# Direct and Indirect Time Dummy Approaches to Hedonic Price Measurement

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**Abstract:** Quality-adjusted price indexes are frequently obtained by estimating how much of the price difference between a disappearing item and its replacement is due to a quality difference. Hedonic regression has become a popular quality-adjustment method among statistical agencies. The use of the time dummy method is still very limited, though. This paper has two aims. First, it shows how this method fits into the matched-model methodology of agencies applying a geometric mean index formula at the elementary aggregation level. Second, the paper argues that the ordinary or 'direct' time dummy approach cannot cope with systematic price effects of new and disappearing products. Several 'indirect' alternatives are discussed in which the time dummy coefficient serves as a common adjustment factor and in which systematic effects of unmatched products are explicitly taken into account. Special attention is paid to the role of the sampling design, in particular to product sampling proportional to expenditure.

**Keywords:** consumer price index; hedonic regression; quality adjustment; sampling.

## 1. Introduction

The changing mix and quality of products pose difficult problems in constructing the consumer price index (CPI). A statistical agency has to make judgments about how much of the price difference between a disappearing product and its replacement is due to a quality difference. One of the methods to arrive at quality-adjusted price changes is hedonic regression. Most economists agree that this method offers the most promising approach to account for changing product quality. A panel of experts that was asked by the U.S. Bureau of Labor Statistics (BLS) "to investigate conceptual, measurement, and other statistical issues in the development of cost-of-living indexes" shared this view (Schultze and Mackie, 2001).[2] Yet the panel recommends the BLS not to immediately expand the use of hedonics because

[1] The views expressed in this paper are those of the author and do not necessarily reflect the policies of Statistics Netherlands.
[2] The panel will be referred to as the Schultze panel. Silver (2002a) reviews its book entitled *At What Price?*. Hausman (2003) provides a critical assessment of the panel's views and recommendations.

there are substantial unresolved econometric, data and other measurement issues that need further attention.

An approach to hedonic modelling which has mainly been applied in the academic literature is the time dummy approach. In this method, data from multiple periods are used to estimate the coefficients of a function relating the logarithm of price to a set of product characteristics and a set of dummy variables for the periods covered. The antilogarithm (exponential) of the time dummy coefficient directly produces a quality-adjusted price index. Recommendation 4-4 of the Schultze panel states that "BLS should not allocate resources to the direct time dummy method (unless work on other hedonic methods generates empirical evidence that characteristic parameter stability exists for some products)". By parameter stability the panel refers to stability over time. In the longer run this kind of stability is indeed not expected to hold. But in the short run, and especially for consecutive months, the constant-parameter assumption does not seem too restrictive. Time series are obtained by multiplying, or 'chaining' as it is usually called, the adjacent-period (bilateral) hedonic index numbers.

Section 2 of this paper reviews the adjacent-period time dummy method and shows that this method can be viewed as an automatic hedonic quality-adjustment method using a geometric mean index formula at the elementary aggregation level – a point stressed before by Triplett (2001) but not always fully appreciated. Section 3 accounts for the possibility that there are systematic differences between the quality-adjusted prices of matched products (sold in adjacent periods) and unmatched products. This is done by incorporating dummy variables for new and disappearing products into the model. The antilog of the time dummy coefficient should not be interpreted as a quality-adjusted price index any longer, but a hedonic quality-adjustment index akin to the time dummy index can be defined. Section 4 discusses imputation methods that attempt to estimate unobservable prices of unmatched products from the extended model. Special attention is paid to the role of the sampling design. We present an hedonic imputation estimator for the superlative Törnqvist price index under product sampling strictly proportional to expenditure in each period. Section 5 considers some practical issues and suggests a simplified, more feasible Törnqvist-type estimator. Section 6 concludes.

## 2. The (Direct) Time Dummy Price Index

### *2.1 A review*

This is not the place for a thorough review of the literature on hedonics; see e.g. Rosen (1974), Triplett (1988), Griliches (1990), and Berndt et al. (1995). It suffices to mention that there has been some debate about the preferred functional form of hedonic models. Many researchers believe that the choice of the functional form is an empirical rather than a theoretical matter. Most empirical findings favour the logarithmic model over its linear counterpart. This is in agreement with Diewert's (2001, 2003) a priori point of view. Among other things, he argues that the residuals from a logarithmic model are less likely to be heteroskedastic. Time dummy approaches to hedonic regression use a logarithmic specification merely for reasons of convenience.

We will compare two time periods: the base period 0 and the current period 1. The (semi-) logarithmic model can be expressed as

$$\ln p_i^t = \alpha^t + \sum_{k=1}^{K} \beta_k^t x_{ik} + \varepsilon_i^t \qquad (t = 0,1), \tag{1}$$

where $p_i^t$ is the price of product $i$ in period $t$, $x_{ik}$ its $k$-th characteristic ($k$= 1,…,$K$), $\beta_k^t$ the corresponding parameter and $\varepsilon_i^t$ an error term with an expected value of zero. Our analysis does not change when some or even all characteristics are logarithmic. All parameters are time dependent as there is no reason to believe they must be constant over time, and model (1) should preferably be estimated on cross-section data for each time period separately. However, we might expect the parameters to be approximately constant in the short run. Thus if period 0 and period 1 are adjacent periods (i.e. months) it seems justifiable to impose a priori restrictions $\beta_k^1 = \beta_k^0 = \beta_k$ for all $k$. This implies that the restricted model

