Price-updating of weights in the CPI
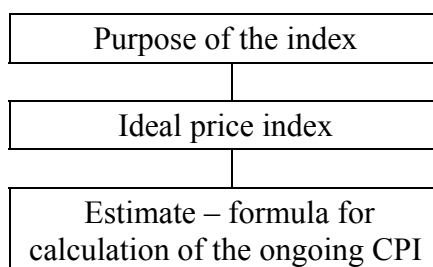
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Abstract

On the basis of the *CPI Manual*[1] the paper discuss the issue of price-updating of expenditure weights for the regular calculation of the CPI. In the first sections the decision of how to calculate the CPI is considered as a three-step procedure: From the purpose of the index to the definition of an ideal or target index, and the decision of which formula to apply for the ongoing calculation of the monthly CPI. In practice most, if not all, countries calculate the ongoing CPI as an arithmetic expenditure weighted average of the elementary aggregate indices, applying weights from a past reference period. In this context it is argued that the question of whether to price-update the expenditure weights or not is important for the interpretation of the CPI and may have significant influence on the measured rate of price change. The question of price-updating is discussed by use of the Lowe and Young indices introduced in the *CPI Manual*. The main findings are summarized in the conclusion in the last section of the paper.

1. Introduction

The question of how to select the index can in principle be dealt with in three steps:

Purpose of the index
Ideal price index
Estimate – formula for calculation of the ongoing CPI

This may be seen as an ideal approach for most survey statistics.[2] In the case of Consumer Price Indices (CPIs) it tells that *firstly* the purpose of the index should be agreed. That is, what is the index supposed to measure? *Secondly*, an ideal measure for the purpose should be selected. *Thirdly*, one has to decide on an index formula, the estimate, for the regular, ongoing calculation of the CPI.

2. The purpose of the index

Most national statistical offices compile only one CPI, which is then often used for different purposes, such as a measure of inflation and for indexation of contracts, wages and pensions etc. However, even if in practice the CPI is used for different purposes, it is useful to discuss and decide the main purpose(s) of the index as this should serve as the basis for in principle all subsequent decisions regarding the compilation of the index.

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[1] ILO/IMF/OECD/UNECE/Eurostat/The World Bank: *Consumer price index manual. Theory and practice*. International Labour Office, Geneva, 2004. Hereafter just referred to as the *CPI Manual*.
[2] Dalen, J.: *Computing elementary aggregates in the Swedish consumer price index*. Journal of Official Statistics, 8, 129-147, 1991.

The purpose of the CPI forms the reference or frame on the basis of which the ideal target index and the estimation formula can be selected. If the purpose of the index is not defined or unclear the selection of calculation method may be somewhat arbitrary and difficult to justify, as may the methods used for the treatment of particular goods or services in the CPI. For example the coverage of goods or services and the treatment of owner-occupied housing depend to some extend on the purpose of the index.

There are two main types of consumer price indices, which also serves in principle two different purposes:

Fixed basket indices. The purpose of this type of an index is to measure the average price change of a representative basket of goods and services, which is kept constant over time. This type of an index is sometimes referred to as a *pure price index*, a *cost of goods index (COGI)* or an *inflation index*.

Cost of living indices (COLIs). The purpose of a COLI is to measure the change in the minimum cost of maintaining a given level of utility, or welfare, over time. The COLI is derived from microeconomic theory and on a set of more or less restrictive assumptions about consumer behaviour and market conditions. On the assumption of utility maximizing consumers, constant preferences and given prices, the COLI is derived as the ratio of the minimum expenditure to maintain a given level of utility at different price regimes.

Both a fixed basket index and a COLI may be expressed as the ratios of expenditures on goods and services in two periods. However, while in the fixed basket approach by definition the quantities are kept constant, they are allowed to vary in a COLI.

The task of deciding on the purpose of the CPI should in principle be quite simple as there are only these two options. However, the index may in practise serve different purposes and there are many different basket indices and many different cost of living indices, and some basket indices are also good estimates of cost of living indices. In practice, it is therefore not straightforward to decide which index formula to apply.

3. The ideal or target index

Once the purpose of the index is defined an ideal index, based on *in principle* obtainable data, may be selected as a target index. Ideally, the target index should be a central and unbiased estimate of the purpose.

The purpose of having a clear target index is twofold. Firstly, it is helpful to have an ideal to target as it provides a reference for the practical and ongoing compilation of the CPI. Secondly, it is necessary to have a measurable target in order to quantify the size of any potential bias. The bias is the difference between what is actually measured and what should be measured.

