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Towards an applicable true cost-of-living index that incorporates housing.

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A dynamic model of consumer behavior that incorporates the demand for housing is specified such that it is consistent with the general purpose of a consumer price index. From this model a true cost-of-living index that includes housing is derived. Being an ideal index it cannot be computed without imposing additional assumptions about the behavior of the consumer, but it is possible to draw conclusions about the prices and weights that should be used in conventional approximations to such an ideal index. It is demonstrated that a price index can be computed that is consistent both with theory and the general purposes of a CPI using conventional approaches and data that are available at most statistical agencies. (C43, D91)

The design of a price index for housing services within a consumer price index is a long lasting controversy not only among academics but also among national statistical institutes and international organizations. The problem lies in the definition of a price index for the services obtained from owner occupied housing, but in principle it is a more general problem that applies to all consumer durables. Conventional index theory that relies on Alexander A. Konüs's famous paper (Konüs, 1924, 1939) is static, while a theory that handles consumer durables has to be dynamic. Solutions adopted in practice vary from one country to another. In some countries there is no index at all for owned housing, in others the price changes of

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these services are represented by the index for rented apartments, and in still others attempts are made to estimate some kind of user-cost.

The literature on price index numbers, see for instance Ralph Turvey(1989) and W. Erwin Diewert(2003, 2004), distinguishes between a few major approaches: The acquisitions approach, the rental equivalence approach, the payments approach and the user cost approach. In most CPIs the acquisition approach is used for all durables but owner occupied housing. For most purposes of a CPI one prefers to measure changes in the price of the services derived from a house in any period. Even if no single household bought a new home, the existing stock of houses still produce services to their owners. The rental equivalence approach is simple and useful if there exits a rental market for equivalent houses. However, in many countries the rental market is very thin, only covering a selected sample of houses. This approach could also be criticized because the rental includes landlord costs that an owner does not incur. The payments approach builds on cash out-payments for the cost of operating an owner occupied dwelling. It ignores depreciation and the opportunity cost of holding the equity in the owner occupied dwelling.

As pointed out in Diewert(2003, 2004) the user cost approach divides the purchase price of a house into two components. One is the services derived from the house in a given period, the other, the end of period value of the house, is an investment. As an investor the house owner wants a return on his investment, something that can be seen as an opportunity cost. This approach involves the estimation of rather tricky concepts such as the depreciation of a house, the opportunity cost of the investment, the subjective interest rate that equalizes amounts in the beginning and the end of a period, purchase prices of constant quality houses and in some user cost definitions expected future purchase price changes as well. With all these difficulties it does not come as a surprise that no consensus approach has emerged, in particular as some attempts to compute an index have not produced results that are plausible.

There are probably several reasons why this problem has not been solved for more than 60 years.[1] The derivation of an ideal index from a dynamic theory that involves intertemporal utility maximization subject to an intertemporal budget constraint and taking the durability of a house into account easily becomes very complex and difficult to apply in practice.[2] To obtain any useful results additional constraints are needed, either constraints on the behavior of the consumer or constraints on the functioning of the market. The latter approach is usually alien to most index designers who are not used to invoke assumptions about the supply side. A recent exception is the paper by Chuan-Zhong Li and Karl-Gustaf Löfgren (2005). They investigated the cost-of-living index problem in a general equilibrium multi-sector growth model. Models that assume a forward looking consumer usually need assumptions about the formation of expectations and the ideal index becomes a function of these expectations that again is alien to most index designers.

The root of the problem is, however, more fundamental. It lies in a rather wide spread misconception about the existence of a true or pure measure of inflation that is model free and independent of what it will be used for. It would then only become a matter of defining the price of a good and plugging it into one of our conventional weighting formulas. Thus, it has been a mistake of the past to try to define the price of housing services independently of a model of consumer choice. The attempts to define a user cost of housing rely on investment theory and not on a theory of consumer behavior and generally no attempts have been made to integrate the two. In this approach the underlying model is thus not well-specified and no true cost of living index can be defined.

