Why Price Index Number Formulae Differ: Economic Theory And Evidence On Price Dispersion

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Abstract: The results from different index number formulae can differ and can do so substantially. The main criteria for explaining such differences, and governing choice between them, are their ability to satisfy desirable test properties—the axiomatic approach—and their correspondence with plausible substitution behavior as predicted from economic theory. Yet the numerical differences between such formulae has been shown to be related to the extent of, and changes in, the dispersion of prices. However, within the index number literature there is, to the author's knowledge, no formal attempt to explain differences between the results from individual formulae in terms of theories and evidence on price dispersion. Explaining differences between formulae in terms of changes in the dispersion of prices benefits from the existence of economic theoretical frameworks to explain such dispersion, and thus improve our understanding of why differences from formulae occur. Such frameworks include search cost and menu cost theories and signal extraction models. This paper outlines the nature of the relationships between formulae in term of price dispersion, then considers economic theories of price dispersion and uses them to model price variation both within months and over time using an extensive scanner data set on television sets amounting to over 70,000 observations over 51 months. It concludes by considering the implications for index number construction.

Keywords: Index Numbers; Relative Price Dispersion; Search Cost; Signal Extraction; Menu Cost; Hedonic Regression.

JEL classifications: C43, C81, D11, D12, D83, E31, L11, L15.

1. Introduction

Choice of formula for the measurement of inflation does matter. In January 1999 the formula principally used for aggregating price changes for the U.S. consumer price index (CPI) at the *lower* level of aggregation was changed from an arithmetic to a geometric mean. The effect of the change has been estimated by the Bureau of Labor Statistics (BLS) (2001) to have reduced the annual rate of increase by approximately 0.2 percentage points. Following estimates from the Boskin Commission's Report on the U.S. CPI (Boskin *et. al*, 1996 and 1998), this implied a cumulative additional national debt from over-indexing the budget of more than $200 billion over a twelve year period up to the mid-1990s. This 'lower' level aggregation only applies to samples of prices from stores of finely-defined goods such as brands of washing machines, varieties of apples. These are the building blocks of a CPI and the choice of formula for their aggregation is an important practical matter.[1] The difference between these formulae can be shown to be primarily determined by the changes in *price dispersion*.

The subsequent Schultze and Mackie (2002) report recommended the use of a trailing superlative index instead of the Laspeyres index since it would capture weighted 'upper level' substitution effects. One such superlative index which has much to commend it (Diewert, 1995), is the Fisher index, a geometric mean of Laspeyres and Paasche. Boskin estimated that upper level substitution accounted for 0.15 percentage points bias in the U.S. CPI. Again changes in *price dispersion* will be seen below to account for some of the difference between these formulae.

Although the Laspeyres formula is commonly thought to be the formula used for the U.S. and other CPIs at the upper level, the expenditure weights for a comparison, say between periods 0 and *t,* relate to a previous time period *b*, as opposed to period 0, since it takes time to compile the information from expenditure surveys for the weights. The resulting practically used index is a Young index which is shown below to be biased (Diewert, 2003). But the extent of the bias depends again on changes in *price dispersion*.[2]

In spite of the importance of price dispersion in explaining the differences between these key formulae, it is axiomatic tests that are used to choose between such formulae along with the economic theory of consumer substitution. The contribution of this paper lies in its analysis of the differences between formula in terms of explanations of the factors governing price dispersion and changes in such dispersion over time. For explanation it draws on the very nature of product heterogeneity in terms of the characteristics, branding and outlet-types goods they are sold in. But it extends the analysis to theoretical frameworks not usually associated with index number work, including search cost and menu cost theories and signal extraction models. The empirical section is directly related to such theory and is based on detailed scanner data from retailers' bar-code readers. More particularly, the empirical work on lower level indices provides a hitherto neglected focus on product heterogeneity to explain the bias in the Dutot index.

Section 2 considers elementary index number formulae. It outlines the three main formulae and their current justification from axiomatic considerations, economic and sampling theory. Section 3 shows how changes in price dispersion are important to any explanation of differences in upper level formulae, whose justification has again been in terms of consumer substitution theory and axiomatic tests. Section 4 and the Annexes provide the numerical relationships between the formulas in terms of how they differ in terms of changes in the variance of their prices. The paper brings to bear in section 5 a quite novel approach to the consideration of the difference between such formulae with a focus on search cost theory, but

[1] The resulting indexes of price changes at athe lower level are combined at the higher level using a base-period weighted arithmetic mean of price changes to form the overall index.

[2] There are other index number issues whose probity are dictated by price dispersion. For example, Ehemann, Katz and Moulton (2002) identify price dispersion as leading to negative values for a subaggregate composed of postive values in a proposed additive system of national accounts by Hillinger (2002).

including menu cost theory, signal extraction models, pass-through rates, price discrimination and consumer inventory models. Section 6 commences the empirical work, based on extensive retail bar-code scanner data for television sets (TVs) with an outline of data, variables and measures and section 7 provides the results. Section 8 concludes with implications for index number compilation. The analysis shows how economic theory rooted in the failure of the law of one price and the persistence of price dispersion can provide insights into differences between index number formula at upper and lower levels. This novel approach[3] complements the still valuable analysis previously considered only in terms of axioms and consumer substitution theory.

2. Lower-Level Formulae and their Rationale: Axioms and Consumer Substitution Theory

2.1 The formulae

The main formulas used in practice (see Dalen (1992) and Diewert (1995 and 2003) for details of other such indices)[4] are given, for m=1,..M items with prices and quantities in period t, p_m^t and q_m^t respectively for t=0, t , by:

The arithmetic mean of price relatives—the *Carli price index* P_C :

$$P_C(p^0,p^t) \equiv \sum_{m=1}^{M} (1/M)\, p_m^t/p_m^0 \tag{1}$$

The relative of the arithmetic means of arithmetic averages)—the *Dutot price index,* P_D:

$$P_D(p^0,p^t) \equiv \sum_{m=1}^{M} (p_m^t/M) / \sum_{m=1}^{M} (p_m^0/M) = \sum_{m=1}^{M} (p_m^t) / \sum_{m=1}^{M} (p_m^0) = \sum_{m=1}^{M} (p_m^t/p_m^0)\, p_m^0 / \sum_{m=1}^{M} (p_m^0) \tag{2}$$

which can be seen to be a base-period price share weighted Carli index.

The geometric mean of price relatives (which is also equal to the relative (ratio) of geometric means of the prices in periods 0 and t) —the *Jevons price index* P_J:

$$P_J(p^0,p^t) \equiv \prod_{m=1}^{M} \left[p_m^t/p_m^0\right]^{1/M} = \frac{\prod_{m=1}^{M} \left[p_m^t\right]^{1/M}}{\prod_{m=1}^{M} \left[p_m^0\right]^{1/M}} \tag{3}$$

The use of the geometric mean is not a novel idea. It was first proposed in 1922 on axiomatic grounds by Irving Fisher, though its adoption was prompted by the Boskin Commission's Report in 1996 based on the economic theory of consumer substitution behavior. These three widely-used simple formulas are for calculating lower level, aggregate, unweighted price changes of matched items over time. These humble formulas are the building blocks of a CPI until weights are used at a higher level.

[3] Balk (2001:2) comments that some insights have been obtained by looking at changes in variances, but only using an approximate "...more or less intuitive economic reasoning." There has been, to the author's knowledge, no formal examination of changes in dispersion in this context.

[4] The main alternative formulae are the harmonic mean of price relatives—the *harmonic* version of equation (2); the relative of the harmonic means; and the geometric mean of the Carli arithmetic mean P_C of price relatives and harmonic mean of price relatives.

2.2 The axiomatic approach, price dispersion and commensurability

The axiomatic approach identifies which formulae are desirable on the basis of their satisfaction of reasonable test properties. The Carli index fails the time reversal test such that $P_C(p^0, p^1)\times P_C(p^1, p^0)\geq 1$; it is *upwards-biased.*[5] The Jevons index satisfies all of the tests as does the Dutot index with the important exception of Commensurability Test, i.e., if we change the units of measurement for each commodity in each outlet, then the elementary index remains unchanged. There is an implication for quality variation here. In practice the quality differences—be they brands, technical specifications, or level of service in outlets—amount to a change in the (utility flow) unit the price is measured in. Thus while each of the matched models will have the same units over time, they may differ across units. This is the concern of the Commensurability Test. A heterogeneous collection of items gives rise to varying units of measurement. This can be seen by identifying the Dutot index in terms of an arithmetic average of weighted price changes the weights being the base period price shares as in (2). If, for example, the quality and prices of say washing machines in the basket are very diverse, then the Dutot index will give more emphasis to models with higher prices, for which there is no immediate justification. Diewert (2003) notes:

> "..in actual practice, there will usually be thousands of individual items in each elementary aggregate and the hypothesis of item homogeneity is not warranted. Under these circumstances, it is important that the elementary index satisfy the commensurability test, since the units of measurement of the heterogeneous items in the elementary aggregate are arbitrary and hence *the price statistician can change the index simply by changing the units of measurement for some of the items*."[their emphasis].

He continues:

> "If there are heterogeneous items in the elementary aggregate, this is a rather serious failure and hence price statisticians should be careful in using this index under these conditions."

There is thus a concern with the *absolute level* of price dispersion and why it arises as well as the *changes* in dispersion. This paper addresses two main issues in this context. First, that the axiomatic approach only supports the Jevons against the Dutot index because the heterogeneity of items/outlets bundled together in the aggregation leads to bias in the Dutot. We seek to explain such heterogeneity and, in the empirical section, examine heterogeneity-controlled prices for the Dutot index. The superiority of a Jevons index against a heterogeneity-controlled Dutot index may not be straightforward.[6] Second, that the differences between results from such index formulae *over time* can be explained by changes in the dispersion of prices. Signal extraction search cost and menu cost theory are brought to bear to explain and model such differences in dispersion, both heterogeneity-controlled and otherwise.

[5] Fisher (1922) famously commented: "In fields other than index numbers it is often the best form of average to use. But we shall see that the simple arithmetic average produces one of the very worst of index numbers. And if this book has no other effect than to lead to the total abandonment of the simple arithmetic type of index number, it will have served a useful purpose." Irving Fisher (1922; 29-30).

[6] The BLS in their CPI argued that only 60% of product areas should be changed to using the Jevons index as justified by a likely approximation to substitution behaviour characterised by an elasticity of substitution equations.

