Evolution of the GEKS index

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Agenda

- The "classical" GEKS-type index idea
- Semi-GEKS indices (**GEKS-L**, **GEKS-GL**)
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The "classical" idea of **GEKS-type** indices

Let us consider the class of multilateral indices which are dedicated to scanner data case. Let us denote sets of products belonging to the same product group in the months 0 and t by G_0 and G_t respectively, and let $G_{0,t}$ denote a set of matched products in both moments 0 and t. The GEKS price index between the months 0 (the base period) and t (the current period) is an unweighted geometric mean of $T+1$ ratios of bilateral price indices $P^{\tau,t}$ and $P^{\tau,0}$, which are based on the same price index formula. **Typically, the GEKS method uses the superlative Fisher (1922) price index**, resulting in the following formula:

$$P_{GEKS}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_F^{\tau,t}}{P_F^{\tau,0}} \right)^{\frac{1}{T+1}}, \qquad (1)$$

where $P_F^{\tau,t}$ denotes the Fisher price index calculated for products from the set $G_{\tau,t}$.

The "classical" idea of **GEKS-type** indices

In the article by Diewert and Fox (2018), the multilateral price comparison method involving the GEKS method based on the **Törnqvist price index** is called the **CCDI** method (sometimes it is denoted by **GEKS-T**). In the paper by Chessa et al. (2017), we can find a hint for selecting a base index formula for the GEKS method: **"the bilateral indices should satisfy the time reversal test"** but it is most often assumed that the price index formula found in the body of the GEKS index is a **superlative formula** (van Loon and Roels, 2018; Diewert and Fox, 2018). For this reason, a GEKS index based on the superlative **Walsh** (1901) index (i.e. the **GEKS-W** index) is also often considered. Out notations:

$$P_{CCDI}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_T^{\tau,t}}{P_T^{\tau,0}} \right)^{\frac{1}{T+1}}, \tag{2}$$

$$P_{GEKS-W}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_W^{\tau,t}}{P_W^{\tau,0}} \right)^{\frac{1}{T+1}}. \tag{3}$$

Semi GEKS index class - not so "classical" GEKS indices

Please also note that there have recently appeared in the literature GEKS-type indices, which are not based on a superlative price index at all, nor on an index that meets the *time reversal test*. For example, the paper by Białek (2022b) proposes a general class of such indices (the GS-GEKS class) and discusses its two special cases, i.e. the **GEKS-L** and **GEKS-GL** index. Under some restrictions regarding the **f** function, the general semi-GEKS index class can be written as follows (Białek, 2022a):

$$P_{GS-GEKS}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{f_{G_{\tau,t}}(q^{\tau}, p^{\tau}, p^{t})}{f_{G_{\tau,0}}(q^{\tau}, p^{\tau}, p^{0})} \right)^{\frac{1}{T+1}}. \quad (4)$$

Semi-GEKS index class -> The GEKS-L index

The GEKS-L index can be defined as follows (Białek, 2022b):

$$P_{GEKS-L}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_L^{\tau,t}}{P_L^{\tau,0}}\right)^{\frac{1}{T+1}} = \prod_{\tau=0}^{T} \left(\frac{\frac{\sum_{i \in G_{\tau,t}} q_i^\tau p_i^t}{\sum_{i \in G_{\tau,t}} q_i^\tau p_i^\tau}}{\frac{\sum_{i \in G_{\tau,0}} q_i^\tau p_i^0}{\sum_{i \in G_{\tau,0}} q_i^\tau p_i^\tau}}\right)^{\frac{1}{T+1}}. \qquad (5)$$

Please note that the GEKS-L index can be treated as the generalization of the Fisher price index formula ($P_F^{0,t}$) to the multi-period case. In fact, in a static item universe G observed over the two period time interval $[0,1]$, we obtain

$$P_{GEKS-L}^{0,1} = \prod_{\tau=0}^{1} \left(\frac{\sum_{i \in G} q_i^\tau p_i^1}{\sum_{i \in G} q_i^\tau p_i^0}\right)^{\frac{1}{1+1}} = \left(\frac{\sum_{i \in G} q_i^0 p_i^1}{\sum_{i \in G} q_i^0 p_i^0} \times \frac{\sum_{i \in G} q_i^1 p_i^1}{\sum_{i \in G} q_i^1 p_i^0}\right)^{\frac{1}{2}} = P_F^{0,1}, \qquad (6)$$

since $G_0 = G_1 = G_{0,1} = G$.

