The Use of Weights in Hedonic Regressions: the Measurement of Quality-Adjusted Price Changes

Mick Silver

Cardiff Business School, Cardiff University, UK

Acknowledgements are due to Erwin Diewert (University of British Columbia) and Verena Dexheimer (Federal Statistical Office, Germany) for helpful comments and the UK Office for National Statistics for their support.

Abstract : Hedonic regressions are used in the measurement of quality-adjusted price changes. Price is regressed on a set of characteristics of a sample of items and the estimated equation is used in a number of ways to undertake adjustments for quality changes. Statistical offices use the coefficients, or predicted prices, to adjust prices for quality when a new item of a different quality replaces an obsolete existing one. This is to ensure the resulting price comparisons are not tainted by quality differences. In academic studies it is more usual to include dummy variables for time in the regression equation, their coefficients providing estimates of the change in price over time having adjusted for quality changes. In these and other approaches it is important that the price (changes) are properly weighted in the calculation. It is axiomatic that the price (change) of an item with relatively low sales should not have the same effect as one with relatively high sales. Yet in spite of the widespread use of hedonic regressions little attention has been given to the proper incorporation of weights, it being simply assumed that a weighted least squares (WLS) estimator is appropriate. First, this paper shows how influence and leverage effects have a distorting effect on the weights under WLS. It second, develops and outlines a number of alternative approaches to the measurement of *weighted* quality-adjusted price changes and the circumstances under which each is most appropriate.

Keywords: Hedonic Regression; Quality-adjustment; Consumer Price Index; CPI; Leverage Effects.

JEL classification: C43, C81, C82, E31, M21.

1. Introduction

It is axiomatic that a measure of the aggregate change in prices should be weighted. Some items may have much larger sales, in terms of relative quantities or values, and they should be given correspondingly more emphasis in the calculation. However, items vary in quality both for a given time period and over time. There is an extensive literature on the theory and use of hedonic regressions to 'control' for such quality differences so that the resulting measures of price changes are unaffected by quality variation (Rosen 1974, Triplett, 1988, Griliches, 1990 and Berndt et al., 1995). One form of such quality-adjusted 'hedonic indices' is the time dummy variable approach as provided by δ_t in a hedonic regression of the (log of)[1] price of m=1,...,M items in periods t=1,…,T, $p_{mt,}$ on their k=1…,K quality characteristics, x_{kmt} , D_t dummy variables and errors ε_m where:

$$\ln p_{mt} = \alpha + \sum_{k=1}^{K} \beta_k x_{kmt} + \sum_{t=2}^{T} \delta_t D_t + \varepsilon_{mt} \qquad (1)$$

While alternative forms of weighting have received a lot of attention in choosing between index number formulae, little attention has been paid to weighted hedonic estimates, in spite of the increasing need for, and use of, such estimates as a result of rapid changes in the quality of items. An ordinary least squares regression estimator for (1) makes no use of weights, something quite unacceptable by normal index number standards if data on weights are available. The use of a weighted least squares (WLS) estimator is an obvious approach, yet its rationale is not clear. Neither is it clear whether relative quantities or sales values should be used as the weights.

A second motivating factor behind this paper is that the coefficients from hedonic regressions are used by some statistical offices to quality-adjust the prices of non-comparable replacements in the compilation of consumer price indices (CPIs) (for example, for the BLS see Liegey and Shepler, 1999). When one of the sample of matched items is no longer available, a replacement is chosen, and this may be of a different quality. The estimated coefficients β_k (or predicted values) from a hedonic regression are used to 'correct' the price of the replaced or original item for the difference in quality, as outlined below. Yet again it is axiomatic that some weighting be applied in the estimation of such coefficients, since again some observations may have relatively high sales and other negligible ones.

This paper shows how a WLS estimator may be inappropriate and outlines and develops a number of approaches to show how, and under what circumstances, different methods should be used for the proper measurement of weighted, hedonic, quality-adjusted indices. Section 2 follows index number theory by asking what the target index should be for a hedonic index. Section 3 is based on Diewert (2002a) and asks which form the weights should take when using a WLS estimator to correspond to a target index. Section 4 develops this framework to show that leverage and influential observations affect the desired weighting structure. Section 5 provides empirical evidence based on scanner data to show that the weights from a WLS estimator may not coincide with those required for the target index formula. Section 6 proposes a number of alternative approaches to WLS, the appropriate measure depending on the extent, if at all, to which data are matched over time. Section 7 concludes.

