sum_{i=1}^{N_t}\left(\varphi_{it}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{i=1}^{N_s}\left(\frac{p_{is}}{\varphi_{is}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \times \left(\sum_{i=1}^{N_s}\left(\varphi_{is}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}.$$

- To calculate the above COLI, we need to
  1. estimate φ_{it}
  2. specify the exogenous reference vector q_{if}

Proposition 1

When $\sigma \neq 1$, the ordinal COLI can be written as

$$COLI\left(s,t,q_f\right)\_B = \frac{\left(\sum_{i=1}^{N_t}\left(p_{it}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\left(w_{it}\right)^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{i=1}^{N_s}\left(p_{is}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\left(w_{is}\right)^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}$$

where w_{it} is the expenditure share of commodity i at time t.

Proposition 2

Suppose m_i is a continuous mean function of q_{im}. If $PI\left(p_s, p_t, q\right)$ is invariant to proportional changes of any one state, passes **the transitivity test** and the state reversal test, then m_i should have the following functional form,

$$m_i = \prod_{m=1}^{M}\left(q_{im}\right)^{1/M}.$$

Supply COLI

We have two COLIs. The one assuming that product turnover occurs due to demand shocks: people loose interest in consuming the commodities,

$$COLI\left(s,t,q_f\right)\_D = \frac{\left(\sum_{i\in\Omega_t}\left(p_{it}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\left(\omega_{it}\right)^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{i\in\Omega s}\left(p_{is}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\left(\omega_{is}\right)^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}$$

When product turnover occurs due to supply reason, we get the supply COLI: in spite of demands among people, product become unavailable

$$COLI\left(s,t,q_f\right)\_S = COLI\left(s,t,q_f\right)\_D \times \frac{\left(\sum_{i\in\Omega_s}\left(\varphi_{is}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{i\in\Omega_t}\left(\varphi_{it}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}} \times \frac{\left(\sum_{i\in\Omega}\left(\varphi_{it}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{i\in\Omega}\left(\varphi_{is}q_{if}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}$$

Identification of Demand and Supply shocks

To estimate the aggregate demand and supply shocks, we need to separate the aggregate shocks D_t and Z_t from the idiosyncratic shocks, ε_{it} and ξ_{it}

Demand function: $\Delta \ln p_{it} = \Delta \ln COLI - \frac{1}{(\sigma-1)}\Delta \ln w_{it} + D_t + \varepsilon_{it}$

