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Methodology for constructing a price index for mobile-telephony services

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There are three difficulties involved in constructing a consumer price index (CPI) for mobile-telephony services:

(*i*) Mobile telephony is a truly new product. Its introduction itself should theoretically cause the index to decline.

(*ii*) The main product itself is simple: making phone calls (a classic product) but from a "mobile" location. However, its pricing is more complex than that of any other consumer product.

(*iii*) The service is often offered with a terminal (handset) whose price is very low ("starter packs").

The first difficulty has been studied (Hausman 1999) but has not received a satisfactory operational solution: how does one measure the increase in utility for consumers arising from the fact that they can extend their scope of consumption choice to a radically new product?

The second explains why most countries have been so late in setting up such an index. Not only are the contracts for using mobile-telephony services constantly changing (as are the costs), but *call pricing* is determined by a complex set of factors:

- different tariffs depending on call type and call periods (packages versus pre-paid cards [vouchers])
- roll-over minutes (packages)
- additional calls (exceeding "inclusive minutes" in package)
- life span (pre-paid vouchers)
- non-linear charges for minutes consumed (packages and vouchers)

The construction of a price index is greatly complicated by product-pricing thresholds such as indivisible minutes for vouchers and packages, higher charges for additional minutes in packages, and finite life span for vouchers. The task requires a fairly detailed description of consumer profiles using *distributions*—rather than averages alone—for call length, total monthly call time, and so on. Consider, for example, the indivisibility of the first call minute (the current practice of all operators). Suppose that operators return to a per-second charge. All other things being equal, this should cause the index to fall. Let us take the *average* call length, for example 60 seconds, in the description of a consumption profile. On that measure, prices remain stable. By contrast, if we refine the profile description by assuming the calls are short (45 seconds), medium (60 seconds), or long (75 seconds), with a uniform probability, then the base-100 price index becomes 100.(45/60 + 60/60 + 75/75)/3 = 100.(0.75 + 1 + 1)/3 = 92, i.e., an 8% price fall.

A classic *Laspeyres index* cannot track such complexity, or the substitution effects due to tariff changes or the introduction of new mobile-telephony products. These effects can be measured only—and imperfectly—by a *unit-value index*, which is what we would implicitly obtain by computing a volume index summing the call minutes for each period, for all products in the aggregate, irrespective of distance and call period.

The best approach is to use a *cost-of-living (COL) index* (also known as *constant-utility index* or *CUI*). The methodology suggested here is inspired by the one used in Germany (Beuerlein 2000) and by OFTEL (n/e/r/a, 1999) in the United Kingdom: in both cases, "consumer profiles" are defined and made to react to a varied and fast-changing supply of products. If they are rational, have perfect information, and can predict their future consumption perfectly, they choose the product that satisfies their mobile-telephony needs most cheaply. However, because of these extremely heroic assumptions and the hypothesis of transition costs when migrating from one product to another, we would strongly qualify the model outlined above.

While the consumer-profile approach does not solve problem (*i*), it does make allowance for the introduction of new mobile-telephony products (which, in theory, sends the index down). But it does not solve problem (*iii*). However, terminals sold at full retail price (rather than in "starter packs") are already monitored in the CPI.

In this paper, we propose a methodological framework for constructing a price index for mobile-telephony services. Its implementation would require gathering and processing a large quantity of information. Its production on a current basis will therefore involve some simplifications.

1. Mobile-telephony services

1.1. The mobile-telephony market in France

The penetration rate of mobile telephony—i.e., the ratio of mobile-telephony subscribers to the total French population—was only 2.2% in 1995. The first two operators were France Télécom Mobiles (FTM) and SFR. The arrival of a third—Bouygues Télécom (BT)—in 1996 and the establishment of a telecommunications regulation authority (ART) in 1997 gave market growth a powerful boost. Given the exponential rate of increase, it is not inappropriate to speak of an explosion. The penetration rate reached 19.2% at year-end 1998 and 49.4% at year-end 2000 (ART 2000). Family ownership of several cell phones—already a significant phenomenon in 1999 (Rouquette 2000)—became more common. All European countries have experienced this mobile-telephony craze. In fact, the spread of cell phones in France is still a long way from the record levels reached in the Scandinavian countries, so there is room for further growth.

While BT, the most recent entrant, continues to gain market share with 17.5% of subscribers at year-end 2000, both its competitors have maintained solid positions: SFR had a 34.3% share at year-end 2000; remarkably, the "historical" operator, FTM, remains the market leader with a nearly stabilized share of 48.2%.

Each operator offers two types of products: packages (monthly call plans) and pre-paid vouchers (cards). The market thus comprises more than twenty different packages and ten different vouchers, which in turn are consolidated into a dozen product ranges. These ranges differ by the scope and cost of services offered and their tariff structure.

Although reliable statistics on the mobile-telephony industry are rare and the market is fast-changing, more consumers choose packages (65% of the total) than pre-paid vouchers. The volume of calls placed per subscriber was on the order of two hours in H1 2000, the two-hour package being the most widespread product. The average monthly expenditure per subscriber in the same period was FRF184, which is very close to the price of the leading monthly two-hour packages.

Another characteristic of the French market is the prevalence of sales of "starter packs" comprising a discounted handset plus a subscription: packs accounted for 83% of purchases according to a survey conducted by SOFRES for ART in 1999. Moreover, subscription contracts can be taken out directly from operators but also via cellular-telephone service providers, which sell packages and vouchers in specialized stores or in mass retail outlets.[1]

1.2. SIM card

The consumption of *mobile-telephony services*, in the form of *pre-paid vouchers* or *packages*, requires:

- *line activation* (i.e., access to an operator's network);

- the assignment of a *call number* (ten digits, the first two being 06).

[1] Cellular-telephone service providers, officially designated in France as *sociétés de commercialisation de services* (SCSs), can be either subsidiaries of the network operators or independent companies.

Both can be performed with a *SIM (Subscriber Identity Module) card*, which also stores:

- information on the consumer (enabling payment by credit card, for example);

- information on the use of mobile-telephony services (such as text messaging and voicemail);

- call time.