Suppose the main purpose of the CPI has been decided to be a measure of "inflation" or pure price changes. It may then be argued that the ideal index should be a basket index of the type:

(1) $$I_{0:t}^{Lo} = \frac{\sum p_t^i q_b^i}{\sum p_0^i q_b^i}$$

The basket index measures the ratio of expenditures of buying the same basket (q^i_b) at two different periods in time, 0 and t. Following the *CPI Manual* this type of a basket index is referred to as a Lowe price index after Joseph Lowe. If $b=0$ the Lowe index turns into a Laspeyres index and if $b=t$ it turns into a Paasche index. Thus, the Lowe index is a more general definition of a basket index since it does not requires the quantities, q^i_b, to refer to any actual basket in a particular period in time.

The immediate question is how the quantities should be chosen. One obvious answer is that the quantities should be as representative as possible for the period the index is supposed to cover. For example, if a country updates its weights every five years and this is done in 2004 and 2009, the quantities should reflect the average quantities from 2004 to 2009. This suggests that some type of average of the quantities from the initial and terminal period might be good estimates. Fortunately there are several of such indices of which the most well known are the Walsh and the Marshall-Edgeworth price indices. The former is defined as:

(2)
$$I^W_{0:t} = \frac{\sum p^i_t \sqrt{q^i_0 \cdot q^i_t}}{\sum p^i_0 \sqrt{q^i_0 \cdot q^i_t}} = \sum w^i_W \cdot \left(\frac{p^i_t}{p^i_0}\right)$$

$$w^i_W = \frac{\sqrt{(w^i_0 \cdot w^i_t)/(p^i_t/p^i_0)}}{\sum \sqrt{(w^i_0 \cdot w^i_t)/(p^i_t/p^i_0)}}$$

where w^i_0 and w^i_t are the expenditure shares $\left(p^i q^i / \sum p^i q^i\right)$ of period 0 and t, respectively. The Marshall-Edgeworth price index is defined as:

(3)
$$I^{ME}_{0:t} = \frac{\sum p^i_t (q^i_0 + q^i_t)/2}{\sum p^i_0 (q^i_0 + q^i_t)/2} = \sum w^i_{ME} \left(\frac{p^i_t}{p^i_0}\right)$$

$$w^i_{ME} = \frac{v^i_0 + (v^i_t/(p^i_t/p^i_0))}{\sum (v^i_0 + (v^i_t/(p^i_t/p^i_0)))}$$

$$v^i_t = \frac{p^i_t q^i_t}{\sum p^i_t q^i_t}$$

Both the Walsh and the Marshall-Edgeworth price indices are special cases of the Lowe index (for $q^i_b = (q^i_0 q^i_t)^{1/2}$ and $q^i_b = (q^i_0 + q^i_t)/2$, respectively). The Walsh and the Marshall-Edgeworth indices and the theory of "pure" price indices are described in more details in the *CPI Manual*, chapter 15.

Suppose instead that the main purpose of the CPI is to measure the cost of living. A cost of living index is defined as

(4)
$$I^{COLI}_{0:t} = \frac{C(U, p^i_t)}{C(U, p^i_0)}$$

where $C(U, p^i_t)$ and $C(U, p^i_0)$ are the cost of maintaining a given reference utility level, U, in period 0 and period t. The theory of cost of living indices is explained in detail in the

CPI Manual chapter 17. As the theory of COLIs is derived from economic theory, this is also referred to as the *economic approach* to index number theory.

It should be noted that there is not one single "correct" COLI, although some COLIs may be better estimates of the actual development in the cost of living than others. Any COLI depends on the assumption about the (individual) consumer's preferences or utility function. In theory, it is possible to assume very different kinds of preferences, which will give rise to equally different price indices.

In order to express the COLI as a function of quantities and prices, assumptions about the underlying preferences have to be made. Under different assumptions about consumer behaviour, the two most known COLIs, the Fisher and the Törnqvist indices, can be derived as:

(5) $$I_{0:t}^F = \left(\frac{\sum p_t^i q_0^i}{\sum p_0^i q_0^i} \frac{\sum p_t^i q_t^i}{\sum p_0^i q_t^i} \right)^{1/2}$$

(6) $$I_{0:t}^T = \prod \left(\frac{p_t^i}{p_0^i} \right)^{\left(w_0^i + w_t^i \right)/2}$$

It is interesting to note that under (slightly) different assumptions about the preferences also the Walsh price index can be derived as a COLI (The *CPI Manual*, chapter 17)

The Walsh, Fisher and Törnqvist price indices can also be arrived at from the so-called *axiomatic approach* to index numbers, as explained in the *CPI Manual* chapter 16. In the axiomatic approach a set of axioms, or tests, are selected and the index number formula which passes the most, or the most important tests are selected as the preferred one.

