<u>Consumer price indices: real world quality measures</u>

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ABSTRACT

The UK National Statistician has responsibility for the professional quality of all outputs falling within the scope of National Statistics. This includes establishing mechanisms for taking into account user needs and clear quality assurance processes. In the context of consumer price indices we must ask what tools are available to enable users to take a view on fitness for purpose and allow producers to manage index development effectively and efficiently. A review of the literature would suggest that national statistical offices use a variety of tools but that it is far easier to point to conference papers advocating the theory than to user or producer decisions driven by the practice. This paper will describe attempts by ONS to systematically review and develop the quality of its consumer price indices. It then draws general lessons from the ONS experience.

The paper consists of three parts: an overview of ONS' Data Quality Assessment Framework and how this is applied to consumer price indices through specific techniques such as the European Foundation for Quality Management's Excellence Model (EFQM) and the international standard ISO9002; a more specific look at the quantitative analyses of the representativity of the existing local collection made through an analysis of the variance of our collected prices and through an examination of the differences in prices collected for the CPI and those from household scanner data and other ONS data on the retail sector; and a discussion of the lessons learnt and the implications for our future strategy for index development.

Keywords: Quality Review Programmes, Excellence Model, ISO 9002, CPI Research Programme, Prioritisation, Sample Optimisation, Neymann-allocation, Stratification, Scanner Data, Sampling Bias.

1.0 Background

Statistical Offices are faced with the continuous challenge of delivering a wide range of good quality statistical outputs and services to meet the needs of what can be a disparate group of users working in an evolutionary environment. For UK National Statistics this challenge and the associated requirement for good quality is recognised in the Framework Document for UK National Statistics, published June 2000. The framework states that the UK National Statistician has a duty:

- "to improve the quality, timeliness and relevance of its service to customers both within government and the wider community";
- "to improve public confidence in official statistics by demonstrating that they are produced to best professional standards and free from political interference".

This responsibility is being fulfilled through, amongst other things;

- the establishment of a review programme of key outputs at least once every five years with the involvement of external expertise;
- promoting high quality statistical output through systematic evaluation and research;
- improved reporting on quality and documentation of methods leading to effective use of statistics ;
- encouraging business areas to adopt quality management tools as appropriate, such as ISO9002 and UK Investors in People, and the Excellence Model developed by the European Foundation for Quality Management as a diagnostic tool to improve quality and performance through regular self-assessment.

This paper describes the processes for achieving quality in UK National Statistics and how they have led to the quality management system adopted for the processing of the monthly Consumer Prices Index and Retail Prices Index. It then goes on to describe in more detail the measurement of statistical quality and how this provides an important input into the methodological improvement of consumer price indices. It takes by way of example recent work undertaken into sampling efficiency and an evaluation of how representative the prices used in ONS' consumer price indices are of consumer purchases as a whole. This work is part of a published programme of methodological investigation and development of CPIs which is managed by a Technical Board on consumer prices indices.

2.0 The Data Quality Assessment Framework

"There is no exact, unambiguous definition for quality. The quality of statistics is a dynamic, multi-dimensional and cross-influential concept. A set of statistics may be of high quality for one purpose, yet the same statistics may prove to be deficient for another. Relevance, accuracy, timeliness, accessibility, comparability and consistency are regarded as constituents of statistical quality. Some can lead to exact, measurable quality indicators, but for others the users of statistics must contend with general and perhaps vague descriptions provided by the producers". Statistics Finland.

The ONS's Data Quality Assessment Framework and the processes for implementing it in the context of consumer price indices (CPIs) have two roots: the international frameworks and processes designed to improve the quality of CPIs and the UK's Framework Document for UK National Statistics in general. This paper considers each in turn before turning to the specific practices adopted for the production of UK consumer inflation indices.

2.1 International Frameworks for Quality Management of Consumer Price Statistics

Although there are several international frameworks they all follow broadly the same format and are supported by the UN Fundamental Principles of Official Statistics[1]. Perhaps the most pertinent and specific international "quality" framework is the International Monetary Fund's Data Quality Assessment Framework for Consumer Prices, drafted in collaboration with the World Bank. This is one of seven Data Quality Assessment Frameworks (DQAFs) covering a range of key economic statistics. DQAFs are used to assess an individual country's data quality and cover institutional environs (e.g. governance), statistical processes as well as processes and the characteristics of the statistical outputs.

Each DQAF has three levels: at the top there are "prerequisites of quality" and five dimensions; assurance of integrity, methodological soundness, accuracy and reliability, serviceability, and accessibility. Next come "elements" i.e. the various aspects of the dimensions; and indicators i.e. how the elements can be measured and compliance proven. For instance, the ability to reconcile statistics over a period of time may be an "indicator" that the consistency "element" under the "dimension" of serviceability is achieved.

The DQAF for Consumer Prices then has "focal issues", which are specific to consumer prices and associated quality features or "key points". For instance, internal consistency may be a "focal issues" and use of a summation method which is insensitive to classification typology may be a "key point". The DQAF has multiple references to international classifications and to

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[1] The Fundamental Principles of Official Statistics, endorsed by the United Nations Statistical Commission, are widely agreed as the appropriate defining principles for a national statistical system. They are set out in full in Annex 1 and provide the underlying foundation of UK National Statistics. The second principle listed in Annex 1 relates to professionalism in choice of method and presentation. The ninth principle relates to the use of international concepts, classifications and methods. The final principle relates to international co-operation and the benefits of such co-operation to improving statistics. The Ottawa Group is a good example of the opportunities and benefits of pro-active involvement. It can be seen that the choice and application of appropriate methodology is a key element which underlies the Fundamental Principles of Official Statistics. They are also a key element in the Framework Document for UK National Statistics (see next section).

recognised best methodological practices, such as the use of COICOP and ILO guidelines and resolutions on the construction of consumer price indices[2].

For the UK, like other EU members, general frameworks of recommendations such as the DQAF are supplemented by European regulations that have the force of law. In particular the regulations which govern the compilation of the Harmonised Index of Consumer Prices and the mechanisms for checking that these are being followed inevitably have a large impact on Quality management for the UK CPI.

2.2 The Framework Document for UK National Statistics

The Framework Document for UK National Statistics defines the UK statistical system and sets out its broad parameters. It establishes the position and roles of National Statistician and the Statistics Commission, departmental Heads of Profession and Ministers (see Annex 2). National Statistics are defined as a series of outputs that can be added to, by ministerial agreement, or from which an output can be taken by the National Statistician if she/he judges it not to meet professional standards. All Consumer Price Indices compiled by ONS are deemed to be national statistics covered by the code.

The Framework comes with associated ONS Codes of Practice to provide the standard setting mechanism for implementing it. These place a strong emphasis on transparency of methodology, documentation of outputs and processes, and provision of information relating to quality in the National Statistics Quality Strategy. For instance, there are codes of practice on quality management, data presentation and dissemination, documentation and use consultation.