2.3 Consumer substitution theory

Consumer substitution theory holds that utility-maximizing consumers substitute away from items with relatively high prices. An index that ignores such effects in its weighting of price changes is open to *substitution bias*. However, lower level indices do not include information on weights so such theory may be argued to not be relevant. Yet Balk (2002) has shown that if the items are selected with probability proportionate to (quantity or value share) size (pps), then the sample unweighted estimator is of a population weighted target index, for which economic theory applies. For example, if sampling is with probability proportionate to base period value shares, then the expected value of a Carli index is a Laspeyres index. A Laspeyres index restricts consumer substitution to be zero and overstates inflation since items with above average price changes are not given less weight since any fall in quantity is not reflected in the weights. However, the expected value of a Jevons index under the same sampling scheme produces a base period weighted geometric mean which Balk (2002) has shown to correspond to consumer substitution behavior consistent with an elasticity of substitution of unity. The incorporation of such substitution effects was the main justification for the BLS switch to the Jevons index.

3. Upper-Level Formulae and their Rationale: Axioms and Consumer Substitution Theory

3.1 The formulae

Laspeyres and Paasche indices are fixed basket indices measuring the price change of a basket of goods whose quantities are either fixed in period 0, Laspeyres:

$$P_L \equiv \frac{\sum_{m=1}^{M} p_m^t q_m^0}{\sum_{m=1}^{M} p_m^0 q_m^0} \qquad (4)$$

or period t, Paasche

$$P_P \equiv \frac{\sum_{m=1}^{M} p_{mi}^t q_{im}^t}{\sum_{m=1}^{M} p_m^0 q_m^t} \qquad (5)$$

A geometric mean of the two is the Fisher index:

$$P_F \equiv (P_L . P_P)^{1/2} \qquad (6)$$

Paasche and Fisher cannot be used in real time because it takes time to compile the quantity weights from expenditure surveys. While the CPI is considered to be a Laspeyres index, this is not the case in fact. The weights are taken from a survey of expenditure patterns in a weight reference period b. It takes time to compile such results so that their final use is for a subsequent comparison between price reference period 0 and period t. The resulting Young index is:

$$P_Y \equiv \sum_{m=1}^{M} s_m^b \left(p_m^t / p_m^0\right) \qquad \text{where } s_m^b \equiv \frac{p_m^b q_m^b}{\sum_{m=1}^{M} p_m^b q_m^b}; \qquad m = 1,...,M. \qquad (7)$$

3.2 The axiomatic approach and consumer substitution theory

Fisher (1922) defined (6) as 'ideal' in terms of its satisfaction of desirable axioms (see also Diewert, 1996). In seminal work Konüs (1924) found Laspeyres and Paasche price indices to provide upper and lower bounds on theoretical cost-of-living indices and Diewert (1976 and 1978) defined a class of superlative formulae, one of which was the Fisher index, to incorporate forms of substitution effects corresponding to flexible functional forms. Such results render a Fisher price index as superior to Laspeyres price index with regard to its ability to incorporate substitution effects, which the Laspeyres fixed basket index cannot, and thus as a better approximation to a cost-of-living index.

4. Numerical Relationships between Frequently Used Index Number Formulae – the Importance of Dispersion

4.1 Lower level indices

The relationships have been developed by Marks and Stuart (1971), Carruthers et al., (1980), Dalen (1992), and Diewert (1995 and 2003). We borrow in this section primarily from Diewert for the exposition.[7] Three things are apparent from the above section 2. First, the Carli index may be biased on axiomatic grounds. Second, the Dutot index is only advisable when there is limited price dispersion arising from quality differences in the item itself or the services provided by the outlet at the time of sale. Finally, that the Jevons index has much to commend it on axiomatic grounds and also, under pps, when consumer substitution behaviour approximates that characterised by a unitary elasticity of substitution. More innocuous is the incentive for governments to encourage a switch to the use of the Jevons index since it will lead to lower inflation than its arithmetic counterparts and thus, lower public (index-linked) debt (Hulten, 2002). We thus consider the numerical relationships between these three formulas.

4.1.1 The relationship between Dutot and Jevons indices

First, it can be shown[8] that the Carli and Jevons satisfy the following inequality:

$$P_J(p^0, p^1) \leq P_C(p^0, p^1) \qquad (8)$$

i.e., the Jevons index is always equal to or less than the Carli index. In fact, the strict inequality in (6) will hold provided that the period 0 vector of prices, p^0, is not proportional to the period 1 vector of prices, p^1.

The inequality (6) does not tell us by how much the Carli index will exceed the Jevons index. Carruthers, Sellwood and Ward, (1980:25) show an approximate relationship between Dutot $P_D(p^0,p^1)$ and Jevons $P_J(p^0,p^1)$ — see also Diewert (1995a:27-28) and Balk (2002: 23-4) and Annex 1 for more detail. Consider

[7] The 2003 reference is from a draft chapter on elementary price indexes by Erwin Diewert in a forthcoming Manual on Consumer Price Indexes to be published by the International Labour Office, Geneva available at www.ilo.org/public/english/bureau/stat/guides/cpi/index.htm

[8] As noted by Diewert (1995) each of the three indices P_H, P_J and P_C is a mean of order r where r equals −1, 0 and 1 respectively and so the inequalities follow from Schlömilch's inequality.

$$P_m{}^t = \bar{p}^t\ (1 + \varepsilon_m{}^t) \qquad for\ t=0,1\ldots t \qquad \text{where} \sum_{m=1}^{M} \varepsilon_m{}^t = 0 \tag{9}$$

and $\bar{p}^t$ is the arithmetic mean of prices in period t. To realize the individual prices the mean is multiplied by each item's deviation from the mean, $(1+\varepsilon_m{}^t)$. Since the Dutot price index is defined as $P_D(p^0,p^1) = \bar{p}^t/\bar{p}^0$ it follows that the Jevons price index is given by:

$$P_J(p^0,p^1) = \prod_{m=1}^{M}\left[\bar{p}^t\ (1+\varepsilon_m{}^t)/\bar{p}^0\ (1+\varepsilon_m{}^0)\right]^{1/M} = \prod_{m=1}^{M}\left[\bar{p}^t/\bar{p}^0\ (1+\varepsilon_m{}^t)/(1+\varepsilon_m{}^0)\right]^{1/M}$$

$$= P_D(p^t,p^0)\, f(\varepsilon^t,\varepsilon^0) \qquad \text{where } f(\varepsilon^t,\varepsilon^0) = \prod_{m=1}^{M}\left[(1+\varepsilon_m{}^t)/(1+\varepsilon_m{}^0)\right]^{1/M} \tag{10}$$

Expanding $f(\varepsilon^t,\varepsilon^0)$ using a second order Taylor series around ε^0 and ε^t, the following second order approximation results:[9]

$$P_J(p^0,p^1) = P_D(p^t,p^0)\big(1 + (1/2)\text{var}(\varepsilon^0) - (1/2)\text{var}(\varepsilon^t)\big) \tag{11}$$

4.1.2 The relationship between Carli and Jevons indices

In equation (2) the elementary index number took the form of a mean of price relatives. Alternative means of price relatives considered in section 2.1 were the arithmetic Carli index P_C, geometric Jevons index P_J, harmonic index P_{HR}, and the geometric mean of P_{HR} and P_C, the P_{HRC} index. Since these are all functions of price ratios it is quite straightforward to establish the mathematical relationships between them (see Dálen (1992) and Diewert (1995) for formal proofs). The interest here lies in the conditions under which these formulae approximate each other and the factors determining their differences. Let the price relatives for a comparison between 0 and t be given by:

$$R_m{}^t = p_m{}^t/p_m{}^0 = \bar{r}^t\,(1+\nu_m{}^t) \tag{12}$$

where ν_m^t is the deviation from the mean r^{*t} of the price relative for item m in period t and $\sum \nu_m{}^t = 0$. Now regard each formula as a function of the deviations $\nu_j{}^t$ and take second order Taylor series approximations around the point $\nu_m{}^t = 0$ for $m = 1,\ldots,M$ and we obtain the following approximations:

$$P_C(p^0,p^t) \equiv \sum_{m=1}^{M}(r_m^t)(1/M) = \bar{r}^t \sum_{m=1}^{M}(1+\nu_m{}^t)(1/M) = \bar{r}^t \tag{13}$$

$$P_J(p^0,p^t) = \prod_{m=1}^{M}\left[\bar{r}^t\right]^{(1/M)} = \bar{r}^t \prod_{m=1}^{M}(1+\nu_m{}^t)^{(1/M)} \cong \bar{r}^t(1 - (1/2)\text{var}(\nu_m{}^t)) \tag{14}$$

This formulation of differences in formulae in terms of the dispersion of price relatives is of use since, as will be shown in section 3, there are theoretical frameworks in economics concerned with explaining the existence and change in price dispersion. Annex 2 outlines the

[9] Similar results can be found in relation to the Carli index and each of the Harmonic mean of price relatives, the Carruthers, Sellwood , Ward and Dalen index (geometric mean of Harmonic and Carli index), and the Balk-Walsh index, the approach having a wider application than to the more widely used Carli, Dutot and Jevons indices (Balk, 2002: 22).

relationship in more detail. Differences between Dutot and Carli can also in part be explained by changes in such variances (Diewert, 1995) and Annex 3.

4.2 Upper level indices

4.2.1 The relationship between Laspeyres and Paasche

Annex 4 shows the extent of the divergence between these two formulae, and by extension between Laspeyres and Fisher, to depend in part on the extent of the dispersion in price relatives where dispersion is considered in terms of the coefficient of variation.

4.2.2 The bias in the Young index

It was noted in section 3 that in practice Laspeyres index is not used at the upper level for a price comparison between periods 0 and t since expenditure weights are unavailable in period 0 due to the tome taken to compile them. Instead the weight reference period refers to an earlier period b, the resulting index being the Young index defined by equation (7). Annex 5 follows Diewert (2003) and shows the Young index to be biased, the extent of the bias depending on the dispersion in price relatives.

4.2.3 A note concerning the Taylor expansion/approximation

The results from the above sections and annexes show the differences between formulae in terms of variances, usually arising from a Taylor expansion around zero. It is an approximation and Annex 6 considers the expansion in more detail.

5. Some economic theory of price dispersion and its change over time

The fact that the decomposition of the differences between index number formulae can be identified, at least partially, in terms of changes in variances allows recourse to economic theory concerned with such changes. A body of well-developed economic theory is available regarding the existence and persistence of changes in, the variances of prices and this theory. The theory is quite different from classical theory, for which price dispersion is an anathema. Jevon’s law of one price predicts that under perfect competition identical items will be sold at the same price. There is a burgeoning theoretical literature that explains price dispersion of homogeneous goods. There is a related literature that links changes in (relative) price dispersion to changes in the mean (unanticipated and otherwise) of such prices. Much of this theory which includes search cost, menu cost and signal extraction models relates to micro-economic behavior. Lach (2002), in his useful contribution using micro data, notes a dearth of related empirical studies blaming it on problems with access to micro-level data.