Any true cost of living index in the Konüs sense must be defined using a model of consumer behavior, but not necessarily the very simple static model that he used. Further more, every scientist knows that in general there are many models of human behavior consistent with the data and that in scientific work the choice of model within this class is driven by the applications of the model. In our case we would like to have a model that is useful for the purposes of a consumer price index.

In this paper I will first review the standard basic theory of a true cost of living index to fix ideas and introduce notation. I will then argue that it is possible to specify a relatively simple dynamic model to which we can pose the questions we need to ask for an ideal consumer price index that incorporates owner occupied housing. This model is reached in two steps, the first of which is just a marginal extension of the classical static model. Depending on the question we ask the model, alternative ideal indices are derived. Such indices cannot be computed without additional strong assumptions, but the expressions obtained provide a guide as to the prices and weights that should be used in approximations to the ideal index. In the end we find that a CPI that approximates a true index and incorporates housing can be computed using conventional approaches.

I. The standard theory of a true cost of living index

Let $\mathbf{p}$ be a vector of n commodity prices and $\mathbf{q}$ the corresponding vector of consumed quantities. Assume the consumer's preferences are indexed by a utility function $U(\mathbf{q})$ with standard properties. The consumer is assumed to maximize U(**q**) with respect to **q**, subject to the budget constraint,

(1) $y=\mathbf{p}'\mathbf{q}$;

where y is the consumer's "income". The solution to this problem is an optimal combination of quantities **q**. In this simple model it is implicitly assumed that there is no saving and asset change and thus that the sum of all expenditures equals income. This ceases to be the case in section III and the rest of the paper.

A dual problem, given the price vector **p**, is to seek the minimal income $y=\mathbf{p}'\mathbf{q}$ needed to attain a certain utility U, i e

(2) $$\underset{\mathbf{q}}{Min}(\mathbf{p}'\mathbf{q}) \text{ subject to } U = U(\mathbf{q});$$

The solution to this problem is a vector $\mathbf{q}^*(\mathbf{p},U)$ and an income $y(\mathbf{p},U)=\mathbf{p}'\mathbf{q}^*(\mathbf{p},U)$. The function $y(\mathbf{p},U)$ is usually referred to as the cost function.

A Konüs index or a true cost of living index is now defined as,

(3) $$I = \frac{y(\mathbf{p_1},U)}{y(\mathbf{p_0},U)} = \frac{\mathbf{p'_1}\,\mathbf{q}^*(\mathbf{p_1},U)}{\mathbf{p'_0}\,\mathbf{q}^*(\mathbf{p_0},U)};$$

for given price vectors $\mathbf{p_0}$ and $\mathbf{p_1}$ and utility U. This index thus tells us what income compensation the consumer needs to give him the same utility U when prices are p_1 as when they are $\mathbf{p_0}$. Another name for this index is thus a compensation index. [3]

It is useful to observe that the properties of the cost function depend both on the properties of the utility function and on the budget constraint. For instance, if commodities are close substitutes the compensating income change in general becomes smaller than if they are not. The instrumental importance of the formulation of the budget constraint is immediately seen from the expression after the second equality sign in expression (3). Additional constraints on the behavior of the consumer, such as rationing would explicitly influence the cost function and thus also the compensation index.

In practice it is usually not possible to compute the ideal compensation index (3) but we have to seek approximations. If the index (3) is set for the maximum utility obtained when prices are $\mathbf{p_0}$, U_0, then we obtain,

$$(4)\qquad I=\frac{y(\mathbf{p_1},U_0)}{y(\mathbf{p_0},U_0)}=\frac{\mathbf{p'_1\,q^*}(\mathbf{p_1},U_0)}{\mathbf{p'_0\,q^*}(\mathbf{p_0},U_0)}=\frac{\mathbf{p'_1\,q^*}(\mathbf{p_0},U_0)}{\mathbf{p'_0\,q_0}};$$

Depending on the shape of the indifference surfaces of the utility function **q*** lies in between $\mathbf{q_0}$ and $\mathbf{q_1}$ in the sense that,

$$(5a)\quad \mathbf{p'_0\,q_0}\le\mathbf{p'_1\,q^*}(\mathbf{p_1},U_0)\le\mathbf{p'_1\,q_1}\ \ if\ \ \mathbf{p'_0\,q_0}\le\mathbf{p'_1\,q_1};$$

and

$$(5b)\quad \mathbf{p'_0\,q_0}\ge\mathbf{p'_1\,q^*}(\mathbf{p_1},U_0)\ge\mathbf{p'_1\,q_1}\ \ if\ \ \mathbf{p'_0\,q_0}\ge\mathbf{p'_1\,q_1};$$

The well-known property that a Laspeyres index is an upper bound to the ideal index (4) follows immediately. The equally well-known property that a Paasche index is a lower bound to an index conditioned on U_1 can be demonstrated in a similar way.