The National Statistics Framework provides the process by which the integrity and quality of the UK statistical system can be underpinned. The Codes of Practice provide the standards. However, there must be a process for quality improvement for National Statistics.

'Quality' as referred to in these documents is defined in terms of the intended use of the outputs. Delivering quality statistical outputs and services essentially involves two related sequential activities:

- Building quality into statistics. At its broadest this covers: defining user needs; translating these into an appropriate statistical methodology; putting that methodology into effect; putting in place the appropriate infrastructure to ensure effective delivery;

- Monitoring the quality of statistics. Essentially monitoring effectiveness in the delivery of outputs. This involves scrutinising the quality of the final output against both user needs and the associated statistical design and monitoring the individual activities in the production process to ensure that these are properly carried through.

[2] More information can be found on the IMF website.

Clearly both activities are a pre-requisite to the effective meeting of user needs although it can be noted that often no distinction is made between them in practice and "quality management" is often used as a generic term to cover both activities.

The quality strategy for UK National Statistics was launched in the Autumn of 2000 and sets out three formal elements:

- A programme of reviews to quality assure key outputs. All key National Statistics Outputs will be reviewed at least once every five years, but in practice for consumer price indices there is a continuous programme of review and development. Key questions to be addressed include:
  - What are the user needs of the outputs?
  - Do the outputs meet user needs?
  - Are the outputs of adequate quality (a broad definition of "quality" is used, taking into account user need)?

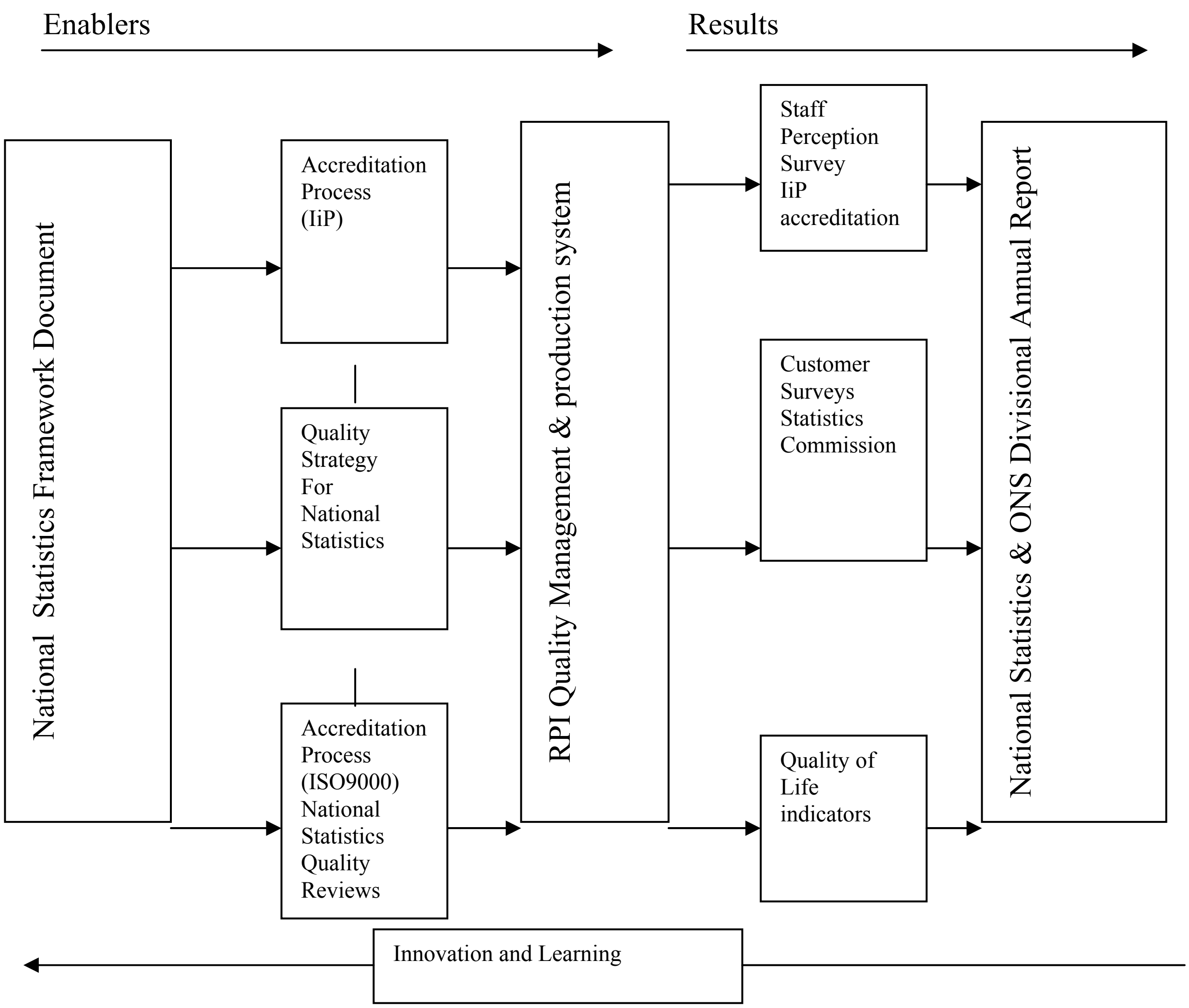
- Documentation of surveys and outputs (essentially metadata for users of National Statistics). This is being carried out with the use of standard templates which meet international obligations such as the metadata needs of the International Monetary Fund.

- Documentation of processes. This covers desk instructions and other background information required by compilers. It provides necessary information for quality management and assurance and for business continuity.

2.3 Quality management of UK consumer price indices

The price indices produced by ONS are the UK's most widely used official statistics with a variety of important uses which impact on the well being of individuals. It is therefore appropriate that we have gone further than any other statistical area on adopting formal mechanisms for quality management. We use the European Foundation for Quality Management's Excellence Model to provide an umbrella for activities aimed at continuous improvement. This is a self-assessment diagnostic tool that focuses on organisations and general business areas. It looks at performance against five criteria covering what the business area does (the enablers: leadership; people; policy/strategy; partnership/resources; processes) and four criteria on what the business area achieves (the results: people results; customer results; society results; key performance results). Evidence based on feedback from focus groups, questionnaires and personal interviews is used to assess performance across the criteria and a resulting action plan for improvement is introduced. This is then included in the business plan. Further details are given at Annex 3.

The Quality Management System adopted for CPIs can be represented in the context of the Excellence Model by the diagram below. The enablers are represented by ISO9002 and Investors in People and the results by the award of the corresponding charter marks and feedback from customer surveys.



Quality measures cover every dimension of the DQAF.

2.3.1 Integrity

The Framework document insures the integrity of outputs. In the case of the CPGI this is reinforced by the need to meet HICP requirements.