Moreover, these three indices are all *superlative indices*. The notion of superlative index number formulas is explained in more detail in the *CPI Manual* p. 319. It is sufficient to note here that it means that they all meet the central tests in the axiomatic approach, or do so to a fairly close approximation, and they all can provide good estimates of a cost of living index. Overall these three index number formulas appear to be among the best:

> "Fisher, Walsh and Törnqvist price indices approximate each other very closely using "normal" time series data. This is a very convenient result since these three index number formulae repeatedly show up as being "best" in all the approaches to index number theory. Hence, this approximation result implies that it normally will not matter which of these indices is chosen as the preferred target index for a consumer price index." (The *CPI Manual*, p 313)

This means that the Fisher, Walsh and Törnqvist price indices comes out as being best according to both the axiomatic approach and the economic approach to index number theory and that for practical purposes the three indices can be expected to give very similar results. The Edgeworth index, while not superlative, can be expected to give results similar to the Walsh index under normal price behaviour.

Unlike Fisher and Törnqvist both Walsh and Marshall-Edgeworth holds the very practical property that they can be calculated as the expenditure weighted arithmetic mean of the
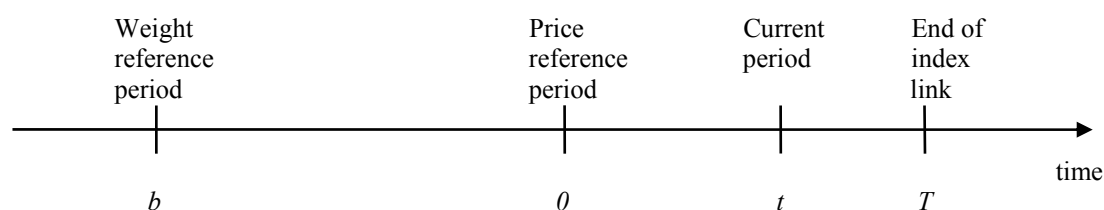
price relatives. In practice this means that they can both be calculated as the elementary aggregate indices weighted together with their expenditure shares.

3. The actual index

In principle the formula for the ongoing calculation of the CPI should be selected as the one, which provides the best estimate of the target index. However, a number of constraints have to be taken into account in practice.

The typical situation is that a monthly CPI has to be calculated as from period 0 and forward for the consecutive months t until period T. The index link period from 0 to T may be a year or more, depending on how often the weights are updated. Assume, further, that the available information about the households consumption expenditures refer to some period, b, prior to period 0.

Figure 1. The typical situation

Firstly, the regular monthly CPI cannot be calculated directly as any of the above-mentioned indices. The quantities or expenditure shares are usually not available at the level of the individual goods and services. What are available, or can be calculated, are the expenditure shares of groups of goods and services at some level of aggregation. Most, if not all, countries calculate their CPI in two steps. In the first step the elementary aggregate indices are calculated for groups of relatively homogenous goods and services. In the second step the higher-level indices are calculated by weighting together the elementary aggregate indices, using the relative expenditure shares of the elementary aggregates as weights. Thus, the formula to be applied for the ongoing calculation of the monthly CPI has to be expressed as a function of elementary aggregate expenditure weights and elementary aggregate indices.

Secondly, when an index going from period 0 to t is to be calculated, the available weighting information will usually refer to a period prior to both 0 and t. Usually the expenditure weights are derived from Household Budget Surveys or National Accounts data (perhaps in combination with other sources). Since it takes a considerable time to process the weighting information, the expenditure weights will be available usually only with a considerable time lag compared to the price reference period of the index.

Thirdly, in many countries the expenditure weights refer to a period of a year or even more. Hence, the periodicity of the weights does not correspond to the monthly periodicity of the prices. It may be noted that if only annual weights are available a "true" monthly Laspeyres price index cannot be calculated in practice, not even retrospectively, as it requires the weight and price reference period to coincide.

Fourthly, users and index compilers may have an interest in being able to analyze and decompose the official index figures in an easy manner. This may support that the CPI be calculated as an arithmetic weighted average of the elementary aggregate indices.

