Higher-Level Aggregation with Long-Term Links

– An Application to the Swedish CPI

Olivia Ståhl
Statistics Sweden

olivia.stahl@scb.se

SCB

Higher-Level Aggregation with Long-Term Links

– An Application to the Swedish CPI

- Long-term links vs revising the series

Higher-Level Aggregation with Long-Term Links

– An Application to the Swedish CPI

- **Long-term links vs revising the series**
- **3 different higher-level aggregation methods**
  - Method used in the Swedish CPI
  - HICP method
  - Previous CPI method (”mixed approach”)

SCB

Higher-Level Aggregation with Long-Term Links

– An Application to the Swedish CPI

- **Long-term links vs revising the series**
- **3 different higher-level aggregation methods**
  - Method used in the Swedish CPI
  - HICP method
  - Previous CPI method (”mixed approach”)
- **Empirical evaluation**
  - Long run effects 2005-2023
  - Year-on-year effects (benchmark comparison)

SCB

Long-term links VS
Retrospective series

$\boldsymbol{t})$ $$C_0^t = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t}$$

Long-term links VS Retrospective series

$\boldsymbol{t})$ $$C_0^t = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t}$$

Revised series

$\boldsymbol{t+1})$ $$C_0^t = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t}$$
$$C_0^{t+1} = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t} \times \boldsymbol{I_t^{t+1}}$$

Long-term links VS Retrospective series

$\boldsymbol{t})$ $$C_0^t = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t}$$

Revised series

$\boldsymbol{t+1})$ $$C_0^t = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t}$$
$$C_0^{t+1} = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t} \times \boldsymbol{I_t^{t+1}}$$

Long-term link approach

$\boldsymbol{t+1})$ $$C_0^t = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t}$$
$$C_0^{t+1} = I_0^1 \times I_1^2 \times \cdots \times I_{t-2}^{t-1} \times \boldsymbol{I_{t-1}^t} \times \boldsymbol{I_t^{t+1}}$$

SCB

Higher-level aggregation methods

- HICP method
- Previous Swedish CPI method
- Current Swedish CPI method

HICP method

$$C_r^{y,m} = I_r^{r,12} \cdot \left[S_{r,12}^{r+1,12} \cdot \ldots \cdot S_{y-2,12}^{y-1,12} \right] \cdot S_{y-1,12}^{y,m}$$

$$S_{y-2}^{y,m} = \frac{\sum_g p_g^{y,m} q_g^{y-2}}{\sum_g p_g^{y-1,12} q_g^{y-2}}$$ *Laspeyres-type*

HICP method

$$C_r^{y,m} = I_r^{r,12} \cdot \left[S_{r,12}^{r+1,12} \cdot \ldots \cdot S_{y-2,12}^{y-1,12} \right] \cdot S_{y-1,12}^{y,m}$$

$$S_{y-2}^{y,m} = \frac{\sum_g p_g^{y,m} q_g^{y-2}}{\sum_g p_g^{y-1,12} q_g^{y-2}}$$

Laspeyres-type

From 2021:

$$S_{y-1,12}^{y,m} = \frac{\sum_g p_g^{y,m} \hat{q}_g^{y-1}}{\sum_g p_g^{y-1,12} \hat{q}_g^{y-1}}$$

Previous CPI method

$$C_r^{y,m} = I_r^{r,12} \cdot \left[L_{r,12}^{r+1,12} \cdot \ldots \cdot L_{y-2,12}^{y-1,12} \right] \cdot S_{y-1,12}^{y,m}$$

$$S_{y-1,12}^{y,m} = \frac{\sum_g p_g^{y,m} \hat{q}_g^{y-1}}{\sum_g p_g^{y-1,12} \hat{q}_g^{y-1}}$$ *Laspeyres-type*

$$L_{y-1,12}^{y,12} = \frac{\sum_g p_g^{y,12} \hat{q}_g^{y}}{\sum_g p_g^{y-1,12} \hat{q}_g^{y}}$$ *Laspeyres/mid-year-type*

Current CPI method

$$C_r^{y,m} = \left[L_r^{r+1} \cdot \ldots \cdot L_{y-3}^{y-2}\right] \cdot S_{y-2}^{y,m}$$

$$L_{y-1}^{y} = \frac{\sum_g p_g^{y} \sqrt{q_g^{y-1} \cdot q_g^{y}}}{\sum_g p_g^{y-1} \sqrt{q_g^{y-1} \cdot q_g^{y}}}$$ *Walsh*

$$S_{y-2}^{y,m} = \frac{\sum_g p_g^{y,m} q_g^{y-2}}{\sum_g p_g^{y-2} q_g^{y-2}}$$ *Laspeyres*