2. Target indices

First it is necessary to establish target index number formulae to which the outcome from the hedonic analysis should correspond. Following Diewert (1976 and 1978), economic theory supports the use of superlative index number formulae. Diewert (1995 and 2002) show that these formulae can also be justified from an axiomatic, fixed basket, stochastic and Divisia approach and in practice give very similar results. The two most widely adopted superlative indices from respective geometric and arithmetic approaches are the Törnqvist price index,

$$P_T \equiv \prod_{m=1}^{M} \left(p_{m1} / p_{m0}\right)^{\left(S_m^0 + S_m^1\right)/2} \qquad (2)$$

[1] Diewert (2002a) argues that the residuals from a logarithmic formulation are less likely to be heteroskedastic.

where q_{mt} are quantities of item m in period t and $s_{mt} \equiv p_{mt}q_{mt} / \sum_{m=1}^{M} p_{mt}q_{mt}$ (t=0,1) their expenditure share. The Fisher index,

$$P_F \equiv \left(\frac{\sum_{m=1}^{M} p_{m1}q_{m0}}{\sum_{m=1}^{M} p_{m0}q_{m0}} \right)^{1/2} \left(\frac{\sum_{m=1}^{M} p_{m1}q_{m1}}{\sum_{m=1}^{M} p_{m0}q_{m1}} \right)^{1/2} = \left(P_L P_P\right)^{1/2} \qquad (3)$$

is the geometric average of the Laspeyres and the Paasche price index. Equations (2) and (3) require price comparisons of matched quality; the same m items are included in the summation in both periods. If these items are of very different qualities then the matching has to be abandoned and quality differences controlled for using (1). The first concern is how to include appropriate weights in a hedonic formulation such as equation (1) to make it correspond to a target index.

3. Weighted least squares (WLS) estimator and superlative formulae

3.1 An index number approach

Diewert (2002a) makes a number of contributions. First, he argues that observations on a model should be accordingly repeated if they sell more for the estimates to be *representative*. A WLS estimator with quantity weights is equivalent to an OLS estimator for which observations are replicated according to the number of times they occur. The use of a quantity WLS estimator effectively treats the observations as transactions, one observation repeated for each transaction.

Second, that value weights are preferred to quantity weights: "The problem with quantity weighting is this: it will tend to give too little weight to cheap models that have low amounts of useful characteristics." (Diewert, 2002a: 8). Third, that for WLS estimates of (1), expenditure share weights should be used as opposed to the value of expenditure, to avoid inflation increasing period 1 value weights resulting in possible heteroskedastic residuals. Finally, when a model is present in both periods, the average expenditure shares, 1/2(s_{m0} + s_{m1}), should be used as weights in the WLS estimator. If only matched models exist in the data, then such estimates will be equivalent to the Törnqvist index (2). If an observation m is only available in one of the periods, its weight should be s_{m0} or s_{m1} accordingly, and the WLS estimator provides a *generalisation* of the Törnqvist index.

3.2 The econometric approach

WLS estimators are generally advised when the errors from estimated models are heteroskedastic and some reference is necessary to WLS in this context.

A WLS estimator of $\gamma_i = \beta x_i + u_i$ minimises $\sum_i w_i(\gamma_i - \beta x_i)^2$

For $\hat{\beta}$ to have the smallest conditional error variance $w_i = 1/\sigma_i$ where $\text{var}(u_i) = \sigma^2$. OLS gives equal weight to each observation while WLS give more weight to observations with less conditional variance, thereby decreasing the sampling variance of the OLS estimator. Thus an observation from a distribution with less conditional variance is more informative (in a predictive sense), than an observation from a distribution with a higher conditional variance. *A priori* we cannot say whether items with larger sales will naturally have a commensurately smaller variance, such analysis being unhelpful for the motivation for using weights in this context. Moreover, the axiomatic concern of giving more weight to prices (price changes) with higher sales is not necessarily met by a methodology whose focus is on minimising (squared) residuals.