Semi GEKS index class -> The GEKS-GL index

The GEKS-L index can be defined as follows (Białek, 2022b):

$$P_{GEKS-GL}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_{GL}^{\tau,t}}{P_{GL}^{\tau,0}}\right)^{\frac{1}{T+1}} = \prod_{\tau=0}^{T} \left(\frac{\prod_{i \in G_{\tau,t}} \left(\frac{p_i^t}{p_i^\tau}\right)^{w_i^{\tau,t}(\tau)}}{\prod_{i \in G_{\tau,0}} \left(\frac{p_i^0}{p_i^\tau}\right)^{w_i^{\tau,0}(\tau)}}\right)^{\frac{1}{T+1}}. \quad (7)$$

Please note that the GEKS-GL index can be treated as the generalisation of the Törnqvist (Törnqvist (1936)) price index formula ($P_T^{0,t}$) to the multi-period case. In fact, in a static item universe G observed over the two period time interval $[0, 1]$, we obtain

$$P_{GEKS-GL}^{0,1} = \prod_{\tau=0}^{1} \left(\frac{\prod_{i \in G} \left(\frac{p_i^1}{p_i^\tau}\right)^{w_i^{\tau,1}(\tau)}}{\prod_{i \in G} \left(\frac{p_i^0}{p_i^\tau}\right)^{w_i^{\tau,0}(\tau)}}\right)^{\frac{1}{1+1}} = \prod_{\tau=0}^{1} \left(\prod_{i \in G} \left(\frac{p_i^1}{p_i^0}\right)^{w_i(\tau)}\right)^{\frac{1}{2}}$$

$$= \prod_{i \in G} \left(\frac{p_i^1}{p_i^0}\right)^{\frac{w_i(0)}{2}} \prod_{i \in G} \left(\frac{p_i^1}{p_i^0}\right)^{\frac{w_i(1)}{2}} = \prod_{i \in G} \left(\frac{p_i^1}{p_i^0}\right)^{\frac{w_i(0)+w_i(1)}{2}} = P_T^{0,1}, \quad (8)$$

since $G_0 = G_1 = G_{0,1} = G$, and consequently $w_i^{\tau,0}(\tau) = w_i^{\tau,1}(\tau) = w_i(\tau)$.

Semi GEKS index class

In the cited paper (Białek (2022b)) it is proved the following theorem:

Theorem

*The GEKS-L and GEKS-GL indices satisfy the transitivity, **identity**, multi period identity, responsiveness, continuity, positivity and normalization, commensurability, price proportionality, homogeneity in prices and homogeneity in quantitites tests.*

Quality adjusted GEKS-type indices

Two new multilateral indices, the structure of which may resemble the idea of the GEKS index at first glance, were proposed in Białek (2023) (these proposals were firstly discussed in Białek (2022a)). However, the structure of the base index of the proposed multilateral formulas differs completely from the adopted convention related to the application of the superlative index. **Moreover, the calculation of the base index will require *quality adjusting*, which in turn is more like the Geary-Khamis index idea**. In fact, the proposed indices are in a sense a hybrid approach, i.e. they constitute a bridge between the quality adjusted unit value method and the GEKS method.

The GEKS-AQU index

The "classical" quality adjusted unit value $QUV^s_{G_s}$ of a set of products G_s in month s can be expressed as follows:

$$QUV^s_{G_s} = \frac{\sum_{i \in G_s} q^s_i p^s_i}{\sum_{i \in G_s} v_i q^s_i}, \tag{9}$$

where prices p^s_i of different products $i \in G_s$ in month s are transformed into "quality-adjusted prices" $\frac{p^s_i}{v_i}$ and quantities q^s_i are converted into "common units" $v_i q^s_i$ by using a set of factors $v = \{v_i : i \in G_s\}$ (Chessa et al., 2017). The term “Quality-adjusted unit value method” (QU method for short) was introduced by Chessa (2015; 2016). The **QU method** is a family of unit value based index methods and its general form can be expressed by the following ratio:

$$P^{0,t}_{QU} = \frac{QUV^t_{G_t}}{QUV^0_{G_0}} \tag{10}$$

The GEKS-AQU index

In practice, consumer response to price changes can be delayed or even accelerated as consumers not only react to current price changes but also use their own "forecasts" or concerns about future price increases. Some interesting study on "unconventional" consumer behaviour, such as stocking and delayed quantity responses to price changes, and its impact on chain drift bias can be found in the paper by von Auer (2019). Since in practice we often observe prices and quantities that are not perfectly synchronised in time, the following form of the "**asynchronous quality-adjusted unit value**" is proposed:

$$AQUV_{G_{\tau,s}}^{\tau,s} = \frac{\sum_{i \in G_{\tau,s}} q_i^{\tau} p_i^{s}}{\sum_{i \in G_{\tau,s}} v_i q_i^{\tau}}, \tag{11}$$

where τ is any period from the considered time interval $[0, T]$. Obviously it holds that $AQUV_{G_{s,s}}^{s,s} = QUV_{G_s}^{s}$.

The GEKS-AQU index

Under the above significations the GEKS-AQU index can be written as:

$$P_{GEKS-AQU}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{\frac{\sum_{i \in G_{\tau,t}} q_i^{\tau} p_i^t}{\sum_{i \in G_{\tau,t}} v_i q_i^{\tau}}}{\frac{\sum_{i \in G_{\tau,0}} q_i^{\tau} p_i^0}{\sum_{i \in G_{\tau,0}} v_i q_i^{\tau}}} \right)^{\frac{1}{T+1}}. \tag{12}$$

If the item universe is static, we obtain

$$\begin{aligned} P_{GEKS-AQU}^{0,t} &= \prod_{\tau=0}^{T} \left(\frac{\frac{\sum_{i \in G} q_i^{\tau} p_i^t}{\sum_{i \in G} v_i q_i^{\tau}}}{\frac{\sum_{i \in G} q_i^{\tau} p_i^0}{\sum_{i \in G} v_i q_i^{\tau}}} \right)^{\frac{1}{T+1}} = \prod_{\tau=0}^{T} \left(\frac{\sum_{i \in G} q_i^{\tau} p_i^t}{\sum_{i \in G} q_i^{\tau} p_i^0} \right)^{\frac{1}{T+1}} \\ &= \prod_{\tau=0}^{T} \left(\frac{\frac{\sum_{i \in G} q_i^{\tau} p_i^t}{\sum_{i \in G} q_i^{\tau} p_i^{\tau}}}{\frac{\sum_{i \in G} q_i^{\tau} p_i^0}{\sum_{i \in G} q_i^{\tau} p_i^{\tau}}} \right)^{\frac{1}{T+1}} = P_{GEKS-L}^{0,t}. \end{aligned} \tag{13}$$

The GEKS-AQU index

Finally, please also note, that theoretically the class of the $GEKS - AQU$ indices is infinite, since different choices of v_i factors lead to different index values. We could, for instance, consider v_i factors defined in the Geary-Khamis multilateral index resulting in a new, hybrid index, which would be a mixture of the GEKS and Geary-Khamis ideas. That would, however, be probably a slow solution. In this paper, we adopt the system of weights v_i corresponding to the augmented Lehr index (Lamboray, 2017; van Loon and Roels, 2018), where

$$v_i = \frac{\sum_{t=0}^{T} p_i^t q_i^t}{\sum_{t=0}^{T} q_i^t}. \tag{14}$$

If we replace all the adjusted prices $(\frac{p_i^s}{v_i})$ with the relative prices $(\frac{p_i^s}{p_i^\tau})$, then we obtain an "asynchronous quality-adjusted price index" (AQI), i.e.

$$AQI_{G_{\tau,s}}^{\tau,s} = \frac{\sum_{i \in G_{\tau,s}} v_i q_i^\tau \frac{p_i^s}{p_i^\tau}}{\sum_{i \in G_{\tau,s}} v_i q_i^\tau}. \tag{15}$$