Current CPI method

$$C_r^{y,m} = \left[L_r^{r+1} \cdot \ldots \cdot L_{y-3}^{y-2}\right] \cdot S_{y-2}^{y,m}$$

$$L_{y-1}^{y} = \frac{\sum_g p_g^y \sqrt{q_g^{y-1} \cdot q_g^y}}{\sum_g p_g^{y-1} \sqrt{q_g^{y-1} \cdot q_g^y}}$$ *Walsh*

$$S_{y-2}^{y,m} = \frac{\sum_g p_g^{y,m} q_g^{y-2}}{\sum_g p_g^{y-2} q_g^{y-2}}$$ *Laspeyres*

From 2021:

$$S_{y-2}^{y,m} = \frac{\sum_g p_g^{y,m} \hat{q}_g^{y-1}}{\sum_g p_g^{y-2} \hat{q}_g^{y-1}}$$

Laspeyres/mid-year-type

Empirical evaluation

➢ **Long run effects 2005-2023**

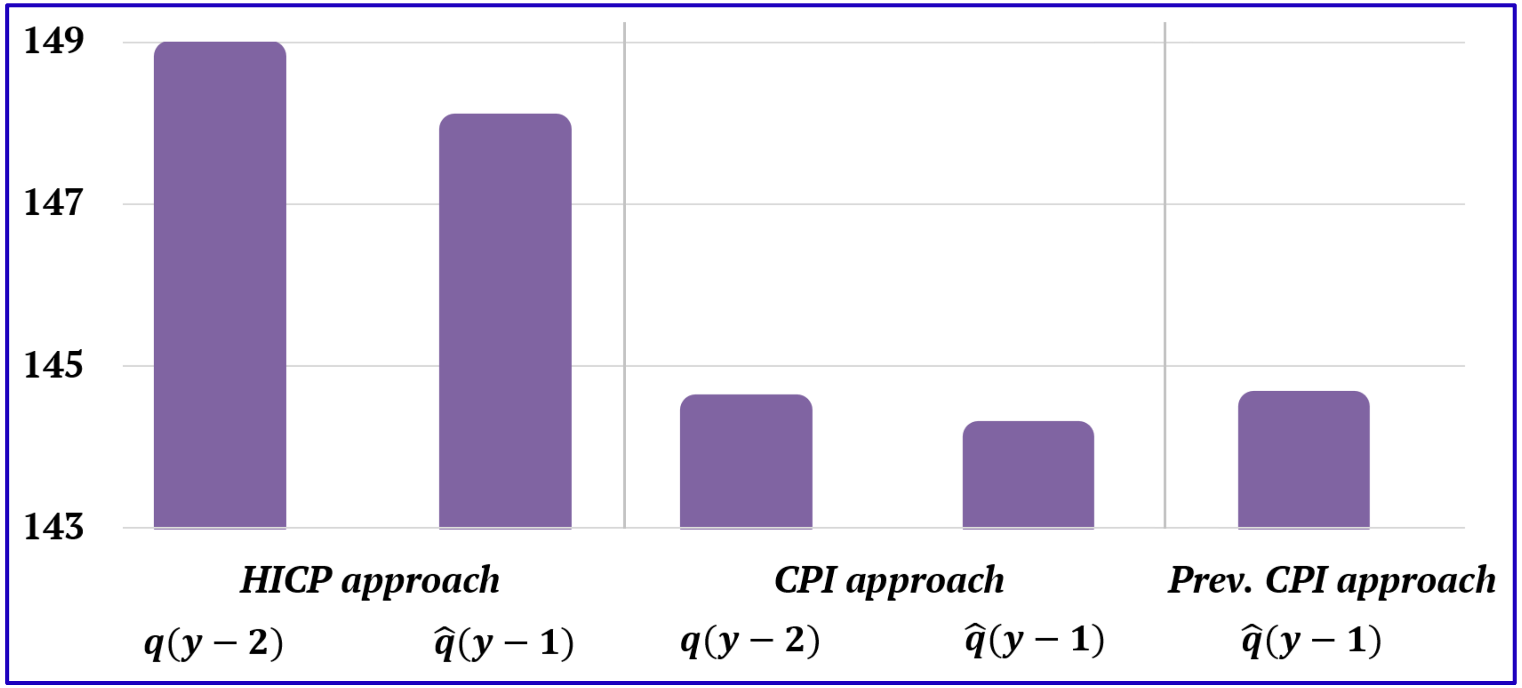
- Chained series compiled for 2005 – 2023
- Compare average index value in 2023

Empirical evaluation

- **Long run effects 2005-2023**
  - Chained series compiled for 2005 – 2023
  - Compare average index value in 2023
- **Year-on-year effects (benchmark comparison) 2007-2022**
  - Y-o-y rates of change compiled from chained series
  - Comparison with ”fisher-like” benchmark
  - *Reweighting* and *pure-price effects*