4. Leverage and influence

It is first noted that an OLS vector of $\boldsymbol{\beta}$ estimates is a weighted average of the individual **p** elements, the prices of individual models,

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{p} \qquad (4)$$

where the matrix **X** are the explanatory variable and $(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ are the implicit weights given to the prices. Equation (4) clearly shows that the $\hat{\boldsymbol{\beta}}$ estimate is a weighted average of prices, **p**. Consider also a WLS estimator where the explicit weights are expenditure shares:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{W}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}\mathbf{p} \qquad (5)$$

It is apparent from (4) and (5) that outliers with unusual values of **X** will have a stronger influence in determining $\hat{\boldsymbol{\beta}}$, than observations which are one of a group clustered in a small area. Furthermore, (5) shows that the imposition of weights **W** allows the influence to vary with **W**. Thus in normal index number formulae such as (2) and (3), the weights given to price changes are expenditure shares, while in the hedonic framework in (1) the results from an expenditure share weighted hedonic regression will also be determined by the residuals and **X** characteristics. An old model of a, for example, washing machine may have unusually poor quality characteristics, and an unusually low price given such characteristics, the relatively high residuals and leverage giving it undue influence in spite of the weights in (5).

There are two concerns. First, for considering the effect of an outlier on the hedonic estimates $\hat{\boldsymbol{\beta}}$, and second, for estimating the coefficients on the time dummy δ_t in equation (1).

4.1 Estimating $\hat{\boldsymbol{\beta}}$

Consider the effect of adding a, for simplicity, single unusual observation belonging to a different data generating process to the OLS regression estimate via equation (4). Following Davidson and McKinnon (1993) we compare $\hat{\boldsymbol{\beta}}$ with $\hat{\boldsymbol{\beta}}^{(t)}$ where the latter is an estimate of $\boldsymbol{\beta}$ if OLS was used on a sample *omitting* the new t^{th} observation. Distinguish between the leverage of the t^{th} observation, h_t and its residual $\hat{u}_t$. The *leverage* for observation t is given by:

$$h_t = \mathbf{X_t}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X_t}^T \qquad \text{where } 0 \le h_t \le 1 \qquad (6)$$

and the difference between the hedonic coefficients with the t^{th} observation omitted and included by:

$$\hat{\boldsymbol{\beta}}^{(t)} - \hat{\boldsymbol{\beta}} = -\left(\frac{1}{1-h_t}\right)(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X_t}^T\hat{u}_t \qquad (7)$$

Where h_t and $\hat{u}_t$ are relatively large the effect of the t^{th} observation on at least some of $\hat{\boldsymbol{\beta}}$ is likely to be substantial. Thus high leverage h_t only potentially affects $\hat{\boldsymbol{\beta}}$, it also requires that $\hat{u}_t$

is not close to zero. It follows that including the t^{th} observation in the regression affects the fitted value for that observation by:

$$\mathbf{X_t}\hat{\boldsymbol{\beta}} = \mathbf{X_t}\hat{\boldsymbol{\beta}}^{(t)} + \left(\frac{h_t}{1-h_t}\right)\hat{u}_t \qquad (8)$$

and therefore the *influence*, or the change in the t^{th} residual by including the t^{th} observation is given by:

$$-\left(\frac{h_t}{1-h_t}\right)\hat{u}_t \qquad (9)$$

It can be shown that h_t must on average equal k/n where there are k explanatory variables and n observations. If all h_t were equal to k/n then every observation would have the same leverage. We can thus explore on an empirical basis the values of $-\left(\frac{h_t}{1-h_t}\right)\hat{u}_t$ and h_t when estimating hedonic regressions.

4.2 Hedonic indices

Hedonic indices can take the form of a hedonic regression with dummy variables included for time as in equation (1). Assume that instead of there being a single new t observations in equations (6) to (9), there are n_t which belong to each period t=0,1, the initial observations belonging to period 0. Bear in mind that in the previous analysis if we could identify the unusual observation, a dummy variable which took the value of 1 for the unusual observations and zero otherwise, would be included in the regression, and then $\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}}^{(t)}$ in (7). Say the period 1 observations are only unusual in the sense that they have a constant shift parameter δ_1 (in logs) applied to them; they are ($\exp(\delta_1)$-1) percent higher than in period 0. Then the dummy variable hedonic indices in equation (1) will capture the quality-adjusted price change. The appropriate weights for a WLS estimator of δ_1 in (1) to equate with the average expenditure-share weighted Törnqvist index (2) are thus expenditure-share weights. As in (5), a potential problem still remains with influential observations, as opposed to the simple shift parameter.