II. A static model of demand for housing

Assume that a consumer can obtain housing either by owning a house or by renting an apartment. Also assume that there is a well functioning market both for owner occupied houses and for rented apartments such that it is always possible to buy and sell a house and find an apartment with no transaction costs. Let us also assume that the consumer is myopic and easily switches from one dwelling to another. The consumer's decision problem can then be formulated in the following way,

$$(6) \quad \underset{\mathbf{q}, q_h, q_r}{Max}(U(\mathbf{q}, q_h, q_r)) \;\; subject\ to\ y + p_h^0 q_h^0 = \mathbf{p}'\mathbf{q} + p_h q_h + p_r q_r;$$

where p_h and p_r are the price of houses and the unit rent of an apartment respectively, and q_h and q_r are the corresponding volumes. **p** and **q** are vectors of prices and volumes of all other commodities. $p_h^0 q_h^0$ is the initial value of any house the consumer might own. The properties of the utility function may be such that the consumer only chooses owned housing or a rented apartment, but there is no reason to exclude the possibility of both owning and renting. In this model an owned house is an asset that enters the budget constraint, but the durability of a house has no direct consumption value, because the consumer knows that he can always buy a new house at no transaction cost or switch to an apartment. For this reason he is able to behave myopically and treat a house like any other good.

The dual of the maximization problem is to minimize the expression on the right hand of the equality sign of the budget constraint with respect to all the q:s, holding utility constant. This yields the ideal index,

$$(7) \quad I = \frac{\mathbf{p^{1}}'\mathbf{q}^{*}(\mathbf{p^{1}}, p_h^1, p_r^1, U) + p_h^1 q_h^{*}(\mathbf{p^{1}}, p_h^1, p_r^1, U) + p_r^1 q_r^{*}(\mathbf{p^{1}}, p_h^1, p_r^1, U)}{\mathbf{p^{0}}'\mathbf{q}^{*}(\mathbf{p^{0}}, p_h^0, p_r^0, U) + p_h^0 q_h^{*}(\mathbf{p^{0}}, p_h^0, p_r^0, U) + p_r^0 q_r^{*}(\mathbf{p^{0}}, p_h^0, p_r^0, U)};$$

This answers the question how much the consumer's total resources, incomes and assets, must change to maintain the utility U at the two sets of prices. If we wish to know what income change is needed, holding assets constant, the answer is,

$$(8) \quad I = \frac{\mathbf{p^{1}}'\mathbf{q}^{*}(\mathbf{p^{1}}, p_h^1, p_r^1, U) + p_h^1 q_h^{*}(\mathbf{p^{1}}, p_h^1, p_r^1, U) + p_r^1 q_r^{*}(\mathbf{p^{1}}, p_h^1, p_r^1, U) - p_h^0 q_h^0}{\mathbf{p^{0}}'\mathbf{q}^{*}(\mathbf{p^{0}}, p_h^0, p_r^0, U) + p_h^0 q_h^{*}(\mathbf{p^{0}}, p_h^0, p_r^0, U) + p_r^0 q_r^{*}(\mathbf{p^{0}}, p_h^0, p_r^0, U) - p_h^0 q_h^0};$$

In this model the consumer treats a house like any good and as a result price changes of houses enter the index. The only difference compared with the previous model is that owning a house is an asset that can be used for consumption purposes including that of buying a new house.