As a consequence, the GEKS-AQI index method can be proposed 12:

$$P_{GEKS-AQI}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{\frac{\sum_{i \in G_{\tau,t}} v_i q_i^\tau \frac{p_i^t}{p_i^\tau}}{\sum_{i \in G_{\tau,t}} v_i q_i^\tau}}{\frac{\sum_{i \in G_{\tau,0}} v_i q_i^\tau \frac{p_i^0}{p_i^\tau}}{\sum_{i \in G_{\tau,0}} v_i q_i^\tau}} \right)^{\frac{1}{T+1}}. \tag{16}$$

The GEKS-AQU and GEKS-AQI indices

The following theorem can be proved (Białek (2023)):

Theorem

*The GEKS-AQU and GEKS-AQI indices satisfy the following tests: the transitivity, **identity**, multi period identity, responsiveness, continuity, positivity and normalisation, price proportionality and weak commensurability. If the item universe is the same in the compared periods* 0 *and* t *then the GEKS-AQU and GEKS-AQI indices satisfy also the homogeneity in prices and homogeneity in quantities tests.*

Generalizations of the GEKS method

It can be shown that the following quadratic mean of order r price index:

$$P^r(p^0, p^t, q^0, q^t) = \frac{\sqrt[r]{\sum_{i=1}^n s_i^0 (\frac{p_i^t}{p_i^0})^{r/2}}}{\sqrt[r]{\sum_{i=1}^n s_i^t (\frac{p_i^t}{p_i^0})^{-r/2}}}. \tag{17}$$

is a **superlative** price index formula for any $r \neq 0$ (Diewert, 1976).
The *implicit* quadratic mean of order r price index:

$$P_{im}^r(p^0, p^t, q^0, q^t) = \frac{\sum_{i=1}^n p_t^t q_i^t}{Q^r(p^0, p^t, q^0, q^t) \sum_{i=1}^n p_i^0 q_i^0}, \tag{18}$$

where the quadratic mean of order r quantity index Q^r can be written as follows:

$$Q^r(p^0, p^t, q^0, q^t) = \frac{\sqrt[r]{\sum_{i=1}^n s_i^0 (\frac{q_i^t}{q_i^0})^{r/2}}}{\sqrt[r]{\sum_{i=1}^n s_i^t (\frac{q_i^t}{q_i^0})^{-r/2}}}. \tag{19}$$

Generalizations of the GEKS method

The *implicit* quadratic mean of order r price index is also a **superlative** price index formula (Diewert, 1976; International Labour Office, 2004). We have: $P_{im}^{1} = P_W$, $P_{im}^{2} = P_F$, $P^2 = P_F$ and $P^r(r \to 0) = P_T$, where P_W, P_F and P_T denote the superlative Walsh (1901), Fisher (1922) and Törnqvist (1936) price index respectively.

The GEKS-IQM index class

Let us define a general GEKS-type index family as follows:

$$P^{0,t}_{GEKS-IQM}(r) = \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{IQM}(r)}{P^{\tau,0}_{IQM}(r)} \right)^{\frac{1}{T+1}}, \tag{20}$$

where $P^{\tau,t}_{IQM}(r) \equiv P^{r}_{im}(p^{\tau}, p^{t}, q^{\tau}, q^{t})$.

The *IQM* indices satisfy the *time reversal test*:

$$P^{0,t}_{IQM}(r) P^{t,0}_{IQM}(r) = \frac{\sum_{i \in G_{0,t}} p_i^t q_i^t}{Q^r(p^0,p^t,q^0,q^t) \sum_{i \in G_{0,t}} p_i^0 q_i^0} \frac{\sum_{i \in G_{0,t}} p_i^0 q_i^0}{Q^r(p^t,p^0,q^t,q^0) \sum_{i \in G_{0,t}} p_i^t q_i^t} =$$

$$\frac{1}{Q^r(p^0,p^t,q^0,q^t) Q^r(p^t,p^0,q^t,q^0)} = \frac{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^t (\frac{q_i^t}{q_i^0})^{-r/2}}}{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^0 (\frac{q_i^t}{q_i^0})^{r/2}}} \frac{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^0 (\frac{q_i^0}{q_i^t})^{-r/2}}}{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^t (\frac{q_i^0}{q_i^t})^{r/2}}} =$$

$$\frac{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^t (\frac{q_i^0}{q_i^t})^{r/2}}}{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^0 (\frac{q_i^t}{q_i^0})^{r/2}}} \frac{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^0 (\frac{q_i^t}{q_i^0})^{r/2}}}{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^t (\frac{q_i^0}{q_i^t})^{r/2}}} = 1.$$

The GEKS-IQM index class

Theorem

For each $r \neq 0$ the multilateral price index $P^{0,t}_{GEKS-IQM}(r)$ satisfies the following tests: the transitivity, multi period identity, responsiveness, continuity, positivity and normalisation, homogeneity in prices and homogeneity in quantities, as well as commensurability.