A simple illustrative example is provided in Annex 2 on influence effects, while in section 5 we turn to examining whether influence matters in our calculations; whether quantity/expenditure share weights and influence differ.

5. Some empirical evidence

It may be argued that older/newer models/brands of a product are likely to have unusual characteristics, prices and thus residuals, and therefore influence over and above their expenditure share weights (Berndt et al., 2001; Pakes, 2002). However, if the hedonic regression controls for the effect on prices of the unusual features the residuals may be low and, via (9), their influence. The alignment of weights and influence is an empirical matter.

Table 1 provides some evidence and shows average and standard deviation leverages (6), h_t residuals $\underline{\hat{u}_t}$ and influence (9) for successive expenditure share weighted quartile quantity and quartile expenditure shares. If weights and influence diverge, the average influence in each quartile would differ. The results are from monthly scanner data on washing machines for 1998. Hedonic OLS regressions were run each month for about 550 models of washing machines, over 6,000 observations. The regressions fitted the data relatively well by the usual criteria, the average R^2=0.80 with the signs on the coefficients according with expectations. Leverage, (absolute) residuals and influence values were calculated each month and the means and standard deviations calculated for each quartile group, the results being averaged over the 12 months. The data are outlined in the Annex and the monthly regressions and monthly results are available from the authors on request.

Table 1 shows for both quantity and value quartile shares a clear inverse relationship between the relative sales (expenditure) shares and mean influences: observations with higher sales share had, for OLS regressions, *less* influence. Their residuals and leverage were lower. F-tests for equality of means over quartile groups was rejected at the 1% level (for value shares leverage, residuals and influence respectively: F=4.45, *p-value*=0.002; F=3.90, *p*=0.004 and F=4.10, *p*=0.003 and for quantity shares F=6.14, *p-value*=0.000; F=4.49, *p*=0.001 and F=4.93, *p*=0.001. Influence and expenditure weighting did not coincide.

Table 1: Summary statistics on influence and residuals for weight distribution

	Mean			**Standard deviation**		
Sales shares	**leverage**	**residuals**	**influence**	**leverage**	**residuals**	**influence**
under Q1: q-share	0.0958	0.1537	-0.0163	0.1215	0.1644	0.0778
Q1 to median: q-share	0.0773	0.1149	-0.0115	0.1022	0.1008	0.0522
Median to Q3: q-share	0.0760	0.1072	-0.0081	0.0975	0.0928	0.0415
Q3 to Q4: q-share	0.0525	0.0966	-0.0061	0.0521	0.0785	0.0125
All: q-share	0.0756	0.1185	-0.0106	0.1001	0.1183	0.0610
under Q1: v-share	0.0870	0.1549	-0.0169	0.1055	0.1693	0.0550
Q1 to median: v-share	0.0847	0.1151	-0.0112	0.1254	0.0990	0.0422
Median to Q3: v-share	0.0765	0.1064	-0.0095	0.0910	0.0933	0.0500
Q3 to Q4: v-share	0.0541	0.0977	-0.0062	0.0540	0.0765	0.0110
All: v-share	0.0756	0.1185	-0.0110	0.1001	0.1183	0.0513
n	6694					
R^2-adj (mean)	0.8017					

6. Proposed aternative approaches

Given concern about the use of WLS for hedonic adjustment a number of alternative approaches are proposed, their suitability being related to the need to be representative of the universe of models being sold.

6.1 Matched models

The desired target measure of price change for a semi-log hedonic function is (2), the superlative geometric Törnqvist index (see Feenstra (1995) who shows that for a Fisher index (3) a linear hedonic form is required). Our trouble with the dummy time variable method (1) is that the OLS estimator excludes the expenditure share weights and the WLS estimator is influenced by factors other than these weights. Consider the following:

$$\left[\prod_{m=1}^{M}\left(\frac{\hat{p}_{m1}}{p_{m0}}\right)^{s_{m1}}\prod_{m=1}^{M}\left(\frac{p_{m1}}{\hat{p}_{m0}}\right)^{s_{m0}}\right]^{1/2} \quad (10a)$$

$$\hat{p}_{m0} \equiv p_{m0}\exp[\sum\beta_0(z_{mk1}-z_{mk0})]$$

$$\hat{p}_{m1} \equiv p_{m1}\exp[-\sum\beta_1(z_{mk1}-z_{mk0})] \quad (10b)$$

or

$$\hat{p}_{m0} \equiv \exp[\alpha_0+\sum\beta_0 z_{mk1}]$$

$$\hat{p}_{m1} \equiv \exp[\alpha_1+\sum\beta_1 z_{mk0}] \quad (10c)$$

where s_{mt} are expenditure shares, p_{mt} are price, and z_{mkt} are k characteristics with associated β_{kt} derived from a semi-log hedonic regressions over m=1…M product varieties (models) for each period t=0,1.

