THE CONSUMER PRICE INDEX AND INCOME ESCALATION

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Abstract: This paper addresses the issue of income escalation in the presence of price inflation. It appears that there is in general no simple measure, such as the consumer price index or a cost-of-living index, that tells households by how much their income should be escalated in order to prevent a welfare decrease.

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1. Introduction

The consumer price index (CPI) is an important measure of price inflation. Roughly stated, it measures price inflation by answering the question: what is today's cost of yesterday's consumption basket, compared to yesterday's cost? This way of measuring price inflation is essentially based upon Edgeworth's influential First Memorandum presented to the British Association for the Advancement of Science in 1887 (and reprinted in Edgeworth 1925). In this Memorandum he considered the question of finding a measure for the "appreciation or depreciation of money". He discussed six different measures and argued - still convincingly, I think - for the "consumption standard" as the principal one. According to the

*) The views expressed in this paper are those of the author and do not necessarily reflect the policies of Statistics Netherlands.

conclusion of the Third Memorandum it

> "takes for the measure of appreciation or depreciation the change in the monetary value on a certain set of articles. This set of articles consists of all the commodities consumed yearly by the community either at the earlier or the later epoch, or some mean between those two sets."

Rephrased in modern language, Edgeworth recommended a (chained) all households Laspeyres, Paasche, or Marshall-Edgeworth price index. Because of operational constraints the CPI is usually of the Laspeyres type.

There are signs, however, that Edgeworth was already aware of a more fundamental - "more philosophical", as he calls it - concept. In a footnote (Edgeworth 1925, p.210) he adds the possibility

> "that the Unit [i.e. the sum of money estimated to be equivalent at present (or at some future time) to what a Unit of money, say a pound, was worth at some past time] should constantly afford the same quantity of utility."

Thus the more fundamental version of the question stated in the opening paragraph is: what is today's cost of yesterday's utility level, compared to yesterday's cost? The operationalization of this question has lead to the economic index theory, the foundations of which were laid down in the years 1920-30. The answer is given by a cost-of-living index. Theoretical work in consumer theory has made clear that it is possible to undo 'utility level' from its cardinal connotations; nowadays, 'utility level' stands for a set of consumption possibilities between which a consuming unit is indifferent. The CPI is now considered as an approximation to a cost-of-living index, which, because of operational constraints, cannot be calculated frequently and precisely.

The CPI (or a set of CPI's for subgroups of households) is used for a variety of purposes. One of those is indexation, a purpose that already figures prominently in Edgeworth's writings. A widespread practice is to use the CPI for the escalation of incomes and all sorts of transfer payments from the government to the households. The purported aim is to protect income and transfer recipients entirely or partially from a welfare decrease due to price inflation. However, according to Triplett (1983),

> "economists generally believe that a cost-of-living index is the appropriate measure for escalation purposes."

Triplett's paper contains a very lucid discussion of the possible ends and means of escalation as such. It is not my intention to repeat that discussion here. There is not much to add to it either. This paper explores only some descriptive index theoretic issues. It is assumed that, whether the escalation is exercised or not, households are interested in a measure that tells them by how much their income should be escalated in order to prevent a welfare decrease. It will appear that in general there does not exist a simple, single measure of this kind. In particular, I will focus on the problem of income escalation in relation to a further use that can be made of the CPI, namely to index the income tax rule. It will then appear that income escalation and tax indexation are reciprocally related to each other. More of the one needs less of the other.

This paper unfolds as follows. Section 2 reviews fairly familiar material. When the aim of escalation is to enable a household, confronted with later period prices, to attain its base period utility level, then the household's budget must be inflated by a cost-of-living index. However, following Triplett (1983) we show that certain complications arise when the budget comes from, say, two sources, one of which changes autonomously. Further complications arise because escalation is usually intended to act upon the household's income rather than upon its budget. This is the subject of section 3. The appropriate escalation factor now appears to be a gross cost-of-living index, which is in general not only a function of the prices but also of the tax rules of both periods. Of special interest is the case in which the government has decided to index the income tax rule. Then the income escalation factor depends not only on the cost-of-living index but also on the index used for updating the tax rule. In section 4 we consider the case where the government also uses the instrument of indirect taxes, and where the aim is not to compensate for indirect tax tariff changes. Section 5 concludes.

