Multilateral Approaches in Inflation Measurement: Why Does the TPD Method Fail Us and Can We Do Something About It?

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Content

Section 1: Introduction

Multilateral price indices

- Many empirical studies demonstrate that chained bilateral price indices generate chain drift bias.
- Multilateral price indices have been advocated as a solution.
- Among the most popular multilateral price indices are the
  - Gini-Éltető-Köves-Szulc (GEKS) approach
  - Time Product Dummy (TPD) regression
  - Geary-Khamis (GK) approach
- Chain drift is not the only source of potential bias.
- How do the multilateral methods perform when
  - the aggregated items or products have individual price trends, and/or
  - data gaps occur?

Section 2: Problem

Example 1: Complete Price Data

Period	1	2	3	4
Product A:	22	22	21	20
Product B:	30	32	37	40
Product C:	49	50	52	54

Table 1: Prices of three products during four periods.

TPD Regression Model

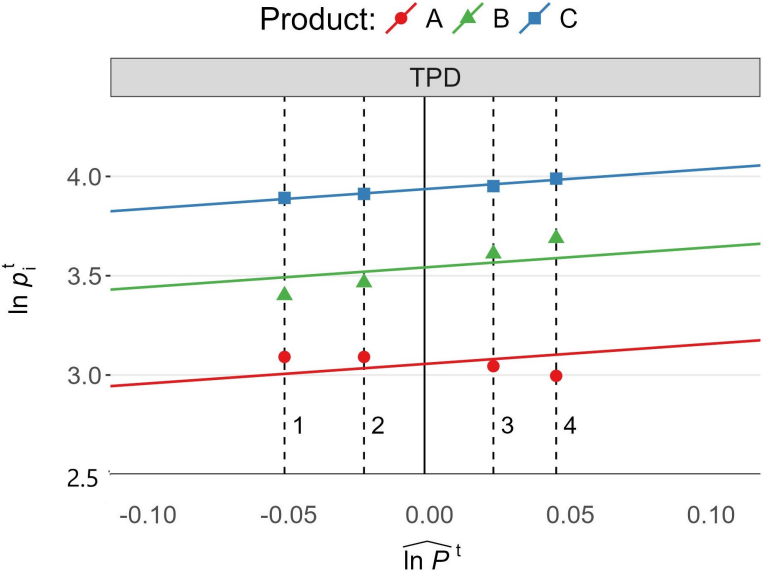
- Notation:
  - p_i^t price of item i in period t
  - P^t is the price level in period t
  - π_i is the general value of product i
  - $u_i^t \sim N\left(0, \sigma^2\right)$ is an error term
- The TPD regression assumes that

$$p_i^t = P^t \pi_i e^{u_i^t}$$

- Written in linear form:

$$\ln p_i^t = \ln P^t + \ln \pi_i + u_i^t$$

TPD Estimates for the Price Data in Table 1



TPD Regression

- In TPD regressions, the slope of the coloured lines is 1 (by assumption).
- This slope is the price level elasticity of the product prices p_i^t.
- When in some period t the general price level increases by 1%, the prices of all items increase by 1%.
- The TPD regression allows for inference.
- However, with product-specific price level elasticities, the TPD regression leads to autocorrelated and heteroskedastic residuals.
- Thus, the TPD regression is inefficient and inference is invalid.

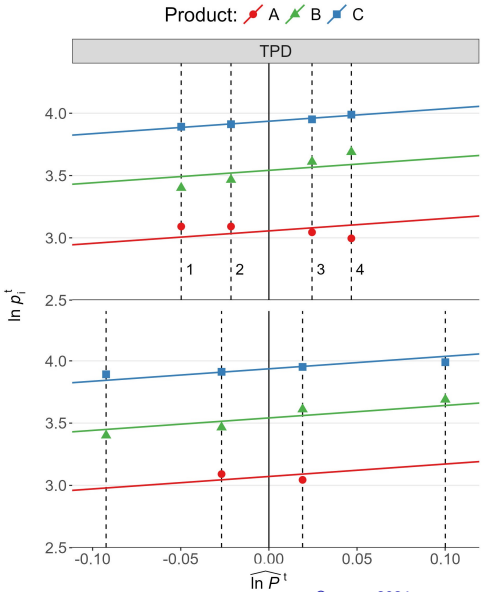
Example 2: Data Gaps

Period	1	2	3	4
Product A:		22	21	
Product B:	30	32	37	40
Product C:	49	50	52	54

Table 2: Missing prices of Product A in periods 1 and 4.