We may obtain some known particular cases of this index class:

$$P^{0,t}_{GEKS-IQM}(1) = \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{W}}{P^{\tau,0}_{W}} \right)^{\frac{1}{T+1}} = P^{0,t}_{GEKS-W} \tag{21}$$

$$P^{0,t}_{GEKS-IQM}(2) = \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{F}}{P^{\tau,0}_{F}} \right)^{\frac{1}{T+1}} = P^{0,t}_{GEKS}. \tag{22}$$

$$P^{0,t}_{GEKS-IQM}(r \to 0) \approx \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{T}}{P^{\tau,0}_{T}} \right)^{\frac{1}{T+1}} = P^{0,t}_{CCDI}. \tag{23}$$

The GEKS-QM index class

Let us define a general GEKS-type index family as follows:

$$P^{0,t}_{GEKS-QM}(r) = \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{QM}(r)}{P^{\tau,0}_{QM}(r)} \right)^{\frac{1}{T+1}}. \qquad (24)$$

where $P^{\tau,t}_{QM}(r) \equiv P^r(p^\tau, p^t, q^\tau, q^t)$.
The QM indices satisfy the *time reversal test*:

$$P^{0,t}_{QM}(r)P^{t,0}_{QM}(r) = \frac{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^0 (\frac{p_i^t}{p_i^0})^{r/2}}}{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^t (\frac{p_i^t}{p_i^0})^{-r/2}}} \frac{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^t (\frac{p_i^0}{p_i^t})^{r/2}}}{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^0 (\frac{p_i^0}{p_i^t})^{-r/2}}} =$$

$$= \frac{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^0 (\frac{p_i^t}{p_i^0})^{r/2}}}{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^t (\frac{p_i^0}{p_i^t})^{r/2}}} \frac{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^t (\frac{p_i^0}{p_i^t})^{r/2}}}{\sqrt[r]{\sum_{i \in G_{0,t}} s_i^0 (\frac{p_i^t}{p_i^0})^{r/2}}} = 1.$$

The GEKS-QM index class

Theorem

For each $r \neq 0$ the multilateral price index $P^{0,t}_{GEKS-QM}(r)$ satisfies the following tests: the transitivity, multi period identity, responsiveness, continuity, positivity and normalisation, weak price proportionality, homogeneity in prices and homogeneity in quantities tests, as well as commensurability.

$$P^{0,t}_{GEKS-QM}(r \to 0) = \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{T}}{P^{\tau,0}_{T}} \right)^{\frac{1}{T+1}} = P^{0,t}_{CCDI}, \tag{25}$$

$$P^{0,t}_{GEKS-QM}(1) \approx \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{W}}{P^{\tau,0}_{W}} \right)^{\frac{1}{T+1}} = P^{0,t}_{GEKS-W}, \tag{26}$$

$$P^{0,t}_{GEKS-QM}(2) = \prod^{T} \left(\frac{P^{\tau,t}_{F}}{P^{\tau,0}} \right)^{\frac{1}{T+1}} = P^{0,t}_{GEKS}; \tag{27}$$

The GEKS-LM index class

The Lloyd-Moulton price index (Loyd, 1975) can be written as follows:

$$P_{LM}^{\tau,t}(\sigma) = (\sum_{i \in G_{\tau,t}} s_i^{\tau} (\frac{p_i^t}{p_i^{\tau}})^{1-\sigma})^{\frac{1}{1-\sigma}}, \tag{28}$$

where the parameter σ denotes the **elasticity of substitution**. It can be shown (Loyd, 1975; Moulton, 1996) that under the assumption of cost minimising the behaviour the index P_{LM} is exact for CES preferences. On the bases of the Lloyd-Moulton index, let us define a general GEKS-type index family as follows:

$$P_{GEKS-LM}^{0,t}(\sigma) = \prod_{\tau=0}^{T} \left(\frac{P_{LM}^{\tau,t}(\sigma)}{P_{LM}^{\tau,0}(\sigma)} \right)^{\frac{1}{T+1}}. \tag{29}$$