The card is protected by a four-digit PIN number chosen by the consumer*. The consumption of mobile-telephony services, therefore, is not linked to a handset but to the SIM card*, which can be installed in another handset before the expiry of a package or pre-paid voucher.

1.3. Formalization of the "mobile-telephony services" product

Products are defined as offerings that combine quantities of *calls* and *services*.

1.3.1. Calls (voice calls to other subscribers)

Voice telephone calls to other subscribers (excluding numbers to which special charges apply) are the main component of mobile-telephony services. They can be classified into different categories according to call destination. By call *destination* we mean the specification of a *geographic* destination—national or international (subdivided into European Union (EU) and other countries—and a type of called number: on the same mobile network, on another mobile network, or on a fixed-line telephone.

1.3.2. Services

The services offered may be classified into three categories: subscription-management services, call-management services, and services involving the transmission of information by means other than voice calls.

Subscription-management services include consumption tracking, itemized billing, roll-over minutes, and customer service.

Call management mainly refers to facilities for making international calls (not always available) or using one's handset abroad ("roaming"), restrictions on outgoing calls, call waiting, caller ID, call forwarding, and unlisted number.

Information transmission (other than by voice calls) consists of voice messaging (answering service) and text messages (SMS): these are short messages, not exceeding 160 characters, for communicating with another mobile on the same network or any other network. "Office" services (sending and receiving faxes, data, and e-mails) will not be tracked, as they mainly concern business users.

1.4. Pricing

1.4.1. Packages

The earliest subscription contracts were set up without packages, as in fixed-line telephony. This pricing system has become totally marginal, having been replaced by a wide range of packages with different life spans and contents.

Today, all packages offered are monthly. Subscribers have to commit themselves for anywhere from 12 to 24 months. A package includes a specified volume of calls and access to different services. The characteristic of the current packages is the minimal difference in call charges by destination, and a free offer of services (when they involve phone calls, the services are sometimes charged). Unused inclusive call minutes may or may not be rolled over free of charge to the following month. Several services are offered and charged extra, generally at a higher and more differentiated cost than the services included in the package. First, they include calls exceeding the inclusive-minutes allowance and specific types of calls—the most blatant example being calls to abroad, or from abroad when the cell phone is used outside France. Second, some services are consistently offered (if at all) on an "out-of-package" basis, such as messaging services and access to information services.

1.4.2. Pre-paid vouchers

Pre-paid vouchers resemble packages as they contain a specified volume of calls that must be used within a fixed time limit. One major difference, however, is the length of the subscriber's commitment: the life span of vouchers is from one to three months. The product is therefore more flexible than packages, being well-suited to intermittent-consumption patterns.

Another interesting feature of vouchers is that subscribers can continue to receive calls for between six to twelve months after the call credit expires. Clearly, the pre-paid voucher is designed for users who make few calls, or who call sporadically, and in particular for users who want to remain able to receive calls. Vouchers also provide greater financial security than packages. In addition to the initial, more limited commitment, they inherently prevent overspending: once the voucher is used up, the expenditure is self-terminating!

Obviously these advantages have their downsides. The first (and not the least) is the cost of the product: pre-paid calls are between 1.5 and 2 times as expensive as package calls. Second, the service offering is more restricted. For example, most pre-paid vouchers do not allow roaming abroad. Third, vouchers differ from packages on one tariff principle, the distinction between call periods: "daytime" (peak hours) and "evenings and weekends" (off-peak hours). Each operator sets its own dividing lines. The call-period distinction is fairly uncommon in packages, whereas it is the rule with pre-paid vouchers.

1.4.3. Call periods

Peak-hour access—when demand is heaviest—is a rare, hence expensive resource. Charging for peak-hour calls at a higher rate than off-peak calls is a means of regulating this factor. One operator, SFR, uses another demand-management method. In its packages and vouchers, peak-hour calls are restricted to 50% of the total call time. For example, a two-hour *Entrée Libre* ("free entrance") package entitles the consumer to two call hours, of which no more than one at peak hours.

1.5. Information sources

As their complexity implies, mobile-telephony services theoretically require access to a wide variety of information in order to construct an efficient price index. We would need a continuous input of data on the distribution of products sold, details on their respective consumption, and on the patterns of user consumption and choice of products on offer. Unfortunately, the existing and easily accessible sources of information are fairly meager. The mobile-telephony industry is still young, and operators prefer the marketing approach—with its emphasis on the short term—to a socio-economic analysis based on heavyweight statistical instruments. When the information is available, it is often organized in ways that are ill-suited to INSEE's goals. Moreover, in a highly competitive market, operators are understandably somewhat reluctant to divulge information they regard as strategic. Lastly, three other factors blur the picture of the personal mobile-telephony services market:

- the distinction between personal and business use is often hard to draw;

- the information on consumption arising from sales via cellular-telephone service providers (SCSs: see note 1) is far sketchier than on consumption by users subscribing directly from operators;

- the consumption patterns of voucher users are far less well known than those of package users: unlike with vouchers, the billing of package charges requires a detailed knowledge of consumption.

Thanks to the contacts established by INSEE with the three operators, it is hoped that the statistical information relevant to the price-index computation will increase. INSEE, for its part, must try to define its needs more precisely—not the easiest thing to do in the initial phase of index development.

At a more summary level, the main source of information on the mobile-telephony market is ART. This independent agency gathers consistent data from operators; it also conducts or commissions studies on the market's functioning, particularly by means of consumer surveys.

2. Determining consumer profiles

2.1. Consumption, consumer profiles, and substitution

To compute a price index for mobile-telephony services, we do not follow the classic approach used for service prices defined in tariff form. In the classic method, total consumption is broken down into micro-consumption items (subscription, type-1 call, ..., type-n call), whose prices and weights are aggregated into a Laspeyres index. Two main factors preclude such an approach: first, it is impossible to evaluate micro-consumption prices because of the package and voucher principle, which aggregates all the expenditures for a single consumer; second, the volatility of the mobile-telephony market—fueled by the ceaseless waves of new or cheaper products—generates product substitutions that lead consumers (if we assume them to be perfectly rational) to select the least expensive of the products suited to their needs. Since we must abandon the notion of tracking micro-consumptions, we need to tackle the problem at the consumer level.