First, assume the data can be organised into matched pairs of the same model over time, akin to the method used by statistical offices in the compilation of CPIs. Then only (10a) is appropriate. Equation (10a) is equivalent to the target index (2) since $\hat{p}_{mt} = p_t$ for t=0,1. However, the monitoring of matched prices not only requires additional information on the matching, but is also potentially biased since the sample of matched models excludes 'old' models available in period 0, but not in 1, and also excludes the 'new' ones introduced in period 1, but of course not available in period 0 (see Silver and Heravi (2002) for evidence on the bias).

6.2 Patched matched models using replacement items

Replacement models for obsolete ones in period 1 may be found, though being new, their quality may differ. A quality adjustment to the original or replacement price can be undertaken as in (10b) using changes in the quality of the two models picked up via changes in their characteristics ($z_{kt} - z_{kt-1}$) which are multiplied by estimates of their associated marginal hedonic values $\beta_{kt.}$, and summed. Thus (10a) and (10b) are appropriate. Note that $\hat{p}_{mt}$ corrects the observed prices $\hat{p}_{mt}$ for changes in the characteristics between the two periods, corresponding to the "explicit quality adjustment" described by Triplett (1990:39); see also Silver and Heravi (2001).

6.3 Patched matched models using imputations

If replacement items are unavailable the missing old or new prices may be imputed using (10c). Thus (10a) and (10c) are appropriate. For example, if the model is in the sample in period 0, but not 1, the imputed price for the model in period 1 is its period 0 characteristics evaluated at period 1 hedonic prices.[2] Equation (10) may be more cumbersome than equation (1), but benefits from employing a pure weighting system, untainted by undue influence.

[2] Diewert (2002a) shows how a further adjustment may be necessary since predicted and actual prices are being compared.

Equation (10a) has been shown to be suitable for matched data, with 10(b) for matched data when an observation is missing in any period, but there is a replacement of different quality and with 10(c) for matched data without replacements using imputations. Yet there remains the question of whether to use a WLS or OLS estimator for the parameters in (10b or c). Two concerns impose: efficiency and representativity. A concern in (10b or c) in using WLS is not the appropriateness of the weighting, for this is captured in (10a), but the efficiency of $\hat{\beta}$. It was argued in section 3.2 on econometric grounds that an appropriate variable for use as weights in WLS was one that was strongly related to the residual variance. The question of choice between value or quantity weights is thus dictated by the strength of their relationship to the conditional residual variation. Thus price changes in (10a) are expenditure share weighted while the weights for (10b or c) are an empirical matter. However, another criterion is how representative the hedonic regression is for the adjustments being made. Assume there are many new observations for a brand, their characteristics being say superior to other brands. The analysis in section 4 shows that they will have further influence on the estimated $\hat{\boldsymbol{\beta}}$ over and above that due to expenditure share weights. If, by and large, the new non-comparable replacements are of this new brand, then this 'bias' is arguably in the right direction, giving more emphasis to the branded observations whose quality is to be adjusted. In principle, degrees of freedom permitting, separate parameters may be estimated for each brand by way of slope and intercept brand dummies as a better mechanism for 'tailoring' the coefficients to the brand-specific prediction.

6.4 Unmatched data, the time dummy hedonic and deletion

A concern with matching, especially over relatively long periods, is that it leads to a deterioration of the sample. For example, Koskimäki and Vartia (2001) attempted to match prices of models of personal computers (PCs) over three two-month periods (spring, summer and fall) using a sample of prices collected as part of standard price collection for the Finish CPI. Of the 83 spring prices only 55 matched pairs could be made with the summer, and then only 16 continued through to the fall (see also Silver and Heravi, 2002). The use of imputations such as (10b) or (10c) on this scale is not desirable. We return to the hedonic time dummy variable approach in equation (1). It allows popular models to be included in the sample in each month without any restriction as to whether they were previously or subsequently included and matched. Bear in mind that the use of replacements in (10b) only allows sampling from the 'dynamic' population when a replacement is required, while that of (10c) is based on imputations from the deteriorating original matched sample. Both are unsatisfactory. The hedonic adjustment in (1) allows re-sampling each month from all models and for items such as consumer durables, where there is a high turnover of models of different qualities, this is a highly desirable property. However, as discussed, the implicit weighting system in an expenditure share WLS estimate of (1) may be inappropriate and the OLS one even more so. As such it is proposed that the time dummy hedonic method (1) is only used when there is substantial sample degradation and that influence measures (9) and residuals be computed and observations with relatively low weights and high influence values be deleted, and (1) re-estimated.