The GEKS-LM index class

Theorem

For each $\sigma \neq 1$ the multilateral price index $P^{0,t}_{GEKS-LM}(\sigma)$ satisfies the transitivity, ***identity****, multi period identity, responsiveness, continuity, positivity and normalisation, commensurability, strong price proportionality, as well as homogeneity in prices and homogeneity in quantities tests.*

$$P^{0,t}_{GEKS-LM}(0) = \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{La}}{P^{\tau,0}_{La}} \right)^{\frac{1}{T+1}} = P^{0,t}_{GEKS-L} \tag{30}$$

and

$$P^{0,t}_{GEKS-LM}(\sigma \to 1) = \prod_{\tau=0}^{T} \left(\frac{P^{\tau,t}_{GLa}}{P^{\tau,0}_{GLa}} \right)^{\frac{1}{T+1}} = P^{0,t}_{GEKS-GL}. \tag{31}$$

Empirical illustration

An empirical illustration of the potential differences between the proposed indices (including representatives of general index classes) and the multilateral indices known from the literature (GEKS, CCDI, TPD, Geary-Khamis) was carried out on the **milk** and **coffee** datasets. These datasets are anonymized sets of real data sets on coffee and milk sales in one of the retail chains operating in Poland. It should be noted that these datasets are available in the R package *PriceIndices* (Białek, 2021) and all results presented here are reproducible.

Empirical illustration

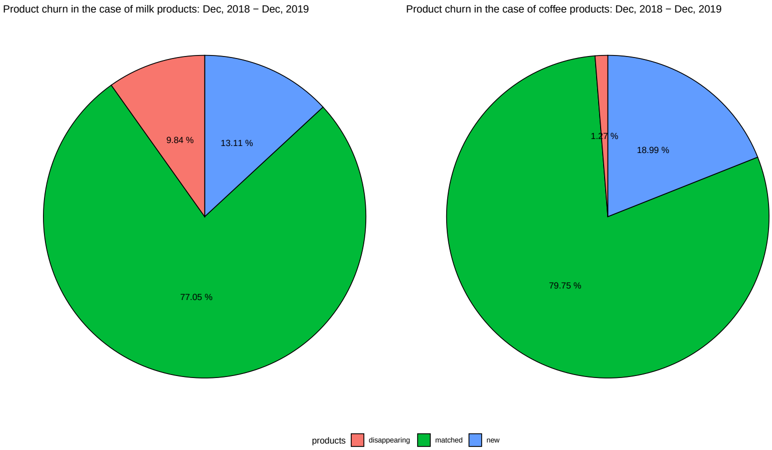


Figure: Product churn in the considered scanner data sets.

Empirical illustration

Figure: Comparison of the GEKS-L, GEKS-GL, GEKS-AQI and GEKS-AQU indices with known multilateral indices.