III. A dynamic model of the demand for housing

A truly dynamic model that involves inter temporal utility maximization, forward planning, the formation of expectations and takes depreciation into account not only adds considerably to complexity but also provides answers to more questions than a simple static model. For instance, we can ask what income is needed tomorrow to compensate for a price change today, or what income is needed today to compensate for a price change tomorrow, etc. Given the rather simple minded question we usually ask a consumer price index: “What income is needed today to compensate for a price change today?”, we don’t really need all that complexity. We need a model that recognizes the depreciation of a house, that there are transactions costs of moving from one dwelling to another which implies that a house does not only represent consumption value today but also tomorrow, and that the consumer borrows and owns assets. Let us now try to specify such a model!

Assume the following utility function,

(9) $U(\mathbf{q}, q_h^0 + \lambda q_m, q_h^1, q_r, g(A^1 - M^1));$

q has the same interpretation as before. It represents all commodities but own housing and the services of a rented apartment. q_h^0 is the initial stock of own housing and $q_h^0 + \lambda q_m$ represents the current consumption value of an owned house including any maintenance and repair. λ is a factor that translates maintenance and repairs into house value. A value different from one allows for more or less value enhancing repairs and maintenance activities.[4]

In principle one could represent the services the consumer obtains from his own house by the product of a depreciation factor and the stock, but for simplicity this factor is absorbed into the utility function.[5] q_h^1 is the terminal stock of own housing representing the quantity of house which has a future consumption value. Through the utility function the consumer attaches a current value to the future services a house is expected to provide. There is thus a trade off in utility between using a house today and using it tomorrow. q_r has the same interpretation as before. Most consumers will have utility functions with properties such that they will either choose a house or an apartment, but we do not exclude the possibility of having both. Finally, $A^1 - M^1$ is net financial assets at the end of a period, i.e. gross assets less mortgages and loans. The function g($A^1 - M^1$) represents the goods these assets can buy in the future as the consumer evaluates them today. Each consumer forms his own expectations about future price increases and any deflation of these assets is included in the function g.

The budget constraint becomes,

$$
\begin{aligned}
&y + A^0 r_A = \\
(10)\quad &\mathbf{p'q} + p_m q_m + r_M M^0 + p_r q_r + \\
&(A^1 - A^0) - (M^1 - M^0) + p_h(q_h^1 - (1-\delta)(q_h^0 + \lambda q_m));
\end{aligned}
$$

$A^0 r_A$ to the left of the equality sign should be interpreted as capital income from financial assets, including unrealized capital gains. The sum of these incomes and other incomes y (labour incomes) can be used for non housing consumption, maintenance and repairs of own house $p_m q_m$, interest payments on mortgages and loans $r_M M^0$, rent, increase in net financial assets, and to change the assets and thus also the consumption of own housing. The last term is the end of period value of the difference between end of period own housing and the beginning of period own housing adjusted for depreciation, maintenance and repairs. δ is a depreciation factor. Please note that beginning of period housing is evaluated at end of period prices which implies that any capital gains on houses are accounted for. Moving predetermined entities which define the total resources of the household to the left of the equality sign, the budget constraint is rewritten in the following way[2],

$$
(11)\quad \begin{aligned} & y + A^0(1+r_A) + p_h(1-\delta)q_h^0 - M^0 = \\ & \mathbf{p'q} + p_m q_m + p_h(q_h^1 - (1-\delta)\lambda q_m) + p_r q_r + r_M M^0 + (A^1 - M^1); \end{aligned}
$$

The second term to the left of the equality sign is the end of period value of financial assets held at the beginning of the period, including realized and unrealized returns and capital gains during the period. The third term is the end of period value of a house owned at the beginning of the period, but after depreciation during the period.