2. The cost-of-living index and the CPI

We consider a single (representative) household, endowed with a utility function $U(x)$, where $x \equiv (x_1, \ldots, x_N) \in R^N_+$ is a quantity vector of market commodities. We assume that the usual regularity conditions hold (see for instance Cornes 1992). The household's decision problem is

$$(1) \quad \max_x \{U(x) \mid px \leq e\},$$

where $p \equiv (p_1, \ldots, p_N) \in R^N_{++}$ is a vector of prices, $e \in R_{++}$ is the household's budget, and $px \equiv \Sigma^N_{n=1} p_n x_n$. Let $x(p,e)$ be the quantity vector that solves (1). Then $V(p,e) \equiv U(x(p,e))$ is the household's indirect utility function. $V(p,e)$ is the maximum utility attainable when the prices are p and the budget is e.

We consider a base period where prices are p^0 and budget is e^0. The maximum utility attainable is then given by $V(p^0, e^0)$. Let in a comparison period the prices be changed into p^1. We now ask for the minimum factor λ with which base period budget e^0 has to be escalated in order to keep the household on the base period utility level $V(p^0, e^0)$, that is we ask for

$$(2) \quad \min \{\lambda \mid V(p^0, e^0) \leq V(p^1, \lambda e^0)\}.$$

Let $V(p^0, e^0) = u^0$. Then $e^0 = C(p^0, u^0)$, where $C(p,u)$ is the household's expenditure (or cost) function. The solution of (2) is

$$(3) \quad P(p^1, p^0, u^0) \equiv C(p^1, u^0)/C(p^0, u^0),$$

that is the Laspeyres-Konüs (L-K for short) *cost-of-living index*. It is wellknown that

$$(4) \quad P(p^1, p^0, u^0) \leq p^1 x^0 / p^0 x^0,$$

where $x^0 = x(p^0, e^0)$. The Laspeyres price index $P^{01}_L \equiv p^1 x^0 / p^0 x^0$ serves usually as the basic model for the *consumer price index* (CPI). A dual version of expression (4) is

(5) $\quad V(p^1, P_L^{01} e^0) \geq u^0$.

This inequality makes clear that escalating the base period budget with the Laspeyres price index overcompensates the household. With the escalated budget it can reach a higher utility level.

The Laspeyres price index can be considered as a first order approximation to the L-K cost-of-living index: expand $C(p^1, u^0)$ as a Taylor series around p^0 and make use of Shephard's Lemma. Alternatively, we can expand $\ln C(p^1, u^0)$ around $\ln p^0$. This procedure yields as a first order approximation to $\ln P(p^1, p^0, u^0)$

(6) $\quad \ln P_{CD}^{01} \equiv \Sigma_{n=1}^{N} w_n^0 \ln(p_n^1/p_n^0)$,

where $w_n^0 \equiv p_n^0 x_n^0 / p^0 x^0$ $(n=1, \ldots, N)$ are the base period budget shares. P_{CD}^{01} is the Cobb-Douglas price index, which is exact for the Cobb-Douglas utility function. Notice that

(7) $\quad P_{CD}^{01} \leq P_L^{01} = \Sigma_{n=1}^{N} w_n^0 (p_n^1/p_n^0)$,

but P_{CD}^{01} is not necessarily larger than $P(p^1, p^0, u^0)$. Economic considerations suggest that the Cobb-Douglas price index is a better approximation to the L-K cost-of-living index than the Laspeyres price index. Thus it is preferable to use the Cobb-Douglas price index as an escalator. Notice that both price indices require the same information.