$$\ln p_i^t = \alpha + \delta D_i + \sum_{k=1}^{K} \beta_k x_{ik} + \varepsilon_i^t \qquad (t= 0,1), \tag{2}$$

may be estimated on the pooled data of both periods, where the time dummy variable $D_i$ takes on the value of one if the $i$-th observation comes from period 1 and zero otherwise. We assume that the errors are independently and identically distributed with constant variances. If these (rather strong) assumptions do not hold, then Ordinary Least Squares (OLS) estimators might be very inefficient. Moreover, a complex structure of the variance-covariance matrix might make it difficult to test whether the parameters are indeed constant over time. Notice that we did not change the notation for the errors, notwithstanding that model (2) is a restricted version of model (1). This should not lead to confusion. The method is known as the adjacent-period time dummy approach. The estimated base period and current period prices of $i$ are $\hat{p}_i^0 = \exp(\hat{\alpha} + \sum_{k=1}^{K} \hat{\beta}_k x_{ik})$ and $\hat{p}_i^1 = \exp(\hat{\alpha} + \hat{\delta} + \sum_{k=1}^{K} \hat{\beta}_k x_{ik})$, respectively. So a *time dummy price index*

$$\hat{P}_{TD} = \hat{p}_i^1 / \hat{p}_i^0 = \exp(\hat{\delta}) \qquad \text{(for all } i) \tag{3}$$

can be computed directly from the estimated model.[3] This is probably why the time dummy method is sometimes called the 'direct' hedonic approach.

Suppose we have a product sample $S^0$ in period 0 and a sample $S^1$ in period 1. It is assumed that hedonic model (2) is estimated by OLS regression on the pooled data of $S^0 \cup S^1$. The regression residuals are defined as $u_i^t = \ln p_i^t - \ln \hat{p}_i^t = \ln(p_i^t / \hat{p}_i^t)$; $t$= 0,1. Due to the inclusion of a constant term and a time dummy, the residuals sum to zero in both periods, or:

$$\sum_{i \in S^0} \ln\left(\frac{p_i^0}{\hat{p}_i^0}\right) = \sum_{i \in S^1} \ln\left(\frac{p_i^1}{\hat{p}_i^1}\right) = 0 . \tag{4}$$

Taking antilogarithms yields

$$\prod_{i \in S^0} \left(\frac{p_i^0}{\hat{p}_i^0}\right) = \prod_{i \in S^1} \left(\frac{p_i^1}{\hat{p}_i^1}\right). \tag{5}$$

If the sample does not change ($S^1 = S^0 = S$, with size $n$), it follows from (5) that

[3] Note that the antilog of $\hat{\delta}$ is not an unbiased estimator of the antilog of $\delta$. Goldberger (1968) provides a correction term. For not too small samples the bias can safely be ignored.

$$\prod_{i\in S}\left(\frac{\hat{p}_i^1}{\hat{p}_i^0}\right)^{\frac{1}{n}} = \hat{P}_{TD} = \prod_{i\in S}\left(\frac{p_i^1}{p_i^0}\right)^{\frac{1}{n}}. \tag{6}$$

Thus, "the price index number formula implied by a dummy variable (logarithmic) regression run on matched models is a ratio of equally-weighted geometric means" (Triplett, 2001). Obviously, the model specification does not matter in this specific case. The Schultze panel also mentions Triplett's work: "Triplett (2001b; 6-7) notes that the dummy variable method, when specified in a double-log or semilog functional form, produces a price index based on the geometric mean formula. Since statistical agencies have begun moving toward using the geometric mean formula to construct elementary item indexes (for other reasons), time dummy approaches have become more consistent with the prevailing methodology." (Schultze and Mackie, 2001, p. 4-19). It is important to realise that the expected value of any elementary price index, and thus of (6), depends on the sampling design. For a discussion of elementary price indexes from a sampling perspective, see Balk (2003).

Often the product sample does change, and this is when quality adjustment comes into play. Products observed in both time periods will be referred to as matched products; $S^1 \cap S^0$ is the matched sample with size $n_M$. We assume that $S^1 \cap S^0 \neq \varnothing$. The sub-sample of products observed during the base period that has disappeared is denoted by $S^{0D}$, and the current period sub-sample of new products is denoted by $S^{1N}$. For reasons of simplicity and because this reflects statistical agencies' usual practices, the sample size $n$ will be kept constant. Hence, the size of the unmatched part of the sample is $n - n_M$. The following relation can be derived from equation (5):

$$\prod_{i\in S^1\cap S^0}\left(\frac{\hat{p}_i^1}{\hat{p}_i^0}\right) = \prod_{i\in S^1\cap S^0}\left(\frac{p_i^1}{p_i^0}\right)\frac{\prod_{i\in S^{1N}}\left(\frac{p_i^1}{\hat{p}_i^1}\right)}{\prod_{i\in S^{0D}}\left(\frac{p_i^0}{\hat{p}_i^0}\right)}. \tag{7}$$

Substituting $\hat{p}_i^1 / \hat{p}_i^0 = \exp(\hat{\delta}) = \hat{P}_{TD}$ for all $i \in S^1 \cap S^0$ in the left-hand side of (7) and some rearranging gives

$$\hat{P}_{TD} = [P_{MJ}]^{f_M}\left[\frac{\prod_{i\in S^{1N}}\left(\frac{p_i^1}{\hat{p}_i^1/\exp(\hat{\delta})}\right)^{\frac{1}{n-n_M}}}{\prod_{i\in S^{0D}}\left(\frac{p_i^0}{\hat{p}_i^0}\right)^{\frac{1}{n-n_M}}}\right]^{1-f_M}, \tag{8}$$

where $P_{MJ} = \prod_{i\in S^1\cap S^0}(p_i^1 / p_i^0)^{1/n_M}$ denotes the geometric mean or Jevons price index for the matched sample; $f_M = n_M / n$ is the fraction of matched products.