5.1 Search Costs and the law of one price: cross-sectional price dispersion

Stigler (1961) argued that optimizing consumers with imperfect information search for additional information such that their (rising) marginal search cost equals the (falling) marginal search benefits. Even in markets with symmetric firms selling homogeneous goods, product prices may differ in equilibrium if there is a positive, but uncertain, probability that a randomly chosen customer knows only one price.[10] This would result in imperfect information that the firm can exploit by charging a higher price (Sorensen, 2002). It is in the

[10] It is not even required that search costs vary across buyers. Heterogeneity of beliefs about the (cumulative distribution function of) prices for buyers with identical search costs is sufficient (Rauh, 2001).

interests of firms to adopt strategies which increase search costs, the effect of which is to increase price dispersion and thus the difference between formulae.
Electronic consumer durables (ECDs) have particular characteristics with regard to search costs. First, they are highly differentiated by brand and features. This may be argued to meet the needs of different segments of the market, but this also hinders search and increases price dispersion. There is competition within stores, between brands and within brands with different features. There is also competition between stores yet, as will be shown, stores do not often stock the same brands or models of a brand to further hinder price comparisons.

It may be argued that advertising serves to reduce search costs. There are intensive media advertising and door-to-door flyers for ECDs. Yet this is usually on a small selection of models. Koch and Cebular (2002) distinguish between advertising expenditure which reduces consumer search costs and decreases mean prices and advertising that focuses on branding to diminish price elasticity of demand thus increase mean prices and their dispersion. Cohen (2002) similarly argues that while greater brand selection increases rivalry and stimulates price competition, it also increases the value of information on prices and features providing scope for poorly informed customers which can dampen price competition.

Second, models are sold in a variety of outlets offering different levels of customer services. Price dispersion may be due to outlet heterogeneity as well as feature and brand heterogeneity. ECDs are sold in electrical multiples/chains (EM), mass merchandisers (department stores) (MM), independents (IND), and mail order catalogues (MAIL).[11] The majority of sales are in EMs which specialize in electrical goods, each chain being made up of hundreds of branches spread across the country selling a similar and large selection of goods at the same prices. The different types of outlets provide different types of service: EMs are often large out-of-town (easy parking), specialist warehouses while MMs are usually in-town department stores selling a much wider range of goods. Sorensen (2000) found store effects to not be a source of price variation for pharmaceutical products. In contrast, Lach (2002) found outlet-type to explain some of price variability for chicken, flour, coffee and refrigerators.[12] In this study outlet types are one feature considered to explain price dispersion.

Third, Sorensen (2000) found the prices of repeatedly purchased prescriptions to be lower and less dispersed than irregularly purchased ones. He argued that the search benefits from repeat prescriptions were higher since the savings could be repeatedly realized. Yet if search savings were accumulated, a store's ranking in the price distribution would be stable over time and Varian (1980) and Lach (2002) provide theory and evidence respectively that this need not be the case. While infrequently purchased items such as ECDs provide less incentive to accumulate information, their being higher priced provides more incentive to reap search benefits (Lach, 2002).

[11] ECD online purchases are rare and while differences in prices, between on-line and regular stores have been studied for books by Brynjdfsson and Smith (1999) and Clay et al.(2002), the results of the studies differ as to which is the cheapest on average.

[12] The prices were for a single identical model of refrigerator (size, brand type and so forth), size and type of chicken, coffee and flour, being collected from on average 38, 37, 14 and 15 stores respectively. The study by Lach (2000) controlled for product heterogeneity by looking at only one item. This study in this paper covers virtually the whole market, as required by theory, with the heterogeneity of the items being controlled for by the hedonic regressions.

Price dispersion can also be explained simply as a result of price discrimination. Yoskowitz (2002) found price dispersion for the purchase of raw water by companies and municipalities. Price discrimination arises from consumer heterogeneity, the Schultze Panel's (2002) Report on the U.S. CPI noting extensive heterogeneity within a stratum of goods: "…different people buy widely different qualities and brands of goods, often shop at different retail outlets and pay different prices for the same product." (Mackie and Schultz, 2002: 224). The Report attributes the product heterogeneity to consumer heterogeneity of tastes[13], differences in age, family composition, geographical location and income, the latter affecting the willingness of an individual to substitute in response to relative prices. Basic marketing teaches managers to segment their markets according to differences in price elasticity by offering products of different quality to different segments and, more particularly, targeting brands to different segments to exploit any consumer surplus. The analysis by quality characteristic and brand picks up such price dispersion.

5.2 The persistence of price dispersion

Over time marginal search costs and benefits will change and consumers will build up their stock of knowledge when costs fall, say on routine shopping trips. With stable prices (no depreciation in knowledge) price dispersion should diminish. Yet while it is in the interests of consumers to build up their knowledge to identify the lowest prices, it is in the interest of stores to prevent this. Varian (1980) explains the persistence of price dispersion by distinguishing between 'shoppers' who pay the lowest price and the remaining consumers with search costs who shop randomly. Price dispersion persists because outlets change their prices (randomly) so as to prevent consumers with search costs from becoming fully informed. Lach (2002) found an intensive process of re-positioning prices over time across stores consistent with Varian's (1980) random pricing model.

5.3 Price dispersion and its mean

As consumers purchase more, they learn more, and it can be argued that price dispersion and the differences between the results from different index number formulas should diminish. Yet the persistence of the failure of the law of one price can also be explained by search cost theory in terms of a relationship between (relative) price dispersion and its mean over time. Classical economic theory at the aggregate level argues that inflation is a monetary phenomenon and should have no effect on relative price distribution. Van Hoomissen (1988) argues that as inflation increases the value of existing information decreases requiring higher search costs to just return to the previous search equilibrium. With price increases, the consumer's understanding of the price distribution is eroded and with higher price increases it is eroded faster. Price dispersion thus persists and varies directly with inflation (Stigler and Kindahl, 1970). However, for infrequently purchased items the store of information should be minimal[14] and any relationship between the dispersion and mean of prices (or relative price changes) requires an alternative theoretical framework, of which there are several.

Signal extraction models hold that relative price variability will increase with inflation as consumers become less able to distinguish between unanticipated inflationary price variation and relative price changes (Barro (1978), Lucas (1973) and Friedman (1977), extensive empirical work including Vining and Elwertowsky (1976), Parks (1978), Balk (1983),

[13] Rauh (2002) has shown that with heterogeneity of taste and search costs alone there will be price dispersion.
[14] Since stores sell a range of infrequently purchased items including fridges, washing machines, dishwashers, stereos, television sets and the like, it might be argued that search information is accumulated on the store, as opposed to the item, giving some credence to the theory.

Domberger (1987), Debelle and Lamont (1993), Reinsdorf (1994) and Silver and Ioannidis (2001)). At higher rates of (unanticipated) mean prices (or relative prices) higher rates of (relative) price dispersion are expected.

Menu cost models find price dispersion occurs when firm's nominal prices are held constant since there are costs to undertaking price changes.[15] Yet there will come a point (lower bound) when the extent of the change in its real price demands a nominal price adjustment to its upper bound. The resulting staggered price changes give rise to a positive relationship between price dispersion and inflation (Sheshinski and Weiss (1977), Bénabou and Gertner (1993), Ball and Mankiw (1994 and 1995) and Levy and Bergen (1997)).[16]

Many ECDs are imported or assembled from imported components. If prices are set in the consumer's local currency then changes in nominal exchange rates do not affect prices; there is zero pass-through of exchange rate changes. Feenstra and Kendall (1997) and Engel and Rogers (2001) found a significant proportion of price dispersion to be due to incomplete exchange rate fluctuations. If nominal exchange rates fluctuate with inflation, so too might price dispersion.

Serial correlation in price dispersion may arise from sales, in which prices are marked down for a short period only to return to their preceding levels. Hong et al. (2002) argued that serial correlation will be induced for *fmgs* such as paper towels as consumers build up inventories at sale prices. While this is not applicable to ECDs, prices are reduced at well known sale times and consumers may delay their purchase until such times. This can be modeled as a seasonal effect.

In conclusion, there are reasons to expect price dispersion for ECDs. First, we explain such cross-sectional price dispersion to model and outlet heterogeneity which has been argued to induce search costs. Use is made of hedonic regressions which relate prices to the quality characteristics of these differentiated models, their brand and outlet-type. The residuals are then related to a more direct search cost variable, the proportion of stores in which the model is sold. Second, our concern is whether the dispersion persists or converges and the distributions of the resulting residuals from each month are compared. Third, time series properties and the relationship between price dispersion and (unanticipated) mean prices are investigated.

5.4 Price skewness and its mean

The relationship between the skewness of relative price changes and inflation (Ball and Mankiw, 1995) has also been the subject of study. The aforementioned lower and upper bounds on prices, within which it is not optimal for price changes to be made, come into play here, but with an asymmetry. If, for example, the distribution of desired relative price changes (shocks) is skewed to the right, the average price level will rise since in the short run firms will respond to the large relative price changes (shocks), the smaller ones falling in the range of inactivity. Similarly the price level would fall for negatively skewed desired relative price

[15] Levy and Bergen (1997) show such costs can be substantial.

[16] More recently the focus of such work has been on the relationship between the skewness of relative price changes and inflation (Ball and Mankiw, 1995). While Balke and Wynne (1996) have argued for a similar relationship using a multisector real-business-cycle model, Bryan and Cecchetti (1999) have dismissed the relationship as a statistical artefact due to small sample bias. Our concern with dispersion only has potential statistical small-sample bias if the data are drawn from a skewed distribution. Our use of the population of observations in any event argues against any such bias induced, spurious relationship.

changes. This holds even if the mean of desired prices remains unchanged. If the distribution of desired prices is skewed, a larger variance magnifies the asymmetry in the tails and thus increased the change in the average price level. Ball and Mankiw's (1995) model thus postulates no independent effect for the variance, but a positive interaction effect for the variance with skewness. We thus have a model whereby the mean price is related to its higher moments for goods where frequent and potentially sizable adjustments are made to desired prices.

It is finally worth noting that other explanations have been given for relationships between the mean and its higher moments of relative price *changes*. Balke and Wynne (1996) have shown how spillover effects from large shocks to a few volatile sectors might generate a positive association between inflation and its higher moments. Bryan and Cecchetti (1996) have argued that the two theories can be distinguished if the periodicity of the data is varied, menu costs being a theory concerned with the short run (see also Debelle and Lamont, 1997). While Balke and Wynne (1996) have argued for a similar relationship using a multisector real-business-cycle model, Bryan and Cecchetti (1999) have dismissed the relationship as a statistical artefact due to small sample bias. Our concern is mainly with dispersion and this only has potential statistical small-sample bias if the data are drawn from a skewed distribution. Our use of the population of observations in any event argues against any such 'small-sample' bias-induced, spurious relationship(see also Ball and Mankiw (1999) and Verbrugge (1998) and for empirical work on skewness see Rodger (2000) and Silver and Ioannidis (1996).