6.5 Unmatched data and the superlative hedonic approach

If data are not matched, they are re-sampled each period to be representative of the universe of new and old models, the time dummy method has been proposed with some deletion of observations with undue influence. But the method remains problematic if a model is has a relatively high weight, so cannot be excluded, but its influence is over and above that due to is weight. One approach is to use calculated influence variable to adjust the weights. An alternative is to use a development to the formulation 10(a) and 10(b) akin to stratified random sampling. First stratification variables are selected which are related to price changes, say screen size and makes for television sets. Then, using (10a), weighted indices are calculated for price changes in these 'core' stratum, that is the prices in (10a) are the (geometric) mean prices in each stratum, for example a Sony 21 inch television set. But over time the quality of items in each stratum will change by other 'non-core' quality characteristics, such as the possession of stereo, wide-screen etc. Such changes in the quality of the average prices being compared are then controlled for in 10(b). This approach was adopted for weighted price comparisons using scanner data for television sets over time and across countries (Silver and Heravi (2001) and Heravi, Heston and Silver, 2003 respectively).

7. Conclusions

The conclusions for estimating hedonic quality-adjusted price changes when weights are available are:

- Some weighting system is better than none, and a WLS estimator is preferred to an OLS one (section 3).

- The use of expenditure share weights for hedonic time dummy regression estimates is preferred to relative quantities (section 3 and 4).

- WLS estimators may not give the appropriate weights required by the target superlative indices, some observations having undue influence effects thus contributing to the effective weights (section 4).

- The empirical work shows an inverse relationship between expenditure share weights and influence (section 5).

- The use of the proposed hedonically-adjusted predicted prices in an explicitly weighted superlative framework (10a) is preferred, the nature of the adjustment depending on whether replacement models are (10b) or are not (10c) available (section 6).

- When matching and imputations/adjustments are undertaken on a matched sample that has substantially deteriorated, the approach in (10) may be inappropriate. Triplett (2002) has warned of such selectivity or 'out-of-sample bias' and Silver and Heravi (2002) have demonstrated its nature and substantial effect. In such circumstances the time dummy variable hedonic approach (1) should be reconsidered. It has the advantage of allowing re-sampling each month from the dynamic universe of items, rather than from just the matched ones or matched/replacement ones. It should, data permitting, be undertaken with a WLS estimator, though observations with relatively low weights and high influence values should be deleted and the regression re-run (section 6).

- Alternatively, price can be re-sampled each period, but explicit weights can be imposed on average price changes of more loosely defined cells of core characteristics or stratum, with quality changes being undertaken within each stratum. This at least imposes an appropriate weighting structure between the strata.

Data Annex for Table 1

The data were monthly observations for four outlet types for 1998 amounting to 7,750 observations (models sold in an outlet type) amounting to 1.5 million transactions worth over £0.5 billion. The price was the unit value of sales of a model in an outlet-type. The characteristics included: (i) Manufacturer (make) – dummy variables for about 20 makes; (ii) type of machine: 5 types – top-loader; twin tub; washing machine (WM); washer dryer (WD) with and without computer; WD with /without condensors; (iii) drying capacity of WD; (iv) height of machines in cms; (v) width ; (vi) spin speeds: 5 main - 800rpm, 1000rpm, 1100rpm, 1200rpm and 1400rpm; (vii)water consumption; (viii) load capacity; (ix) energy consumption (kWh per cycle); (x) free standing, built-under and integrated; built-under not integrated; built-in and integrated; (xi) vintage; (xii) outlet-types: multiples, mass merchandisers, independents, multiples.