The ideal indices may be calculated retrospectively when the necessary information becomes available, which practice is applied in a few countries. These indices can then be published and/or be used for analytical purposes. However, for the ongoing monthly CPI the question remains how this should be calculated from 0 to T, given that the expenditure weights refer to a period of a year, or more, prior to the price reference period 0? The above mentioned points seem to support that the ongoing CPI should be calculated as the expenditure weighted arithmetic average of the elementary aggregate indices:

(7) $$I_{0:t} = \sum w^j I_{0:t}^j$$

$I_{0:t}$ denotes the overall CPI, or any higher-level index, from period 0 to t, w^j is the expenditure weight of the elementary aggregates, and $I_{0:t}^j$ is the corresponding elementary aggregate indices. Other formulas might be suggested, such as for example the expenditure weighted geometric mean of the elementary aggregate indices. To some degree such an index would take into account substitution behaviour. On the other hand, it cannot be interpreted as a fixed basked index.

However, suppose that it has been decided to calculate the CPI as in (7), which seem to be the approach in most countries. What are the options then? From 0 to t the regular CPI can be calculated only in two ways, namely by weighting together the price relatives from 0 to t with the expenditure shares from period b, or with the expenditure shares price-updated from b to 0.

If the weights are price updated this corresponds to the calculation of a Lowe index:

(8)
$$I_{0:t}^{Lo} = \frac{\sum p_t^i q_b^i}{\sum p_0^i q_b^i} = \sum \frac{p_0^i q_b^i}{\sum p_0^i q_b^i} \frac{p_t^i}{p_0^i} = \sum w_{b(0)}^i \frac{p_t^i}{p_0^i},$$
$$w_{b(0)}^i = \frac{w_b^i \left(p_0^i / p_b^i \right)}{\sum w_b^i \left(p_0^i / p_b^i \right)}, \quad w_b^i = \frac{p_b^i q_b^i}{\sum p_b^i q_b^i}$$

In the Lowe index the individual price ratios are weighted together with their hybrid expenditure shares, i.e. the period b quantities valued at period 0 prices. The hybrid expenditure shares can be calculated by price-updating the period b expenditure shares from period b to 0. The practical counterpart of (8) where the CPI is calculated by weighting together the elementary aggregate indices ($I_{0:t}^j$) is simply:

(8') $$I_{0:t}^{Lo} = \sum w_{b(0)}^j I_{0:t}^j, \quad w_{b(0)}^j = \frac{w_b^j I_{0:b}^j}{\sum w_b^j I_{0:b}^j}$$

The Lowe index is a *fixed basket index* which from period to period (month to month) measures the expenditure of buying the same (annual) basket of goods and service. The index in (8) measures the changing cost of buying the period b basket in period t in relation to the cost of buying the same basket in period 0.

The index in (8) is not a Laspeyres index, as the weight and price reference periods do not coincide. However, the Lowe index can be written as the ratio of two Laspeyres indices, one from b to 0 and one from b to t:

$$(9) \quad I_{0:t}^{Lo} = \frac{\sum p_t^i q_b^i}{\sum p_0^i q_b^i} = \frac{\sum p_t^i q_b^i}{\sum p_b^i q_b^i} \bigg/ \frac{\sum p_0^i q_b^i}{\sum p_b^i q_b^i} = \frac{\sum w_b^i \cdot \left(p_t^i / p_b^i \right)}{\sum w_b^i \cdot \left(p_0^i / p_b^i \right)},$$

$$w_b^i = \frac{p_b^i q_b^i}{\sum p_b^i q_b^i}$$

Hence, if the weights are price-updated from period *b* to *0*, then from period *0* to *t* the CPI will show the same rate of change as a Laspeyres price index with period *b* as weight and price reference period. The only difference is that the price-updated index in (9) is rescaled to period *0* as reference period. In other words, price-updating the weights from b to 0 means that the CPI will give the same rate of changes as if the weights had been applied and kept constant from period b.

If the weights are not price-updated, this corresponds to the calculation of a Young index defined as:

$$(10) \quad I_{0:t}^{Yo} = \sum w_b^i \left(\frac{p_t^i}{p_0^i} \right), \quad w_b^i = \frac{p_b^i q_b^i}{\sum p_b^i q_b^i}$$

In this index, named after English economist Arthur Young, and described in detail in the *CPI Manual* chapter 15, pp. 275-78, the individual price relatives are weighted together with the expenditure shares from period *b*. In practice, a Young CPI would be calculated simply as:

$$(10') \quad I_{0:t}^{Yo} = \sum w_b^j I_{0:t}^j$$

The Young index is a *fixed weight index*. Here, focus is that the weights shall be as representative as possible for the average expenditure shares for the index link period. A fixed weight index is not necessarily a fixed basket index, i.e. it does not necessarily measure the change in the cost of buying a fixed basket such as the Lowe index. Only if *b* chance to equal *0* or *t* is the Young index also a fixed basket index.