Let us now consider the situation in which total budget comes from two sources, one of which changes autonomously (for instance, income from interest) while the other has to be escalated. In this case we must solve λ from the equation

(8) $\quad u^0 = V(p^0, e_1^0 + e_2^0) = V(p^1, \lambda e_1^0 + e_2^1)$.

The solution is

(9) $\quad \lambda = P(p^1, p^0, u^0) + (e_2^0/e_1^0)[P(p^1, p^0, u^0) - e_2^1/e_2^0]$.

The first thing to notice is that the escalator λ now depends on two factors, the cost-of-living index $P(p^1,p^0,u^0)$ and the change of the autonomous component e_2^1/e_2^0. If the growth of the autonomous component is smaller than the cost-of-living index, then the remaining component has to be escalated with a factor which is larger than the L-K cost-of-living index. Notice further that if e_1^0 is escalated with the Laspeyres price index there can still be overcompensation, namely when

$$(10)\quad P_L^{01} - P(p^1,p^0,u^0) > (e_2^0/e_1^0)[P(p^1,p^0,u^0) - e_2^1/e_2^0].$$

The lefthand side of this inequality is a measure of the substitution bias of the Laspeyres price index. Thus if the substitution bias is large enough, escalation with the Laspeyres price index need not be harmful.

3. The gross cost-of-living index

In the previous section we considered the household's budget e as an exogenous variable. However, it is more realistic to consider the household's income $y \in R_{++}$ as such. The relation between income and budget is assumed to be

$$(11)\quad e = y - T(y),$$

where $T(y)$ is the amount of (income) tax. For simplicity's sake we assume that there are no tax deductible expenditures and that the household does not save. With respect to the tax rule T we assume that (i) T is continuously differentiable, (ii) T is convex, (iii) $T(0) \leq 0$, and (iv) $0 < T'(y) < 1$. Assumptions (ii) and (iii) imply that the marginal tax rate is greater than the average tax rate,

$$(12)\quad T'(y) \geq T(y)/y.$$

An equality sign in (12) would correspond to a linear tax, and a strict inequality sign to a progressive tax. Assumption (iv) combined with (12) implies that $y - T(y) \in R_{++}$ and that $y - T(y)$ is a strictly increasing function of y.

To start with, we assume that between base period and comparison period the tax rule has not changed. We ask for the factor with which base period income y^0 has to be escalated in order to keep the household on the base period utility level, that is we ask for the solution λ of the equation

$$(13) \quad u^0 = V(p^0, y^0 - T(y^0)) = V(p^1, \lambda y^0 - T(\lambda y^0)).$$

This solution is a particular instance of the *gross cost-of-living index*, discussed in Balk (1994). It is immediate that if T is linear then $\lambda = P(p^1, p^0, u^0)$, the L-K cost-of-living index. Consider the non-linear case. The solution λ of (13) has to satisfy

$$(14) \quad P(p^1, p^0, u^0) = [\lambda y^0 - T(\lambda y^0)]/[y^0 - T(y^0)].$$

Suppose that $p^1 \geq p^0$ (elementwise). Then $\lambda y^0 \geq y^0$, because $y - T(y)$ is strictly increasing. Since T is convex, $T(y^0)/y^0 \leq T(\lambda y^0)/\lambda y^0$, or $T(\lambda y^0) \geq \lambda T(y^0)$. Combining this with (14), we obtain

$$(15) \quad 1 \leq P(p^1, p^0, u^0) \leq \lambda.$$

Similarly, if $p^1 \leq p^0$ then

$$(16) \quad 1 \geq P(p^1, p^0, u^0) \geq \lambda.$$

Thus if between base period and comparison period the prices have increased (decreased) and the household's income has been escalated with the L-K cost-of-living index, then the household has been undercompensated (over-compensated).