The logical approach consists in defining classes of products regarded as substitutable and in selecting consumption profiles—i.e. "typical" consumers—in each class. Each profile is defined by the precise characteristics of the products consumed and by data on the profile's detailed consumption over several months. The typical consumer is representative of a distinct group of consumers who engage in product substitutions from one month to the next. We calculate unit-value indices (i.e., for average prices) for each group: this implies the heroic assumption that we can obtain real-time information on product substitutions. The last step is to assign weights to the profiles and then calculate a Laspeyres index from the profile indices.

2.2. Stratification of mobile-telephony consumers

2.2.1. Substitution classes

The only distinction that will be used here is the one between packages and vouchers. The description of these two types of product in §1.4 effectively showed that their characteristics are different enough to preclude comparison.

By contrast, we will regard the products offered by the three operators as substitutable. This choice is not a foregone conclusion and it can be challenged: network size, technical quality (outage frequency, reception quality, etc.), the quality of service (advice on choosing a product, after-sales service), and the choice of handsets available in the "starter packs" are not identical for FTM, SFR, and BT. However real, these differences seem minimal to us today—unlike in the early days of mobile telephony.

The substitutability hypothesis chosen here will, in fact, be weakened by two factors:

- the commitment entailed by the purchase of each product, which deters over-hasty migrations to another operator's products;

- consumer loyalty to operators, which is stronger than a perfect-competition model would suggest. We return to this issue in the final section of our paper.

2.2.2. Consumer profiles

For each of the two classes, we distinguish 6 consumer profiles according to the volume and call-period distribution of the main consumption item, i.e., national telephone calls. The stratification criteria chosen are the following:

- Total call time per month. We define three classes of consumers: Q1 (light), Q2 (medium), and Q3 (heavy).

- Time distribution of calls: mainly daytime (peak hours, PH), mainly evenings and weekends (off-peak, OP). In all, the stratification comprises 12 classes:

Table 1 - Consumer profiles

		Q1	Q2	Q3
Packages	PH	1	2	3
	OP	4	5	6
Pre-paid vouchers	PH	7	8	9
	OP	10	11	12

We can calculate *average values* for each of the 12 categories of mobile-telephony users. The first concern *national telephone calls*, the second concern *other calls* and *related services*. This division implicitly takes into account the differences in consumption between consumers of "other calls" and services: heavy consumers subscribing to a package (profiles 3 and 6) use these marginal consumptions more than intermittent users who have bought pre-paid vouchers (profiles 7 and 10).

For national telephone calls

(1) Monthly number of call minutes: average and standard deviation

(2) Percentage of call minutes

- at peak hours
- at off-peak hours

(3) Percentage of call minutes[2] to

- a *fixed* phone
- a *mobile* phone
    - on same network
    - on another network

(4) Total monthly number of calls: average and standard deviation.

[2] Voicemail retrieval is excluded because it is offered free of charge in all products.

For other calls and services

(1) Expenditure tracking ("consumption tracking")

(2) Itemized billing: yes or no (packages only)

(3) Call waiting: yes or no

(4) Caller ID: yes or no

(5) Customer-service call

(6) Number of SMS messages sent

(7) Roaming.

For the "calls component," these 12 profiles can be tabulated as follows:

Table 2 - Consumer profiles and related consumptions

			Profiles						
NATIONAL CALLS			1	2	3	…	T	…	12
Total average monthly duration of national calls	(in minutes)						Λ_T		
Share (%) of total monthly call duration consisting of calls made at	peak hours						$\pi_T(hp)$		
	off-peak hours						$\pi_T(hc)$		
Share (%) of total monthly call duration consisting of calls made to	a fixed phone						$\delta_T(fi)$		
	a mobile phone	on same network					$\delta_T(me)$		
		on another network					$\delta_T(au)$		
Total monthly number of calls							N_T		
OTHER CALLS AND SERVICES									
Expenditure tracking							$\tau_T(sd)$		
Itemized billing							$\tau_T(fd)$		
Call waiting							$\tau_T(da)$		
Caller ID							$\tau_T(pn)$		
Customer service							$\tau_T(sc)$		
SMS	(number)						NS_T		
International calls (EU)	(in minutes)						$\Lambda S_T(ae)$		
Roaming (EU)	(in minutes)						$\Lambda S_T(ar)$		

The monthly call duration and monthly number of calls for each profile are therefore represented by random distributions Λ_T and N_T. These are discussed in greater detail in

Section 3. A service, for example the customer service, is represented by a rate $\tau_T(sc)$ between 0 and 1. It measures the proportion of consumers in profile *T* who consume this service.

3. Analysis of national-call consumption

In this section, the analysis is confined to national calls (first part of table 2). The notations are those used in table 2. Generally speaking, $X_T^{p,m}(t,d)$ designates the value of a variable *X* (ex., number of minutes consumed) for a profile *T*, a product *p*, or a month *m*, during period *t* (peak or off-peak) toward *d* (fixed phone, mobile phone on same network, etc.).

The use of a product by a profile-*T* consumer in month *m* is expressed as the consumption of a number of minutes $C_T^m(t,d)$ in period *t* toward destination *d*.

For a profile *T* and for each month *m*, the consumption of call minutes in a period *t* to a destination *d* is deduced from the total consumption Λ_T^m of minutes (all periods *t* and all destinations *d* in the aggregate):

$$C_T^m(t,d) = \pi_T(t)\delta_T(d)\Lambda_T^m \qquad (1)$$

In this equation, $\pi_T(t)$ denotes the proportion of minutes consumed in *t* and $\delta_T(d)$ the proportion of minutes consumed toward destination *d*. We assume this distribution to be independent of the product (package or voucher) used and of the month. Moreover, equation (1) reflects the assumption that call periods and call destinations are unrelated.