From *0* to *t* the Young index shows the development in the consumption expenditure if the expenditure shares are kept constant as from period *b*. This does not correspond to the changing cost of any actual basket. However, if the expenditure shares are representative for the index link period, the Young index can be seen as an estimate of a basket index, for example the Walsh or the Marshall-Edgeworth index, which are Lowe indices but can both be re-written in the same form as the Young index.

The ideal indices aims to measure the average price change from *0* to *T* and are all based on information about the consumption pattern in the two periods. Whether a Young or Lowe index is the better estimate for an ideal index depends on whether w_b^i or $w_{b(0)}^i$ is the best estimate of the average expenditure shares from *0* to *T*. This is easily seen for the Walsh and the Marshall-Edgeworth indices, which can be expressed in the same form as both the Lowe and Young indices and differ only as regards the weights component. It is less obvious for the Fisher and the Törnqvist indices. However, both of these try to average the influence of the consumption pattern in period *0* and *T*, and can in practice be expected to give results similar to the Walsh index.

Whether the original or the price-updated expenditure weights are the best estimates of the average expenditure shares depends on the households' response to change in the relative prices. The expenditure shares may of course change because of a number of other factors; changes in income and preferences, and the appearance of new products, for example. However, the price-updating of the weights only takes into account the effect of changes in the relative prices. If the households are most likely to hold fixed expenditure shares, i.e. the price elasticity of demand is around one, the Young index is the best estimate. If the households hold fixed quantities, indicating zero price elasticity of demand, the Lowe index is the best estimate.

Thus, the question is whether, on average, price elasticities at the elementary aggregate level are closer to one or zero? This is an empirical question, although normal consumer behaviour suggests that in general some substitution should be expected. Reliable estimates for price elasticities at a detailed level of groups of goods and services are difficult to estimate. However, for illustrative purposes elasticities for main groups of goods and services from two macroeconomic models are shown in table 1 and 2.

Table 1. Price elasticities of demand estimates, Norway

	Short-term	Medium-term	Long-term
Food	-0,12	-0,26	-0,52
Beverages	-0,93	-0,94	-0,94
Tobacco	-0,67	-0,70	-0,77
Electricity	-0,04	-0,13	-0,31
Fuel for housing	-0,37	-0,58	-0,60
Fuel for transport	-0,51	-0,60	-0,89
Other non-durable goods	-0,86	-0,96	-1,18
Clothing and footwear	-0,56	-0,68	-0,87
Purchase of own vehicle	-0,80	-1,51	-1,59
Other durable goods	-0,28	-0,73	-1,05
Other services	-0,35	-0,49	-0,75
Transport services	-0,92	-0,96	-1,05
Consumption abroad	-0,78	-0,97	-1,43
Weighted average	-0,52	-0,69	-0,92

Source: Boug, Paal, Yngvar Dyvi, Per R. Johansen, Björn E Naug: *Modag – en makroökonomisk model for norsk ökonomi*, 2005. Statistisk Sentral Byraa, Statistics Norway, Oslo.

Table 2. Price and income elasticities of demand estimates, Denmark

	Price elasticities		Income elasticities	
	Short-term	Long-term	Short-term	Long-term
Food	-0,09	-0,19	0,54	0,22
Beverages and tobacco	-0,09	-0,50	0,90	0,63
Other non-durable goods	-0,40	-0,81	1,79	1,04
Fuel	-0,20	-0,70	1,12	0,91
Transport	-0,16	-0,94	0,63	1,22
Durable goods	-0,36	-0,93	1,94	1,22
Services	-0,21	-0,91	0,58	1,14
Tourist expenditures	-0,26	-1,27	1,49	1,71
Weighted average	-0,22	-0,79		

Source: Danmarks Statistik: *Adam – en model af dansk ökonomi*, 1996. Copenhagen, Denmark.

The models are constructed for the purpose of macroeconomic analysis and forecasts, not for estimation of price elasticities. Hence, the estimated elasticities are subject to some uncertainty and should not necessarily be taken at their exact face value. Nevertheless, the estimates are in line with what could be expected; low elasticities for necessities and larger for 'luxury' goods, and larger in the long-term than in the short-term.

It is not possible to say much about the elasticities for the short-term as they seem to diverge. For the medium-term and long-term the elasticities in the two models are, by and large, of the same magnitude, taking the uncertainty of the estimates into account, and closer to one than zero.