Let us take a closer look at equation (8). The second factor between square brackets can be viewed as a quality-adjusted geometric mean price index for the unmatched part. To show this, suppose that a statistical agency selects a replacement product $j \in S^{1N}$ for a disappearing product $i \in S^{0D}$ in order to maintain the initial sample size. The agency observes a difference in quality and adjusts the current period price of replacement *j* as follows:

$$p_j^{1adj} = p_j^1 \exp\left[\sum_{k=1}^{K} \hat{\beta}_k (x_{ik} - x_{jk})\right] = \hat{p}_i^0 \left[\frac{p_j^1}{\hat{p}_j^1 / \exp(\hat{\delta})}\right]. \tag{9}$$

Comparing the second expression on the right-hand side of (9) with (8) reveals that the time dummy method automatically produces a geometric mean price index based on hedonic quality adjustment. This has been mentioned before by e.g. Triplett (2001) and Silver and Heravi (2002). Notice that $p_j^{1adj} = p_j^1$ if $x_{jk} = x_{ik}$ for all *k*. That is, when the disappearing product and its replacement have identical characteristics, no adjustment will be made. It does not matter which product from the set of new products is attached to a certain disappearing product.

## 2.2 Sampling issues

Nothing has been said so far about the sampling design, i.e. about how products are selected. Yet this is a crucial issue because it determines the population price index the time dummy index effectively aims at (the target index). Our starting point is a sample $S^0$ drawn proportional to base period expenditure. This design resembles the initial sampling procedures of some statistical agencies, notably that of the U.S. BLS. Suppose for the moment that there are no disappearing products, neither in the sample nor in the universe (population). The sample geometric mean price index $\prod_{i \in S^0} (p_i^1 / p_i^0)^{1/n} = \hat{P}_{TD}$, given by equation (6), then is an approximately unbiased estimator of

$$P_{GL} = \prod_{i \in U^0} \left(\frac{p_i^1}{p_i^0}\right)^{s_i^0}, \tag{10}$$

where $s_i^0$ is the base period expenditure share of product *i* and $U^0$ the base period universe of products belonging to the product group in question. The bias is positive and will approach zero when the sample size grows. Formula (10) is sometimes called the geometric Laspeyres price index.

Silver (2002b) criticises unweighted (OLS) regression. He concludes that " …. a WLS estimator is preferred to an OLS one", and that "the use of expenditure share weights for hedonic time dummy regression estimates is preferred to relative quantities". This is certainly true for sample data obtained from simple random sampling but not for data obtained from sampling proportional to expenditure. In the latter case, an expenditure-weighted least squares estimator places too much weight on high-expenditure products. WLS may be useful for estimating hedonic imputation indexes (which are discussed in section 4) if the errors are heteroskedastic, even under product sampling proportional to expenditure.

Next, we look at the case where some products have disappeared and are replaced by other products. We might view the adjusted current period price of replacement *j*, given by quality-adjustment procedure (9), as an estimator $\hat{p}_i^{1*}$ of the unobservable price $p_i^{1*}$ of the disappearing product *i*. Hence, under initial sampling proportional to base period expenditure, the time dummy index might loosely be interpreted as a quality-adjusted estimator of the geometric Laspeyres price index (10) when some products disappeared. A problem arises because the statistical agencies' replacement procedures typically do not rely on probability sampling. This not only makes variance estimation difficult, it affects the expected value of the time dummy price index in an unknown way.

## 3. Systematic Effects of Unmatched Products

### *3.1 Extending the model*

Silver and Heravi (2002), Triplett (2001), and various other authors have argued that unmatched (i.e. new and disappearing) products may have unusual prices given their characteristics. Stated otherwise, the 'law of one quality-adjusted price' might not hold. We are particularly interested in those situations where the quality-adjusted prices of unmatched and matched products differ in a systematic way. For example, the observed (current period) prices of new products might be higher than we would expect from an hedonic model – consumers face hidden price increases, it is sometimes said. In this particular case new products most likely exhibit positive residuals. Quality-adjustment procedure (9), and hence the time dummy index (8), seemingly takes this possibility into account. This can easily be shown by rewriting (8) as

$$\hat{P}_{TD} = P_{MJ} \left[ \exp(\bar{u}_N^1 - \bar{u}_D^0) \right]^{\frac{1}{f_M} - 1}, \tag{8'}$$

where $\bar{u}_N^1 = \sum_{i \in S^{1N}} u_i^1 / (n - n_M)$ and $\bar{u}_D^0 = \sum_{i \in S^{0D}} u_i^0 / (n - n_M)$ are the average residuals for the new and disappearing products, respectively. Thus if $\bar{u}_N^1 > 0$ (and $\bar{u}_D^0 \approx 0$), then the time dummy index $\hat{P}_{TD}$ will be greater than the matched-product geometric mean index $P_{MJ}$. There is an inconsistency, however. Equation (3) tells us that the estimated price relative is the same for every product. Consequently, quality-adjusted geometric mean price changes of matched and unmatched products should not differ, apart from random fluctuations, provided that model (2) holds.[4] The advantage of using the time dummy index instead of the matched-product geometric mean index is an expected gain in efficiency (a lower standard error) because the former relies on a larger number of observations.