3.1. Randomness of monthly consumption duration

The call durations Λ_T^m are random in each month *m*. The total number of call minutes varies from month to month, and the average value of this number is, in itself, insufficient to measure the impact of a change in a tariff parameter on a price index. One example will make this clear.

Let us consider a unlimited-commitment package,[3] consisting of 60 minutes/month, at 1 franc per "inclusive" and "additional" minute. Take a profile with an average consumption of 60 minutes par month. Let us assume the price of additional minutes rises from 1 to 2 francs.

If we describe the profile strictly in terms of its average monthly call duration, prices remain stable.

If, however, we refine the hypotheses by assuming the profile's monthly consumption to be random—45 minutes, 60 minutes or 75 minutes with the same probability for each—then the profile's consumption of additional time is 5 minutes on average (or in mathematical-mean terms): 0 minutes if the monthly call time is 45 or 60 minutes (2/3rds probability), 15 minutes with a 1/3rd probability. Thus, the average total monthly expenditure grows from 60 + 5 = 65 francs before the price rise to 60 + 2*5 = 70 francs after the price rise, making a price increase of 7.7%.

If we now factor in the roll-over minutes—which are equal, at most, to the package inclusive time (1 hour)—the stock of available minutes can take the following values: 60, 75, 90, 105 or 120 minutes. For example, the 120-minute call time will be reached after four consecutive periods of consumption of 45 minutes. Let α be the probability that the available stock will be only 60 minutes ($0 < \alpha < 1$). In the following month, we have 15 additional minutes (out of a total consumption of 75 minutes) with a 1/3rd probability, generating an average excess cost of (1/3)*15*1 = 5 francs before the increase and (1/3)*15*2 = 10 francs after. If the stock exceeds 60 minutes (probability equal to 1-α), there are no additional minutes.

[3] As we shall see, this hypothesis is not very restrictive.

Consequently, the price index, set at base 1 for the initial period, is:

$$I(\alpha) \;=\; \frac{(1-\alpha)60+\alpha 70}{(1-\alpha)60+\alpha 65} \;=\; \frac{60+10\alpha}{60+5\alpha}$$

This is an *increasing* function in α. In sum, prices still increase with the roll-over, but by less than the 7.7% figure we had found before factoring in the roll-over effect ($\alpha = 1$).

3.2. Actual consumption and consumption billed by operators

In reality, it is not the *actual* call-time consumption $C_T^m(t,d)$ that will play a role, but the minutes *billed by operators*. Let us consider a product *p*. The call time $f^p(\Delta)$ billed by an operator differs from the actual call time Δ :

$$f^p(\Delta) \;=\; \Delta_1^p \;+\; \left\{ Ent\left[\frac{(\Delta-\Delta_1^p)^+}{\Delta_2^p}\right] \;+\; \frac{(\Delta-\Delta_1^p)^+}{\left|\Delta-\Delta_1^p\right|} \right\}\Delta_2^p \qquad (\forall\ \Delta > 0) \qquad (2)$$

where Δ_1^p denotes the minimum call time billed and Δ_2^p the minimum additional time billed (*Ent* stands for the whole part and $x^+ = \sup(x,0)$). The calls do not all have the same duration. Let N_T^m be the total monthly number of calls per profile and Φ_T the distribution of this *number* of calls by duration. In month *m*, profile *T* makes $(\Phi_T(\Delta)d\Delta)N_T^m$ calls ranging in duration from Δ to $\Delta + d\Delta$. We assume this distribution to be deterministic and independent of the month examined, the product, the call period, and the destination. This assumption is justified by the very large number of calls made each month. We have:

$$\Lambda_T^m \;=\; \left(\int_0^{+\infty} \Delta\Phi_T(\Delta)d\Delta\right)N_T^m \qquad (3)$$

Thus, the total call time *billed* by the operator is:

$$\Lambda_T^{p,m} = \left(\int_0^{+\infty} f^p(\Delta)\Phi_T(\Delta)d\Delta\right)N_T^m \qquad (4)$$

The billed monthly call time for profile *T* in *t* to destination *d* therefore depends on the product *p* used:

$$C_T^{p,m}(t,d) = \pi_T(t)\delta_T(d)\Lambda_T^{p,m} \qquad (5)$$

In consequence, according to (4) and (5), the same is true of the $\Lambda_T^{p,m}$ and $C_T^{p,m}(t,d)$ values. Equations (3) and (4) show that the total call time *billed* is proportional to the actual time:

$$\Lambda_T^{p,m} = \frac{\int_0^{+\infty} f^p(\Delta)\Phi_T(\Delta)d\Delta}{\int_0^{+\infty} \Delta\Phi_T(\Delta)d\Delta}\,\Lambda_T^m \qquad (6)$$

3.3. Managing the time allowance: packages

The monthly expenditure $M_T^{p,m}$ resulting from the use of a package *p* by a profile *T* is equal to the monthly subscription F^p plus additional minutes billed at the unit price $\pi^{p,dep}$. Measuring the number of additional minutes is complicated by roll-over (offered by all operators), i.e., unused minutes from the current month can be carried forward to the following month. The distinction between peak hours and off-peak hours in some packages (SFR, for example) makes this assessment even more difficult.

<u>3.3.1. Roll-over minutes</u>

The purchase of a package *p* includes a total monthly number $C^p(t)$ of call minutes (a "time allowance") for each period *t*. The time allowance in month *m* can be increased by the previous month's unused minutes $R_T^{p,m-1}(t)$. This roll-over depends on the product, the month (the $C_T^{p,m}(t)$ values are random[4]), and the profile. The number of minutes available $\overline{C}_T^{p,m}(t)$ in a given month *m* during a period *t* thus also depends on that month and on the profile:

$$\overline{C}_T^{p,m}(t) = C^p(t) + R_T^{p,m-1}(t) \qquad (7)$$

If the package is not a renewal, the first month $R_T^{p,m-1}(t) = 0$.