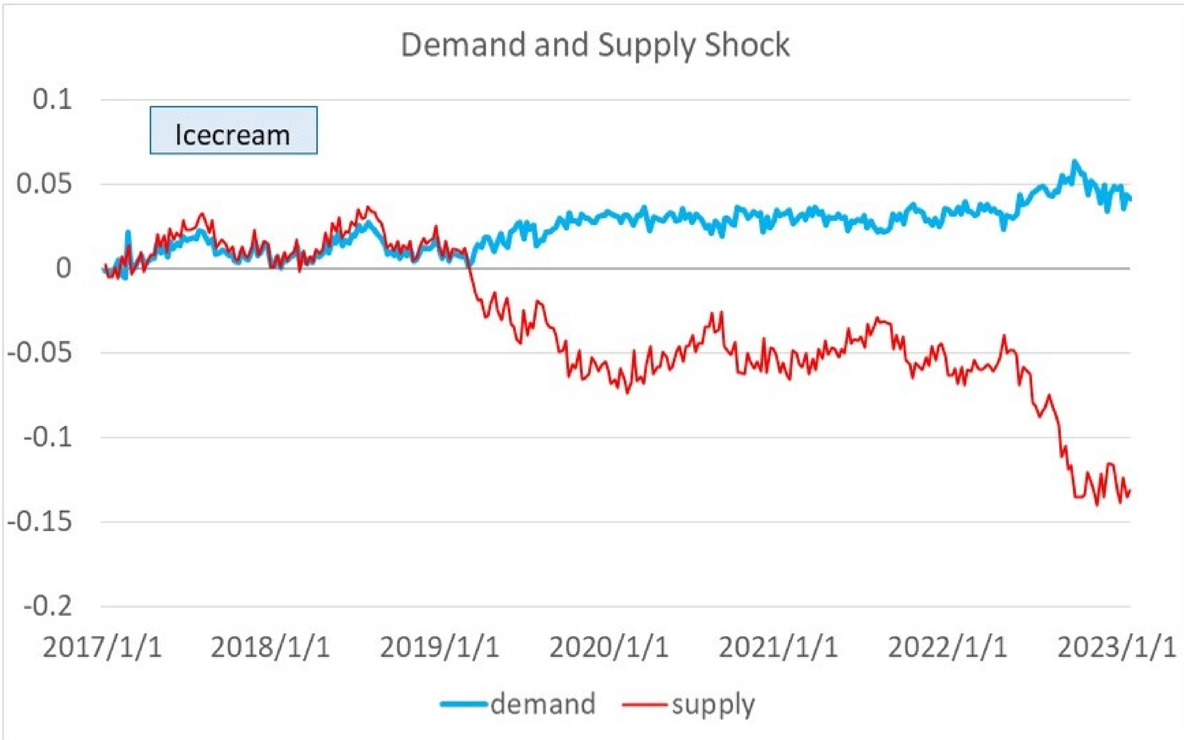
Supply function: $\Delta \ln p_{it} = \frac{1}{\omega}\Delta \ln q_{it} - Z_t + \xi_{it}$

We adopt the following identification assumptions,

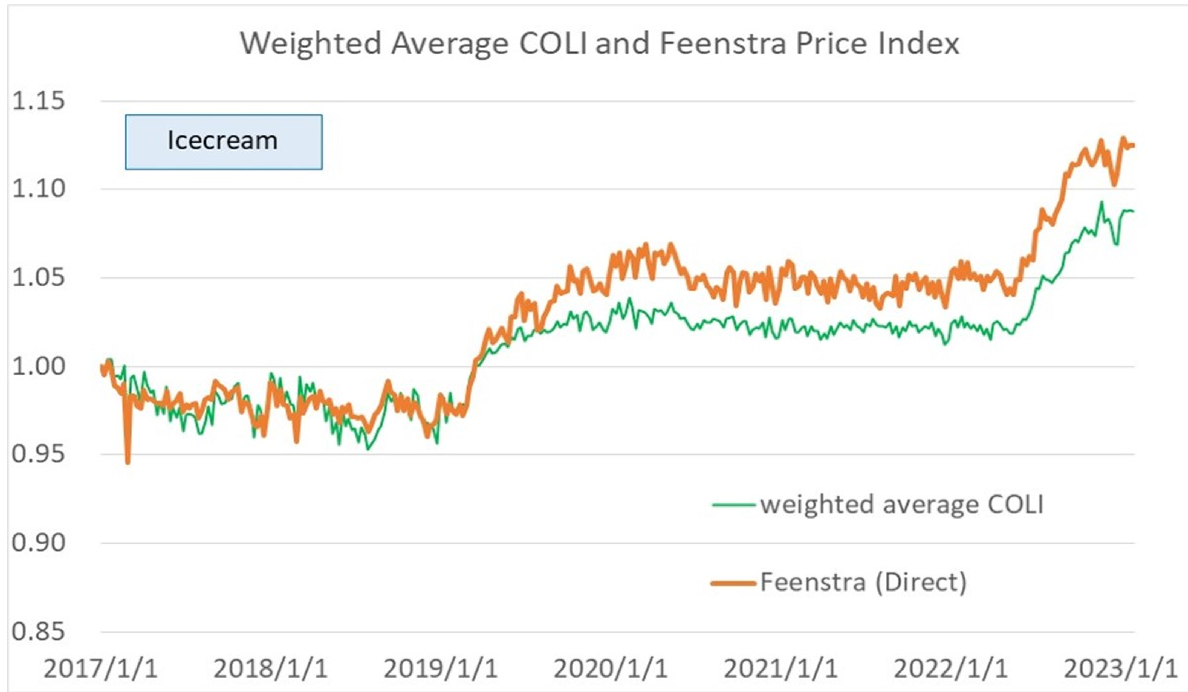
$$\sum_{i=1}^{N_c} sv_{it}\xi_{it} = \sum_{i=1}^{N_c} sv_{it}\varepsilon_{it} = 0$$