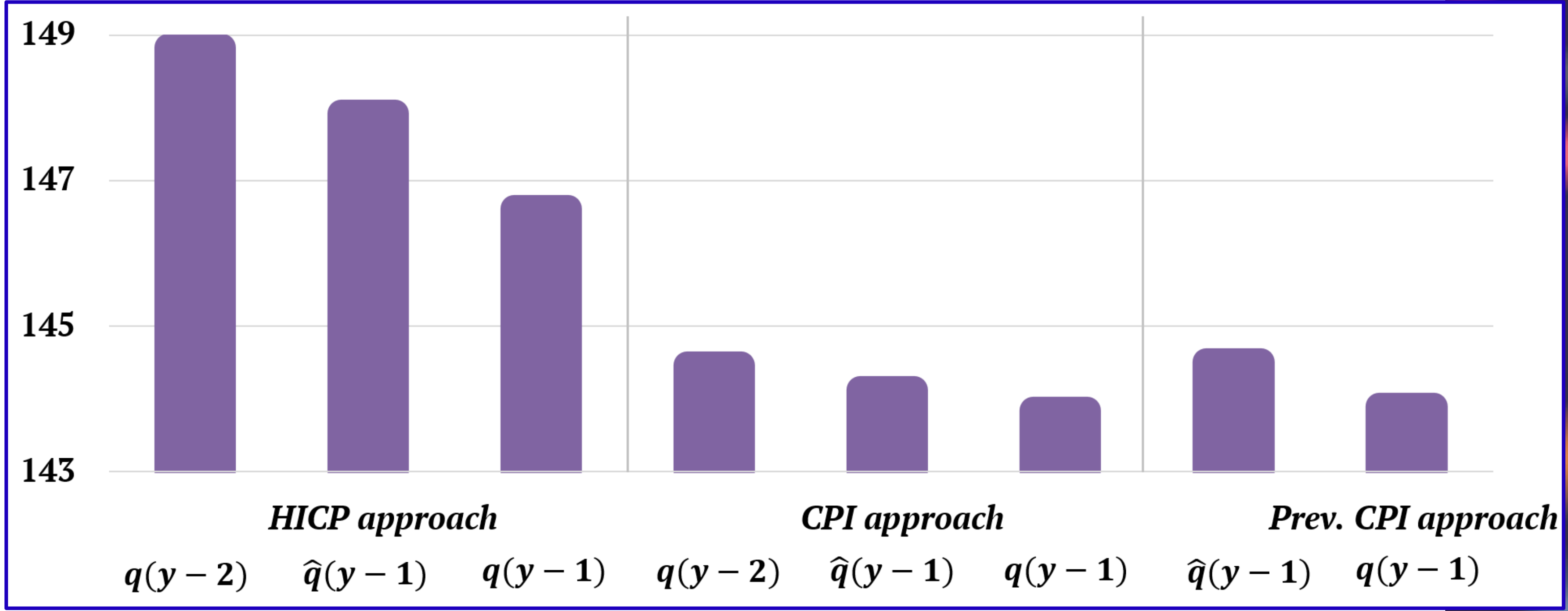
Long-run effects

Average index value in 2023:



Long-run effects

Average index value in 2023:



SCB

Year-on-year effects

Year-on-year rate of change:

$$\widetilde{\Delta}^{y,m}_{y-1,m} = \frac{C_r^{y,m} - C_r^{y-1,m}}{C_r^{y-1,m}} = \frac{C_r^{y,m}}{C_r^{y-1,m}} - 1$$

Year-on-year effects

Year-on-year rate of change:

$$\widetilde{\Delta}_{y-1,m}^{y,m} = \frac{C_r^{y,m} - C_r^{y-1,m}}{C_r^{y-1,m}} = \frac{C_r^{y,m}}{C_r^{y-1,m}} - 1$$

Benchmark:

$$\theta_{y-1,m}^{y,m} = \sqrt{\frac{\sum_g \, p^{y,m}\tilde{q}^{y-1,m}}{\sum_g \, p^{y-1,m}\tilde{q}^{y-1,m}} \cdot \frac{\sum_g \, p^{y,m}\tilde{q}^{y,m}}{\sum_g \, p^{y-1,m}\tilde{q}^{y,m}}} - 1$$

where $\tilde{q}_g^{y,m}$ = weighted average of quantities in a period of 12 months