We now turn to some first order approximations to λ. Rearranging (14) yields

$$(17) \quad \lambda = [C(p^1, u^0) + T(\lambda y^0)]/[C(p^0, u^0) + T(y^0)].$$

We expand $\ln[C(p^1, u^0) + T(\lambda y^0)]$ as a Taylor series around $(\ln p^0, \ln y^0)$ and use Shephard's Lemma. Neglecting second and higher order terms we obtain after some rearranging

(18) $\ln \lambda \approx a^0 \ln P_{CD}^{01}$,

where $a^0 \equiv [1 - T(y^0)/y^0]/[1 - T'(y^0)]$ is the base period value of the inverse of the residual (income) progression, that is the elasticity of after tax income with respect to pretax income. Notice that assumption (iv) and (12) imply that $a^0 \geq 1$. Alternatively, we can expand $C(p^1,u^0) + T(\lambda y^0)$ as a Taylor series around (p^0,y^0) and retrace the foregoing steps. We then obtain

(19) $\lambda \approx a^0(P_L^{01} - 1) + 1$.

The expressions (18) and (19) throw more light on the inequalities (15) and (16).

We now assume that between base period and comparison period not only the prices change but also the tax rule. Thus instead of (13) we consider

(20) $u^0 = V(p^0,y^0-T(y^0)) = V(p^1,\lambda y^0-T^1(\lambda y^0))$.

The solution λ is the gross cost-of-living index $P_G(p^1,p^0,u^0)$, considered by Balk (1994). Expression (20) is equivalent to

(21) $P(p^1,p^0,u^0) = [\lambda y^0-T^1(\lambda y^0)]/[y^0-T(y^0)]$.

Suppose that $T^1(y) \geq T^0(y)$ for all $y \in R_{++}$. In the same way as above we can show that if $p^1 \geq p^0$ then

(22) $1 \leq P(p^1,p^0,u^0) \leq \lambda$.

Thus if between base period and comparison period the prices as well as the tax have increased and the household's income has been escalated with the L-K cost-of-living index, then the household has been undercompensated.

It is interesting to consider the case of an indexed tax rule. We call a tax rule indexed when for $t=0,1,\ldots,T$

(23) $T^t(y) = \pi^t T(y/\pi^t)$ for all $y \in R_{++}$ and a certain $\pi^t \in R_{++}$.

In particular, (23) implies that

(24) $T^1(y) = (\pi^1/\pi^0)T^0(y/(\pi^1/\pi^0))$ for all $y \in R_{++}$.

Using the convexity of T^0 it is easy to see that

(25) $\pi^1 \geq(\leq)\ \pi^0 \Rightarrow T^1(y) \leq(\geq)\ T^0(y)$ for all $y \in R_{++}$.

Substituting (24) into (21) and using the same reasoning as leading to (15) we obtain the following result:

(26) $P(p^1,p^0,u^0) \geq(\leq)\ \pi^1/\pi^0 \Rightarrow P(p^1,p^0,u^0) \leq(\geq)\ \lambda$.

This result corresponds to Proposition 9 of Balk (1994). In particular we see that if between base and comparison period the tax rule is indexed with the L-K cost-of-living index, then the L-K cost-of-living index is also the appropriate income escalator.

Under an indexed tax rule it is possible to derive a first order approximation to the escalation factor λ. Rearranging (21) with (24) substituted yields

(27) $\lambda = [C(p^1,u^0) + \pi^1 T(\lambda y^0/\pi^1)]/[C(p^0,u^0) + \pi^0 T(y^0/\pi^0)]$.

Expanding $\ln[C(p^1,u^0) + \pi^1 T(\lambda y^0/\pi^1)]$ as a Taylor series around $(\ln p^0, \ln \pi^0, \ln y^0)$, using Shephard's Lemma, neglecting second and higher order terms, and doing some rearranging, we finally obtain

(28) $\ln \lambda \simeq a^0 \ln P^{01}_{CD} + (1 - a^0)\ln(\pi^1/\pi^0)$.

