Insurance and quality adjustment: excess and option-cost method

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Abstract: This paper shows an option cost method for excess quality adjustment in the field of insurance index. Concept, estimator and estimates are given. Estimates are rather old but come from true statistical production.

Keywords: Index, quality adjustment, insurance, option-cost, excess, lognormal distribution, gamma distribution.

Excess is the first part of a damage, which stays to the policyholder burden.

Excess might a fixed amount. An excess of 1000 € means that the policyholder receives nothing for an 800-€ damage and receives 5000 € for a 6000-€ damage.

Excess might be proportional or proportional with limit. 10% with a maximum of 1000 € means that the policyholder would receive 720 € for an 800-€ damage.

A variation of excess is clearly a variation of quality. Often insurance companies increase the excess instead of raising the premium.

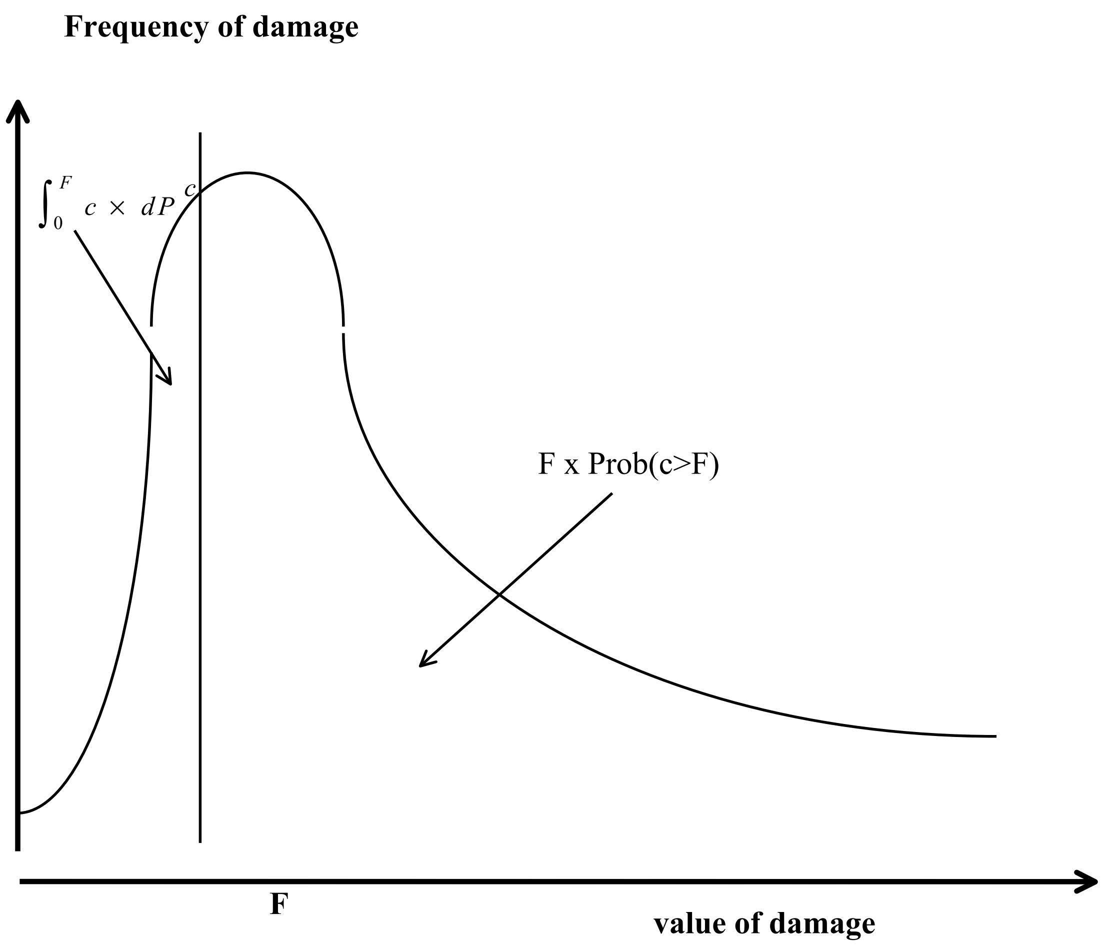
It is possible to assess the expected cost for a policyholder due to excess. With this estimate it is possible to quality adjust the premium with an option cost method.

[1] The views expressed in this paper are those of the author and do not necessarily reflect the policies of Eurostat.