2.3.2 Methodological Soundness

2.3.2.1 Methodology in practice

The Methodology used is shown in a technical manual published on the ONS website. Where practices differ from international recommendations the differences are recorded and reasons given. Implementation of the methods described in the Manual is documented in exhaustive detail on the Intranet. ISO9002 and Investors in People (IiP) form the backbone of the CPI Quality Management System to ensure that the Methodology we aspire to use is actually carried out in practice.

- **The ISO9002 accreditation process**. The International Standard ISO 9002 states that "The supplier shall establish, document and maintain a quality system as a means of ensuring that the product conforms to specified requirements". The view is taken that whilst ISO 9002 does not set standards or guarantee quality it does provide a readily available framework within which quality could be continually improved. In addition, the fact that it is internationally recognised is seen as an advantage. ISO9002 has been applied to the monthly production cycle from price collection to index production and to index development. However, it has not been applied to methodology. ISO9002 is used in **defining standards as** a benchmark for monitoring performance**; describing the processes** for business continuity and training; **producing the evidence** that instructions are being followed**.** The key aspects of the system are threefold. Each aspect should be seen as inter-dependent and an integral part of the whole:

    - **documentation of the monthly production processes** from the collection of prices to the data processing to publication of the index;
    - **auditing** of local price collection and monthly processing including post-hoc price checks in the field;
    - **A regular review system**, which focuses both on the monthly production process and longer-term issues. This includes monthly reviews with staff incorporating training sessions.

    An outline of the Quality Management System is given at Annex 4. ISO9002 is effective at ensuring our indices are produced to a high standard according to current methodology but extends no further than auditing. It does not facilitate methodological development.

- **Investors in People** is a corporate initiative using an externally recognised national standard that sets a level of good practice for improving an organisation's performance through its people. It puts in place a two-way link between business plans at every level and training and development plans for the employees who have to deliver the business. The documenting of procedures as part of the Quality Management System helps to identify training needs and these have been incorporated into personal development plans of individuals and in-group Training Plans. In addition it has facilitated more effective pre-training and post-training evaluation procedures to monitor the quality of training and its value against business needs and also provide an important input into quality management. Thus IiP is being used as an effective "quality" tool in the context of the CPI Quality Management System and ISO9002. Monthly "quality" (training) days are an integral part of it. The ONS as a whole first became an accredited IiP organisation in 2000.

2.3.2.2 *Commodity reviews*

Methods for measuring the prices of specific items and proposals for including new items are reviewed during an annual process. Suggestions for improvements are produced, tested, and peer reviewed.

2.3.2.3 *Methodological Research*

Like other national statistics in the UK, CPIs are subject to a continuous programme of methodological investigation and development which is more demanding than the requirements laid down in the Framework for National Statistics which stipulates five-yearly reviews. The programme is managed by an internal Technical Board which draws on expertise from the wider community of price statisticians and economists. It is also subject to official external scrutiny by Parliament through various select committees and also from investigations initiated by the Statistics Commission and by users and commentators more generally, for example in the media and by academia. Each year an article is published which reviews the outcomes of the previous year's research work (including methodological improvements which are being implemented) and looks ahead to the research agenda for the next year. In this way there is a great deal of high-profile public accountability. In a European context there is the added discipline resulting from independent compliance monitoring by Eurostat of adherence to EU regulations on the compilation of the Harmonised Index of Consumer Prices (HICP).

The CPI research programme facilitates continuous methodological improvement of indices and complements ISO9002. It has its origins in the investigation of the issue of potential bias brought to prominence by the Boskin Report. At any particular time the work carried out is determined by a number of factors including the results of previous research. It is also responsive to external factors such as new requirements from the EU (for example more rigorous rules for quality adjustment or an increase in index coverage) and from other users (for example, a request from the ECB for an HICP-constant tax index) and from the challenges thrown up by an evolution in the retail market requiring action to be taken to ensure the continuing relevance of indices (for example, the increasing use of internet shopping). More details are given at Annex 5.

The methodological techniques developed and implemented over the past few years- including hedonic quality adjustment and local probability sampling of product varieties (both include the regular use of scanner data) have now achieved acceptance and become incorporated into the regular work of the ONS. Future research priorities are being assessed in the light of the opportunities raised by the ONS's Statistical Modernisation programme and the re-tendering of the prices collection contract; the priorities identified in the Allsopp review of regional statistics (which identified unmet demand for regional economic data); recent academic research; and newly available data sources. Areas currently being researched include:

- Strengthening sampling procedures at all levels to ensure that price quotes collected are representative of retail expenditure;

- Examining the measurement of the price evolution in difficult services such as retail banking or owner-occupier housing costs in the HICP;

- Examining alternative data sources, in particular checkout scanner data.

The research programme is overseen by boards of users from inside and outside ONS.

2.3.3 Accuracy and Reliability

The data compilation process includes several layers of checks to ensure that all unusual price movements are scrutinised. In addition ten percent of local collections are back-checked each month to check for response error in local price collection. There are targets for response error and the market research company which collects the prices is subject to financial penalties if it misses them and financial bonuses for exceeding them. This provides a monetary incentive for improved performance and performance has increased during the course of the contract.

The most difficult aspect of quality to assess is the sampling and non sampling errors associated with our collection. Our efforts to do this are reported in section 3.

2.3.4 Serviceability

Indices are available monthly. They are fully consistent internally and over time.

2.3.5 Accessibility

Statistics and metadata on methodology are available at no charge on the ONS Website. A dedicated team is available to answer user queries in office hours and there is a 24 hour recorded telephone message enquiry service giving the latest figures.

3.0 Assessment of Sampling and Non Sampling Errors in the UK CPI

3.1 Background.

The target of the CPI and RPI in the UK is to measure price changes for consumption goods and services across time for all relevant households throughout the UK. In order to achieve this goal a sample of the prices which consumers pay is taken. The sampling process is highly complex and has developed over many years. In the succeeding paragraph we briefly sketch the sampling methods used in the UK CPI and RPI[3].

First, each year a set of *items* is chosen which are used to represent given categories of expenditure – the expenditure data is obtained mainly from the Expenditure and Food Survey (EFS) and supplemented by information from other sources. An example of an item in the *Footwear* expenditure category is 'Men's Training Shoe – Sportswear'. The sample of items is chosen *purposively* (i.e. items are chosen according to the best judgement of ONS statisticians) with consideration usually given to incorporating items for which the expenditure is relatively large within an expenditure category. There are approximately 715 items currently included in the CPI. Some items are chosen as "central" items for which products are selected by ONS staff in head office and priced using telephone or internet enquiries while the rest are "local" items where product selection and price collection is done separately in each collection by staff employed by a contractor to visit stores in locations throughout the country. For the "local" items, various *locations* are selected within the 12 standard UK regions. This is the second element of the sampling process. A location is a shopping centre, large grouping of shops, or high street. The number of locations selected in a region is proportional to the overall level of consumer expenditure in a region. Within a region the locations are selected using a Probability Proportional to Size (PPS) sample where the size measure is number of employees in the outlets at each location. There are currently around 145 locations included in the UK CPI. Third, at each location the outlets are again sampled using PPS where the size measure is floor space, a proxy for turnover. These are the outlets where prices for the selected items will be surveyed. There are roughly around 1000 outlets currently visited each month for price collection. Fourth, once the basic sampled unit is selected (*item*, *region*, *location*, *outlet*) a price collector visits the specific outlet and chooses a *variety* of the item to price for inclusion in the index. Apart from a few exceptions relating to hi tech fast evolving goods, the variety is chosen by the collector purposively. Finally, the price is then used in index computation, usually stratified by item as well as some combination of region and shop-type. Shop weights are often applied within a stratum and weights are used in the index formula to aggregate the different strata together.