- In this case, the TPD regression generates an upward biased estimate of the price level change between periods 1 and 4.

TPD Estimates for the Price Data in Table 2



Implications

- **Result 1:** When observations of the product(s) with the flatest price trend(s) are missing, the overall price change as measured by the TPD regression is upward biased. Downward bias arises when observations of the product(s) with the steepest price trend(s) are missing.

- **Result 2:** The GK method is not the solution because it usually approximates the TPD regression.
- Is the GEKS method the solution?

GEKS Method

Period	1	2	3	4
Product A:	22	22	21	20
Product B:	30	32	37	40
Product C:	49	50	52	54

Table 3: Prices of three products during four periods.

- Notation: $P^{1,2}$ indicates the Jevons index for base period 1 and comparison period 2.
- The price change between periods 1 and 4 as measured by the GEKS-Jevons method is

$$[(\underbrace{P^{1,1}}_{=1} P^{1,4})(P^{1,2}P^{2,4})(P^{1,3}P^{3,4})(P^{1,4}\underbrace{P^{4,4}}_{=1})]^{1/4}$$

- Without Product A, the values of the Jevons indices $P^{1,2}$, $P^{1,3}$, $P^{1,4}$, $P^{2,4}$, and $P^{3,4}$ would increase ($\Rightarrow$ upward bias).
- **Result 3:** The GEKS-Jevons method is not the solution.

Section 3: Solution

NLTPD Regression

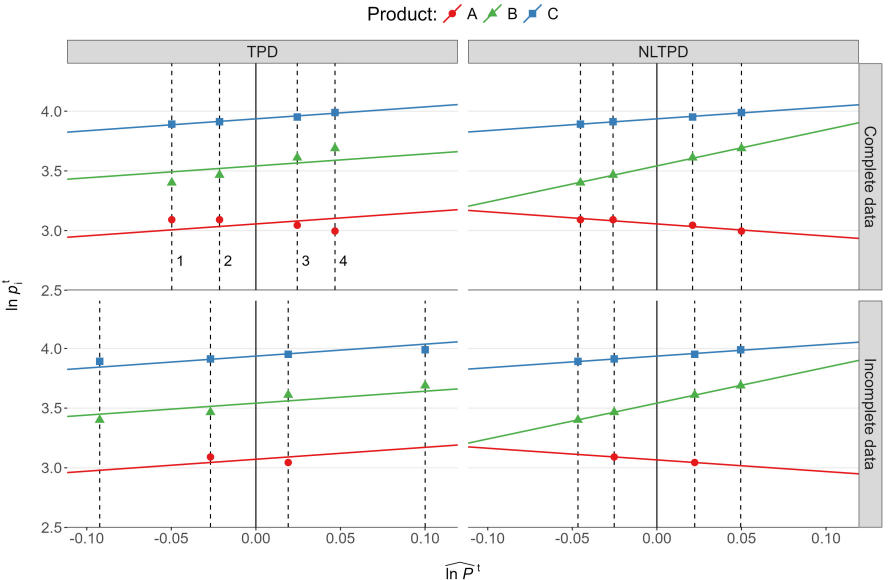
- For multilateral interregional price comparisons, Auer and Weinand (2022) propose a regression method that can be adapted to multilateral intertemporal price comparisons.
- This new method is denoted as *Non-Linear-Time-Product-Dummy* (NLTPD) regression:

$$\ln p_i^t = \delta_i \ln P^t + \ln \pi_i + u_i^t$$

- The δ_i-values determine the slopes of the coloured lines (price level elasticities).
- Instead of estimating the δ_i-values, the TPD regression sets $\delta_i = 1$:

$$\ln p_i^t = \ln P^t + \ln \pi_i + u_i^t$$

TPD- Versus NLTPD Estimates



Advantages of the NLTPD Regression

- **Result 4:** The NLTPD regression has a better model fit.
- **Result 5:** The NLTPD estimates remain stable even when there are systematic data gaps.