The income elasticities, although not to be discussed here, seem to confirm the traditional pattern (the richer we get, the less we spend, relatively, on food and other necessities, and the more on luxury products). The point to be made here is simply that the difference in income elasticities may add to the errors in the weights. The longer the period from the weight reference period to the price reference period, and in particular so in periods of strong economic growth, the larger changes in expenditure pattern may have occurred, but are not reflected in the expenditure weights – price-updated or not.

Conceptually the Lowe index may appear to be more straightforward than the Young index as it simply refers to the changing cost of buying the same reference basket of goods and services. From the viewpoint of economic interpretation the Lowe index appears also to be consistent as it assumes fixed quantities and zero elasticity of substitution throughout, from period b to T. On the other hand, it is not obvious what should be the target index of a Lowe index.

The target could be to measure the change in costs from 0 to t of buying the past period b reference basket. In this case things are straightforward as the estimate equals the target and there is no bias. Also the Laspeyres price index with period 0 as weight and price reference period could be the target. However, the Lowe index from 0 to t with weights from period b will tend to exceed the Laspeyres index from 0 to t if consumers substitute, and the more so the further back in time is period b. It is generally recognized that a Laspeyres index has an upward bias compared to a COLI because consumers do substitute as a response to changes in relative prices. It follows that a Lowe index will tend to overestimate the cost of living even more than the Laspeyres index.

It seems reasonable to relate the target of the CPI to the period that it covers. If the relative quantities from period b are good estimates of the average quantities for the index link period from 0 to T, the Lowe index is also a good estimate of an ideal basket index. However, this assumes that the relative quantities are constant which is not very likely.

The Young index is perhaps less clear conceptually, and it does not correspond to the cost of any actual basket. It may be argued that in calculating a Young index, an elasticity of substitution of one is assumed from b to 0, while from 0 to T no substitution is allowed. To be consistent, therefore, the CPI should be calculated as the expenditure weighted geometric average of the elementary aggregate indices, since this would correspond to assume *also* in the link period an elasticity of substitution of one.

However, the Young index can be applied irrespective of what may happen to the expenditure shares in the real world. The expenditure weights in the Young index may be seen as estimates of the average expenditure weights for the index link period from 0 to T, and the index can be an estimate of an ideal basket index. The economic approach only tells that if consumers substitute and keep constant expenditure shares, then a CPI calculated as the expenditure weighted geometric average of the elementary aggregate indices, using the period b expenditure shares as weights, would provide an estimate for a COLI.

As the Young index allow for some substitution from *b* to *0,* while quantities are kept constant in the Lowe index, it may be argued that the traditional Laspeyres bias to some degree is reduced in the Young index as compared to the Lowe index. Thus, to omit price-updating of the weights may be one practical way in which to reduce this type of bias.

It should be mentioned that the Lowe index satisfy more axioms, or tests, than the Young index (the *CPI Manual,* p. 310-11). In particular Lowe satisfies the *time reversal test* and the *circularity test,* which the Young index fails. However, the relevance and importance of meeting these tests in practice has to be weighted against the expectation of which index will provide the best estimate of the target index.

The difference between the Young and Lowe indices can be illustrated by subtracting the one from the other:

(11)
$$I_{0:t}^{Lo} - I_{0:t}^{Y} = \sum w_{b(0)}^{i}\left(p_t^i / p_0^i\right) - \sum w_b^i\left(p_t^i / p_0^i\right)$$
$$= \sum \left(w_{b(0)}^{i} - w_b^i\right)\left(p_t^i / p_0^i\right)$$

In the Lowe index the price changes from *0* to *t* are weighted together by the price-updated weights while in the Young index the price changes are weighted together by the unadjusted weights. Thus, the Lowe index will give more weights to those elementary aggregates the prices of which have increased by more than average from *b* to *0* and less weights to those where the prices have increased by less than average.

Therefore, if there are long-term trends in the prices, so that prices which have increased relatively from *b* to *0* continues to do so from *0* to *t*, and prices which have fallen from *b* to *0* continues to fall, the Lowe index will exceed the Young index.

For example if real wages are increasing in the long run, the prices of services are likely to increase more than the prices of goods. As illustration for the "old" 15 EU-member states from 1996 to 2005 prices of goods increased by 13% while services prices increased by the double (see table 3). In this case, the price-updated weights for services will exceed the unadjusted weights, so that the relative price increase of services will have a bigger weight in the Lowe index compared to the Young index. The price-updated weights for goods will be smaller than the unadjusted weights, so that the relative price decrease of goods will have a smaller weight in the Lowe index compared to the Young index. This indicates a long-run tendency for the Lowe index to exceed the Young index.