6. Empirical Work: Data and Measures

6.1 Data

The empirical work utilizes monthly scanner data for television sets from January 1998 to March 2002. The scanner data was supplemented by data from price collectors from stores without bar-code readers, though this was a negligible. The observations are for a model of the product, for which there was a transaction, in a given month in one of four different outlet types: multiples, mass merchandisers, independent and catalogue. For example, an observation in the data set for January 1998 includes the unit value (£275.80), volume (5,410 transactions) and quality characteristics (including possession of Nicam stereo and fastext text retrieval facilities) of the Toshiba 2173DB 21 inch television set sold in multiples only. *For the 51 months of January 1998 to March 2002 there were 73,020 observations which covered 10.8 million transactions worth £3.9 billion*.

6.2 Variables

The variable set *on each observation* included: **Price,** the unit value of a model in a month/outlet-type across all transactions. For example, there were 22,485 basic 14" TVs sold in 'mass merchandisers' outlets in December 1998, a seasonal 'blip' to meet the demand for Christmas presents. The 22,485 transaction prices are simply summed and divided by the number of transactions to yield the single observation: the price of this model in this store this month - £97.50 (see Balk, 1996 for the statistical properties of unit values). **Volume** is the sum of the transactions during the period. Many of the models sold in any month had relatively low sales. There were 38 **brands**—37 dummy variables benchmarked on Sony; the **characteristics** included (i) size of screen—dummy variables for about 19 screen sizes; possession of (ii) Nicam stereo; (iii) wide screen; (iv) on-screen text retrieval news and information panels from broadcasting companies, in order of sophistication: teletext, fasttext and top fasttext – 3 dummy variables; (v) reception system—6 types; (vi) monitor style; (vii) with Dolby Pro, Dolby SUR./DPL., Dolby Digital sound—3 dummy variables; (viii) Flat & Square, Super-Planar tube—2 dummy; (ix) s-vhs socket; (x) with satellite tuner, analogue/digital—2 dummy variables; (xi) digital; (xii) with DVD playback or DVD recording—2 dummy variables; (xiii) with rear speakers; (xiv) without PC-internet/PC+internet; (xv) with real flat tube; (xvi) 100 hertz, doubles refresh rate of picture image; (xvii) vintage and DIST—the percentage of stores in which the model was sold. **Outlet types are** multiples, mass merchandisers, independents and catalogue. The hedonic regressions were based on about 100 variables.[17]

6.3 Measures of dispersion and average prices

6.3.1 Weights

Our concern with differences between (weighted) *upper-level* indices gives rise to an empirical concern with difference between *weighted* variances. The empirical micro literature on price dispersion and the law-of-one price is dominated by the use of unweighted measures and include Clay et al. (2002), Lach (2002), Hong et al. (2002), Engel and Rodgers (2001), Cohen (2000), Sorensen (2000) and Beaulieu and Mattey (1999). But this use is only because weighted measures are often unavailable and this is a serious shortcoming of existing studies. An increase in price, for example, in one store will lead, *ceteris paribus*, to a fall in quantity and weight, unweighted measures exaggerating price dispersion.[18] Since Annexes 4 and 5 have shown that the extent of any difference between Laspeyres and Paasche and the bias in the Young index to be at least in part dictated by differences in the *weighted* dispersion in prices over time, results for weighted dispersion measures are calculated and analyzed in the empirical section.

Our concern with differences between (unweighted) *lower-level* indices gives rise to an empirical concern with difference between *unweighted* variances. While the empirical work can be rightly undertaken in this manner, three points are worth noting. First, that prices are often collected by statistical agencies (at least initially) for major selling/typically purchased items for the CPI sample. If, as for the U.S. CPI, sampling is with probability proportionate to value share in the base period, our concern is with weighted indices. Under such sampling the expected value of a Carli index is a Laspeyres index (Balk, 2002) and the results for weighted indices apply. Second, and related to this, other sampling systems might be replicated using the data. In many countries 'typically purchased' items (at least in the price reference period) are sampled and unweighted variances based on cut-off sampling may be more appropriate for the empirical work.

Third, the analysis of dispersion at the weighted level is motivated by the economic theories of search cost, menu costs and signal extraction models. The differences between lower-level indices were shown in section 4 to be concerned with differences in unweighted price dispersion. Thus while the empirical work can ascertain patterns of unweighted price dispersion to explain the differences between these formulae, it may be argued that it should not draw on theory which relates to market behavior which includes prices *and* quantities. Against this such theories can be seen as theories of market failure in price setting: menu cost

[17] There is some variability in this over time with DVD, rear speakers, top fasttext, Dolby digital and SUR>/DPL> (as opposed to just Dolby Pro sound), 100 hertz and integrated PC not being used until January 2000; 11 variables excluded as not being relevant.

[18] Spurious correlations between dispersion and its mean have been argued by Bryan and Cecchetti (1999) to arise when unweighted measures are used.

theory predicts that retailers will have costs of price adjustment and not undertake such adjustments unless the price change is outside of some bounds, thus leading to price dispersion—a case that can be argued for all models of a good irrespective of their sales quantities. Similarly if a proportion of the population has search costs some retailers can enjoy a surplus on some/all of their models which will again lead to price dispersion irrespective of sales quantities. And again mistakes in anticipating inflation will lead to erroneous decisions by economic agents, an argument that will lead to price dispersion. In all cases the welfare effects require quantities to be taken into account. But all that we require here is that the analysis of unweighted changes in the dispersion of prices be motivated by the aforementioned theoretical frameworks.

Finally, we have a genuine interest in the behavior of a heterogeneity-controlled Dutot index since the Dutot index has fine axiomatic only being dismissed from the analysis because it fares badly when the items are relatively heterogeneous. The analysis of price dispersion as a means to minimize such heterogeneity by statistical mechanisms as opposed to the selection of a limited matched sample is of interest.

6.3.2 Parametric measures of absolute and relative dispersion

The weighted standard deviation and coefficient of variation are given as absolute and relative parametric measures respectively by:

$$SD_w^t = \left[\sum_{m=1}^{M} w_m^t \left(p_m^t - \overline{p}^t \right)^2 \right]^{1/2} \text{ and } CV_w^t = SD_w^t / \overline{p}_w^t \qquad (15)$$

where the aggregation is over $m = 1, \ldots M$ models in a given month and outlet-type, w_i is their sales (value) share p_m is the price and $\overline{p}_w^t$ the mean price = $\sum_{m=1}^{M} w_m^t p_m^t$. Unweighted measures are similarly defined. Non parametric/robust measures are not used since the index number formulae relationships are based on parametric measures.

7. Empirical Work: Results

For the empirical work two things are of interest. First, the extent of price dispersion in any period. Price collectors are required to collect prices of similar items for defined 'representative items.' The more similar the items, the less the dispersion in prices and the closer together the results of the different indices. Search cost theory tells us that the product heterogeneity may itself be a device to increase search costs and allow further 'real' dispersion. An understanding of the differences between formulae thus requires an understanding of the heterogeneity of prices and the derivation of estimates of heterogeneity-controlled prices and their dispersion. Second is the *explanation of changes* in dispersion over time. This was shown in sections 2 and 3 to be at the core of explaining differences between index number formulae and search cost, menu cost and signal extraction models will be used to underlie this empirical work. It was also shown in section 2 that the Dutot index is particularly sensitive to product heterogeneity, since it fails the commensurability test. This work will also compare Dutot, Carli and Jevons (matched) indices for *heterogeneity-controlled* prices as against uncontrolled, raw prices.

In both instances highly detailed scanner data from the bar-code readers of retailers will be used. Such data cover the market of transactions and provide information on the price-determining quality characteristics responsible for much of the price dispersion.

7.1 Explaining price variation

Table 1 provides summary statistics on raw price dispersion. First, the extent of dispersion in raw prices is substantial. The coefficients of variation (CV= standard deviation $(SD)/\bar{x}$) averaged 0.85. Second, the standard deviation can be seen to have increased substantially: by just over 20 percent for the unweighted measure. Yet the unweighted CV is relatively stable and shows much of this increase in price dispersion to be accounted for by inflation. Table 1 shows weighted dispersion increased by 40 percent compared with the 20 percent for the unweighted dispersion. Commonly purchased models have quite disparate price movements. Yet the weighted CV also increased substantially over the first three years, inflation not being an immediately obvious explanation for the increased dispersion in raw prices, but subsequently fell.

Measuring price dispersion under product differentiation requires controls for the brand and technical characteristics of the model and, since different outlet-types provide different services, the outlet-types in which the model is sold. Table 2 is based on a hedonic regression for observation *m* in period *t*:

$$\ln \text{Price}^{t}_{m} = \beta_0 + \sum_{i=1}^{11} \beta_i \text{Month}^{t}_{m} + \gamma Year^{t}_{m} + \sum_{j=1}^{48} \delta_j \text{ Charac}^{t}_{m} +$$

$$\sum_{k=1}^{37} \lambda_k \ B\text{rand}^{t}_{m} + \sum_{l=1}^{3} \theta_l \text{ Outlettype}^{t}_{m} + \varepsilon^{t}_{m} \qquad (16)$$

Table 2 provides a nested decomposition of price variation to explain some of the existing price dispersion and identify the remaining dispersion—the heterogeneity-controlled prices—in terms of the residuals of the regression. The observations for such a pooled regression were the prices, characteristics and brands of individual models of TVs in a specific outlet-type over the 51 months of January 1998 to March 2003—just over 73,000 observations on 37 brand dummies, 3 outlet-type dummies, 19 screen size dummies, 6 tube-type dummies and 23 further characteristics as outlined in sections 6.1 and 6.2 above. The coefficients were almost invariably statistically significant and their signs accorded with *a priori* expectations.[19] The $\bar{R}^2$ for the estimated equation (3) shows that over 90 percent of variation in price was explained by the model. Month and time provided little explanatory power; it was product heterogeneity vis-à-vis product characteristics, brand and outlet-type variation that accounted for most of the price variation. Multicollinearity precludes our assigning variation separately to brands, characteristics or outlet-types, however, characteristics do most of the work: a regression on month, trend and brands only accounted for 0.35 percent of variation ($\bar{R}^2 = 0.0035$) and similarly low for month, trend and outlets. The regression successfully controlled prices for the heterogeneity of their features. The residuals, $\left|\hat{\varepsilon}^{t}_{m}\right|$, are estimates of heterogeneity-controlled prices. Mean variation in prices was reduced by over 50 percent by the regression, and the standard deviation of the heterogeneity-controlled prices was about one-fifth that of the actual price dispersion (Table 2). Bear in mind it is not just the variation in technical characteristics, brands and the services from outlet-types that explain the price dispersion. If this were the case the law-of-one-price should hold for the residuals. The sheer multiplicity of models hinders search giving rise to price dispersion, though we return to this later.

[19] Details available from authors.