where sv_{it} is the Sato-Vartia weights, which makes the Feenstra's index as our special case when no demand shocks occur.

Demand and Supply Decomposition in Ice Cream



Cost of Living Index for Ice Creams



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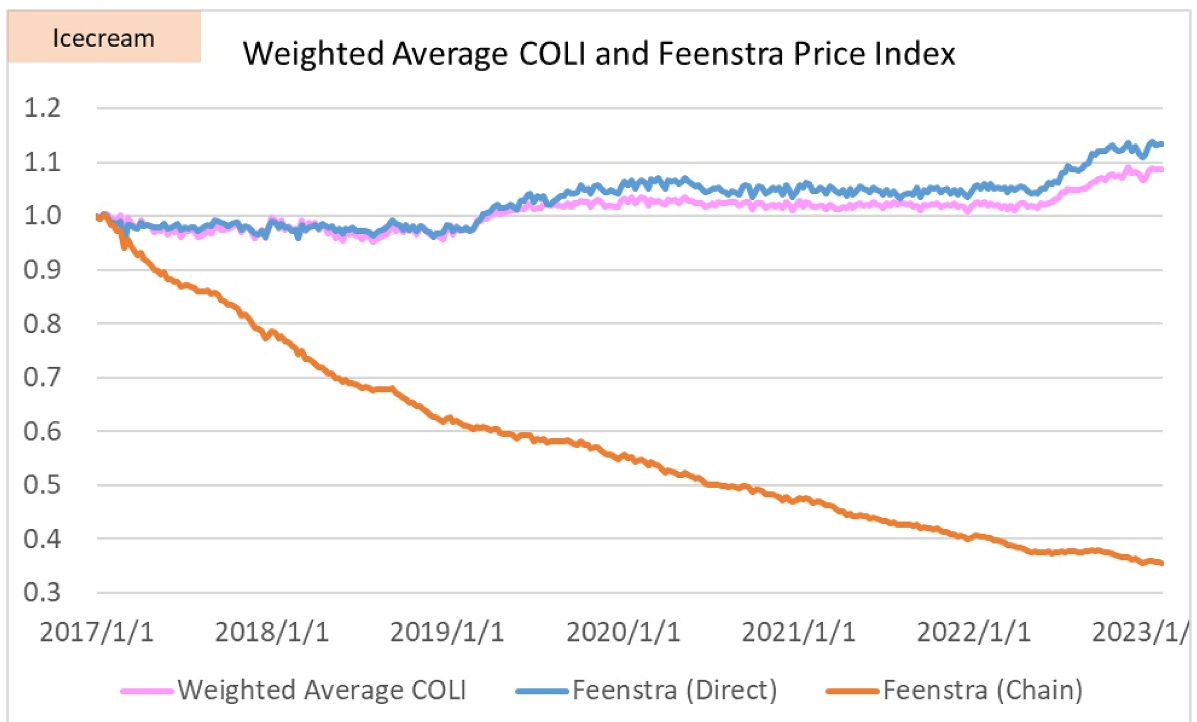

Conclusion

Our COLI is transitive, easy to construct, and provides estimates of demand and supply shocks. It can also reflect the effects of "shrinkflation," which is challenging for GEKS or GK indices. Additionally, our COLI can be estimated even when the elasticity of substitution is less than one, making it suitable for upper-level aggregation.