Table 3. HICP, EU-15 (1996 = 100)

	1997	1998	1999	2000	2001	2002	2003	2004	2005	Ann. % change
HICP all-items	101,7	103,0	104,2	106,2	108,6	110,8	113,0	115,2	117,7	1,8
Information processing equip.	89,4	76,7	65,0	52,6	42,2	35,2	28,2	23,5	19,7	-16,2
Goods	101,2	101,9	102,7	104,7	106,5	107,9	109,4	110,9	112,8	1,3
Services	102,8	105,2	107,3	109,2	112,3	116,0	119,2	122,4	125,6	2,5

Source: Eurostat database. Information processing equipment = COICOP 9.1.3.

Some empirical findings support that the Lowe index is likely to exceed the corresponding Young index. For example the average annual change from 2003 to 2004 for the Danish CPI and the Danish HICP has been calculated with expenditure weights from 1999 and with these weights price-updated to December 2002. For both the CPI and the HICP the

average annual rate of change calculated by use of price-updated weights exceeded the rate of change obtained from applying the original expenditure shares by 0,2 pct. point.[1] As the Danish CPI and HICP increased by only 1,2% and 0,9%, respectively, from 2003 to 2004, this is a relative large difference which may be the result of the rather long price-updating period of 3,5 years from 1999 to December 2002.

If expenditure weights are price-updated, it should be noted that automatic updating may lead to bias in the weights for goods or services with unusual price changes. The typical example is PCs and other IT products (such as printers, digital cameras, DVD players etc.) as shown in table 3. If the expenditure weights for such products the prices of which have fallen significantly compared to the average price changes, while at the same time the consumption expenditures continues to stay at the same level or even increase, the price updated weights may seriously underestimate the actual expenditure share and contribute to an upward bias in the overall CPI.

Assume, for illustration, that information processing equipment (COICOP 9.1.3) has a weight of 1 pct. in the overall CPI and that, on average, prices for this group falls by 16,2 pct. per year while the overall CPI increases by 1,8 pct. per year. If the weights are price-updated two years, the weight of information processing equipment is reduced to 0,68 pct. and after three years to 0,56 pct.

The arguments and examples above indicate that the decision of whether to price-update the expenditure weights or not may have a significant influence on the measured rate of price changes. There are different practices in different countries, but there is no overview of which national statistical offices apply which practice. It follows that also the international comparability may be influenced by different practices in different countries. It thus appears that there may be a need for further research and discussion in this area. For example, it might be interesting to have an overview of practices in different countries, and national statistical offices might try on experimental basis to calculate the CPI with both types of weights to obtain empirical evidence,

6. Bias

If an ideal index is specified it is possible to obtain an expression of the bias in the actual index compared to the target index by subtraction the two index formulas. Assume, for illustration, that the Walsh index has been selected as target index. Let r^i be the price relatives from period 0 to period t, $r^i = p^i_t / p^i_0$. Subtracting the Walsh index from the Young index and following the same method as in the *CPI Manual*, chp. 15, we get the following expression:

(12)
$$\begin{aligned} I^Y_{0:t} - I^W_{0:t} &= \sum w^i_b \cdot r^i - \sum w^i_W \cdot r^i \\ &= \sum (w^i_b - w^i_W) r^i \\ &= \sum (w^i_b - w^i_W)(r^i - r^*) + r^* \sum (w^i_b - w^i_W) \\ &= \sum (w^i_b - w^i_W)(r^i - r^*) \end{aligned}$$

where $\sum w^i_W = \sum w^i_b = 1$ and r^* is the average price change from 0 til $t, r^* = \sum w^i_W \cdot r^i = I^W_{0:t}$.

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[1] Danmarks Statistik: *Forbruger- og nettoprisindekset. Dokumentation. 2005.* Copenhagen, Denmark.

The Young index is thus equal to the Walsh index plus the covariance between the differences in the budget shares and the differences between the individual price changes and the average price change. The size and the sign of the bias will depend on how the budget shares change when the relative prices change. This, in turn, is a question about consumer behaviour, i.e. how consumers react on changes in relative prices.

Assume that the demand is elastic, i.e. that the price elasticity of demand is greater than 1, or that there are large substitution effects. If the price of an item increases so that $(r^i - r^*)$ is positive, the expenditure share will decrease over time so that also $(w^i_b - w^i_W)$ is positive. The covariance will then also be positive. Therefore, if demand is elastic and there are long-term trends in prices, the Young index will show a bigger price increase than the Walsh index.