The dual problem to the maximization of the utility function subject to this budget constraint is the minimization of the expression to the right of the equality sign subject to the level of utility. The corresponding ideal index is,

$$
(12)\quad I = \frac{\mathbf{p^{(1)\prime}q^{*}} + p_m^{(1)} q_m^{*} + p_h^{(1)}(q_h^{1*} - (1-\delta)\lambda q_m^{*}) + p_r^{(1)} q_r^{*} + r_M^{(1)} M^0 + (A^{1*} - M^{1*})}{\mathbf{p^{(0)\prime}q^{**}} + p_m^{(0)} q_m^{**} + p_h^{(0)}(q_h^{1**} - (1-\delta)\lambda q_m^{**}) + p_r^{(0)} q_r^{**} + r_M^{(0)} M^0 + (A^{1**} - M^{1**})};
$$

The quantities q* etc are functions of the prices with top superscript (1) while q** etc are functions of the prices with top superscript (0). This index tells us what change in total resources is needed to maintain the same utility U for the two sets of prices

$$
(13)\quad \{\mathbf{p^{(t)}}, p_h^{(t)}, p_r^{(t)}, p_m^{(t)}, r_M^{(t)}\} \; for \; t = 0,1
$$

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[2] $r_M M^0$, interest paid on mortgages and loans, is kept on the right side of the equality sign as consumption expenditure. In reality mortgages and loans are not only changed at the end of a period but could be changed any time during the period. One might then prefer to see interest paid as the product of an interest rate and a weighted average of beginning and end of period outstanding mortgages and loan. In order not to complicate the notation further the formulation of eq. (11) is kept.

If we would rather know what change in total income is needed, the ideal index becomes,

$$(14)\quad I = \frac{\mathbf{p}^{(1)\prime}\mathbf{q}^{*} + p_m^{(1)}q_m^{*} + p_h^{(1)}(q_h^{1*} - (1-\delta)(q_h^{0} + \lambda q_m^{*})) + p_r^{(1)}q_r^{*} + r_M^{(1)}M^{0} + (A^{1*} - M^{1*}) - (A^{0} - M^{0})}{\mathbf{p}^{(0)\prime}\mathbf{q}^{**} + p_m^{(0)}q_m^{**} + p_h^{(0)}(q_h^{1**} - (1-\delta)(q_h^{0} + \lambda q_m^{**}) + p_r^{(0)}q_r^{**} + r_M^{(0)}M^{0} + (A^{1**} - M^{1**}) - (A^{0} - M^{0})};$$

This expression demonstrates what price changes should go into an index and what kind of weights one should use. We note that the price change on owned houses should be included in its own right with a weight that is proportional to the change in own housing. Using the conventional non-rigorous transformation from a single individual to an aggregate index the aggregate weight should be proportional to the sum of the value of all newly produced one family houses less demolition. The change in interest rate on mortgages and loans should only be weighted with a weight proportional to the size of mortgages and loans, not by the value of the housing stock or anything else. This index also includes a variable that represents the change in net financial assets, a variable that we are unused to finding in a consumer price index, see the discussion about this issue in section IV below.

Still another question to ask this model is what change in labour income y is needed to compensate for the price changes. An index corresponding to this question is obtained if $r_A A^0$ is subtracted both from the numerator and the denominator of expression (14). Such an index can either be conditioned on a given rate of return r_A, or one can choose to include r_A in the price sets that are compared. In the latter case $r_A^0 A^0$ is subtracted from the denominator of equation (14) and $r_A^1 A^0$ from the numerator.

Transaction costs were used to explain why a consumer considers the future consumption value of an owned house, but they were never explicitly introduced in the budget constraint above. This is easily done. All that is needed is to add the term

$$(15)\quad p_T\left[(q_h^1-(1-\delta)(q_h^0+\lambda q_m))\neq 0\right];$$

to the right of the equality sign of equation (10). p_T is the "price of moving" while the expression within brackets is a dummy variable that takes the value one if a consumer sells his house and buys another one.[6] The ideal price indices (12) and (14) will then also include this component. Because we have assumed a unit price of moving, the corresponding aggregate weights will simply be the total number of consumers who changed house. A more conventional type of index for this subgroup of services could be obtained if the model were to allow for differences in volume and quality of moving services. In practice these services would usually be included among other transport services.