This complex mix of purposive and probabilistic sampling makes it extremely difficult even to calculate meaningful estimates of the sampling errors in the collection of prices for the items and outlets chosen. The problem of assessing the non-sampling error in price changes (as opposed to expenditure) caused by our choice of items and outlets is even more difficult. Without conclusions about the link between our procedures and the accuracy of our outputs it is impossible to identify how those procedures can be improved.

The next two sections describe two strategies to investigate questions of variance and bias. First we look at the studies of the variance of price quotes within the current "local" collection. Next

[3] For further detail see "Consumer Price Indices: Technical Manual", Office for National Statistics, 2005.

we describe initial work on the relationship between measured prices and prices relating to the total population of consumer purchases drawn from household scanner data.

3.2 Studies of the Variability of Quotes in the Local collection

There have been three phases to CPGI research programme's research into the variability of its local collection. In 1999 we commissioned Randy Sitter of Simon Fraser University to develop measures of sampling error and aggregation (as opposed to sampling) bias. Several papers were written in 1999-2001. In 2004 we commissioned ONS' methodology directorate to carry on the work with more realistic assumptions. Both work programmes produced variance estimates for parts of the index and were able to identify possibilities for reducing it through allocating more quotes to items with higher variability. In both cases, however, there were doubts about the extent to which the authors had allowed for the detailed sampling procedures we actually follow in practice. Moreover, opportunities to use the results to improve sampling procedures were also limited by operational factors associated with the long term price collection contract, leaving the main opportunity for change the imminent re-tendering of the latter. Thus the re-tendering of the price collection contract prompted us to take up the work again in 2005 using resources from ONS' consumer prices Division's own research Branch. It is this work that is described below.

3.2.2 The underlying sampling issues.

The study investigated how "price" quotes collected in shops should be allocated in the CPI to provide the best measure of inflation. The approach taken was to apply a Neymann-type allocation rule where quotes are allocated on the basis of the weight in the index and the variability of the stratum. It follows on from some earlier internal work by Ashworth and Green (2005) of the Survey Methods Division of ONS into how the CPI sample should be allocated amongst the various COICOP Sections[4]. Their results advocated that a large redistribution of quotes was warranted in order to either, lower the cost of collection for the same index quality or to raise the quality of the index. In this paper we extend their basic results with a more detailed empirical application. We also provide an explanation and justification of the conceptual approach used and an initial indication of how the work is being taken forward to include stratification of variables such as shop-type.

3.2.2.1 *The theoretical framework and sample allocation criteria*

The study adopted the "classic" approach to the sample allocation problem dating back to Neymann and also used by Ashworth and Green in their preliminary study. This leads to the expected and uncontroversial result that variance of the index is inversely proportional to the

[4] COICOP (**C**lassification **O**f **I**ndividual **Co**nsumption by **P**urpose).

sample size and as the stratum variance will differ by strata the overall variance of the index can be influenced by how the sample are allocated. This flows from the formula:

$$Var[I^{0t}] = \sum_{n=1}^{N} w_{nt} \frac{\sigma_{nt}^2}{M_{nt}}$$

where:

the weights, w_{nt}, relate to stratum n at time t and sum to one; the overall sample size in stratum n at time t is M_{nt} and $\hat{\sigma}_{nt}^2 = \frac{1}{(M_{nt}-1)} \sum_{m=1}^{M_{nt}} \left(\frac{p_m^t}{p_m^0} - \hat{\alpha}_{nt} \right)^2$, the unbiased estimator of the variance of stratum n at time t.

The detailed derivation using a stochastic model is given at Annex 6.

The optimal sample allocation depends on the sample allocation criteria. It should be noted that in the current exercise it was decided that the variance of the index should be minimised (subject to some cost constraint) rather than the variance of the rate of change. This may be considered by some to be rather obtuse given the fact that most users of CPIs are specifically interested in the annual rate of change, for example the Bank of England inflation target and the updating of benefits are both expressed in these terms. However in reality the position is that:

- The argument over whether index values or rates of change should be used is not clear cut[5].
- The expression of the variance of the annual rate of change (i.e. inflation rate) and its estimation introduces an added complication and the use of additional simplifying assumptions.
- Because the at a point-in-time variance will be by far the biggest element in the variance of the rate of change, the results for sample optimisation are unlikely to differ very much in practice between the two approaches.

This is addressed later in the paper.

Taking a cost function $\sum_{n=1}^{N} c_{nt} M_{nt} = C_t$

where:

c_n denotes the (marginal) cost of collecting an additional quote for stratum n and C_0 is total available resources for sample collection. Note a simplifying assumption has been made that marginal cost equates to average cost.

Then using the method of Lagrange it can be shown that the optimal allocation can be given by the equation:

[5] Follow-up work, not reported here in any detail, was based on price movements. See later section.

$$M_{nt} = \frac{\left(\dfrac{w_{nt}\sigma_{nt}}{\sqrt{c_{nt}}}\right)}{\sum_{n=1}^{N}\left(\dfrac{w_{nt}\sigma_{nt}}{\sqrt{c_{nt}}}\right)}\left(\sum_{n=1}^{N} M_{nt}\right)$$

In the absence of information on the cost of collecting quotes in different strata, it is further assumed that the (marginal/average) costs are equal across strata, $c_1 = c_2 = \ldots = c_N$. In this case we have the simplified function:

$$M_{nt} = \frac{w_{nt}\sigma_{nt}}{\sum_{n=1}^{N} w_{nt}\sigma_{nt}}\left(\sum_{n=1}^{N} M_{nt}\right)$$

Further details of the derivation are given in Annex 7.

3.2.2.2 Practical application to the Consumer Prices Index (CPI)

There are two main issues which need to be addressed before applying the framework outlined above to the UK CPI:

- The approach developed above relates to AR (the Average of Relatives) price index which is used for some elementary indices in the RPI. However, a geometric mean formula is used to aggregate prices at the elementary (stratum) level in the CPI.

- The approach relates to calculating variances for prices relative to the base price. With inflation we would expect the variance to get larger as the base price gets further way in time. This is unlikely to pose a major problem in the allocation of quotes[6], as the allocation rule is applied for a common time period and each of the strata are symmetrically affected, but it is problematic in terms of interpreting the variances.