Section 4: Simulation

Example 3: Generating a Benchmark

- Bias of a multilateral price index is its deviation from a benchmark.
- Real world data cannot provide an unassailable benchmark.
- By contrast, careful simulations may provide such a benchmark.

	p_i^r	x_i^r	p_i^t	x_i^t
Product A:	20	6	44	3
Product B:	40	3	11	9
Product C:	10	9	22	6

Table 4: Prices and quantities of three products during periods r and t.

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	p_i^r	x_i^r	p_i^s	x_i^s	p_i^t	x_i^t
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Product B:	40	3	*10*	*9*	11	9
Product C:	10	9	*20*	*6*	22	6

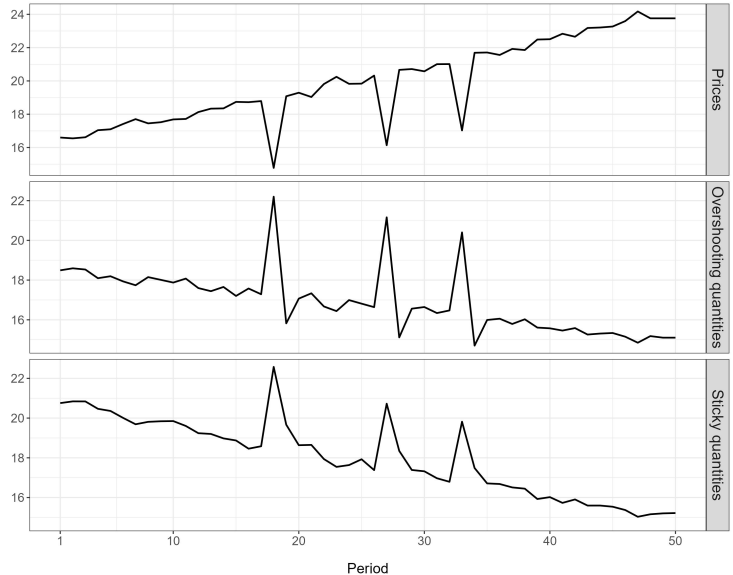
Table 4: Prices and quantities of three products during periods r and t.

- If households have a symmetric appreciation of all products, the price level of period r is identical to the price level of the hypothetical scenario 0.
- Between periods r and t, the price level increases by 10 percent.

Price Tableau

- 30 products during 50 periods.
- The prices in period 1 are randomly generated.
- The prices in period 48 are a permutation of the prices in period 1, but 10 percent higher (as in Table 4).
- The prices of period 48 prevail also in periods 49 and 50.
- For each product, the prices between periods 1 and 48 represent a linear trend with a random component that fluctuates around 0.
- Sales occur randomly. The probability of a sale increases during periods 1 to 24 and falls during periods 25 to 48.

Prices and Quantities



Quantity Tableau

- Quantities are derived from utility maximizing households.
- A quarter of the households stockpile during sales.
- The other households smooth consumption to reduce adjustment costs.
- Quantities in period 50 are a permutation of the quantities in period 1 (same permutation as for the prices).
- Thus, we get a situation like

	p_i^1	x_i^1					p_i^{50}	x_i^{50}
Product 1:	20	6	...	...	*40*	*3*	44	3
Product 2:	40	3	...	...	*10*	*9*	11	9
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Product 30:	10	9	...	...	*20*	*6*	22	6