The model can also be extended to include taxes. Suppose, for instance, that interest paid on mortgages are deductible against capital incomes, that these are taxed at a flat rate τ, and that there is a real estate tax τ_h that is applied to a tax base that is proportional (β) to the market value of the house. The budget constraint (10) then becomes,

$$(16)\quad \begin{aligned}&y+A^0r_A(1-\tau)=\\&\mathbf{p'q}+p_mq_m+r_M(1-\tau)M^0+p_rq_r+\\&(A^1-A^0)-(M^1-M^0)+p_h(q_h^1-(1-\delta)(q_h^0+\lambda q_m))+\tau_h\beta p_hq_h^1;\end{aligned}$$

It follows that the "prices" that will enter the ideal indices (12) and (14) will change a little. The interest paid on mortgages should be adjusted for any changes in the capital tax rate, and the housing price for any changes in the real estate tax (tax rate and tax base). If the general purpose of the index does not allow compensation for changes in the tax system, one can condition on a given tax system. The weights of such an index will, however, be different. In particular, the mortgage interest rate should be weighted with the sum of all outstanding mortgages multiplied by one minus the tax rate.

IV. On implementing an approximation to a true index

The design of an approximation to the true indices suggested above, such as a consumer price index, requires a few policy related decisions all of which depend on the most important uses of the index. One policy issue is how one would wish to represent taxes and tax changes in the index. The discussion above included three tax parameters: The capital income tax rate τ, the real estate tax rate τ_h, and a parameter β that defines the base of the real estate tax.[7] Depending on the tax structure one might also for instance wish to include explicitly value added and other commodity taxes and income taxes on labour incomes. If there is a wealth tax the base of which includes the tax assessed value of the consumer's home, one might also wish to treat this tax as part of the expenditure for housing. The policy issue is whether an index such as the CPI should capture changes in the tax parameters or not. In most countries the CPI treats changes in the value added tax and in other commodity taxes as price changes, while income and wealth taxes do not enter the computations.[8] In the following it is assumed that it is consistent with most uses of the CPI to have changes in the three parameters τ, τ_h, and β affect the index number. Note, if one took a different decision and conditioned on some specific values of these parameters, the implication is that although changes in the parameters will not influence the price relatives, their values will determine the size of the corresponding weights. Although value added taxes and other commodity taxes are not explicitly introduced one can think of prices as gross of these taxes and thus include them implicitly.

The alternative indices suggested differed in the income base for which they were defined. A second policy issue is thus to decide if we want an index that measures the compensating adjustment of the consumer's total resources (equation 12), total income (equation 14), labour income or some fourth income concept. This is a question related to the third issue of how to handle the financial assets that enter the true index.

It is of course possible to measure and include changes in financial wealth in the index as the expression for the true index suggests, but considering how a CPI is used most users would probably not want to allow changes in the net financial wealth of the household sector to influence the CPI. A simple alternative is just to suggest that we are interested in the income change needed to compensate for price changes given that the consumer's net financial wealth is unchanged. This is a well-defined question to ask the model, but it may or may not be a good assumption about the behaviour of people. Households certainly behave differently with respect to their portfolio choice depending on their particular situation. Using this assumption the term that represents the change in net financial wealth drops out of the numerator and denominator of eq. (14), but if the household does not choose to maintain its financial wealth unchanged in current prices, one is not able to observe the q*:s or the q**:s. If the change in net financial wealth was constrained to a number different from zero we would in general have the same problem.

However, it is possible to define an index which tells us what change in income is needed net of a given change in financial wealth to maintain the same level of utility for two different price vectors. Let the change in financial wealth be the change the consumer chooses with the price vector of expression (13) for t=0, i e $(A^{1**} - M^{1**}) - (A^0 - M^0)$. Allowing for taxes and rearranging eq. (16) so the expression for the change in financial assets is subtracted from the income expression to the left of the equality sign, the resulting expression to the right of the equality sign becomes,

(17)
$$\mu_t(\mathbf{Q}) = \mathbf{p^{t\prime}q} + p^t_m q_m + r^t_M (1-\tau^t) M^0 + p^t_r q_r + p^t_h(q_h - (1-\delta)(q^{t-1}_h + \lambda q_m)) + \tau^t_h \beta^t p^t_h q_h;$$

where **Q** is the vector {**q**, q_m, q_r, q_h). Because we think of a consumer maximizing utility for alternative price vectors but subject to the constraint that net financial wealth is $(A^{1**} - M^{1**}) - (A^0 - M^0)$, the notation t is introduced to denote different sets of prices. The corresponding true cost-of-living index for two price vectors t=0,1 and the utility level U_0 becomes,