Annex 2: Illustrative example

Consider Table 2 which contains illustrative data for price (*P*), a quality characteristic *x* which is on average the same over the two time periods, $t = 0,1$. Consider the first 18 observations in Table 2. They are generated for periods 0 and 1 respectively from:

$$P_0 = 100(1.06)^{x_0} \quad \text{and} \quad P_1 = 100(1.06)^{x_1}$$

$$\text{i.e.} \quad \ln P_0 = \ln(100) + x_0 \ln(1.06) \quad \text{and} \quad \ln P_1 = \ln(105) + x_1 \ln(1.06) \qquad (11)$$

Throughout the range of *x* the price change for those observations to 5%. The last two observations are generated at $\bar{x} = 5$, observation 19 in period 1, $P_1^{19} = 115(1.06)^5$ having a 15% price increase compared with $P_0^{20} = 100(1.06)^5$. The first 18 observations are equally weighted accounting for 72% of the weight, the remaining two observations each accounting for 14%.

Table 2: Illustrative Data

Observation	Weight	*P*	*x*	*t*
1	0.04	106.000	1	0
2	0.04	112.360	2	0
3	0.04	119.102	3	0
4	0.04	126.248	4	0
5	0.04	133.823	5	0
6	0.04	141.852	6	0
7	0.04	150.363	7	0
8	0.04	159.385	8	0
9	0.04	168.948	9	0
10	0.04	111.300	1	1
11	0.04	117.978	2	1
12	0.04	125.057	3	1
13	0.04	132.560	4	1
14	0.04	140.514	5	1
15	0.04	148.945	6	1
16	0.04	157.881	7	1
17	0.04	167.354	8	1
18	0.04	177.395	9	1
19	0.14	153.896	5	1
20	0.14	133.823	5	0

Consider the unweighted case. Since the observations are on average of constant quality over time, the aggregate geometric mean price change is $(1.05)^{18/20}(1.15)^{2/20} = 1.0596$, i.e. 5.96%. An OLS regression of P on x and t with P^{19} evaluated at $\bar{x} = 5$ finds a coefficient of 0.057886 and a percentage price change of $(e^{0.057886} - 1)100$ of 5.96. [The coefficients from a semi-logarithmic form are not unbiased and require a correction of ½ (standard error)^2-Goldberger (1968). However, the adjustment in this example is negligible]. The dummy time variable method works. However, if the higher price observation in period 19 had unusual characteristics, say x^{19}=9 or 15 with P_1^{19} accordingly evaluated at a 15% increase using (11), then the OLS price change estimate would be wrong at 5.86 and 5.55% respectively. In each of these three cases the influence of P_1^{19} is extremely high; at x=5, the influence of observation 19, i^{19} =0.0091, its mean $\bar{i}$ = 0.0012 and standard deviation s_i = 0.002, and for x=9: i^{19}, $\bar{i}$ and s_i are 0.0178, 0.002 and 0.0039, and for x=15: 0.044, 0.0036 and 0.0096 respectively. $i^{19}/\bar{i}$ increases from 7.5, to 8.6 and 12.0 as x takes on more extreme values, from 5,9 to 15.

Table 3: Results from $P = \beta_0 c + \beta_1 x + \beta_2 T + \beta_3 D + \varepsilon$ illustrative data

		coefficient (β)	Standard Error	Percentage change: $(e^{\beta} - 1)100$
	constant (c): β_0	4.60517	0.011623	0
	x: β_1	0.058269	0.0019108	6.00
OLS (z_{19} = 5)	T: β_2	0.057886	0.00936	5.96
(z_{19}=9)	T : β_2	0.056912	0.00887	5.86
(z_{19}=15)	T : β_2	0.053988	0.00716	5.55
OLS (z_{19} = 5)	T : β_2	0.048789	0.00000	5.00
	D: β_3	0.090972	0.00000	9.52
OLS (z_{19} = 5)	T : β_2	0.074261	0.01401	7.71

The further inclusion of a dummy variable D=1 for observation 19 leads to a regression model with Rbar^2=1.00 and coefficients on T and D of 0.048789 and 0.090972 (Table 3). The equation is perfectly specified, $y_m = \hat{y}_m m$; the increase in period 1 being exp(0.048789)=1.05 excluding observation 19 and exp(0.048789+0.090972)=1.15 for observation 19 itself. Yet these results by themselves do not allow us to deduce the quality-adjusted price. Table 2 also includes data on relative expenditure weights, assumed to be constant over time. The Törnqvist index is $(1.05)^{0.72}(1.15)^{0.28}$ = 1.07709, i.e. a 7.71% increase. A WLS regression also yields an estimate of 7.71% when P_1^{19} is evaluated at x=5, as in Table 1. Again the estimate will change as x changes, but moreso because of its increased weight. For x=9 and x=15, the WLS price increase is only 7.02 and 5.87% respectively.