The approach needs to be slightly modified for the geometric mean. The use of the geometric mean leads to a different expression of variance using an approximation from a Taylor Series. This results in the expression $M_{nt} = \dfrac{w_{nt}\phi_{nt}\exp(\hat{\beta}_{nt})}{\sum_{n=1}^{N} w_{nt}\phi_{nt}\exp(\hat{\beta}_{nt})}\left(\sum_{n=1}^{N} M_{nt}\right)$

where:

[6] Unless there is a marked variation in the price evolution of some items within a section or particular stratum.

The prices are generated by the following model.

$$\log\left(\frac{p_m^t}{p_m^0}\right) = \beta_{nt} + \varepsilon_{nmt}, \qquad n = 1,...,N,\ m = 1,...,M_{nt}$$

This is the logarithmic equivalent of the model:

$$I_n^{G,0t} = \prod_{m=1}^{M_{nt}} \left(\frac{p_m^t}{p_m^0}\right)^{\frac{1}{M_{nt}}}, \qquad n = 1,...,N$$

Further proof is given in Annex 8.

It can be seen that the only difference between this approach and that for the arithmetic mean of price relatives is the inclusion of a level effect for the change in the index.

The interpretational difficulty in dealing with the time element associated with variances for prices relative to the base price (we would expect that $Var[\hat{\alpha}_{nt}] > Var[\hat{\alpha}_{nt-1}]$ {similarly for the geometric case}) can be overcome by using the variance between adjacent months:

$$I^{t-1t} = \frac{I^{0t}}{I^{0t-1}}$$

Further details are given in Annex 9.

3.2.2.3 The empirical results

The primary results for the allocation exercise undertaken for each month February to December for the years 2002 and 2003 are shown in Table 1 both for the arithmetic mean and the geometric mean. The table reports the *average* quote allocations at the CPI-section level over the 2 years. The reporting of the results at the section level is purely illustrative as the allocation exercise itself was undertaken at the lowest possible level of aggregation, i.e. at the higher-level stratum level within sections. There are around 4,000 to 5,000 strata for the items analysed in this paper. It is assumed that in each stratum at least one quote must be collected. It should be noted that as the important aspect of variance is its size relative to that for other strata., the weighted average variance for a section s which contains the strata $n = 1,...,N_s$ is calculated and the results are averaged over time. The resulting statistic indicates the relative differences in variance.

Turning to the allocation results, which are shown in Table 1, a number of important findings emerge.

- Both the arithmetic and geometric methods for allocating the sample imply major reallocations required of quotes amongst the sections in order to achieve an allocation of maximum efficiency. Applying the arithmetic method would lead to a reallocation of almost 34,000 quotes on average while the geometric method would reallocate almost 34,000 quotes out of around 73,000 quotes in total.

- An optimal allocation will more than halve the variance of the indices. Thus a comparison of the variance of the index before and after reallocation shows that the standard deviation (multiplied by 100) of the monthly change in the index on average was 0.0403 and 0.0598 using the geometric and arithmetic methods respectively prior to the reallocation and 0.0189 and 0.0209 afterwards.
- The gains from efficient reallocation are much larger than from increasing the number of quotes. For example the fall in variance in going from 75,000 quotes to 150,000 is only around 30 percent. This is small compared with the gains from efficient reallocation.

Allocation Method	Sample Size	GM Standard Deviation $\times$100	AR Standard Deviation $\times$100	GM Ratio of Standard Deviations $\times$100	AR Ratio of Standard Deviations $\times$100
Actual/Current	73,309	0.0403	0.0598	2.13	2.86
Optimal	73,300		0.0209		1.00
	73,296	0.0189		1.00	
	50,000	0.0229	0.0253	1.21	1.21
	80,000	0.0180	0.0198	0.95	0.95
	100,000	0.0160	0.0176	0.85	0.84
	120,000	0.0146	0.0161	0.77	0.77
	150,000	0.0130	0.0144	0.69	0.69

- The optimal sample allocations are only to a limited degree dependent on whether the arithmetic or geometric method is used. This is encouraging. The CPI & RPI use the same sample of prices. The former is constructed using geometric means and the latter using arithmetic means. The results suggest that there is limited conflict between the two. A sample which is optimal for one will be close to being optimal for the other.
- There is some volatility in the allocations across time for both arithmetic and geometric means[7]. This can be seen in figures 1 and 2, which plot the optimal allocations over time for the 10 sections which currently have the largest allocation of quotes.

The sections which require the largest change in quotes according to both the arithmetic and geometric methods are:

Sections requiring large <u>increases</u> in quotes:

- GARMENTS (30102), + 6,000 quotes.
- RENTS (40100), + 900 quotes.
- FURNITURE, FURNISHINGS (50101), + 2,800 quotes.
- HOUSEHOLD TEXTILES (50200), + 1,000 quotes.
- MAINTENANCE AND REPAIRS, + 900 quotes.

[7] Reflects to some extent seasonal factors but also the price evolution in the market place.

- APPLIANCES, ARTICLES AND PRODUCTS FOR PERSONAL CARE (120102), + 800 quotes.

Sections requiring large decreases in quotes:

- TOBACCO (20200): – 800 quotes.
- FOOTWEAR, INCLUDING REPAIRS (30200), – 1,500 quotes.
- TOOLS AND EQUIPMENT FOR HOUSE AND GARDEN (50500), – 800 quotes.
- OTHER SERVICES IN RESPECT OF PERSONAL TRANSPORT EQUIPMENT (70204), – 800 quotes.
- RESTAURANTS AND CAFES (110101), – 3,700 quotes.

Figure 1: Optimal Allocation using Arithmetic Method.