(18)
$$I_{01} = \frac{\mu_1(\mathbf{Q_1^*})}{\mu_0(\mathbf{Q_0})};$$

where $\mathbf{Q^*_1}$ is the vector of quantities that minimizes the expenditures (for the given change in financial wealth) at prices p^1, …etc. necessary to attain U_0, and $\mathbf{Q_0}$ the vector that minimizes the expenditures at prices p^0, …etc. This index would seem to correspond relatively well to what is wanted for the purposes of a CPI. It tells us how much income must increase net of base period financial savings to maintain the base period utility.

While in general $\mathbf{Q^*_1}$ is not observable $\mathbf{Q_0}$ is and it is thus possible to obtain a Laspeyres approximation to (18). An index of Laspeyres-type becomes

(19)
$$I_{01} = \frac{\mu_1(\mathbf{Q_0})}{\mu_0(\mathbf{Q_0})};$$

These two indices apply to a single consumer. If each consumer's index is weighted by the consumer's share of the sum of the expenditures of all consumers in the base period 0, an aggregate index of Laspeyres-type becomes,

(20) $$I_{01}^{A} = \frac{\sum_{i} \mu_{1i}(\mathbf{Q_{0i}})}{\sum_{i} \mu_{0i}(\mathbf{Q_{0i}})};$$

Summing expression (17) over all consumers and rearranging gives us the price relatives and the aggregate weights of this index. The numerator of equation (20) can be written in the following form.

(21) $$\begin{aligned}\sum_{i}\mu_{1i}(\mathbf{Q_{0i}}) = {} & \sum_{i}\sum_{j}\frac{p_j^1}{p_j^0}p_j^0 q_{ij}^0 + \sum_{i}\frac{p_m^1}{p_m^0}p_m^0 q_{im} + \\ & \sum_{i}\frac{r_M^1(1-\tau^1)}{r_M^0(1-\tau^0)}r_M^0(1-\tau^0)M_i^0 + \sum_{i}\frac{p_r^1}{p_r^0}p_r^0 q_{ir}^0 + \\ & \sum_{i}\frac{p_h^1}{p_h^0}p_h^0(q_{hi} - (1-\delta_i)(q_{hi}^{t-1} + \lambda q_{im})) + \\ & \sum_{i}\frac{\tau_h^1\beta^1 p_h^1}{\tau_h^0\beta^0 p_h^0}\tau_h^0\beta^0 p_h^0 q_{ih};\end{aligned}$$

We find that the index for aggregate housing is built up of four price relatives involving four "prices": The price of maintenance and repairs p_m, the interest rate after tax on mortgages and loans $r_M(1-\tau)$, the price of rented apartments p_r, the price of owner occupied houses p_h, and real estate taxes $\tau_h\beta p_h$. Given the definition of β, $\beta p_h q_h$ is the tax-assessment value of a house. The price relative of real estate taxes will thus change if at least one of the tax rate, the tax-assessed value or the price on owner occupied houses change.

In practice all these prices differ across the population of consumers. One will thus have to compute elementary price indices for all these components using conventional index formulas for elementary indices.

The corresponding weights also follow from equation (21). The weight for the price relative of p_r is the sum of all expenditures for rented apartments, and for p_m the sum of all expenditures for maintenance and repairs. The weight for the price of owner occupied houses reduces to the sum of the values of new built houses at the end of the base year less the value of demolished houses, because for each house that is not demolished it holds that,

(22) $$q_h^t = (1-\delta)(q_h^{t-1} + \lambda q_m);$$

The weight for the real estate taxes is the sum of all real estate taxes paid by private households, and finally the weight for the interest rates on mortgages is the sum of all interest paid on mortgages for private housing after capital income tax.[9]

If it is not consistent with the purpose of the index to allow changes in the tax parameters to determine the price relatives, one may choose to fix the tax parameters at certain values, for instance the values of the base period. It then follows from equation (21) that the price relative for interest on mortgages depends only on the interest rates while the weight is still the sum of all interest paid after tax. Similarly, the price relative of the real estate tax depends only on the prices of owner occupied houses while the weight remains the same as before.