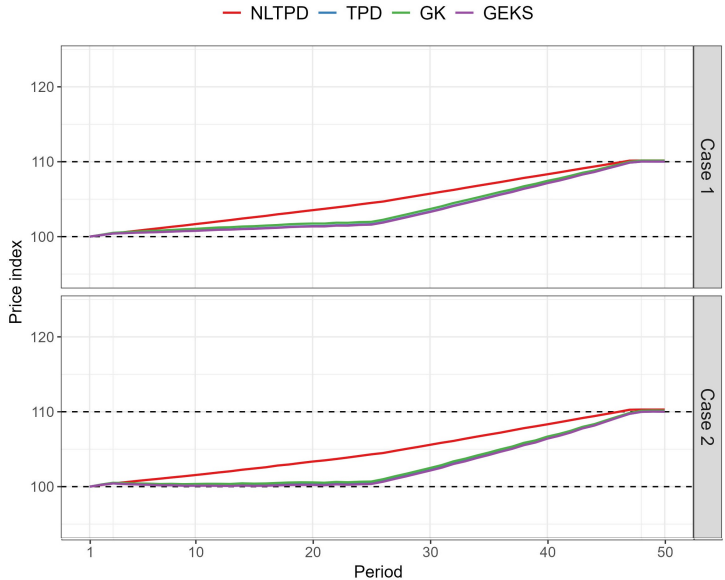
- Between periods 1 and 50, the price level increases by 10 percent.

Measuring Bias

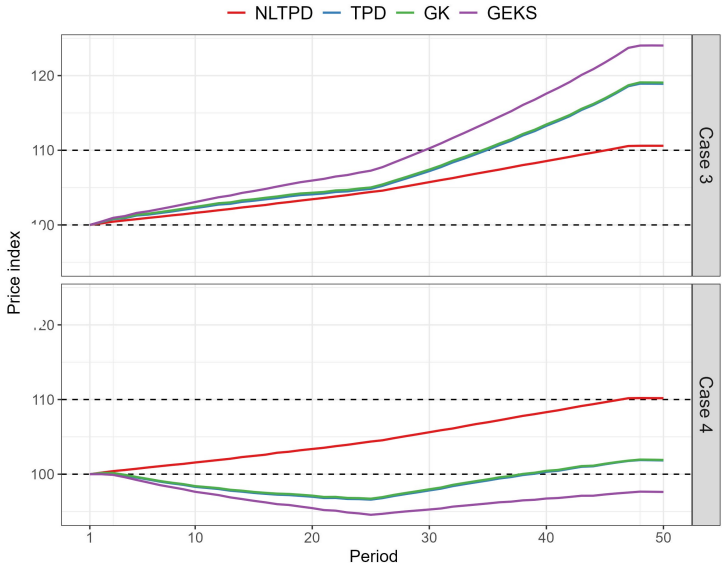
- The price level in period 1 is set to $P^1 = 100$.
- Thus, the price level in period 50 is $P^{50} = 110$.
- A deviation of the multilateral price index, $\widehat{P}^{50}$, from 110 is considered as bias.
- For each multilateral price index, 1000 samples are computed.
- For the 1000 resulting $\widehat{P}^{50}$-values
  - the mean deviation from 110 and
  - the standard deviation

  are computed.
- Four cases:
  - Case 1: No gaps
  - Case 2: Gaps completely at random
  - Case 3: Gaps negatively correlated with price level elasticities (δ_i)
  - Case 4: Gaps positively correlated with price level elasticities (δ_i)

Simulation Results for Cases 1 and 2



Simulation Results for Cases 3 and 4



Section 5: Concluding Remarks

- When products exhibit individual price trends, multilateral price indices (GEKS, TPD, GK) are inefficient and inference is invalid.
- When, in addition, systematic data gaps exist, these indices are biased.
- The NLTPD regression avoids these problems.
- The NLTPD regression is superior to TPD, GEKS, and GK, when the product-specific price level elasticities, δ_i, can be reliably estimated and when the variance of these elasticities is not too small.