As a matter of fact, least squares regression will usually provide biased parameter estimators if the expected value of $\varepsilon_i^t$ is non-zero for some *i*. Or to put it another way: when the law of one quality-adjusted price systematically fails, the model has been misspecified. So if the quality-adjusted prices of the unmatched products *systematically* differ from those of the matched ones, then model (2) should be adapted accordingly. We will retain our basic assumption saying that the characteristics parameters remain constant and add dummy variables (allowing the intercept term to shift) for new as well as for disappearing products. The extended model for *t*= 0,1 becomes:

$$\ln p_i^t = \alpha + \delta D_i + \lambda_D^0 D_{iD}^0 + \lambda_D^1 D_{iD}^1 + \lambda_N^0 D_{iN}^0 + \lambda_N^1 D_{iN}^1 + \sum_{k=1}^{K} \beta_k x_{ik} + \varepsilon_i^t, \tag{11}$$

[4] A situation of equal quality-adjusted prices for all products, apart from a random error term, most likely holds in competitive markets. It is therefore not very surprising that empirical studies using the adjacent-period time dummy approach often find (chained) indexes quite similar to matched-product geometric mean indexes in fairly competitive markets. For such a study on Dutch CPI data for computers, see Van Mulligen (2002).

where the dummy $D_{iD}^{t}$ takes on the value of one if product $i$ disappeared in period $t$+1 and zero otherwise, and where the dummy $D_{iN}^{t}$ takes on the value of one if $i$ is new in period $t$ and zero otherwise. It is assumed that products can be observed in at least two consecutive periods. Products that are new in period 0, for instance, are assumed not to disappear in period 1. Scanner data indeed suggest that products are rarely sold during one month only, so this assumption is not really a restrictive one.

This paper looks at bilateral price change; prices in the current period (1) are compared with prices in the preceding period (0). The specification of (11) may seem a bit curious at first sight because (in some sense) it looks beyond those periods. The model not only takes into account systematic effects of the unmatched products – that is, products that are either new in period 1 or disappear after period 0 – but also systematic effects of matched products that are new in period 0 and those that disappear in period 2.[5] This has been done for reasons of symmetry. For example, if in period 1 the quality-adjusted prices of new products systematically exceed those of the other products, there is reason to expect that something similar exists for new products in period 0. The extended model should account for such effects as well. Note that during period 1 no one knows what products will disappear in period 2. Equation (11) therefore cannot be estimated in real time, but here we act as if we did have this knowledge. We will return to this issue in section 5.

The OLS parameter estimators of (11) will be indicated by a tilde. Compared with the 'true' model (11), the simple model (2) suffers from omitted variables, yielding biased OLS estimators and biased predicted values. It is important to recognise that equation (11) models the whole population. This means, for example, that $D_{iN}^{1}$ must equal 1 only if product $i$ has not been sold in period 0. Because statistical agencies lack this kind of information, $D_{iN}^{1}$ will be set to 1 in practice if $i$ is new in the sample. So there is a sample-selection problem, and $\tilde{\lambda}_{N}^{0}$, $\tilde{\lambda}_{N}^{1}$, $\tilde{\lambda}_{D}^{0}$ and $\tilde{\lambda}_{D}^{1}$ may suffer from sample-selection or 'outside-the-sample' bias.

### *3.2 A hedonic quality adjustment price index*

As we have seen, the time dummy index cannot cope with systematic effects of new and disappearing products. It is possible, however, to incorporate these effects into an index based on hedonic quality adjustment. We apply a quality adjustment procedure similar to (9) but estimated from the extended model (11): $p_{j}^{1adj} = p_{j}^{1} \exp[\sum_{k=1}^{K} \tilde{\beta}_{k}(x_{ik} - x_{jk})]$, where $j \in S^{1N}$ replaces $i \in S^{0D}$. Instead of the first expression on the right of (9), we now have

$$p_{j}^{1adj} = p_{j}^{1}\left[\frac{\exp(\tilde{\delta} + \tilde{\lambda}_{N}^{1} - \tilde{\lambda}_{D}^{0})}{(\tilde{p}_{j}^{1} / \tilde{p}_{i}^{0})}\right]. \qquad (12)$$

Using procedure (12) in a geometric mean framework leads to the following explicit *hedonic quality adjustment price index*:

[5] Using OLS regression, the residuals for the matched products sum to zero in both periods. It can easily be demonstrated that the antilogarithm of the estimated time dummy coefficient would be exactly equal to the matched-product geometric mean index if the latter effects had not been incorporated into the model.