<u>3.3.2. Measuring the time allowance and monthly expenditure</u>

We show (appendix 1) the existence of segmented linear functions Φ, Ψ_{hc} and $\Psi_{hp} : R_+^4 \rightarrow R$ such that, for any package *p*, any profile *T*, and any month *m*:

$$M_T^{p,m} = F^p + \pi^{p,dep}\Phi(C_T^{p,m}(hc), C_T^{p,m}(hp), \overline{C}_T^{p,m}(hc), \overline{C}_T^{p,m}(hp)) \qquad (8)$$

$$\overline{C}_T^{p,m}(hc) = C^p(hc) + \Psi_{hc}(C_T^{p,m-1}(hc), C_T^{p,m-1}(hp), \overline{C}_T^{p,m-1}(hc), \overline{C}_T^{p,m-1}(hp)) \qquad (9)$$

$$\overline{C}_T^{p,m}(hp) = C^p(hp) + \Psi_{hc}(C_T^{p,m-1}(hc), C_T^{p,m-1}(hp), \overline{C}_T^{p,m-1}(hc), \overline{C}_T^{p,m-1}(hp)) \qquad (10)$$

The functions Φ, Ψ_{hc} and Ψ_{hp} are defined for all $x, y, \overline{x}, \overline{y} \in R_+^4$ as follows:

[4] $C_T^{p,m}(t)$ denotes billed consumption of minutes in *t*, for all destinations *d* combined.

if $x+y \leq \bar{x}+\bar{y}$:

	$y \leq \bar{y}$	$y > \bar{y}$
$x \leq \bar{x}$	$\Phi(x, y, \bar{x}, \bar{y}) = 0$ $\Psi_{hc}((x,y,\bar{x},\bar{y})=\bar{x}-x$ $\Psi_{hp}(x,y,\bar{x},\bar{y})=\bar{y}-y$	$\Phi(x, y, \bar{x}, \bar{y}) = y - \bar{y}$ $\Phi_{hc}((x,y,\bar{x},\bar{y})=\bar{x}-x$ $\Psi_{hp}(x,y,\bar{x},\bar{y})=0$
$x > \bar{x}$	$\Phi(x, y, \bar{x}, \bar{y}) = 0$ $\Psi_{hc}((x,y,\bar{x},\bar{y})=0$ $\Psi_{hp}(x,y,\bar{x},\bar{y})=(\bar{x}+\bar{y})-(x+y)$	

if $x+y>\bar{x}+\bar{y}$:

$x \leq \bar{x}$	$\Phi(x, y, \bar{x}, \bar{y}) = y - \bar{y}$ $\Psi_{hc}(x,y,\bar{x},\bar{y})=\bar{x}-x$ $\Psi_{hp}(x,y,\bar{x},\bar{y})=0$
$x > \bar{x}$	$\Phi(x, y, \bar{x}, \bar{y}) = (x+y)-(\bar{x}+\bar{y})$ $\Psi_{hc}(x,y,\bar{x},\bar{y})=0$ $\Psi_{hp}(x,y,\bar{x},\bar{y})=0$

The difference in treatment of the *x*s (off-peak hours) and *y*s (peak hours) is due to the fact that the off-peak hours can be taken from the stock of peak hours available when the initial allowance of off-peak hours is used up, whereas the opposite is not possible.

The sequences $\left(\overline{C}_T^{p,m}(hc)\right)_m$ and $\left(\overline{C}_T^{p,m}(hp)\right)_m$ are therefore determined simultaneously by recurrence from (9) and (10). We can then use them to calculate the monthly expenditure $M_T^{p,m}$ with the aid of (8). However, many consumers renew their subscription a certain number of consecutive times, i.e., a number of months Γ^p equal to a multiple of the package call allowance θ^p. The expected average monthly expenditure $D_T^{p,m}$ must be assessed over the period *m*, *m* + 1, ..., *m* + Γ_p-1:

$$D_T^{p,m} = \frac{1}{\Gamma^p}\sum_{l=0}^{\Gamma^p-1} E(M_T^{p,m+l}) \qquad (11)$$

(*E* stands for the mathematical mean).

3.4. Managing the time allowance: vouchers

3.4.1. Distinguishing call periods by price

Two of the three operators, FTM and BT, use prices to break down consumption by call period. With π^t as the price of the minute in *t*, and *t* = *hc* (off-peak) or *hp* (peak), we have:

$$D_T^{p,m} = E(M_T^{p,m}) = \pi^{p,hc} E(C_T^{p,m}(hc)) + \pi^{p,hp} E(C_T^{p,m}(hp)) \qquad (12)$$

When the voucher is used up, profile *T* buys another, without generating a specific cost. Profile *T* may, however, be constrained by the voucher's life span θ if the consumption is too low. The explanation is as follows. Let G^p be the voucher price. If the monthly expenditure $D_T^{p,m}$ determined with (12) is below G^p/θ, the voucher is not used up when it expires. In this case, the monthly expenditure is:

$$D_T^{p,m} = G^p/\theta \qquad (13)$$

3.4.2. Distinguishing call periods by quantity

The operator SFR uses a system to distinguish call periods by quantity. The voucher priced G^p allows the consumer to use the total time allowance:

$$C^p = C^p(hc) + C^p(hp)$$

at the same price per minute regardless of period:

$$\pi^p = G^p/C^p \qquad (14)$$

Users must, however, observe the distinction between call periods, as in §3.3. The calculation of $D_T^{p,m}$ will depend here on the ratio:

$$C_T^{p,m}(hc)/C_T^{p,m}(hp)$$

which is fixed and equal to $\pi_T(hc)/\pi_T(hp)$, consistently with (5).