While characteristics, brands and outlet-types were found to be one source of price dispersion, we now consider a more explicit modeling of search to explain price dispersion *within* months. In section 6.2 DIST was defined for each model as the percentage of (television) shops stocking or selling that model that the model. Unfortunately such data were only available for the first 24 months and we confine this analysis to this period.[20] A model sold in fewer shops is less likely to have a comparable model available in any search carried out, thus precluding direct comparisons and allowing some premium margin to be charged, an expected negative sign to help explain the remaining variation. To establish whether heterogeneity–controlled price variation can in part be explained by this search cost variable we first estimated separate hedonic regressions of the form in equation (16) for each month. The mean $\bar{R}^2$ for the 24 regressions was 0.89 with a minimum of 0.85 (and for weighted regressions 0.96 and 0.95 respectively), normality of residuals (Jacques-Bera) was by and large rejected for the OLS and WLS estimators,[21] though the null of homoskedasticity was generally not rejected[22] for both estimators (Breusch-Pagan).[23] We second, regressed heterogeneity-controlled prices on DIST:[24]

$$\ln \tau_m^t = \alpha_0^t + \alpha_1^t \, DIST_m^t + \nu_m^t \qquad (17)$$

where τ_m^t are the (exponents of the) residuals from hedonic regressions akin to equation (16) but run each month. The results are in Table 3, though only for the first 24 months such data not subsequently being part of routine data provision by the supplier. For an OLS estimator Table 3 shows a consistently positive, statistically significant relationship. The more stores a model is sold in the *higher* its heterogeneity–controlled price. This runs contrary to our expectation from search cost theory. One explanation for the positive sign is that the OLS results give equal weight to models with very few sales, that is to models near the end of their life cycle. Such models have been shown to have (Silver and Heravi, 2002a) low prices relative to their specifications and are no longer sold in all the stores they were previously sold in. Similarly Silver and Heravi found new more widely distributed models to have a premium snob price relative to their specifications. The WLS estimator for equation (17) virtually ignores the low selling old models with their low quality-adjusted prices to give more ambiguous results for most of the period, except for towards the end when consistently

[20] Even for this period data on DIST were more limited than the rest of the data, comprising nearly 22,000 observations (as opposed to 29,000).

[21] Though for the WLS estimator the null of the (component) test for skewness (Davidson and MacKinnon, 1993) was not rejected in one-third of the months—details available from authors.

[22] The Breusch-Pagan test failed to reject the null of homoskedascity for all but three months using the OLS estimator, though rejected in one-third of the months using WLS (by value)—details available from authors.

[23] This deviation from the normality assumption and some heteroskedasticity may not permit correct inferences to be drawn on the coeficients. However, a heteroskedasticity-consitent covariance matrix estimator (HCCME) was used following White (1980) to allow asymptotically correct tests to be undertaken. A wild bootstrap estimator is commonly applied to models with heteroskedastic and skewed residuals due to small-sample bias in the HCCME. Davidson and Flachaire (2001) show that the wild bootstrap is only necessary to alleviate small-sample bias; the HCCME estimator is appropriate for the large sample tests in this study.

[24] DIST might well have been included in the hedonic regression equation except for *a priori* expectations that the residuals, heterogeneity–controlled prices, may be correlated with it, as indeed we found. Modern stores hold relatively little stock with advanced replacment ordering systems unrelated to price. The level of stocks was used each month as instruments for instrumental variable hedonic regressions that included DIST. The results from such regressions found DIST significant at a 5% level in 6 of the 24 months and at a 10% level in 8 months. In all such regressions the coefficients on DIST had a positive sign. A series of Hausman test for each month found the null of no relationship between the errors and DIST was rejected in 50% of the months using OLS but in only 20% of the months using WLS.

positive results are apparent from Table 3. When we look at better selling models there may be economies of scale in distribution lowering price dispersion.[25] Thus for the difference between unweighted index number formulae, for which our concern is unweighted OLS results, heterogeneity-controlled price dispersion can be further explained by DIST.

Thus three things have been found to explain price variation and thus have the potential to control price dispersion within months: (i) the technical features, brand and outlet type of the models, (ii) for unweighted measures, the extent to which models are sold in different stores, the fewer the stores the lower the (quality-adjusted) prices and (iii) the use of weights.

7.2 Changes in price variation

Section 4 showed that it is the *changes* in dispersion that underlies the differences between the results from index number formulae. Table 4 and Figure 1 shows the unweighted and weighted standard deviation of residuals over time. Note that the residuals are estimated from separately estimated regressions each month and thus are normalized by their respective semi-log hedonic regression in each month to have a mean of unity. As such the standard deviation of the residuals is the *de facto* coefficient of variation; the standard deviation normalized by the mean. This is our measure of dispersion. The derivations of the differences between formulae in the annexes are based on normalized dispersion and the measurement and modeling naturally follows from this. Table 4 and Figure 1 shows an increase in such dispersion of nearly *100 percent* over the 51 months, *and this is after being controlled for heterogeneity and mean (quality-adjusted) inflation*. Note that quality-adjusted mean prices were falling over this period,[26] and that this should contribute to a fall in the standard deviation. So the adjustment by the mean implicit in the measure increases dispersion over time to account for this. But the resulting series can be seen to trend upwards; this relative concept of dispersion is increasing even after accounting for the fall in the mean. Also shown in Table 4 and Figure 1 is substantial volatility in the series. So dispersion is increasing accompanied by volatility, but so too must the difference between the formulae and this is after we control for heterogeneity and average price changes. So can we explain such changes?

The residuals from WLS hedonic regressions have been weighted by their relative expenditure shares to reflect their economic importance. Table 4 shows this weighted heterogeneity-controlled price dispersion to be about two-thirds that of its unweighted counterpart, and it too shows a striking increase, over 75 percent. Figure 1 shows much more volatility in the unweighted (OLS-based) measure, especially towards the end of the series.[27] Thus weighting reduces dispersion, the change in dispersion, and thus the differences between the results from formulae, and volatility of dispersion. Though for weighted and unweighted

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[25] To corroborate the influence of weights on the relationship the OLS estimates were run for a sample of models selling 200 or more inh an outlet-type in any month. There was no relationship for dispersion on DIST for these larger selling models in any of the 24 months.

[26] The measurement of the change in *mean* heterogeneity-corrected prices is problematic since the expected values of the residuals, or their logs ,will be zero or unity by construction and not vary over time. A chained index has been calculated using hedonic regressions with the same specification as (3) above, but the regressions are based on sets of two successive monthly stacked data where *Month* is a dummy variable which takes the value of one if it is the second month and zero otherwise, and the estimated coefficients on *Month* are linked by successive multiplication to form a chained index of the heterogeneity-controlled mean price. The index (results available form authors) fell by about 30 percent over the period.

[27] There is also a shift at January 2000 but this may be due to a change in the format of the data and slightly more detailed variable definitions available from this month onwards.

measures the evidence is of a substantial increase in dispersion even after we have explained technical, brand and outlet-type variation.

7.3 Explaining changes in residual price dispersion

In this section we seek to explain changes in residual price dispersion over time both in relation to its time series properties, search cost related theories, signal extraction models (its anticipated and unanticipated mean) and menu cost theory.

Search cost theory argues that as inflation increases, the value of existing information decreases requiring higher search costs just to return to the previous search equilibrium (Van Hoomissen, 1988). It would predict increased dispersion from an increasing mean, but also increasing dispersion from a falling mean since in both cases the stock of knowledge would depreciate as average prices change. Bearing in mind average (heterogeneity-adjusted) prices were falling (*ff*.26) it would predict that dispersion normalized on the mean would rise. We test for trend stationarity in our mean-adjusted measure of dispersion and take non-rejection of a unit root as evidence to reject stationarity. Unit root test results are given in Table 5 and although not conclusive, the weight of evidence is towards accepting I (1) as expected.[28] The drift from Figure 1 is upwards in both cases confirming a positive trend in dispersion over the mean. If search was frequent then consumers would become more knowledgable and dispersion would decrease. However, given the frequency of search for consumer durables is more limited than for frequently purchased goods the finding is not surprising.

Search cost theory would also predict that as the total number of models of television sets on sale increases, there would be higher search costs and thus dispersion, and thus a negative sign on the estimated coefficient for 'number of models' in a regression of dispersion on the latter. This would hold in spite of their infrequent purchase.

Signal extraction models require an indicator of unanticipated inflation. Anticipated inflation was predicted from ARIMA models for unweighted and weighted H-C means. AR(1) processes fitted the series best in both instances. Unanticipated inflation (H-C $\text{UNANTICIP}_{\text{weighted}}$ and H-C $\text{UNANTICIP}_{\text{unweighted}}$) was generated as the residuals. To test signal extraction models heterogeneity-controlled price dispersion was regressed unanticipated inflation. The estimated coefficient on unanticipated inflation should have a positive sign following signal extraction theory that increased dispersion arises from the inability of economic agents to properly anticipate inflation, such inability increasing with inflation. However, the evidence of a positive relationship is far from conclusive. For example, Hesselman (1983), and Silver (1988) found negative relationships for the UK. Silver (1988) argued that the coefficient may have a negative sign as economic agents become more cautious in their price-setting and price-taking under uncertainty. Buck (1990), found negative association for Germany (using 19th century data), but positive for the US using data for the same period. Reinsdorf (1994) found a negative relationship for 65 categories of goods in 9 US cities, though this was for price levels. Reinsdorf (1994) found his explanation for a *negative* relationship from consumer search cost theories with unexpected inflation inducing more search due to consumers' incomplete information about price distributions.

[28] We also differenced and subsequently repeated the unit root tests to ensure I(2) was rejected in favour of I(1). The *p*-values for (augmented) weighted asymmetric tau tests on weighted and unweighted differenced dispersion were 0.00578 and 0.00249 respectively rejecting I(2).

Silver and Ioannidis (2001) found negative relationships for a range of European countries using a consistent methodology.

Menu cost theories can be tested by examining the relationship between the dispersion and the mean for data with a bimonthly frequency and that with a monthly frequency. Larger price changes and dispersion should materialize in the latter, though the limited time series here precludes this study for the time being. More straightforward is to include a variable on the proportion of *models (*for unweighted dispersion) or *expenditure* (for weighted dispersion) in catalog stores ($CATALOG_{unweighted}$ and $CATALOG_{weighted}$ respectively), the latter having higher menu cost than other stores. A positive sign would be expected on the estimated coefficient.