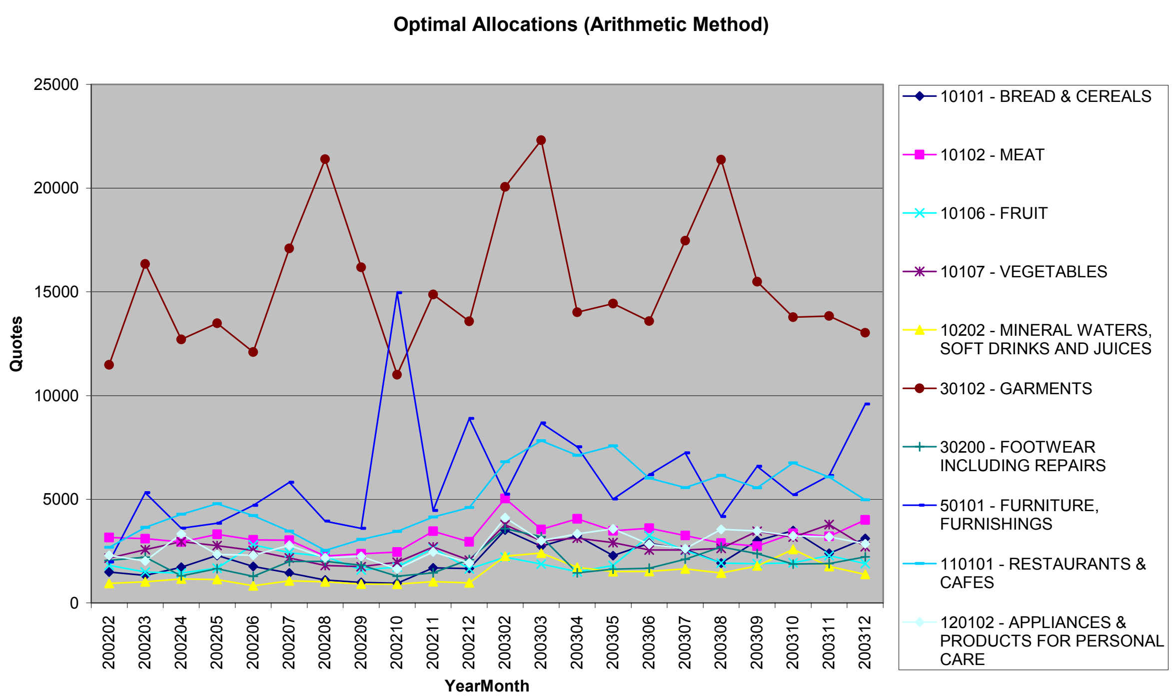
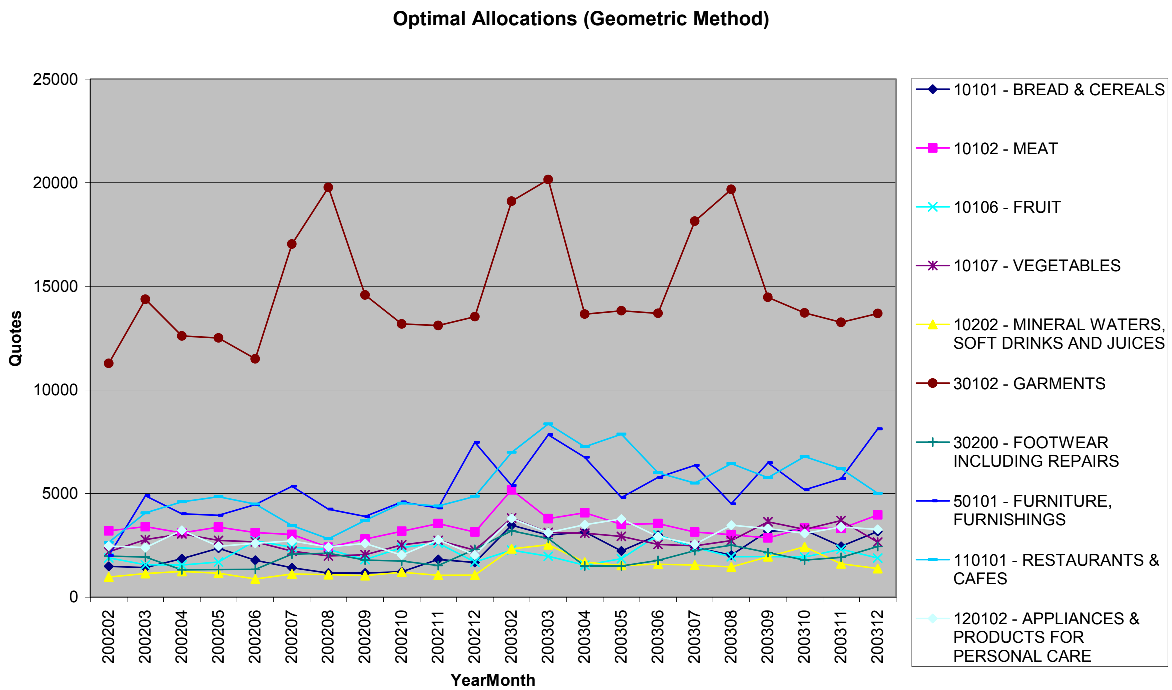


Figure 2: Optimal Allocation using Geometric Method.



3.2.2.4 Conclusions and Further thoughts

The research reported here represents the first stages of investigations on sample design prior to re-tendering of the local price collection contract and focuses on how to allocate a sample once a set of strata have been defined and selected. It makes a number of simplifying assumptions and the results are, of course, dependent on the statistical rigorousness of the current sample. As mentioned in the introduction, it also ignores the prices which are collected centrally by Head Office staff from, for example, central returns provided by retail chains and utilities. It does nevertheless point to inefficiencies in our current sample which can be addressed by a re-balancing with potential savings in collection costs or increases in precision.

The next stage has been to allocate the overall variance of the index to the various components associated with the choice of items, locations and product varieties. This gives a much more detailed insight of sample structure which can inform decisions relating to stratification and multi-stage sampling- issues which were not addressed in the first stage of the research. It is work in progress and it is hoped to report further on this in due course. But the initial results indicate that we do not currently achieve the optimum relative numbers of items, locations and quotes[8].

4.0 Studies into the relationship between measured prices and prices relating to the total population of consumer purchases using household scanner data

4.1 Introduction

4.1.1 Aims

The aim of this aspect of the research programme is to establish whether the RPI provides a reasonable representation of actual consumer expenditure. It is not feasible to collect prices for every product and variety of product purchased by consumers so an item or group of items is chosen on the assumption that it has a similar price trend to others within that particular product group. This research ought to help make better informed decisions when introducing a new representative item to the basket through analysing their recent price trends[9].

4.1.2 The TNS Dataset

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[8] Ashworth and Green's based study assumes somewhat simplistically that Price Relatives for each stratum equal the stratum index and a random term. In effect the location and outlet pattern are taken as given. Thus the work takes everything about the existing CPI sampling structure as given except the number of quotes per COICOP section. This is of limited use in practise as sampling costs depend on the number of locations and outlets much more than the absolute number of quotes.

Compared with the earlier study following the Ashworth and Green approach, the additional work on CPI sampling procedures- not reported in detail here- necessarily makes additional constraining assumptions in terms of imposing more structure on the variance, in order to provide a framework for sample optimisation where numbers of locations and shops can be varied. It focuses on the price movement and starts with the working assumption that:

Price movement (t/t-1) = COICOP by region effect + random location effect +random item effect + random product effect (assumes independence).

By expressing index variance as a function of the number of locations per region, items per COICOP class and quotes per item plus the variances associated with location, item, and product effects, estimators can be computed for the variances associated with the location, item and product effects which can then be used in the expression for index variance to minimise variance with respect to locations, items and quotes to get an optimum allocation.