Case 1: $C_T^{p,m}(hc)/C_T^{p,m}(hp) \geq C^p(hc)/C^p(hp)$

Profile-*T* users consume proportionally more in off-peak hours than in peak hours. They will therefore initially use up the voucher's "off-peak hours" time allowance, but this will not be a problem because the additional off-peak hours will be found in the "peak hours" time allowance. Thus the distinction between call periods in no way constrains profile *T*: this is a situation of unlimited consumption with a price per minute of π^p. Hence:

$$D_T^{p,m} = E(M_T^{p,m}) = \pi^p E(C_T^{p,m}) \qquad (15)$$

Case 2: $C_T^{p,m}(hc)/C_T^{p,m}(hp) < C^p(hc)/C^p(hp)$

Profile-*T* users consume proportionally more in peak hours than in off-peak hours. They will therefore initially use up the voucher's "peak hours" time allowance, without being able to consume the additional off-peak hours. The voucher consumption will therefore be determined by that of peak hours. Hence:

$$D_T^{p,m} = E(M_T^{p,m}) = G^p E(C_T^{p,m}(hp))/C^p(hp) \qquad (16)$$

which we can also write:

$$D_T^{p,m} = E(M_T^{p,m}) = \pi^p E(C_T^{p,m})\left[\frac{C_T^{p,m}(hp)}{C_T^{p,m}} / \frac{C^p(hp)}{C^p}\right] \quad (17)$$

In (17), the term in brackets exceeds 1 and the product of the first two terms represents the expenditure calculated in case 1 in (15): when voucher use is not optimal, the cost is higher. In practice, $C^p(hp))/C^p = 0.5$. For a "pure" peak-hour user, $C_T^{p,m}(hp) = C_T^{p,m}$, so $D_T^{p,m} = 2\pi^p E(C_T^{p,m})$. The price is double that of optimal-use case 1.

The effect of the constraint due to the voucher's limited life span is the following:

Case 1:

If $C^p/\theta > E(C_T^{p,m})$ then $D_T^{p,m} = G^p/\theta$ (18)

Case 2:

If $C^p(hp)/\theta > E(C_T^{p,m}(hp))$ then $D_T^{p,m} = G^p/\theta$ (19)

4. Product substitution

4.1. "Active" and "passive" product ranges

In the search for an optimal product for a given profile, there is an additional difficulty: the existence of products that are no longer sold but are still owned by many consumers, who can keep (and use) them as long as they like. This problem arises in tracking prices of other products as well, particularly insurance.

We can analyze the issue as follows. Each month *m*, two sorts of products *p* live side by side:

- products that are not sold in *m*, but whose owners in *m*-1 can continue to (or are obliged to) consume in *m*; the total set of these products is written $E^{a,m}$;

- products sold in *m* (sold or not in *m*-1). $E^{n,m}$ denotes their set ($E^{a,m} \cap E^{n,m} = \varnothing$).

Each month *m*, therefore, the population of profile-*T* consumers is distributed among the products belonging to $E^{a,m} \cup E^{n,m}$. To put it differently, in a month *m*, each product *p* belonging to $E^m = E^{a,m} \cup E^{n,m}$ is consumed by a proportion $f_T^{p,m}$ of profile-*T* consumers (which may be equal to zero).

4.2. Change in profile consumption between two consecutive months

For a given profile *T*, the relationship between $f_T^{p,m}$ and $f_T^{p,m-1}$ is complex.

The product *p* owned in *m*-1 belongs to $E^{a,m}$

Two sub-cases are possible:

- In *m*, the expenditure $D_T^{p,m}$ generated by *p* is less than or equal to the expenditure on a product belonging to $\hat{E}_T^{n,m}$. In this case, the profile keeps this product *p* in *m*. The set of these products is written $E_T^{a^+,m}$:

$$E_T^{a^+,m} = \left\{ p \in E^{a,m},\ D_T^{p,m} \le \hat{D}_T^{n,m} \right\}$$

- In *m*, the expenditure $D_T^{p,m}$ is <u>greater than</u> the price of a product in $\hat{E}_T^{n,m}$. In this case, the profile stays with this product *p* with a probability of 1 - π_T^p, or leaves this product for a product in the $\hat{E}_T^{n,m}$ set with a probability of π_T^p. Probability π_T^p is called a "migration." The set of these products is written $E_T^{a^-,m}$:

$$E_T^{a^-,m} = \left\{ p \in E^{a,m},\ D_T^{p,m} > \hat{D}_T^{n,m} \right\}$$

Note: $E_T^{a^+,m} \cup E_T^{a^-,m} = E^{a,m}$.

<u>The product *p* owned in *m*-1 belongs to $E^{n,m}$</u>

Two sub-cases are again possible:

- In *m*, the price $D_T^{p,m}$ of *p* is <u>equal</u> to the price of a product in $\hat{E}_T^{n,m}$:

$$D_T^{p,m} = \hat{D}_T^{n,m}$$

i.e., $p \lfloor\ \hat{E}_T^{n,m}$. In this case, the profile keeps this product *p* in *m*.

- The product *p* owned in *m*-1 has a price $D_T^{p,m}$ in *m* that <u>exceeds</u> the price of a product in the $\hat{E}_T^{n,m}$ set:

$$D_T^{p,m} > \hat{D}_T^{n,m}$$

i.e., $p \in \hat{E}_T^{n,m}$. In this case, the profile stays with this product *p* with a probability of 1- π_T^p π_T^p, or leaves this product for a product in the $\hat{E}_T^{n,m}$ set with a probability π_T^p.

Let $S_T^{p,m}$ be the number of profile-*T* consumers owning a product *p* in month *m*. We assume that:

$$\sum_{p \in E^m} S_T^{p,m} = \sum_{p \in E^{m-1}} S_T^{p,m-1} \qquad (m = 1, \ldots 12)$$

Hence:

$$S_T^{p,m} = S_T^{p,m-1} \qquad \text{if } p \in E_T^{a^+,m}$$

$$S_T^{p,m} = (1 - \pi_T^p) S_T^{p,m-1} \ \text{if } p \in E_T^{a^-,m} \cup (E^{n,m} - \hat{E}_T^{n,m})$$

$$S_T^{p,m} = S_T^{p,m-1} + \frac{1}{\left|\hat{E}^{n,m}\right|} \sum_{p' \in E_T^{a^-,m} \cup (E^{n,m} - \hat{E}_T^{n,m})} \pi_T^{p'} S_T^{p',m-1} \qquad \text{if } p \in \hat{E}_T^{n,m}$$

The term $1/\left|\hat{E}^{n,m}\right|$ embodies the hypothesis that new consumers or those leaving a product *p* are uniformly distributed among the optimal products sold in *m*, i.e., the products in the $\hat{E}_T^{n,m}$ set.