This result corresponds to Proposition 12 of Balk (1994). Recall that $a^0 \geq 1$, and hence $1 - a^0 \leq 0$. In general it will be the case that $\ln(\pi^1/\pi^0) \geq 0$. Thus the second term in the righthand side of (28) will be negative. Expression (28) implies that if the tax rule is indexed by the Cobb-Douglas price index, $\pi^1/\pi^0 = P_{CD}^{01}$, then the income escalator is approximately equal to the same Cobb-Douglas price index, $\lambda \approx P_{CD}^{01}$. To go a step further we notice that if $\ln(\pi^1/\pi^0) = (a^0/(a^0 - 1))\ln P_{CD}^{01}$, then $\ln \lambda \approx 0$. Thus income escalation is not necessary when the tax rule is appropriately indexed.

The extension of the foregoing to two income components, one of which is to be escalated, is straightforward. Instead of (20) we consider

$$(29)\quad u^0 = V(p^0, y_1^0 + y_2^0 - T^0(y_1^0 + y_2^0)) =$$

$$V(p^1, \lambda y_1^0 + y_2^1 - T^1(\lambda y_1^0 + y_2^1)).$$

If we again assume that the tax rule is indexed, it is possible to derive a first order approximation to the escalation factor λ. Suppose that y_1 is labor income and y_2 is non-labor income. Then λ is the factor with which labor income has to be escalated in order to keep the household on the same utility level as before. If the hours of work do not change, λ is equivalent to the socalled real wage index (for which a better name would be 'minimum wage index').

4. The net consumer price index

The consumer prices of commodities include indirect taxes like value added tax and various excise duties. Let $\tilde{p}^1$ be the comparison period prices which ceteris paribus would prevail under the base period indirect tax tariffs. These prices are of course only theoretical constructs. For the computation of the indirect tax adjusted comparison period prices one usually takes only the first order effects into account. Let now the objective be to solve λ from

$$(30)\quad u^0 = V(p^0, y^0 - T(y^0)) = V(\tilde{p}^1, \lambda y^0 - T^1(\lambda y^0)).$$

That is we look for the factor λ with which base period income has to be escalated in order to retain the household in the situation with indirect tax adjusted comparison period prices on the base period utility level. It is immediate that when $p^1 \geq \bar{p}^1$ then

$$(31) \quad V(p^1, \lambda y^0 - T^1(\lambda y^0)) \leq V(\bar{p}^1, \lambda y^0 - T^1(\lambda y^0)),$$

that is the household experiences with escalated income still a welfare loss.

If the tax rule is indexed we obtain, by retracing the steps leading to expression (28),

$$(32) \quad \ln \lambda \simeq a^0 \Sigma_{n=1}^N w_n^0 \ln(\bar{p}_n^1/p_n^0) + (1 - a^0)\ln(\pi^1/\pi^0).$$

The first term of the righthand side of (32) contains the logarithm of the Cobb-Douglas price index with indirect tax adjusted comparison period prices instead of plain comparison period prices. The Laspeyres analogue, $\Sigma_{n=1}^N w_n^0 (\bar{p}_n^1/p_n^0)$, is called the *net CPI*. Comparison of the CPI and the net CPI reveals, at least to the first order, the effect of indirect tax tariff changes. From (32) we conclude that if the tax rule is indexed with the net Cobb-Douglas price index then the appropriate income escalator is approximately equal to the net Cobb-Douglas price index.

5. Conclusion

The CPI is frequently used to escalate incomes and transfer payments in the presence of price inflation. The aim of such an escalation is to protect households from a welfare decrease. Given this aim, it has been shown in the previous sections that the CPI is not an appropriate instrument. Replacing the CPI by a cost-of-living index is no solution either. Depending on the actual situation, the desired income escalator appears to be a more or less intricate function of the cost-of-living index and other indicators. Thus there is no single index that will do the job.

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