Assume instead that demand is inelastic i.e. that the price elasticity of demand is less than 1, or that there are only small substitution effects. If the price of an item increases so that $(r^i - r^*)$ is positive, the expenditure share will increase over time so that $(w^i_b - w^i_W)$ becomes negative. The covariance will then be negative. Therefore, if demand is inelastic, and there are long-term trends in prices, the Young index will show a smaller price increase than the Walsh index.

If demand is neutral elastic, i.e. the price elasticity of demand is close to one, the budget shares will be close to each other, so that the Young and the Walsh index will give similar results. Finally, if all prices change in the same proportion, the two indices will also show the same result, irrespective of differences in the expenditure weights. This is, of course, neither realistic nor any surprise (if all prices increase by 5%, both indices also increase by 5%), but it shows that the bias may be expected to increase the larger the difference in the relative price changes.

Potential bias is likely to increase the longer the time from *b* to *0* and from *0* to *t*. This indicate that in order to reduce bias the weights should not be too dated when they are introduced in the index calculation, and the index link period should not be too long, i.e. the weights should be updated regularly.

7. Conclusion

It is useful to discuss and decide the main *purpose* of the index as the purpose may influence the treatment of various goods and services in the index and it is necessary in order to interpret the CPI correctly. The selection of an *ideal index* is useful as it can provide a measurable target for the CPI and it is required to quantify potential bias. It also helps to focus on the information required for the compilation of the CPI.

If the purpose of the CPI is to measure pure price changes or "inflation", the Walsh index appears to be the preferred one according to the *CPI Manual*. If the purpose of the CPI is to measure the cost of living, the Fisher, Walsh and Törnqvist indices appear to be "best". In practice, they can be expected to give very similar results. This means that ideal fixed basket indices and cost of living indices are close to each other in practice.

However, the ideal indices cannot be applied for the regular calculation of the monthly CPI since the required weighting data is usual not available in real time. Secondly, many countries prefer to use expenditure weights based on annual data, while prices are

recorded on a monthly basis; in the ideal indices quantities and prices refer to the same time unit. Therefore, some estimate or approximation is needed.

Most countries calculate the CPI as the expenditure weighted arithmetic mean of the elementary aggregate indices. In this context, the question of whether to price-update the expenditure weights or not is important, both for the interpretation of the CPI and for the measured rate of price changes. The problems associated with price-updating can be described and analyzed in terms of the Lowe and Young indices, as introduced in the *CPI Manual*. The traditional Laspeyres approach is less suited for this purpose and it does not describe appropriately the actual practice in statistical offices.

The purpose of the index and the choice of the ideal index do not give any decisive answer on whether to price-update the weights or not. The Lowe index with price-updated weights is conceptually clear and measures the changing cost of buying a past reference basket of goods and services. It is upward biased compared to a cost of living index if consumers substitute. For the same reason it is also upward biased compared to an ideal fixed basket index, because of the time lag between the weight reference period and the index link period. However, if the relative quantities remain constant, the Lowe index will be a god estimate of an ideal basket index. This will be the case if, on average, the elasticity of substitution is close to zero.

The Young index, which uses the unadjusted expenditure weights, does not measure the changing cost of buying an actual basket of goods and services. However, the weights can be seen as estimates of the "true" weights in the index link period. It can approximate an ideal basked index, and hence in practice also a cost of living index, if the unadjusted weights are good estimates of the average expenditure weights. This will be the case if, on average, the elasticity of substitution is close to one.

The Lowe index will exceed the Young index if there are long-term trends in relative prices. It keeps quantities constant from the weight reference period onwards, while the Young index allow for some substitution from the weight reference period to the price reference period. Hence, using the unadjusted weights may reduce upward bias. If weights are price-updated automatically, special attention should be given to goods and services with unusual price movements.

It is not possible to decide the sign of potential bias of a Young index compared to an ideal index, the Walsh index, for example. If the elasticity of substitution is close to one, the Young index will be close to the Walsh index. If demand is elastic, Young will exceed the Walsh index, and if demand is inelastic the Young index will be less than the Walsh index.

Potential bias is likely to increase the longer the time from the weight reference period to the price reference period, and from the price reference period to the current period. Therefore, the weights should be updated regularly and be as representative as possible for the index link period.

Whether the expenditure weights are price-update or not influences the measured rate of price changes, and there are different practices in different countries, which may affect international comparability. It can be concluded that there is a need for further research and discussion, both on theoretical and conceptual issues as well as the empirical implications.