1. Expected cost due to excess

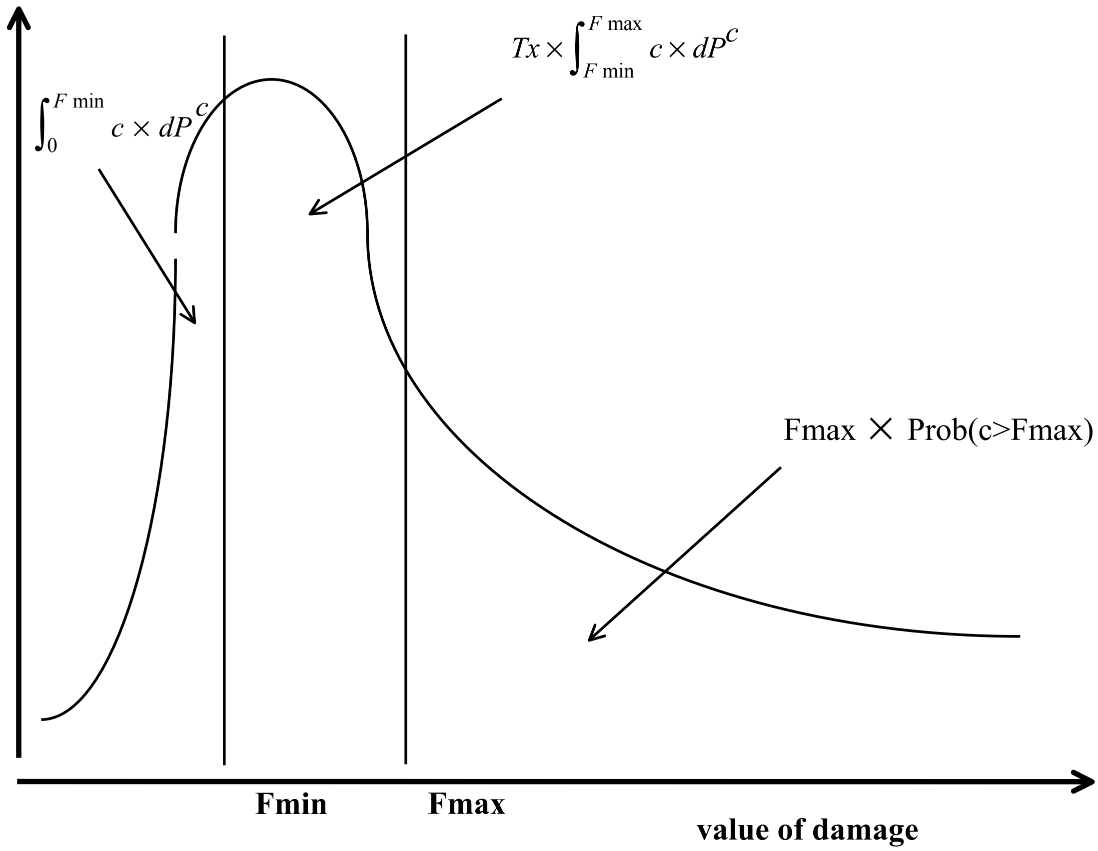
1.1 Fixed excess F

F is the excess and C is the value of damage random variable and $dP^{\,c}$ is the joint probability density function.



$$\text{Excess average cost} = \int_0^F c \times dP^c + \text{F} \times \text{Prob(c>F)}$$

1.2 Proportional (Tx) excess with minimum (Fmin) and (Fmax)



$$\text{average cost due to excess} = \int_0^{F\min} c \times dP^C + Tx \times \int_{F\min}^{F\max} c \times dP^C + \text{Fmax} \times \text{Prob(c>Fmax)}$$

2. Distribution of damage

Actuary and other Staff of insurance companies evaluate or use these frequencies and distributions of damages. This is the core of their work. Also with household survey, National Institute of Statistics can collect the necessary information to estimate these distributions.

The experience shows that damages are always following a lognormal distribution except for fire. In case of fire, value of damage can be very high more often because a fire can spread to another place and increase the destruction. The frequency distribution for fire must have a heavy end. The Gamma distribution is possible.

With a major insurance confederation: APSAD, and an INSEE household survey (PCV 96), damage parameters had been estimated in France in 1995 and 1996 for cars and household insurances.

2.1 "Cars"

2.1.1 Distribution of damage (expressed in Euro)

- *Thefts of cars and fires*: lognormal distribution with m=7.84 et Γ=0.98 (Household survey PCV 96)

- *Thefts of accessories*: lognormal distribution with m=5.91 et Γ=0.96 (PCV 96)

- *Plate glass*: lognormal distribution with m=5.40 et Γ=0.83 (PCV 96)

- *Car crashes*: lognormal distribution with m=7.09 et Γ=1.08 (PCV 96 and insurance confederation APSAD). The expected value is exp(m+Γ^2/2), around 2150€.