It compliments the work reported here but more directly relates to practical sampling issues by recognising practical issues of stratification and by focusing on price movements

[9] This dataset could also be used to provide a more reliable weighting structure. The current weights come from a variety of sources, the most important of which is the Expenditure and Food Survey (EFS). This is a continuous survey of around 7,000 households, compared with 15,000 in the TNS panel so, theoretically, the TNS panel could give us a more accurate picture of consumer expenditure patterns. There is also scope for regional and shop type weights to be improved as the TNS data can be stratified for these purposes. Following on from the work on sample optimisation, the TNS data ought to give us an insight into what happens to the variance when we are collecting more quotes for each item/group.

For several years CPGI has used household scanner data, particularly for electrical items, to try to make our collection more representative of expenditure patterns. However this data covers a relatively small proportion of COICOP categories and does not cover all outlets expenditure and we have not found it clean enough to provide alternative measures of price changes. Recently we have purchased a dataset detailing transactions from a sample panel of 15,000 households from the market research company, TNS. The dataset is stratified in that it covers all ages, gender and social class, so that the proportions in their sample match that of the population. It also covers every region of the UK.

The dataset provides a breakdown of household expenditure between October 2002 and October 2005. It only covers a fraction (150 out of 1000) of total RPI expenditure as the focus is towards groceries although other household goods, such as cleaning products and toiletries, are included.

Each household on the panel is required to scan in all of their shopping purchases within their own home. The dataset contains a thorough breakdown of each individual transaction, detailing bar codes, product codes, product descriptions, market categories, week of transaction and the price of the goods at the till.

There are issues over the reliability of some of the TNS statistics but, although different in nature, in practice they are not different from other scanner data. For example, when extracting monthly data files from the household panel, it is clear that there are substantially less observations for December. This may be attributed to the members of the panel neglecting to scan in their items as they have more pressing concerns in the lead up to Christmas. Consequently, we might expect to see more volatile price changes in December.

Another issue is how to manipulate the dataset so that it matches our reference population. Currently, the RPI excludes pensioners living on state benefits and also the top 4% of the population in terms of household income. Excluding the top 4% of earners is potentially problematic. TNS stratifies its households by social grading and it can be expected that quite a large number (but not all) of those included in the highest social grade which includes higher managerial staff and professionals will be outside the top 4% of earners. For the purposes of this investigation, households of all social grade have been included. Despite these obvious difficulties the dataset still provides our best opportunity of comparing our own collection with the population as a whole.

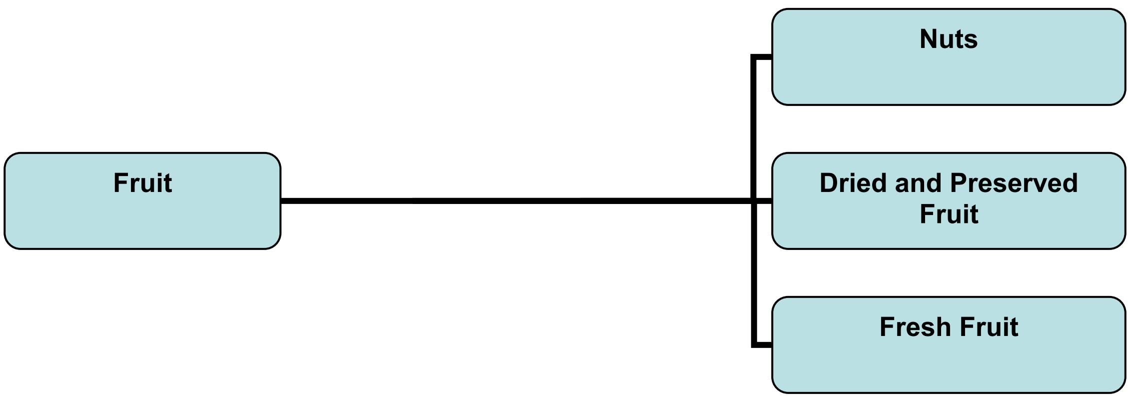
4.1.3 Work carried out to date

The results reported here are very much a work in progress. The TNS data has been used to calculate a Laspeyres index which we can compare directly with our RPI figures. The parallel TNS and RPI indices run between October 2002 and October 2005 and have been used to establish whether there is any discernible difference between the indices. The base period has been set at Jan 2004 = 100.

There is very little common ground in the market categories used by TNS and the COICOP classification we use for the CPI. This complicated the extraction of data as each TNS product has to be allocated to the relevant COICOP category by looking at the various attributes recorded for it. Matching up product codes with product description has allowed us to create tables by COICOP+ categories, a further breakdown of the COICOP classifications, as illustrated below.

It was decided that COICOP classes covered too broad a range of items. A COICOP+ level of aggregation was chosen as it represented a small enough group of items which could be investigated further, if necessary.

COICOP classification *COICOP+ categories*



An index has been constructed for each of these Coicop+ categories, the results of which are shown in Figure 3.

A Kolmogorov-Smirnov test was carried out on each of the Coicop+ groups to establish the significance of the difference between RPI and TNS indices.

4.2 Analysis

4.2.1 Summary

The results of the investigation have raised as many questions as answers. While average prices for the dataset as a whole move in line with those in our collection, the table below shows significant differences at the COICOP+ level.