Table 6 shows the regression results to explain variation in changes in weighted and unweighted heterogeneity-controlled residuals normalized on their means. That Table 5 found some series to be I(1) held out the possibility of long-run cointegrating relationships. Engle-Granger (tau) cointegration tests were undertaken finding non-rejection of the null whichever variable was chosen as the dependent variable. The results for using the unweighted and weighted dispersion as the dependent variables respectively were:

Dep.Var.	TestStat	P-value	Num.lags
$SD_{unweighted}$	-3.08894	0.38296	6.00000
$SD_{weighted}$	-3.65520	0.14725	4.00000

Non-rejection of null that the residuals of the cointegrating relationship has a unit root is evidence of noncointegration precluding estimation of an error correction model. However, where the null of unit roots was rejected first differences were used in the regression. For unweighted dispersion the coefficient on the 'number of models' sold is positive and statistically significant as predicted from search cost theory. No corresponding result is found for weighted dispersion. Unanticipated inflation has the expected negative coefficient for unweighted and weighted dispersion the relationship being more consistent for weighted dispersion there being evidence of it being affected by multicollinearity. There is also evidence of a *negative* relationship for both weighted and unweighted dispersion for the incidence of models/sales in catalog stores. We argued for a positive relationship from a menu cost stance on the grounds that the costs and resulting delays in making adjustments would lead to relatively large changes when they took place. Catalog stores are more prone to such delays. Yet during the print run of the catalog there should be less fluctuations and it may be that this overshadowed the price spikes from the adjustments. More generally while there is evidence of multicollinearity, Table 6 shows some explanatory power of economic variables in explaining price dispersion.

8. Index number implications

Since differences arising from index number formulae can be significant and since they are to varying extents determined by the variance in prices and its changes, the motivation to explain such variation is appropriate, especially since there is an economic theory of price dispersion to ground the work in. A number of implications arise from the study.

First, the discrepancy between elementary indices increases as the dispersion in prices increases. Since much of the variation can be explained by the heterogeneity of the brand, technical characteristics of the good, the requirement is for well-defined, item specifications (albeit at the cost of coverage - Silver and Heravi, 2002b). Bias in the Dutot index arises from such heterogeneity, in spite of its otherwise good axiomatic properties. We calculated a heterogeneity-controlled (matched) Dutot index using the residuals from the hedonic regression and compared it with a Dutot index without such controls, as well as Carli and Jevons matched indices. The results are in Table 7. First, formula does matter; there is 15 per cent fall according to Carli and Dutot, but 20 percent fall given by the Jevons index. Second, the Dutot index, like Jevons, performs well from the test approach, but suffers from its failure of the commensurability test particularly with regard to product heterogeneity. The bias (as measured here) of the Dutot index against the heterogeneity-corrected Dutot index is about 1.5 percentage points upwards over the period.

Second, there was found to be considerable price dispersion in this product area which is not unusual for highly differentiated consumer durables (Table 1). Brand, characteristics and outlet-type together explained much of such variation (Table 2). Minimizing price variance for price index number compilation requires either the use of hedonic indices or detailed specifications for a selection of 'representative' items and care in judging replacements to be 'comparable' when an item goes missing if its characteristics are different. When a model is missing the price collector may judge another model to be of comparable quality and compare its prices with those of the old model. There is too much price variation associated with product heterogeneity to be lax about any leniency in such selections.

Third, as soon as weights were applied the dispersion was reduced (Table 2) and thus the difference between formulae. Selections of more popular models serve to not only make the index more representative, but also to reduce the disparity between the results from different formulae.

Fourth, for unweighted indices, product heterogeneity aside, models sold in more stores (DIST) have higher prices (Table 3). Selection of items should take into account a model's coverage of stores for the sample to be representative.

Fifth, we found an increase in dispersion of nearly *100 percent* over the 51 months, *and this was after being controlled for heterogeneity and mean (quality-adjusted) inflation*. Such differences lead to formulae differences and require explanation (Table 4). Some of the explanation could be identified via the trend, the upwards drift in the series. The drift was more volatile and accentuated for weighted dispersion than unweighted dispersion (Figure 1).

Sixth, differences in dispersion over time accord with aspects of search cost theory, menu cost theory and signal extraction models. Such frameworks were shown to explain some of the variation in dispersion over time (Table 6) and thus the increasing differences between the results of index number formulae. This applied both to weighted dispersion (formulae) and unweighted dispersion (formulae), though more successfully to the former.

Annex 1: The relationship between Dutot and Jevons indices

The first approximate relationship that will be derived is between the Carli index P_C and the Dutot index P_D. For each period t, define the *arithmetic mean of the M prices* pertaining to that period as follows:

(A1.1) $$p^{t*} \equiv \sum_{m=1}^{M} \frac{1}{M} p_m^t \qquad t = 0,1.$$

Defining the multiplicative deviation of the m^{th} price in period *t* relative to the mean price in that period, e_m^t, as:

(A1.2) $$p_m^t = p^{t*}\left(1+e_m^t\right); \qquad m = 1,...,M\,;\; t = 0,1.$$

Note that (A1.1) and (A1.2) imply that the deviations e_m^t sum to zero in each period; i.e., we have:

(A1.3) $$\sum_{m=1}^{M} e_m^t = 0; \qquad t = 0,1.$$

The Dutot index can be written as the ratio of the mean prices, p^{t*}/p^{0*}; i.e., we have:

(A1.4) $$P_D(p^0, p^t) = \frac{p^{t*}}{p^{0*}}$$

Substitute equations (A1.2) into the definition of the Jevons index, (3) using (A1.4):

$$P_J(p^0, p^t) = \prod_{m=1}^{M} \sqrt[M]{\frac{p_m^{t*}(1+e_m^t)}{p_m^{0*}(1+e_m^0)}} = \frac{p^{t*}}{p^{0*}} \prod_{m=1}^{M} \sqrt[M]{\frac{(1+e_m^t)}{(1+e_m^0)}} = \mathrm{P_D}(\mathrm{p}^0, \mathrm{p}^t)\, \mathrm{f}(\mathrm{e}^0, \mathrm{e}^t).$$

(A1.5)

where $e^t \equiv [e_1^t,...,e_M^t]$ for t = 0 and 1 and the function f is defined as follows:

(A1.6) $$\mathrm{f}(\mathrm{e}^0, \mathrm{e}^t) \equiv \prod_{m=1}^{M} \sqrt[M]{\frac{(1+e_m^t)}{(1+e_m^0)}}$$

Expand $f(e^0,e^t)$ by a second order Taylor series approximation around $e^0 = 0_M$ and $e^t = 0_M$. Using (A1.3), it can be verified[29] that we obtain the following second order approximate relationship between P_J and P_D:

(A1.7) $$P_J(p^0,p^t) \approx P_D(p^0,p^t)[1 + (1/2M)e^0 \cdot e^0 - (1/2M)e^t \cdot e^t]$$
$$= P_D(p^0,p^t)[1 + (1/2)\mathrm{var}(e^0) - (1/2)\mathrm{var}(e^t)]$$

[29] This approximate relationship was first obtained by Carruthers, Sellwood and Ward (1980; 25).

where var(e^t) is the variance of the period t multiplicative deviations; i.e., for t = 0,1. Since e^{t*} = 0 using (A1.3):

$$\text{(A1.8)} \qquad \text{var}\,(e^t) \equiv \frac{1}{M}\sum_{m=1}^{M}(e_m^t - e_m^{t*})^2 = \frac{1}{M}\sum_{m=1}^{M}(e_m^t)^2 = \frac{1}{M}e^t e^t$$

Diewert notes that "Under normal conditions, the variance of the deviations of the prices from their means in each period is likely to be approximately constant and so under these conditions, the Jevons price index will approximate the Dutot price index to the second order." He footnotes 'normal conditions' with the caveat that: "If there are significant changes in the overall inflation rate, some studies indicate that the variance of deviations of prices from their means can also change. Also if M is small, then there will be sampling fluctuations in the variances of the prices from period to period." Our concern is with former.

Annex 2: The relationship between Jevons and Carli indices

Both of these indices are functions of the relative prices of the M items being aggregated. This fact is used in order to derive some approximate relationships between these indices[30]. Define the m^{th} price relative as $r_m \equiv p_m^t / p_m^0$ for $m = 1,...,M$ and the *arithmetic mean of the m price relatives* as

(A2.1) $$r^* \equiv \frac{1}{M}\sum_{m=1}^{M} r_m = P_C(p^0,p^t)$$

where the last equality follows from the definition (3) of the Carli index. Finally, define the *deviation* e_m *of the mth price relative* r_m *from the arithmetic average of the M price relatives* r^* as follows:

(A2.2) $$r_m = r^*(1+e_m); \qquad m = 1,...,M.$$

Note that (A2.1) and (A2.2) imply that the deviations e_m sum to zero; i.e., we have:

(A2.3) $$\sum_{m=1}^{M} e_m = 0$$

Now substitute equations (A2.2) into the definitions of P_J, above, in order to obtain the following representations for these indices in terms of the vector of deviations, $e \equiv [e_1,...,e_M]$:

(A2.6) $$P_J(p^0,p^t) = \prod_{m=1}^{M} \sqrt[M]{r_m} = r^* \prod_{m=1}^{M} \sqrt[M]{1+e_m} = P_C(p^0,p^t) \prod_{m=1}^{M} \sqrt[M]{1+e_m}$$

The multiplicative factor which defines the difference between the Jevons and Crli indices is approximated using a second order Taylor series expansion around the point $e = 0_M$ by :

(A2.7) $$\prod_{m=1}^{M} \sqrt[M]{1+e_m} \approx 1 - (1/2M)e \cdot e = 1 - (1/2)\mathrm{var}(e) ;$$

Thus, since var(e) must be positive, to the second order the Carli index P_C is argued to *exceed* the Jevons index, by $(1/2)r^*\mathrm{var}(e)$, which is one half the variance of the M price relatives $p_m^t/p_m^{0.}$

[30] It is very straightforward to do the same with a harmonic mean of price relatives.

Annex 3: The relationship between Dutot and Carli indices
Following Diewert (1995a, 27)

(A3.1) $$P_D(p^0,p^t)\equiv\sum_{m=1}^{M}(r_m^{\,t})*p_m^0/\sum_{m=1}^{M}p_m^0=\sum_{m=1}^{M}r_m^{\,t}/m+\sum_{m=1}^{M}r_m^{\,t}\left[p_m^{\,0}/\sum_{m=1}^{M}p_m^{\,0}-1/M\right]$$

$$=P_C(p^0,p^t)+\sum_{m=1}^{M}r_m^{\,t}\left[p_m^{\,0}/\sum_{m=1}^{M}p_m^{\,0}-\left(\sum_{m=1}^{M}p_m^{\,0}/M\right)/\sum_{m=1}^{M}p_m^{\,0}\right]$$

$$=P_C(p^0,p^t)+\sum_{m=1}^{M}r_m^{\,t}\left[p_m^{\,0}/\sum_{m=1}^{M}p_m^{\,0}-\bar{p}_m^{\,0}/\sum_{m=1}^{M}p_m^{\,0}\right]$$

which is $P_C(p^0,p^t)$ plus the covariance of normalized $r_m^{\,t}$ and p_m^0. The correlation coefficient between price relatives and base period prices is defined as $\rho(r_m^{\,t},p_m^0)$ which is equal to the covariance of (r_m^t,p_m^0) divided by the product of the variances of the individual variables. Therefore:

(A3.2) $$P_D(p^0,p^1) = P_C(p^0,p^1) + m[\mathrm{var}(r_m^{\,t})*\mathrm{var}(p_m^0)]^{1/2}\rho(r_m^t,p_m^0)$$

Since the variances must be positive, the sign of ρ determines which of these formulae will give results with higher values. The correlation would be expected to be negative as higher base period prices for similar items should have lower price increases. Thus $P_C(p^0,p^t)$ is expected to exceed $P_D(p^0,p^t)$. The two formulae will give the same results if the $\mathrm{var}(r_m^t) = 0$, that is, all price relatives are the same, or the $\mathrm{var}(p_m^0) = 0$, all (normalized) base period prices are the same, or if $\rho(r_m^t,p_m^0) = 0$, there is no correlation between price relatives and base period prices. As either of these depart from zero, the difference between the results from the two formulae will increase. Any difference due to the above factors can be seen to be magnified as m, the number of prices increases.