Dividing by $\sum_{p \in E^m} S_T^{p,m}$ we get:

$$
\begin{aligned}
f_T^{p,m} &= f_T^{p,m-1} && \text{if } p \in E_T^{a^+,m} \\
f_T^{p,m} &= (1-\pi_T^p) f_T^{p,m-1} && \text{if } p \in E_T^{a^-,m} \cup (E^{n,m} - \hat{E}_T^{n,m}) \\
f_T^{p,m} &= f_T^{p,m-1} + \frac{1}{\left|\hat{E}_T^{n,m}\right|} \sum_{p' \in E_T^{a^-,m} \cup (E^{n,m} - \hat{E}_T^{n,m})} \pi_T^{p'} f_T^{p',m-1} && \text{if } p \in \hat{E}_T^{n,m}
\end{aligned}
\qquad (20)
$$

4.3. Determining probabilities of migration between products

Using the formulas above (20), we can calculate the frequencies $f_T^{p,m}$ as a function of the values $f_T^{p,m-1}$ known in the previous period. Knowing the actual distributions in base month 0, and therefore the $f_T^{p,0}$ values, we can deduce the $f_T^{p,m}$ values by recurrence. However, we still need to estimate the product-migration probabilities, expressed in simple form by the π parameter in the (20) equations.

Migrations between products are determined by several factors:

- first, the change must be *worthwhile*, i.e., it must not consist in abandoning an optimal product;

- the financial benefit of the switch must be large enough to offset the non-monetary drawbacks of the change (paperwork, new number, etc.);

- the consumer must be *aware* that the change is worthwhile;

- the consumer must be *rational*;

- the consumer must not be *restricted* by the commitment inherent in the purchase of each product (duration of contract for a package, total cost of voucher for a pre-paid voucher). This restriction may be perceived as a powerful monetary constraint: a consumer has very little to gain by canceling a package before maturity to take another, cheaper one.

The probability π of a collective migration is therefore, itself, the product of individual probabilities:

$$\pi = \pi_i \pi_c \qquad (21)$$

The π_i component reflects the irrationality of consumers, their lack of information on the products offered, or the excessive complexity of the offering. The π_c component results from the contractual nature of the packages or the financial commitment involved in purchasing a voucher. In both cases, the user consumes a product that does not belong to the $E_T^{a^+,m} \cup \hat{E}_T^{n,m}$ set.

The probability π depends, of course, on the class: it is greater for vouchers than for packages. It would, however, be scarcely realistic to assume π is independent of *p*. More specifically, there is a great difference between substitution involving (a) products offered by the same operator and (b) products offered by different operators. In the first case, the cost for packages is zero (for example, subscribers can change package durations at any time, free of charge or for a minimal fee, if they stay within the same range of products) and the cost for vouchers is fairly low; in the second case, the cost is much higher, since the migration involves the commitment constraints specific to each product.

Determining π is a crucial step in the index construction. There are two alternative approaches:

- The first consists in relying on observations of consumer substitution behavior. For example, there are surveys indicating the average time a consumer keeps the same product. Dataquest Cellular Services estimates this period at five years. On these assumptions, π = (1/5)(1/12) = 1.66% per month for a package. Each month, 1/12th of all consumers have reached the end of their packages, hence π_c = 1/12. Therefore, even if we had assumed π_i = 1, π would not have exceeded 1/12.

- The second method—which can, in fact, be combined with the first—rests on the comparison between the estimated and actual $f_T^{p,m}$ values. As these are determined by the selected π values, we can make a 12-month comparison *a posteriori* between the estimated and actual $f_T^{p,12}$ values. Using statistical adjustment procedures (such as iteration and linearization), we can adjust the π parameters so as to obtain the most realistic $f_T^{p,12}$ distribution possible.

5. Calculating the price index

5.1. Profile index

Each month *m*, each profile-*T* user consumes a product *p*. The total expenditure by *T* consumers is:

$$D_T^m = \sum_p N_T^{p,m} D_T^{p,m} \quad (22)$$

where $N_T^{p,m}$ is the number of profile-*T* consumers consuming product *p* in month *m*. The average price paid *per T consumer* is:

$$P_T^m = \sum_p f_T^{p,m} D_T^{p,m} \quad \text{where} \quad f_T^{p,m} = \frac{N_T^{p,m}}{\sum_p N_T^{p,m}} \quad (23)$$

The price index for profile *T* between base month 0 and month *m* is:

$$I_T^{m/0} = \frac{P_T^m}{P_T^0} = \frac{\sum_p f_T^{p,m} D_T^{p,m}}{\sum_p f_T^{p,0} D_T^{p,0}} \quad (24)$$

5.2. Overall index

The overall index $I^{m/0}$ is an annually-chained Laspeyres index, with the elementary indices $I_T^{m/0}$ and the weights w_T^0, which represent the *T* profiles' consumption shares in the base period:

$$I^{m/0} = \sum_{T=1}^{12} w_T^0 I_T^{m/0} \qquad (25)$$

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Appendix 1: Evaluation of time allowance and monthly expenditure for packages

This appendix defines equations (8), (9), and (10) of Section 3.3.2. For this purpose, we distinguish between different cases according to whether the consumer does or does not exceed (a) the total time allowance, and (b) the allowance by call period (peak/off-peak).