$$\tilde{P}_{HQA} = [P_{MJ}]^{f_M} \left[ \frac{\prod_{i \in S^{1N}} \left( \frac{p_i^1}{\tilde{p}_i^1} \right)^{\frac{1}{n-n_M}}}{\prod_{i \in S^{0D}} \left( \frac{p_i^0}{\tilde{p}_i^0} \right)^{\frac{1}{n-n_M}}} \right]^{1-f_M} \left[ \exp(\tilde{\delta} + \tilde{\lambda}_N^1 - \tilde{\lambda}_D^0) \right]^{1-f_M} . \tag{13}$$

This approach can be called an 'indirect' time dummy method, where the time dummy coefficient serves as a common adjustment factor. As a result of adding dummies for new and disappearing products, the OLS residuals for $i \in S^{0D}$ and $i \in S^{1N}$ satisfy

$$\sum_{i \in S^{0D}} \ln\left( \frac{p_i^0}{\tilde{p}_i^0} \right) = \sum_{i \in S^{1N}} \ln\left( \frac{p_i^1}{\tilde{p}_i^1} \right) = 0 . \tag{14}$$

From taking antilogs of (14) it follows that the second factor on the right-hand side of (13) is equal to 1, yielding

$$\tilde{P}_{HQA} = [P_{MJ}]^{f_M} \left[ \exp(\tilde{\delta} + \tilde{\lambda}_N^1 - \tilde{\lambda}_D^0) \right]^{1-f_M} . \tag{15}$$

Let us compare (15) with the following alternative expression for the time dummy price index (8):

$$\hat{P}_{TD} = [P_{MJ}]^{f_M} \left[ \exp(\hat{\delta} + \bar{u}_N^1 - \bar{u}_D^0) \right]^{1-f_M} , \tag{8'')$$

where $\bar{u}_N^1$ and $\bar{u}_D^0$ are the average residuals for the new and disappearing products, based on the ordinary model (2). There is a striking similarity between expressions (8'') and (15). The values of $\hat{\delta}$ and $\tilde{\delta}$, as well as the values of $\bar{u}_N^1 - \bar{u}_D^0$ and $\tilde{\lambda}_N^1 - \tilde{\lambda}_D^0$, might differ only slightly. In practice therefore the time dummy index could approximate the hedonic quality adjustment index quite well, but we cannot be sure of this.

One difficulty remains. If initial product sampling had been performed proportional to base period expenditure, so that the geometric Laspeyres index is the target, it seems a bit odd to take systematic effects of new products into account as these products do not belong to the base period universe (but to the current period universe instead). In our opinion hedonic imputation price indexes make much more sense in this respect.

## 4. Hedonic Imputation Price Indexes

### *4.1 Sampling proportional to base period expenditure*

Our starting point is again a sample $S^0$, which has been drawn proportional to base period expenditure. First we will assume that systematic price effects of unmatched products do not occur. We would like to impute the unobservable (fictitious) current period prices of disappearing products. In section 2.2 we noted that the time dummy price index can be interpreted in this manner. The term hedonic imputation is generally used for those methods that estimate unobservable prices directly from the hedonic model itself, though, and those methods will be investigated here. The imputation procedure should measure what the current period prices would have been – given the bundle of characteristics incorporated in the products – had the products still been sold, that is, had they been matched products instead. Using hedonic model (2), those prices are $\hat{p}_i^{1*} = \exp(\hat{\alpha} + \hat{\delta} + \sum_{k=1}^{K} \hat{\beta}_k x_{ik})$ for $i \in S^{0D}$.

A disadvantage might be substantial variability of the 'quality-adjusted' price index for the unmatched products due to a comparison of estimated (current period) prices with actual (base period) prices. There are several ways to deal with this problem. Silver (2002b) proposes to delete so-called influential outliers. However, unless their quality is in serious doubt, throwing away observations is not a very attractive idea. We prefer to enhance stability by replacing the observed base period prices by their predicted values based on model (2). We then obtain $\hat{p}_i^{1*} / \hat{p}_i^0 = \exp(\hat{\delta})$ for $i \in S^{0D}$. The geometric mean formula yields a *double imputation price index*:

$$\hat{P}_{DI} = [P_{MJ}]^{f_M} \left[\exp(\hat{\delta})\right]^{1-f_M} = [P_{MJ}]^{f_M} \left[\hat{P}_{TD}\right]^{1-f_M}, \tag{16}$$

which is also the result of an 'indirect' time dummy approach.

The double imputation price index is a weighted geometric average of the matched-product geometric mean index and the time dummy index. This method is intuitively appealing since it restricts hedonic modelling explicitly – rather than implicitly, as in the time dummy index (8) – to unmatched products, while leaving the price relatives of matched products unaffected. Notice that under the present sampling design $P_{MJ}$ is an approximately unbiased estimator of the matched-product geometric Laspeyres price index; the expected value of $f_M$ is approximately equal to the base period expenditure share of the matched products.