Note that the relationships in this section have been phrased as long-run ones, between periods 0 and *t.* As time progresses it might be expected that the correlation $\rho(r_m^t,p_m^0)$ weakens though the variance of price relatives $\mathrm{var}(r_m^{\,t})$ may increase.

Annex 4: The relationship between Laspeyres and Paasche
The results are due to Bortkiewicz (1922, 1924) reproduced in Allen (1975, 62-64).

The weighted means of price and quantity relatives are Laspeyres price and quantity index numbers:

(A4.1) $P_L = \sum_{m=1}^{M} w_m^0 \frac{p_m^t}{p_m^0} / \sum_{m=1}^{M} w_m^0$ and $Q_L = \sum_{m=1}^{M} w^0 \frac{q_m^t}{q_m^0} / \sum_{m=1}^{M} w_m^0$ where $w^0 = p^0 q^0$

Thus the notation defines Laspeyres price and quantity indices to respectively be P_L and Q_L. Paasche price and quantity indices can be similarly defined and denoted as P_P and Q_P and the value index as V^{0t}. It is easily demonstrated that P_L x Q_P = V^{0t} and P_P x Q_L = V^{0t} so that:

(A4.2) $$\frac{P_P}{P_L} = \frac{Q_P}{Q_L}$$

This is the ratio of Paasche to Laspeyres formulae we seek to explain. The weighted variances for Laspeyres price and quantity indices defined in (1) are:

(A4.3) $$\sigma_p^{\ 2} = \sum_{m=1}^{M} w_m^0 \left\{ \frac{p_m^t}{p_m^0} - P_L \right\}^2 / \sum_{m=1}^{M} w_m^0 \text{ and}$$

$$\sigma_q^{\ 2} = \sum_{m=1}^{M} w_m^0 \left\{ \frac{q_m^t}{q_m^0} - Q_L \right\}^2 / \sum_{m=1}^{M} w_m^0$$

The weighted covariance times $\sum w_m^0$ is:

$$\sum_{m=1}^{M} w_m^0 \left\{ \frac{p_m^t}{p_m^0} - P_L \right\} \left\{ \frac{q_m^t}{q_m^0} - Q_L \right\}$$

$$= \sum_{m=1}^{M} w_m^0 \frac{p_m^t q_m^t}{p_m^0 q_m^0} - P_L \sum_{m=1}^{M} w_m^0 \frac{q_m^t}{q_m^0} - Q_L \sum_{m=1}^{M} w_m^0 \frac{p_m^t}{p_m^0} + P_L Q_L \sum_{m=1}^{M} w_m^0$$

$$= \sum_{m=1}^{M} w_m^0 \frac{p_m^t q_m^t}{p_m^0 q_m^0} - P_L Q_L \sum_{m=1}^{M} w_m^0$$

We divide by $\sigma_p \sigma_q \sum_{m=1}^{M} w_m^0$ to get the weighted correlation coefficient r between price and quantities:

(A4.4) $$r = \frac{\sum_{m=1}^{M} w_m^0 \frac{p_m^t}{p_m^0} \frac{q_m^t}{q_m^0}}{\sigma_p \sigma_q \sum_{m=1}^{M} w_m^0} - \frac{P_L}{\sigma_p} \frac{Q_L}{\sigma_q}$$

Using $w^0 = p^0 q^0$ and (A2.1)

(A4.5) $$\frac{\sum_{m=1}^{M} w_m^0 \frac{p_m^t}{p_m^0} \frac{q_m^t}{q_m^0}}{\sum_{m=1}^{M} w_m^0} = \frac{\sum p_m^t q_m^t}{\sum p_m^0 q_m^0} = V^{01} = P_P Q_L$$

This brings in the Paasche price index. Substituting in (A4.4):

(A4.6) $$r = \frac{P_P}{\sigma_p}\frac{Q_L}{\sigma_q} - \frac{P_L}{\sigma_p}\frac{Q_L}{\sigma_q} = \frac{P_L}{\sigma_p}\frac{Q_L}{\sigma_q}\left\{\frac{P_P}{P_L} - 1\right\}$$

Rearrangement gives the required common ratio (A2.1) of the Paasche to the Laspeyres index numbers:

(A4.7) $$\frac{P_P}{P_L} = \frac{Q_P}{Q_L} = 1 + r\frac{\sigma_p}{P_L}\frac{\sigma_q}{Q_L}$$

To interpret (A4.5), note that the operative terms are the *coefficient of correlation r* between price and quantity relatives, multiplied by two *coefficients of variation*, i.e. the standard deviations from (A4.3) as ratios of the means (A1.2). The coefficients of variation are positive so that the sign of r to fix the *direction* of the divergence of the Paasche from the Laspeyres index. The Paasche index is the greater if $r>0$ and the Laspeyres index if $r<0$. From (2) it follows that the direction of the divergence of the quantity index numbers is the same as that of the price index numbers.

The *extent* of the divergence, in whichever direction it is, depends partly on the strength of the correlation r and partly on the *dispersion* of the price and quantity relatives as shown up in the coefficients of variation.

Annex 5: The relationship between Young and rectified Young

While the Laspeyres index is well-known, it is not used in index number compilation by statistical agencies. This is because expenditure weights for the price reference period 0, for a comparison between periods 0 and *t,* take time to be compiled and relate to an earlier weight reference period *b*. The resulting Young index is :

(A5.1) $$P_Y \equiv \sum_{m=1}^{M} s_m^b \left(p_m^t / p_m^0\right)$$

where $$s_m^b \equiv \frac{p_m^b q_m^b}{\sum_{m=1}^{M} p_m^b q_m^b}; \qquad m = 1,...,M.$$

A problem with this index is that it fails the time reversal test: the index between periods 0 and *t* exceeds its time antithesis—it has an upwards bias. Diewert (2003) compares the Young index with its reciprocal to ascertain the bias and finds that to the accuracy of a certain second order Taylor series approximation, the following relationship holds between the direct Young index, $P_{Y(0,t)}$ and its time antithesis, $P_{Y(0,t)}$:

(A5.2) $$P_{Y(0,t)} \approx P_{Y(0,t)} + P_{Y(0,t)} \text{ var}(e) \qquad \text{where var}(e) \equiv \sum_{m=1}^{M} s_m^b \left[e_m - e^*\right]^2$$

The deviations e_m are defined by $1+e_m = r_m/r^*$ for $m = 1,...,M$ where the r_m and their weighted mean r^* are defined by:

(A5.3) $$r_m \equiv p_m^t / p_m^0; \qquad m = 1,...,M;$$

(A5.4) $$r^* \equiv \sum_{m=1}^{M} s_m^b r_m$$

which turns out to equal the direct Young index, P_Y. The weighted mean of the e_m is defined as

(A5.5) $$e^* \equiv \sum_{m=1}^{M} s_m^b e_m$$

which turns out to equal 0. *Hence the more dispersion there is in the price relatives p_i^t/p_i^0, to the accuracy of a second order approximation, the more the direct Young index will exceed its counterpart that uses month t as the initial base period rather than month 0.*

Annex 6: A note concerning the Taylor expansion/approximation

The results from the above sections and annexes show the differences between formulae in terms of variances, usually arising from a Taylor expansion around zero. It is an approximation and Annex 6 considers the expansion in more detail. The variances in equation (10) contain squares and cross-products and these cross-products are over time across products as well as for a given time period across items.

Say there are only m=4 items over two periods, 0 and *t,* as depicted in Figure A6.1 below. The e_j^t and e_j^0 are the normalized errors so that in any period -0.2 for example, is 20% below the unitary mean and +0.2 is 20% above it.

(A6.1) $$\prod_{m=1}^{M=4}\left[(1+\varepsilon_m^t)/(1+\varepsilon_m^o)\right]^{1/4} \approx 1+\frac{1}{4}\left[\sum_{m=1}^{4}e_m^t-\sum_{m=1}^{4}e_m^0\right]-\frac{1}{4^3}\left[6\sum_{m=1}^{4}\left(e_m^t\right)^2-10\sum_{m=1}^{4}\left(e_m^0\right)^2\right]$$

$$-\frac{1}{4^2}\left[\sum_{m=1}^{4}\sum_{\substack{n=1\\ m\neq n\cap t\neq 0}}^{4}e_m^0e_n^t+\sum_{m=1}^{4}\sum_{\substack{n=1\\ m=n\cap t\neq 0}}e_m^0e_n^t-\sum_{m=1}^{4}\sum_{\substack{n=1\\ m\neq n}}^{4}e_j^0e_k^0-\sum_{m=1}^{4}\sum_{\substack{n=1\\ m\neq n}}^{4}e_m^te_n^t\right]$$

$$+\frac{1}{4^4}\left[14\sum_{m=1}^{4}\left(e_m^t\right)^3-30\sum_{m=1}^{4}\left(e_m^0\right)^3\right]+\ldots\ldots$$

Note that the terms in the first square brackets sum to zero by definition in (9). The next two terms denote the difference between the normalized variances. Consider the cross-product term $e_j^0e_k^1$ for *m≠n* and Figure A6.1 below. There should be ${}^8C_2=28$ cross-products of which 12 are for *m≠n* and *t≠*0 – these are comparisons over time between different items. Say prices fluctuate around a normalized mean, so that in period 0, items 1 and 3 are above average (+ve) and items 2 and 4 below average (-ve), and the positions are in reversed period 1. For large *M* these cross-products will cancel. However, the next term are the 4 changes over time, $e_1^0e_1^1$, $e_2^0e_2^1$, $e_3^0e_3^1$ and $e_4^0e_4^1$ for which will all be negative, followed by the 6 cross-products across items in *each* of period 0 and 1 respectively $m\neq n\cap t\neq 0$, which should cancel to zero from Figure A6.1 for large m. Finally there are 8 cubic terms which will naturally be followed by their cross products.[31] The difference between the formulae also depends upon changes in the skewness of the price deviations.