<u>Case 1</u>: $C_T^{p,m}(hc) + C_T^{p,m}(hp) \leq \overline{C}_T^{p,m}(hc) + \overline{C}_T^{p,m}(hp)$

Sub-case 1.1: $C_T^{p,m}(hc) \leq \overline{C}_T^{p,m}(hc)$

- If $C_T^{p,m}(hp) \leq \overline{C}_T^{p,m}(hp)$ then:

$$D_T^{p,m} = F^p$$

with F^p the monthly charge for the package. Minutes are rolled over to month *m*+1 in peak hours as well as off-peak hours:

$$R_T^{p,m}(hc) = \overline{C}_T^{p,m}(hc) - C_T^{p,m}(hc)$$

$$R_T^{p,m}(hp) = \overline{C}_T^{p,m}(hp) - C_T^{p,m}(hp)$$

where $R_T^{p,m}(t)$ denotes the roll-over minutes in period *t* of month *m*+1.

- If $C_T^{p,m}(hp) > \overline{C}_T^{p,m}(hp)$ then:

$$D_T^{p,m} = F^p + \pi^{p,dep}(C_T^{p,m}(hp) - \overline{C}_T^{p,m}(hp)).$$

Minutes are rolled over to month *m*+1 only in off-peak hours:

$$R_T^{p,m}(hc) = \overline{C}_T^{p,m}(hc) - C_T^{p,m}(hc)$$

$$R_T^{p,m}(hp) = 0.$$

Sub-case 1.2: $C_T^{p,m}(hc) > \overline{C}_T^{p,m}(hc)$

This gives the following breakdown:

$$C_T^{p,m}(hc) = \overline{C}_T^{p,m}(hc) + (C_T^{p,m}(hc) - \overline{C}_T^{p,m}(hc))$$

with, according to the general assumption in case 1,

$$0 < (C_T^{p,m}(hc) - \overline{C}_T^{p,m}(hc)) \leq (\overline{C}_T^{p,m}(hp) - C_T^{p,m}(hp)).$$

The minutes $C_T^{p,m}(hc) - \overline{C}_T^{p,m}(hc)$ are therefore taken from the stock of peak minutes. Hence:

$$D_T^{p,m} = F^p.$$

Minutes are rolled over to month *m*+1 only in peak hours:

$$R_T^{p,m}(hc) = 0$$

$$R_T^{p,m}(hp) = \left[\overline{C}_T^{p,m}(hc) + \overline{C}_T^{p,m}(hp)\right] - \left[C_T^{p,m}(hc) + C_T^{p,m}(hc)\right].$$

Case 2: $C_T^{p,m}(hc) + C_T^{p,m}(hp) \; > \; \overline{C}_T^{p,m}(hc) + \overline{C}_T^{p,m}(hp)$

Sub-case 2.1: $C_T^{p,m}(hc) \; \le \; \overline{C}_T^{p,m}(hc)$

This gives us $C_T^{p,m}(hp) \; > \; \overline{C}_T^{p,m}(hp)$. Thus, in the breakdown

$$C_T^{p,m}(hp) \; = \; \overline{C}_T^{p,m}(hp) \; + \; (C_T^{p,m}(hp) \; - \; \overline{C}_T^{p,m}(hp)),$$

the consumption $C_T^{p,m}(hp) \; - \; \overline{C}_T^{p,m}(hp)$ exceeds the package (the stock of off-peak hours cannot be drawn upon). Hence:

$$D_T^{p,m} \; = \; F^p \; + \; \pi^{p,dep}(C_T^{p,m}(hp) \; - \; \overline{C}_T^{p,m}(hp)).$$

Minutes are rolled over to month *m*+1 only in off-peak hours:

$$R_T^{p,m}(hc) \; = \; \overline{C}_T^{p,m}(hc) \; - \; C_T^{p,m}(hc)$$

$$R_T^{p,m}(hp) \; = \; 0 \; .$$

Sub-case 2.2: $C_T^{p,m}(hc) \; > \; \overline{C}_T^{p,m}(hc)$

- If $C_T^{p,m}(hp) \; \le \; \overline{C}_T^{p,m}(hp)$ then

$$C_T^{p,m}(hc) - \overline{C}_T^{p,m}(hc) \; > \; \overline{C}_T^{p,m}(hp) \; - \; C_T^{p,m}(hp) \; \ge \; 0 \, .$$

Thus, in the breakdown of minutes consumed in off-peak hours:

$$C_T^{p,m}(hc) \; = \; \overline{C}_T^{p,m}(hc) \; + \; \left[\overline{C}_T^{p,m}(hp) \; - \; C_T^{p,m}(hp)\right] \; + \; \left[\left(C_T^{p,m}(hc) - \overline{C}_T^{p,m}(hc)\right) \; - \; \left(\overline{C}_T^{p,m}(hp) \; - \; C_T^{p,m}(hp)\right)\right].$$

The first two terms are "covered" by the package. The second term is drawn from the stock of peak minutes, which is consequently depleted. The third term exceeds the package. Hence:

$$D_T^{p,m} \; = \; F^p \; + \; \pi^{p,dep}\left[\left(C_T^{p,m}(hp) + C_T^{p,m}(hc)\right) \; - \; \left(\overline{C}_T^{p,m}(hp) \; + \; \overline{C}_T^{p,m}(hc)\right)\right].$$

No minutes are rolled over to month *m*+1, either in off-peak hours or in peak hours:

$$R_T^{p,m}(hc) \; = \; 0$$

$$R_T^{p,m}(hp) \; = \; 0 \, .$$

- If $C_T^{p,m}(hp) \; > \; \overline{C}_T^{p,m}(hp)$ then

$$D_T^{p,m} \; = \; F^p \; + \; \pi^{p,dep}\left[\left(C_T^{p,m}(hp) + C_T^{p,m}(hc)\right) \; - \; \left(\overline{C}_T^{p,m}(hp) \; + \; \overline{C}_T^{p,m}(hc)\right)\right].$$

No minutes are rolled over to month *m*+1, either in off-peak hours or in peak hours:

$$R_T^{p,m}(hc) \; = \; 0$$

$$R_T^{p,m}(hp) \; = \; 0 \, .$$