Year-on-year effects

"Total error" for a particular period (*y,m*):

$$\epsilon_{y,m}^{T} = \widetilde{\Delta}_{y-1,m}^{y,m} - \theta_{y-1,m}^{y,m}$$

$$RMSE = \sqrt{\frac{\sum_{y=2007}^{2022} \sum_{m=1}^{12} \left(\epsilon_{y,m}^{T}\right)^2}{192}}$$

$$AV = \frac{\sum_{y=2007}^{2022} \sum_{m=1}^{12} \epsilon_{y,m}^{T}}{192}$$

$$MAX = \max_{y,m}\left(\left|\epsilon_{y,m}^{T}\right|\right)$$

Year-on-year effects

Method		RMSE	AV	MAX
HICP	$q(y-2)$	0.23	0.19	0.48
	$\hat{q}(y-1)$	0.18	0.14	0.36
CPI	$q(y-2)$	0.14	0.00	0.42
	$\hat{q}(y-1)$	0.17	0.01	0.43
Prev. CPI	$\hat{q}(y-1)$	0.10	0.00	0.31

Year-on-year effects

Pure-price effect:

$$\rho_{y-1,m}^{y,m}(b) = \frac{\sum_g p_g^{y,m} q_g^b}{\sum_g p_g^{y-1,m} q_g^b} - 1 \quad \text{(with } b\text{=}y\text{-2 or } y\text{-1)}$$

Reweighting effect:

$$\omega_{y-1,m}^{y,m} = \frac{C_r^{y,m}\left[p_g^{y,m}=p_g^{y-1,m},\forall g\right]}{C_r^{y-1,m}} - 1 \text{, where}$$

$C_r^{y,m}\left[p_g^{y,m}=p_g^{y-1,m},\forall g\right]$ equal to $C_r^{y,m}$ *but* with all prices equal to those one year ago

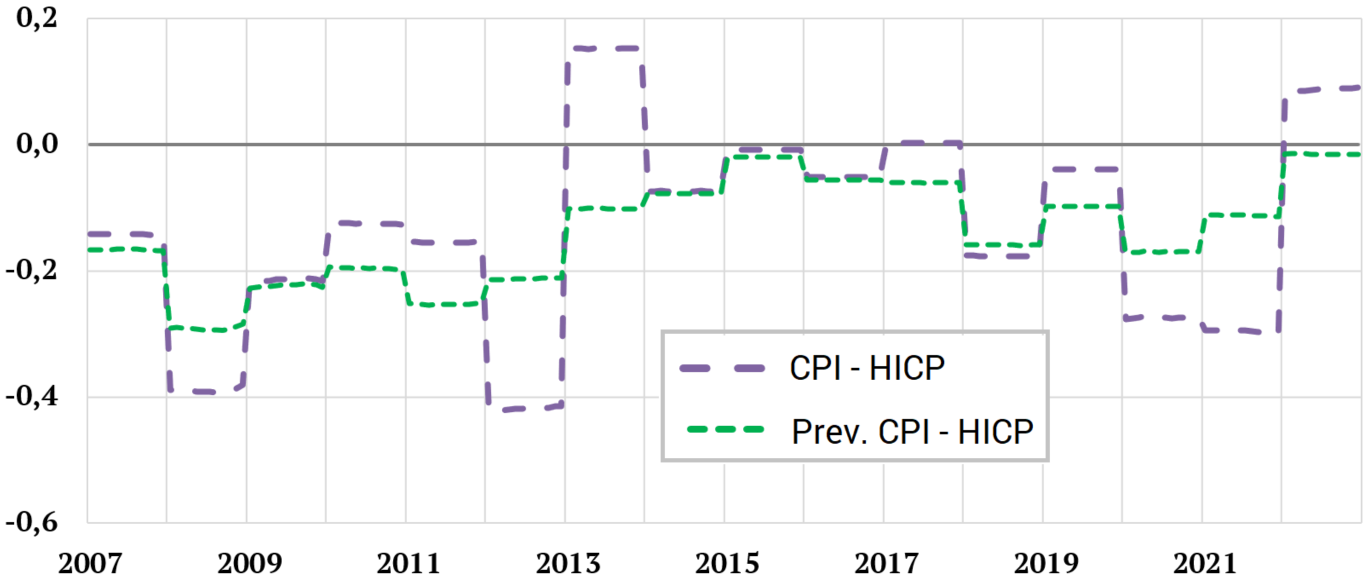
(→ Decomposition: $\Delta_{y-1,m}^{y,m} \approx \rho_{y-1,m}^{y,m} + \omega_{y-1,m}^{y,m}$)

Year-on-year effects

Reweigting effect for CPI methods will include level-shift each year

[CPI / Previous CPI / method y-o-y] – [HICP-method y-o-y]

(<u>all</u> with $\hat{q}(y-1)$ basket in short-term link)



Thank you for your attention!

Olivia Ståhl
Statistics Sweden

olivia.stahl@scb.se

SCB