- *Civil liability in car crashes (bodies and cars)*: lognormal distribution with m=6.67 and Γ=1.0 (APSAD 94)

2.1.2 Frequencies of damage (Insurance confederation APSAD 95 and household survey 96)

- *Thefts of cars and fires* : 2.35 % (burning is about 0.2 %)
- *Thefts of accessories* : 0.70 %
- *Plate glass* : 9.40 %
- *Car crashes* : 10.5 %
- *Civil liability in car crashes* : 6.00 %

2.2 "Houses"

2.2.1 Distributions (in Euro)

- *fires*: Gamma distribution with r=1/2 and a=1/1220 (APSAD 93). The end of distribution is rather heavy, because some fire can cause important value of damage)
- *water damage*: lognormal distribution with m=6.59 and Γ=1.0 (PCV 96)
- *Plate glass*: lognormal distribution with m=6.57 and Γ=0.97 (PCV 96)
- *Thefts*: lognormal distribution: with m=7.50 and Γ=1.04 (PCV 96)
- *Civil liability*: lognormal distribution with m=6.61 and Γ=1.18 (PCV 96)

2.2.2 Frequencies (APSAD and PCV)

- *Fires* : 1.1 %
- *Water damage* : 3.5 %
- *Plate glass* : 2.5 %
- *Thefts* : 1.2 %
- *civil liability* : 1.5 %

3. Cost-option estimator of excess

3.1 with fixed and absolute excess

- if value of damage $_i \rightarrow \mathrm{N}(m_i, \Gamma_i)$ with excess F_i,

$$\cos t^{i}_{\sin istre} = \exp(m_i + \frac{\Gamma_i^2}{2}) \times \Phi_n(\frac{\log F_i - (m_i + \Gamma_i^2)}{\Gamma_i}) + F_i \times \left[1 - \Phi_n(\frac{\log F_i - m_i}{\Gamma_i})\right]$$

with Φ_n distribution function of standardized normal distribution (Gauss, 0, 1).

- if value of damage $_i \to \gamma(p_i, \theta_i)$ with excess F_i,

$$\cos t^{i}_{\sin istre} = \frac{p_i}{\theta_i} \times \Phi_\gamma(\theta_i F_i; p_i + 1) + F_i \times \left[1 - \Phi_\gamma(\theta_i F_i; p_i)\right]$$

with Φ_γ distribution function of Gamma distribution.

3.2 with fixed but relative excess

- if damage $_i \to \mathrm{N}(m_i, \Gamma_i)$ with excess F_i,

$$\cos t^{i}_{\sin istre} = \exp(m_i + \frac{\Gamma_i^2}{2}) \times \Phi_n(\frac{\log F_i - (m_i + \Gamma_i^2)}{\Gamma_i})$$

with Φ_n distribution function of standardized normal distribution.

- if damage $_i \to \gamma(p_i, \theta_i)$ with excess F_i,

$$\cos t^{i}_{\sin istre} = \frac{p_i}{\theta_i} \times \Phi_\gamma(\theta_i F_i; p_i + 1)$$

with Φ_γ distribution function of Gamma distribution.

3.3 with proportional excess without maximum and minimum

- if damage $_i \to \mathrm{N}(m_i, \Gamma_i)$ with excess F_i (%),

$$\cos t^{i}_{\sin istre} = F_i \times \exp(m_i + \frac{\Gamma_i^2}{2})$$

- if damage $_i \to \gamma(p_i, \theta_i)$ with excess F_i (%),

$$\cos t^{i}_{\sin istre} = F_i \times \frac{p_i}{\theta_i}$$

3.4 with proportional excess limited by minimum and maximum

- if damage $_i \rightarrow \mathrm{N}(m_i, \Gamma_i)$ with $F_{\min,i}$ minimum excess, $F_{\max,i}$ maximum excess, α_i rate between maximun and minimum.

$$\mathrm{cost}^{i}_{\sin istre} = \exp(m_i + \frac{\Gamma_i^2}{2}) \times \left[(1-\alpha_i) \times \Phi_n(\frac{\log F_{\min,i} - (m_i + \Gamma_i^2)}{\Gamma_i}) + \alpha_i \times \Phi_n(\frac{\log F_{\max,i} - (m_i + \Gamma_i^2)}{\Gamma_i}) \right]$$
$$+ F_{\max,i} \times \left[1 - \Phi_n(\frac{\log F_{\max,i} - m_i}{\Gamma_i}) \right]$$

with Φ_n distribution function of standardized normal.

4. It is not negligible

Experience shows that adjustment for excess worths few percents on elementary index, and very often for an entire company, which is not negligible.

This result is consistent with a fast comparison made with frequency of damage, excess and gross premiums.

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