Empirical illustration

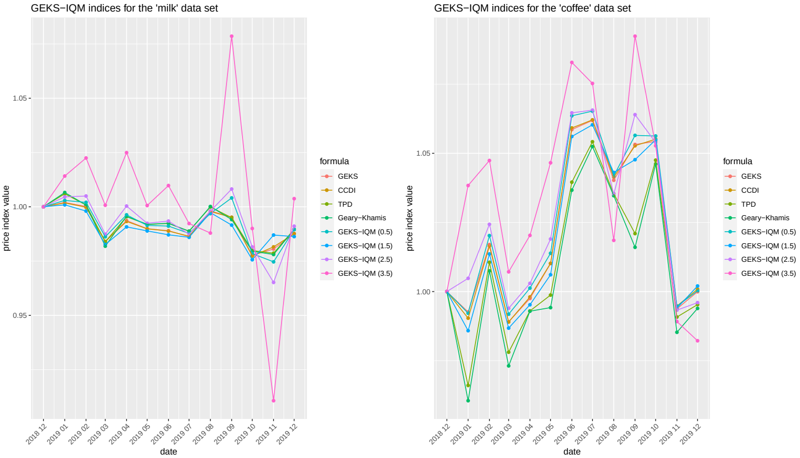
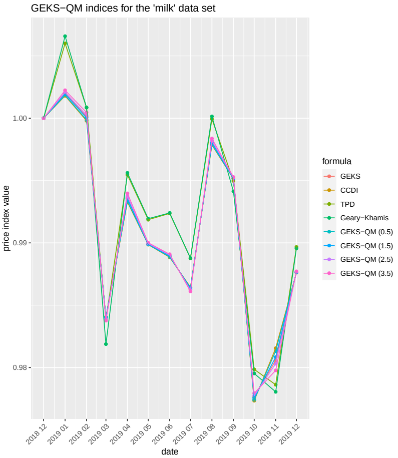


Figure: Comparison of the selected GEKS-IQM indices with known multilateral indices.

Empirical illustration



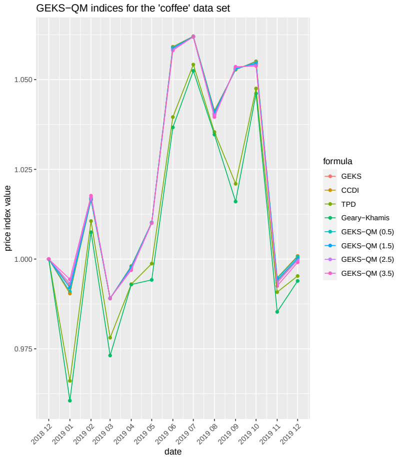
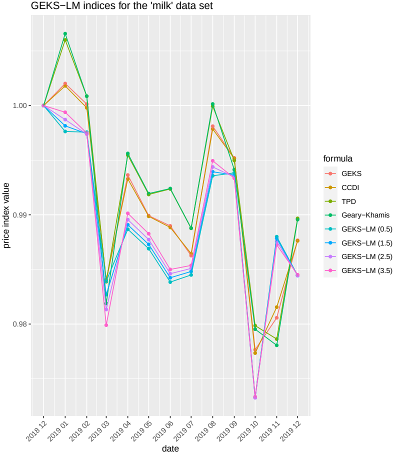


Figure: Comparison of the selected GEKS-QM indices with known multilateral indices.

Empirical illustration



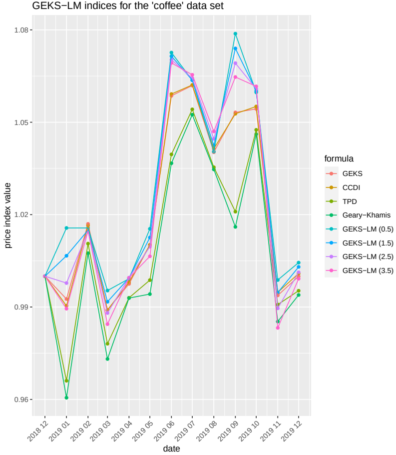


Figure: Comparison of the selected GEKS-LM indices with known multilateral indices.

Conclusions

- The values of the GEKS-L, GEKS-GL, GEKS-AQI and GEKS-AQU indices are close to each other (**Cluster 1**), further - the GEKS and CCDI indices approximate each other (**cluster 2**), and the TPD and Geary-Khamis indices have similar values (**Cluster 3**). In our study, it turned out that values of indices from the **Cluster 3** are always between index values from **Clusters 1** and **Cluster 2**.
- Indices from the GEKS-IQM class begin to unreasonably lag behind typical multilateral indices for the parameter $r > 2$. On the other hand, indices from the GEKS-QM class are stable due to the value of the parameter r
- Indices from the GEKS-LM class form a separate cluster of values and they are moderately sensitive to the choice of σ parameter. From a theoretical (axiomatic) point of view, these indices seems to be the most valuable (e.g., they satisfy the *identity test*). The use of these indices requires estimating the elasticity of substitution (e.g. by using a panel regression model explaining the behavior of quantities based on prices).

Acknowledgement

Thank you very much for your attention!

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