Next we assume that systematic price effects of unmatched products do occur. The unobservable current period prices of disappearing products now have to be imputed according to hedonic model (11). Those prices are $\tilde{p}_i^{1*} = \exp(\tilde{\alpha} + \tilde{\delta} + \sum_{k=1}^{K} \tilde{\beta}_k x_{ik})$, since we want to measure what they would have been had the disappearing products been matched products instead. Using the geometric mean framework again, the imputation index reads

$$\tilde{P}_{GL} = [P_{MJ}]^{f_M} \left[ \frac{\prod_{i \in S^{0D}} \left(\exp(\tilde{\alpha} + \tilde{\delta} + \sum_{k=1}^{K} \tilde{\beta}_k x_{ik}\right)^{\frac{1}{n-n_M}}}{\prod_{i \in S^{0D}} \left(p_i^0\right)^{\frac{1}{n-n_M}}} \right]^{1-f_M}. \tag{17}$$

Equation (14) implies $\prod_{i \in S^{0D}} (p_i^0)^{1/(n-n_M)} = \prod_{i \in S^{0D}} (\tilde{p}_i^0)^{1/(n-n_M)}$. So $\tilde{P}_{GL}$ automatically becomes a *double* imputation index. Using $\tilde{p}_i^0 = \exp(\tilde{\alpha} + \tilde{\lambda}_D^0 + \sum_{k=1}^{K} \tilde{\beta}_k x_{ik})$ for $i \in S^{0D}$, (17) reduces to

$$\tilde{P}_{GL} = [P_{MJ}]^{f_M} \left[\exp(\tilde{\delta} - \tilde{\lambda}_D^0)\right]^{1-f_M}. \tag{18}$$

When $\lambda_D^0$ approaches zero we expect to find $\tilde{\delta} \approx \hat{\delta}$ and thus $\tilde{P}_{GL} \approx \hat{P}_{DI}$, particularly for not too small samples. This result strengthens our choice made above for double instead of single (one-sided) imputation.[6] $\widetilde{P}_{GL}$ can be viewed as an estimator of the geometric Laspeyres price index (10) under sampling proportional to base period expenditure when some products are no longer available. In (10) we define the unobservable current period prices for $i \in U^{0D}$ as $p_i^1 = p_i^{1*}$, where $U^{0D}$ denotes the disappearing part of $U^0$. The procedure to select replacement products is less important, provided that model (11) holds.

### *4.2 Estimating the Törnqvist price index*

Suppose next that our starting point was a sample $S^1$ drawn proportional to current period expenditure. Products that have not been sold in the base period are 'replaced backwards' by products that did sell. Again, the actual 'replacement procedure' is of minor importance. We now wish to compute an imputation price index by imputing the unobservable base period prices for new products. The resulting imputation price index can be expressed as

$$\widetilde{P}_{GP} = [P_{MJ}]^{f_M} \left[\exp(\widetilde{\delta} + \widetilde{\lambda}_N^1)\right]^{1-f_M} . \tag{19}$$

$\widetilde{P}_{GP}$ is an estimator of the 'geometric Paasche price index'

$$P_{GP} = \prod_{i \in U^1} \left( \frac{p_i^1}{p_i^0} \right)^{s_i^1} , \tag{20}$$

where $s_i^1$ is the current period expenditure share of product $i$ and $U^1$ the current period product universe. In (20) we define the unobservable base period prices for $i \in U^{1N}$ as $p_i^0 = p_i^{0*}$, where $U^{1N}$ denotes the new part of $U^1$. It follows that the geometric average of (18) and (19), i.e.

$$\widetilde{P}_T = [P_{MJ}]^{f_M} \left[ \exp(\widetilde{\delta} + \frac{\widetilde{\lambda}_N^1 - \widetilde{\lambda}_D^0}{2}) \right]^{1-f_M} , \tag{21}$$

is an approximately unbiased estimator of the Törnqvist price index

$$P_T = [P_{GL} P_{GP}]^{\frac{1}{2}} = \prod_{i \in U^0} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{s_i^0}{2}} \prod_{i \in U^1} \left( \frac{p_i^1}{p_i^0} \right)^{\frac{s_i^1}{2}} \tag{22}$$

under independent sampling proportional to expenditure in both periods.

The Törnqvist index is defined on a variable set of products or a 'dynamic universe' (Dalén, 2001). This index can be seen as a generalisation of the ordinary, static universe Törnqvist index, which is adjusted for quality changes in the sense that all unobservable prices are imputed. De Haan (2002) describes a similar generalisation of the Fisher price index. Estimator (21) makes no explicit use of $\widetilde{\lambda}_D^1$ and $\widetilde{\lambda}_N^0$. One should not conclude that the corresponding dummy variables $D_{iD}^1$ and $D_{iN}^0$ can be dropped from model (11), since $\delta$ would then no longer be estimated unbiasedly; see also footnote 4. Comparing equations (16)

[6] De Haan and Opperdoes (2002) applied hedonic imputation methods in a Fisher index framework, and found that single imputation sometimes led to implausible results.

and (21) makes clear that the hedonic quality adjustment index (16) is an upward (downward) biased estimator of the Törnqvist index if $\lambda_N^1 > \lambda_D^0$ ($\lambda_N^1 < \lambda_D^0$).

The interpretation of the 'backward imputation' approach used in (19) and (21) can be problematic because production of the new products may not have been possible with the technology of the earlier period. Triplett (2001) remarks: "The price index quality adjustment problem sometimes makes it necessary to estimate a price in period t for a new computer that first became available in period t+1. One cannot ignore the fact that the new computer was not actually available in period t, and the possible reasons why it was not available. The special assumptions necessary to validate "backcasting" a price for a machine that was not in fact produced should be kept in mind." But we cannot escape from 'backcasting' when defining and estimating the Törnqvist price index (22), or any other type of symmetric index for that matter, in a dynamic-universe context.