References

Berndt, E.R., Grilches, Z., and Rappaport N.J. (1995) Econometric Estimates of Price Indexes for Personal Computers in the 1990s, *Journal of Econometrics*, 68, 243-68.

Berndt ER, Ling D and Kyle MK (2002), The Long Shadow of Patent Expiration: Do RX to OTC Switches Provide an Afterlife? In Mathew Shapiro and Rob Feenstra (eds.), *Scanner Data and Price Indexes*, National Bureau of Economic Research, Studies in Income and Wealth, vol. 61 Chicago: University of Chicago Press.

Davidson R and MacKinnon JG (1993), *Estimation and Inference in Econometrics*, Oxford, Oxford University Press.

Diewert, W.E. (1976) Exact and Superlative Index Numbers, *Journal of Econometrics*, 4 115-45.

Diewert,W.E., (1978), Superlative Index Numbers and Consistency in Aggregation, *Econometrica*, 46, 883-900.

Diewert, W.E. (1995), Axiomatic and Economic Approaches to Elementary Price Indices, Discussion Paper No. 95-01, Department of Economics, University of British Columbia, Vancouver, Canada.

Diewert W.E. (2002) Harmonized Indexes of Consumer Prices: Their Conceptual Foundation, *European Central Bank Working Paper Series* No. 130, March 2002, 1-89.

Diewert W.E. (2002a). Hedonic Regressions: A Review of Some Unresolved Issues, *Mimeo*, Department of Economics, University of British Columbia.

Feenstra RC (1995), Exact Hedonic Price Indices. *Review of Economics and Statistics*, vol. LXXVII, pp. 634-54.

Goldberger, A.A. (1968), The Interpretation and Estimation of Cobb-Douglass Functions, *Econometrica,* 35, 3- 4, pp.464-472.

Griliches, Z, (1990), Hedonic Price Indices and the Measurement of Capital and Productivity: Some Historical Reflections. In *Fifty Years of Economic Measurement: The Jubilee Conference of Research in Income and Wealth*, *Studies in Income and Wealth*, Vol.54, NBER (eds. ER Berndt and JE Triplett), pp.185-206. Chicago University of Chicago Press.

Heravi S., Heston, A. and Silver, M. (2003 forthcoming), Using Scanner Data to Estimate Country Price Parities: An Exploratory Study, *Review of Income and Wealth*, March.

Koskimäki T and Vartia Y (2001), Beyond Matched Pairs and Griliches-Type Hedonic Methods for Controlling for Quality Change in CPI Sub-Indices: Mathematical Considerations and Empirical Examples on the use of Linear and Non-Linear Models with Time-Dependent Quality Parameters, *Proceedings*; 6th Meeting of the (Ottawa) International Working Group on Price Indices, Canberra, Australia 2-6 April.

Liegey, P. R. and Shepler N. (1999), Adjusting VCR prices for quality change, *Monthly Labor Review*, September 1999, Vol. 122, No. 9, 22-37.

Pakes A (2002), A Reconsideration of Hedonic Price Indices with an Application to PCs, *NBER Working Paper* No. 8715, NBER, Cambridge MA.

Rosen, S. (1974), Hedonic Prices and Implicit Markets: Product Differentiation and Pure Competition, *Journal of Political Economy*, 82, 34-49.

Silver, M and Heravi, S (2001), Scanner Data and the Measurement of Inflation, *The Economic Journal*, 11 June, 384-405.

Silver, M and Heravi, S. (2002), Why the CPI Matched Models Method May Fail Us: Results from an Hedonic and Matched Experiment Using Scanner Data, *European Central Bank (ECB) Working Paper* No. 144, Frankfurt: ECB.

Triplett, JE (1988), Hedonic Functions and Hedonic Indices, in *The New Palgraves Dictionary of Economics,* pp. 630-634.

Triplett JE (2002), *Handbook on Quality Adjustment of Price Indexes for Information and Communication Technology Products*, OECD Directorate for Science, Technology and Industry, Draft, OECD, Paris