Figure A6.1

	Period	
m	0	1
	(+ve) e_1^0	(-ve) e_1^t
	(-ve) e_2^0	(+ve) e_2^t
	(+ve) e_3^0	(-ve) e_3^t
	(-ve) e_4^0	(+ve) e_4^t

[31] Balk (2002) shows an alternative derivation where the difference is identified as beiung dependenyt on the covariance between (the log mean of each period's) relative prices and price relatives. Such a covariance can be decomposed into the variances of the items on the Taylor expansion below.

The focus remains on changes in the variance to explain differences in the formulae, though fluctuations in the levels for individual items in the manner depicted in Figure 1 will also have an effect, as may changes in the cubic (skewness) terms.

It is not immediately obvious as to how to reconcile (A6.1) with (10). However, if a common denominator of 4^3=64 is used the variances have a weights of -6/64 and +10/64 in periods *t* and 0 respectively and the cross products of (-12/16, -4/16, +6/16 and +6/16) respectively which sums to -16/64. All items at the second order thus have weights summing to -12/64=1/4, i.e., the 1/*M* in (10).

Note the asymmetry in the weights for the variances in (A6.1). If the variances were the same the higher positive weight given to period 0's variance would lead to $P_J(p^0,p^1) > P_D(p^t,p^0)$, though the cross-products might ameliorate the situation, especially the negative influence of price cumulated price changes under the scenario in Figure 1 outlined above. Any increase in the variances over time might tip the expression for the difference between the variances in (A6.1) to be negative, and the expansion to be less than unity so that $P_J(p^0,p^1) < P_D(p^t,p^0)$ as is apparent from (10).

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Table 1: Descriptive statistics on average monthly price dispersion*

Measure of dispersion	1998	1999	2000	2001	2002
Absolute					
SD unweighted	364.05	389.40	419.10	436.58	443.14
SD_w weighted	269.92	316.72	343.79	375.56	381.84
Relative					
CV unweighted	0.83	0.86	0.85	0.83	0.82
CV_w weighted	0.78	0.86	0.90	0.86	0.86

*The figures for the standard deviations (SD) are calculated for each month, the annual figures being simple averages of the 12 monthly SD measures. The CV annual averages follow accordingly. In 2002 there were only 3 months data the averages being for the months to March 2002.

Table 2: Decomposition of price variation

			Absolute value of residuals $\left\|\hat{\varepsilon}_m^t\right\|$	
	$\overline{R}^2$	**(p-value)**	**mean**	**standard deviation**
Ordinary least squares				
constant			2.11	1.00
Month and Trend	0.004	0.0000	2.11	1.00
Month, Trend and Charac	0.88	0.00000	1.24	0.36
Month, Trend, Charac and Brands	0.91	0.00000	1.19	0.278
Month, Trend, Charac, Brands and Outlet-type	0.92	0.00000	1.18	0.279
Weighted least squares*				
constant			2.35	1.28
Month and Trend	0.004	0.00000	2.34	1.33
Month, Trend and Charac	0.87	0.00000	1.25	0.37
Month, Trend, Charac and Brands	0.91	0.00000	1.20	0.29
Month, Trend, Charac, Brand and Outlet-type	0.92	0.00000	1.19	0.28

* $\overline{R}^2$ are based on untransformed variables.

Table 3: Results for regression of H-C prices on DIST

	OLS estimator		WLS estimator (values)			OLS estimator		WLS estimator (values)	
	Coefficient	*t*-statistic	Coefficient	*t*-statistic		Coefficient	*t*-statistic	Coefficient	*t*-statistic
Jan-98	0.0026	5.37	-0.000393	-0.98	Jan-99	0.0017	3.10	-0.0011	-2.31
Feb-98	0.0022	3.99	-0.000334	-0.86	Feb-99	0.0018	3.06	-0.001	-2.06
Mar-98	0.0034	6.95	-0.000303	-0.84	Mar-99	0.0012	2.13	-0.00051	-1.09
Apr-98	0.0030	6.25	-0.000359	-0.86	Apr-99	0.0023	3.81	-0.00079	-1.31
May-98	0.0032	6.13	-0.000405	-0.85	May-99	0.0032	5.17	-0.00042	-0.72
Jun-98	0.0037	6.72	-0.00021	-0.51	Jun-99	-0.1211	-1.25	-0.02261	-0.61
Jul-98	0.0029	4.89	-0.000338	-0.69	Jul-99	0.0020	3.62	-0.00115	-2.74
Aug-98	0.0027	4.86	-0.00053	-1.05	Aug-99	0.0023	3.66	-0.00137	-2.90
Sep-98	0.0025	4.52	-0.00055	-1.22	Sep-99	0.0030	5.16	-0.00123	-2.53
Oct-98	0.0027	4.62	-0.000684	-1.51	Oct-99	0.0022	4.14	-0.00118	-2.45
Nov-98	0.0022	4.08	-0.000437	-0.73	Nov-99	0.0016	3.07	-0.00095	-2.11
Dec-98	0.0022	3.98	-0.000899	-1.54	Dec-99	0.0021	3.55	-0.00169	-3.28

Table 4: Heterogeneity-controlled dispersion

Period	Unweighted	Weighted		Unweighted	Weighted
Jan-98	0.141	0.097	Jan-00	0.170	0.129
Feb-98	0.137	0.096	Feb-00	0.169	0.133
Mar-98	0.154	0.104	Mar-00	0.193	0.144
Apr-98	0.163	0.108	Apr-00	0.211	0.150
May-98	0.181	0.116	May-00	0.169	0.137
Jun-98	0.163	0.110	Jun-00	0.223	0.161
Jul-98	0.152	0.114	Jul-00	0.209	0.156
Aug-98	0.166	0.134	Aug-00	0.219	0.163
Sep-98	0.166	0.123	Sep-00	0.233	0.179
Oct-98	0.149	0.112	Oct-00	0.228	0.176
Nov-98	0.160	0.116	Nov-00	0.248	0.183
Dec-98	0.178	0.116	Dec-00	0.250	0.151
Jan-99	0.186	0.128	Jan-01	0.212	0.149
Feb-99	0.179	0.124	Feb-01	0.209	0.143
Mar-99	0.167	0.123	Mar-01	0.212	0.151
Apr-99	0.174	0.123	Apr-01	0.215	0.150
May-99	0.167	0.123	May-01	0.201	0.154
Jun-99	0.180	0.123	Jun-01	0.274	0.169
Jul-99	0.186	0.136	Jul-01	0.246	0.174
Aug-99	0.239	0.162	Aug-01	0.262	0.186
Sep-99	0.250	0.162	Sep-01	0.289	0.196
Oct-99	0.227	0.161	Oct-01	0.357	0.207
Nov-99	0.242	0.171	Nov-01	0.284	0.213
Dec-99	0.334	0.169	Dec-01	0.359	0.214
			Jan-02	0.305	0.193
			Feb-02	0.289	0.180
			Mar-02	0.279	0.171

Table 5: Unit root tests

	Augmented weighted Symmetric tau		Augmented Dickey-Fuller		Phillips-Perron	
	Coefficient	*p*-value	Coefficient	*p*-value	Coefficient	*p*-value
H-C Normalised $SD_{weighted}$	-3.96	0.01	-2.79	0.20	-7.64	0.61
H-C Normalised $SD_{unweighted}$	-2.61	0.23	-2.54	0.31	-26.54	0.02
H-C $UNANTICIP_{weighted}$	-3.60	0.02	-3.53	0.04	-53.30	0.00
H-C $UNANTICIP_{unweighted}$	-4.30	0.00	-4.15	0.01	-55.59	0.00
Number of models	-2.75	0.17	-2.47	0.34	-23.72	0.03
$CATALOG_{weighted}$	-2.03	0.63	-1.83	0.69	-8.57	0.54
$CATALOG_{unweighted}$	-1.66	0.84	-1.35	0.88	-6.20	0.73

Table 6: Regression results

	1		2		3		4	
Regression of:	Coef	*t*-statistic	Coef	*t*-statistic	Coef	*t*-statistic	Coef	*t*-statistic
ΔH-C normalised $SD_{unweighted}$ on:								
constant	0.003	0.54	0.001	0.03	0.004	0.94	0.001	0.20
ΔNumber of models	0.0001	2.06**	0.001*	1.83	0.000**	2.32		
H-C $UNANTICIP_{unweighted}$	-0.200	0.56	-0.510	1.66			-0.64**	2.11
$\Delta CATALOG_{unweighted}$	-0.235	1.59			-0.279**	2.27		
$\overline{R}^2$	0.14		0.11		0.15		0.06	
ΔH-C normalised $SD_{weighted}$ on:								
constant	0.002	1.36	0.0014	0.85	0.002	1.39	-0.0013	0.83
ΔNumber of models	-0.0000	0.09	0.137	0.65	-0.0000	1.01		
H-C $UNANTICIP_{unweighted}$	-0.012	0.08	-0.290	2.33**			-0.295**	2.40
$\Delta CATALOG_{weighted}$	-0.110	2.82***			-0.112*	3.81		
$\overline{R}^2$	0.20		0.08		0.21		0.09	

Tests are two-tailed and *, **, *** denotes statistically significant at a 5%, 1% and 0.1% level respectively.

Table 7: Index number formulae results

	Heterogeneity-controlled			
Period	Dutot	Dutot	Carli	Jevons
Jan-98	100.00	100.00	100.00	100.00
Feb-98	99.33	99.72	102.15	99.40
Mar-98	97.90	97.75	101.72	97.86
Apr-98	96.86	96.35	100.39	96.92
May-98	95.96	95.47	96.52	95.38
Jun-98	94.01	93.85	95.01	92.91
Jul-98	93.23	93.58	93.99	92.09
Aug-98	91.97	91.70	96.74	91.33
Sep-98	89.28	89.13	90.55	88.51
Oct-98	88.50	88.55	90.03	87.49
Nov-98	86.59	86.04	87.91	85.36
Dec-98	85.16	84.97	89.52	84.30
Jan-99	84.67	v84.96	86.30	83.44
Feb-99	98.94	99.78	99.77	98.73
Mar-99	96.73	96.83	97.39	96.10
Apr-99	95.86	96.69	96.45	95.20
May-99	94.25	94.35	94.85	93.29
Jun-99	92.33	93.22	93.13	91.32
Jul-99	91.79	93.15	93.08	91.19
Aug-99	89.76	90.67	91.22	88.72
Sep-99	88.63	89.01	89.65	87.33
Oct-99	86.45	87.84	88.16	83.91
Nov-99	89.18	89.81	90.85	87.52
Dec-99	84.21	85.29	85.79	80.94

Figure 1: Heterogeneity-controlled dispersion