It should be noted that proportional to expenditure sampling independently in both time periods might be an inefficient sampling strategy, resulting in an unnecessarily small matched sample. Some $i \in S^{1N}$, while not belonging to the matched sample, could actually belong to the matched universe $U^1 \cap U^0$. Despite the fact that their base period and current period prices can be observed, at least in principle, estimator (21) uses the predicted values.

So far it has been assumed that a difference in quality can only exist when products have different sets of characteristics, and systematic effects of new and disappearing have been treated as real price changes. For durable goods, such as televisions and computers, this assumption is a reasonable one. For certain other product groups (for example novels), on the other hand, consumers may appreciate new products over old ones with otherwise the same characteristics. Model (11) would still hold, but in this case 'newness' and 'oldness' become quality aspects themselves. One probably wants to control for those aspects as well and incorporate the dummy variables for new and disappearing products into the quality-adjustment procedure. It is straightforward to show that all imputation indexes discussed above now equal $\exp(\tilde{\delta})$. Unlike in the case of the time dummy index, the expected values of $\exp(\tilde{\delta})$ and the matched-product geometric mean index differ, precisely because model (11) holds instead of model (2). We will not pursue this special case any further here, and next turn to some practical considerations when systematic effects of unmatched products are deemed real.

## 5. Practical Considerations

The analysis in section 4 suggests we should make a choice between the imputation indexes (18), (19) and (21). As noted before, the sampling design is the crucial issue at stake. Let us start with (18), which can be viewed as an estimator of the geometric Laspeyres price index under sampling proportional to base period expenditure.[7] This design resembles the initial sampling procedures of some statistical agencies, but it does not reflect the way in which products are selected for adjacent months at later points in time. Estimator (19) has no

[7] Under *systematic* sampling proportional to size some high-expenditure products may have a probability of 1 to be in the sample. The products belonging to this so-called self-selecting part must be weighted according to expenditure in order to obtain approximately unbiased estimators. See for example De Haan et al. (1999). We will not address this complication here.

practical relevance whatsoever. This estimator has merely been shown in order to arrive at (21).

Under sampling proportional to expenditure in both periods (21) is to be preferred from a theoretical perspective because it estimates the superlative (adjacent-period) Törnqvist price index. In its pure form this sampling design would be infeasible for statistical agencies. They might nevertheless try to mimic it. At present, many agencies select a replacement product similar to the 'old' one. While this method is understandable from a pragmatic side because it seemingly reduces the need for quality adjustment, it should not be recommended. To keep the sample up-to-date, a better procedure would be to select products that (are expected to) have a significant market share. A more offensive strategy would be to replace a part of the sample each month instead of waiting until forced replacements occur. Doing this such that high-expenditure products will have a higher inclusion probability than low-expenditure products mimics product sampling proportional to expenditure to some extent, so that estimator (21) gets a meaningful interpretation. The number of unmatched products decreases, of course, which makes hedonic regression particularly helpful.

Still the fraction of unmatched products could be small. In fact an attrition rate of 0.1 would be unusually large, not only for CPI samples but also for the universe – except perhaps in case of high-tech goods like computers. With a small unmatched sample, the standard errors of $\widetilde{\lambda}_N^1$ and $\widetilde{\lambda}_D^0$ might be very large. The variance of the price index will then be substantial. Using Taylor linearization a formula for the approximated variance of (21), conditional on the design matrix of *x*-values (and thus on $f_M$), is easily derived from the variance-covariance matrix supplied by most statistical packages. One must realise that those packages typically assume simple random sampling, which gives rise to bias in the (co)variance estimators. It would be much more difficult to estimate the unconditional variance.

Things get even more complicated because in practice one does not yet know what products will disappear in period 2. This makes real-time estimation of model (11) – particularly of $\lambda_D^1$ – impossible. However, using the sample-replacement rule suggested above there seems to be no need to include dummies for disappearing products since products will have left the sample well before they actually disappear from the market.[8] This rule might nevertheless create some 'outside-the-sample bias'. An additional (and, admittedly, rather strong) assumption we shall make is $\lambda_N^1 = \lambda_N^0 = \lambda_N$. We therefore propose the following simplified version of (11):

$$\ln p_i^t = \alpha + \delta D_i + \lambda_N D_{iN} + \sum_{k=1}^{K} \beta_k x_{ik} + \varepsilon_i^t \qquad (t = 0{,}1), \tag{23}$$

where the dummy $D_{iN}$ takes on the value of one if *i* is new in either period 0 or 1 and zero otherwise. Denoting the parameter estimators of the dummy variables in (23) by $\breve{\delta}$ and $\breve{\lambda}_N$, estimator (21) now reduces to

[8] For research purposes Statistics Netherlands has bought scannerdata from GfK for some durable goods. A first inspection of the data for televisions and washing machines revealed that most products did not show substantial price changes just before they left the market. This supports the case for not including dummy variables for disappearing products. Silver and Heravi (2002), on the other hand, using similar data on washing machines, did find evidence of lower quality-adjusted prices for disappearing products.

$$\breve{P}_T = [P_{MJ}]^{f_M} \left[ \exp(\breve{\delta} + \frac{1}{2}\breve{\lambda}_N) \right]^{1-f_M} . \qquad (24)$$

The best way to proceed after having estimated model (23) seems to test whether $\lambda_N$ differs from zero. A cautious approach is called for. Testing at low significance levels almost inevitably leads to rejecting the null hypothesis $H_0 : \lambda_N = 0$, and the usual 5% level might be too low. If the null hypothesis cannot be rejected, model (2) should be estimated in a second stage and the (double) imputation index (16) should be computed.