We thus conclude that an aggregate index of housing can be computed as a weighted average of a few sub-indices in much the same way as any other price index.[10]

V. Conclusions

A dynamic model of consumer behavior that incorporates the demand for housing was specified such that it is consistent with the general purpose of a consumer price index. From this model it was possible to derive a true cost-of-living index that includes housing. Being an ideal index it cannot be computed without imposing additional assumptions about the behavior of the consumer, but it is possible to draw conclusions about what prices and weights that should be used in conventional approximations to such an ideal index.

We find that:

- House prices should be included with weights proportional to the sum of the values of new built houses at the end of the base year less the value of demolished houses,
- Interest rates on mortgages should be included with weights proportional to the sum of outstanding mortgages. The weights should not include down payments or the non-mortgaged part of the house value.
- If the tax system allows for deductions of interest paid, either the price relatives of interest rates on mortgages or the weights should be adjusted accordingly. The price relatives should be adjusted if it is desirable to compensate the consumer for tax changes, while only the weights should be adjusted if that is not the case.
- If there are real estate taxes and it is desirable to have an index that compensates for changes in them, a subindex of the real estate tax paid on a house should be included with weights equal to the sum of all real estate taxes paid.

These conclusions follow from the model used. Obviously another model could lead to different conclusions. Although the model is dynamic, it is so in a rather restricted way. One might almost say that it is a dynamic model in disguise of a static model, but as such it responds to the questions we usually ask a consumer price index.

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[1] The problems of a price index for housing services were discussed already in the beginning of the 1950s in the Standing Committee of the Swedish CPI and a public commission was appointed by the government the proposals of which became the basis for the current treatment of housing in the Swedish CPI. These issues have then returned several times to the agenda of the committee. In 1997a new commission was appointed. It suggested radical changes in the price index for the services of owner occupied houses (SOU 1999:124) which have not been implemented.

[2] In his discussion of consumer durables in a cost-of-living index Robert Pollak(1989) takes a rather pessimistic view: "To summarize: it is possible to construct a meaningful partial cost-of-living index if the intertemporal utility and budget functions are separable. In a frictionless world, the budget function is separable if either rental markets or capital and secondhand markets are perfect. Whether preferences are separable is an empirical question which has not yet been systematically investigated. But we do not live in a frictionless world, and casual observations suggest that the budget function is not separable; this effectively precludes construction of a meaningful one-period cost-of-living index." (p.188)

[3]Sten Malmquist (1953) used the terminology "compensation index".

[4] For simplicity in order not to complicate all formulas we also include in q_m the services and material needed to operate a house such as heating. It is not always easy to make a clear distinction between these goods and services and those used for maintenance. For instance, if a house is not heated in a Nordic climate it will quickly wear down.

[5] In fact, the services obtained from a given house will in general differ from one consumer to another depending on the consumers' preferences. It follows that the services a consumer obtains from a house need not be the same as a market determined depreciation.

[6] This expression neglects the unlikely case of selling a house and buying another one of exactly the same size and quality and still having transaction costs.

[7] In Sweden these parameters are τ =0.3 (0.21), τ_h =0.015 and $\beta \leq 0.75$.

[8] In the current Swedish CPI changes in the value added tax and in other commodity taxes are treated as price changes. Changes in the parameter τ_h are accounted for too, but not changes in τ and β. (??)

[9] In most indices based on the user-cost approach the weight is not only determined by outstanding mortgages but by the market value of the house. The argument is that there is an alternative cost to the house owner of the capital corresponding to the non-mortgaged part of a house. Why don't we get the same weight in this model? One interpretation is that in this model the consumer derives utility already today knowing that he will have a house that will yield services also tomorrow.

[10] No distinction has been made between owner occupied houses and condominiums. It should be possible to treat condominiums in the same way as owner occupied houses with the difference that there is no real estate tax paid directly by the owner and that outside maintenance, interest on common mortgages and administration is paid to the co-operative. (The charges to the co-operative also include reduction of the principal that in principle should not be included in the consumption expenditures of the co-op members. In practice it might be difficult to single out this item.)