The question arises whether estimators like (24) or (16) can successfully be applied in practice using price data that are collected for the compilation of the CPI. Suppose the statistical agency needs at least *n* price observations to estimate a geometric mean price index with sufficient accuracy. A necessary requirement for using model (23) on the pooled data of adjacent periods is $n \geq (K+3)/2$. This requirement will usually be met. What is needed as well is sufficient variation in characteristics and prices. This requires the use of wide product descriptions for selecting specific items in the outlets, contrary to Statistics Netherlands' practices. For example, Statistics Netherlands collects about 30 prices monthly for each of 9 specific television sets (*n*= 270). Although the overall sample size is not a bottle-neck for implementing time dummy methods, current data gathering procedures should be redesigned towards using wide product descriptions and, above all, collecting data on the characteristics of all sampled products. Statistical agencies may find the latter requirement especially burdensome. It can be argued, on the other hand, that every sophisticated quality-adjustment method needs data on product characteristics, so they should have been collected anyway. Such data are becoming increasingly available via the internet. Other data sources might be the manufacturers, the importers, or even the retailers where prices are collected.

Expression (24) suggests we do not have to bother so much about the stability of the characteristics' coefficients. Multicollinearity by itself raises no problems. Although the variance of the hedonic index will be higher than it would have been otherwise, the choice of the set of characteristics included in the model should not depend on statistical but on economic reasoning. Those characteristics must be included that are related to the product's performance. According to Triplett (2001) it requires "careful thought, sometimes subtle analysis, and most importantly, knowing one's product". Including irrelevant variables does not affect the unbiasedness of the OLS estimators for the relevant variables. So increasing $R^2$ by including non-performance variables does not affect the unbiasedness of $\breve{\delta}$ and $\breve{\lambda}_N$, and hence does not introduce quality-adjustment bias. But there is a problem of inefficiency. Including irrelevant variables generally raises the standard errors of the coefficients on the relevant ones. In particular, if in model (23) the dummy variable $D_{iN}$ is highly correlated with the unjustly included variables, $\delta$ and $\lambda_N$ would be estimated unnecessarily inefficient.

## 6. Conclusions

It is well established in the index number literature that the direct time dummy approach produces a geometric mean price index. The prices of newly introduced products are synthetically matched to the prices of replaced products via hedonic quality adjustment. One of our conclusions states that when there are no systematic price effects of new and disappearing products, the time dummy index should not deviate in a systematic way from the matched-product geometric mean index – provided that the characteristics parameters are constant (which is our basic assumption). The time dummy approach can still be justified on the grounds that it might lower the standard error of the index by increasing the number of observations, albeit at the expense of possible bias caused by an imperfectly specified hedonic model.

The impact of modelling can be reduced by using an hedonic imputation approach. This approach estimates unobservable prices of unmatched product directly from the hedonic model itself, and does not try to make a synthetic match (quality adjustment) between a disappearing product and a replacement. Hedonic imputation methods allow us to take the sampling design explicitly into account and to estimate target (population) indexes like the geometric Laspeyres index. Moreover, hedonic imputation indexes can account for systematic effects of unmatched products, whereas the time dummy index cannot. To this end, the hedonic model should be extended with dummy variables for new and disappearing products. An imputation index has been proposed, which approximates the population Törnqvist index under sampling proportional to expenditure in both periods. Using a sample replacement rule whereby new products enter the sample timely and replace obsolete products well before they actually disappear from the market has an obvious advantage compared with traditional approaches in which the sample is kept fixed until forced replacements occur. Nevertheless, some 'outside-the-sample' bias can arise if products had unusual prices just before they disappear.

The analysis outlined in this paper depends on the appropriateness of the logarithmic specification of the hedonic model and on the assumption of constant parameters. The constant-parameter assumption has been criticised frequently, recently by the Schultze panel. For adjacent periods, though, this assumption does not seem too unrealistic. In any case it is much less restrictive than using coefficients relating to months or even years ago, which is what statistical agencies often do when applying hedonic regression. The advantage of pooling data from adjacent months is a substantial gain in efficiency, and this property could make time dummy methods feasible on actual CPI data without a need to enlarge the sample.

The Schultze panel raises another point of concern worth mentioning. "Finally, most uses of hedonics ….. predict the price that would have prevailed in period *t* for a variety or model not actually offered for sale in that period. While this seems sensible, it is problematic at the theoretical level: under imperfect competition, if an additional variety or model had actually been offered for sale, the prices of the other products might also have changed" (Schultze and Mackie, 2001, p. 4-40). The panel is right, obviously. Yet the criticism is somewhat misdirected because it pertains to quality adjustment methods in general, not just to hedonics. Predicting unobservable prices of unmatched products (while leaving the prices of matched products unaffected) is essentially what quality adjustment is about, either with or without using hedonics. Taking the panel's opinion literally would make it virtually impossible to compile quality-adjusted price indexes.

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