Figure 3: Comparison of TNS and ONS Price Indices

Coicop+ Groups	TNS Coicop+ Weight	RPI Coicop+ Weight	COICOP GROUP	TNS Coicop Weight	RPI Coicop Weight	TNS Average Annual % change	RPI Average Annual % change	No. of TNS Products	No. of TNS Quotes	No. of RPI Items	No. of RPI Quotes*	Correlation of 12mnth Inflation
Rice	0.30%	0.25%	Bread and Cereals	8.67%	10.10%	-3.33%	-5.30%	884	216,844	1	11,128	0.52
Bread and Non-Choc. Biscuits	6.99%	7.48%				2.00%	-0.12%	19,721	5,617,864	14	146,605	0.15
Chocolate Biscuits	0.79%	1.37%				0.43%	0.14%	793	681,778	2	25,348	-0.03
Pasta Products	0.48%	0.91%				-1.33%	-1.61%	2,055	740,197	2	26,065	0.41
Cereals	0.11%	0.09%				6.29%	6.33%	241	139,409	1	10,724	0.89
Beef	2.28%	2.70%	Meat	17.90%	13.78%	3.90%	0.44%	1,084	574,572	5	86,301	0.54
Pork	1.05%	0.92%				1.83%	2.37%	993	289,643	2	39,547	0.53
Lamb	0.84%	1.39%				3.59%	2.04%	1,674	144,775	5	64,531	0.73
Poultry	5.41%	3.36%				0.73%	0.35%	8,184	1,619,472	7	95,420	0.30
Other Meat	8.32%	5.42%				0.36%	0.94%	13,180	3,989,964	9	147,109	-0.35
Fish	2.45%	1.21%	Fish	3.31%	2.25%	-0.96%	-0.64%	4,494	749,823	2	24,221	0.37
Processed Fish	0.86%	1.04%				0.57%	1.50%	1,349	463,377	3	36,505	0.01
Milk	3.91%	3.45%	Milk, Cheese and Eggs	9.99%	9.02%	3.66%	3.61%	958	2,172,969	3	25,399	0.88
Milk Products	2.17%	2.79%				0.78%	0.30%	3,206	1,743,017	5	47,238	0.01
Cheese	3.09%	2.08%				2.61%	1.29%	6,257	1,709,712	6	58,451	0.48
Eggs	0.82%	0.69%				5.58%	3.60%	433	570,014	2	21,594	0.81
Butter	0.51%	0.69%	Oils and Fats	1.59%	1.39%	2.76%	2.68%	172	326,847	2	21,486	0.88
Oils	1.07%	0.69%				1.28%	-0.08%	842	762,297	2	25,262	0.37
Fruit	4.94%	4.10%	Fruit	5.88%	4.80%	-0.01%	-0.88%	3,699	3,267,992	12	178,485	0.58
Dried Fruit	0.58%	0.28%				1.45%	2.34%	2,479	493,103	1	10,582	0.73
Nuts	0.37%	0.42%				-0.21%	0.07%	1,473	213,433	1	10,838	0.76
Raw Potato	1.25%	1.39%	Vegetables	10.66%	10.11%	5.73%	2.35%	1,564	885,963	6	55,782	0.93
Crisps	0.90%	1.55%				-0.19%	0.80%	1,081	641,275	3	32,665	0.53
Processed Potatoes	0.74%	0.61%				-1.47%	-1.51%	839	525,076	1	14,530	0.13
Veg	7.78%	6.57%				1.53%	2.36%	13,411	7,965,656	20	254,068	0.84
Sugar	0.83%	0.69%	Sugar, Jam, Honey, Syrups, Chocolate and Confectionery	5.41%	8.73%	3.73%	3.89%	1,881	664,247	2	25,368	0.66
Confectionery	3.23%	6.79%				-0.12%	2.77%	7,776	1,923,423	10	99,174	0.32
Ice Cream	1.34%	1.24%				-0.24%	-0.44%	3,383	696,724	2	17,430	-0.09
Coffee	1.04%	0.65%	Coffee, Tea and Cocoa	1.81%	1.46%	-0.84%	-0.11%	1,120	322,427	2	25,393	0.84
Tea	0.71%	0.69%				-2.02%	-4.33%	828	309,783	2	19,471	0.20
Other hot drinks	0.06%	0.12%				-0.41%	2.23%	318	38,509	1	9,424	-0.41
Fruit & veg juices	2.29%	1.82%	Mineral Waters, Soft Drinks & Juices	4.75%	7.50%	-0.83%	-1.94%	3,787	1,527,431	4	38,489	-0.34
Other soft drinks	2.46%	5.68%				0.24%	0.52%	3,613	1,572,912	9	108,029	-0.42
Spirits off trade	3.68%	3.70%	Spirits	3.68%	3.70%	-0.66%	-0.52%	1,354	216,728	4	51,902	0.19
Wine	5.24%	6.64%	Wine	5.24%	6.64%	0.26%	1.01%	3,594	534,950	7	89,170	0.25
Beer	3.48%	3.05%	Beer	3.48%	3.05%	-0.74%	-2.08%	1,906	299,785	5	51,826	0.44
Cleaning and maintenance	4.99%	3.97%	Clea	4.99%	3.97%	-2.25%	-1.23%	10,651	2,572,532	9	102,151	0.15
Pharmaceutical Goods	1.34%	1.96%	Pharmaceutical Goods	1.34%	1.96%	-3.36%	-0.83%	3,306	267,562	4	33,013	0.76
Pet Foods and Care	2.56%	2.62%	Pet Foods and Care	2.56%	2.62%	-0.27%	0.15%	5,997	1,981,733	3	40,014	0.75
Personal Care Articles	8.75%	8.92%	Personal Care Articles	8.75%	8.92%	-2.37%	-0.20%	20,679	2,701,109	17	192,753	0.47
			OVERALL TREND			0.40%	0.55%					
			TREND USING RPI WEIGHTS			0.38%	0.55%					
			TREND USING TNS WEIGHTS			0.40%	0.46%					

***Estimate derived from Apr 05 - Apr 06 quote data**

Figure 4: Average Annual % Change in the Index for each COICOP+ category

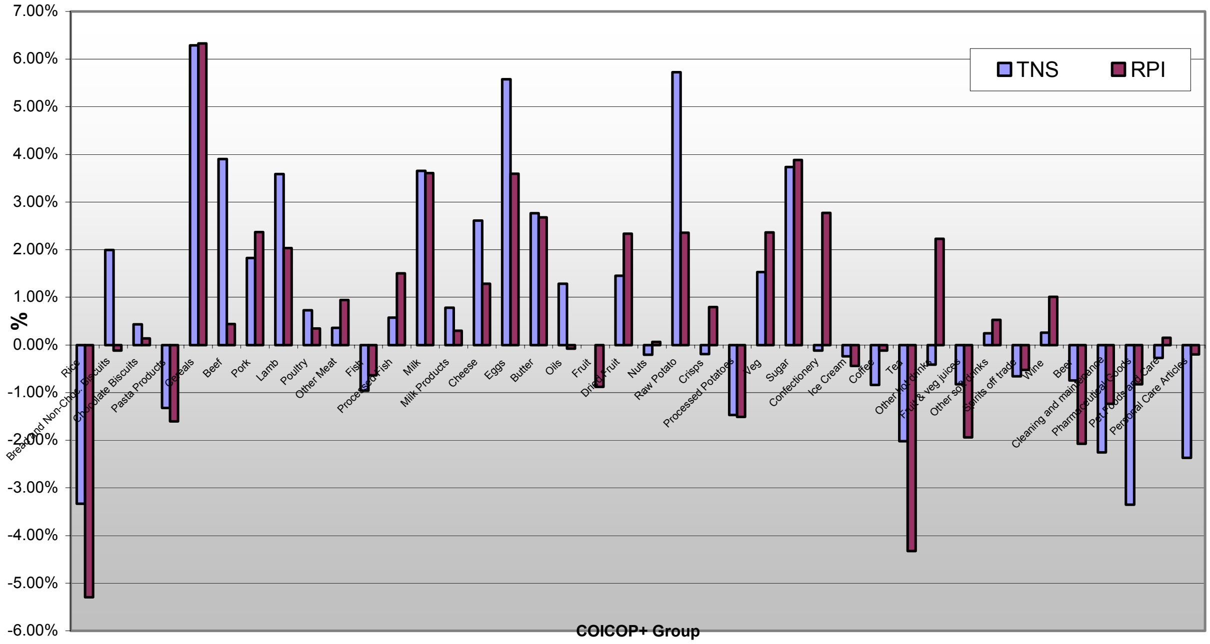
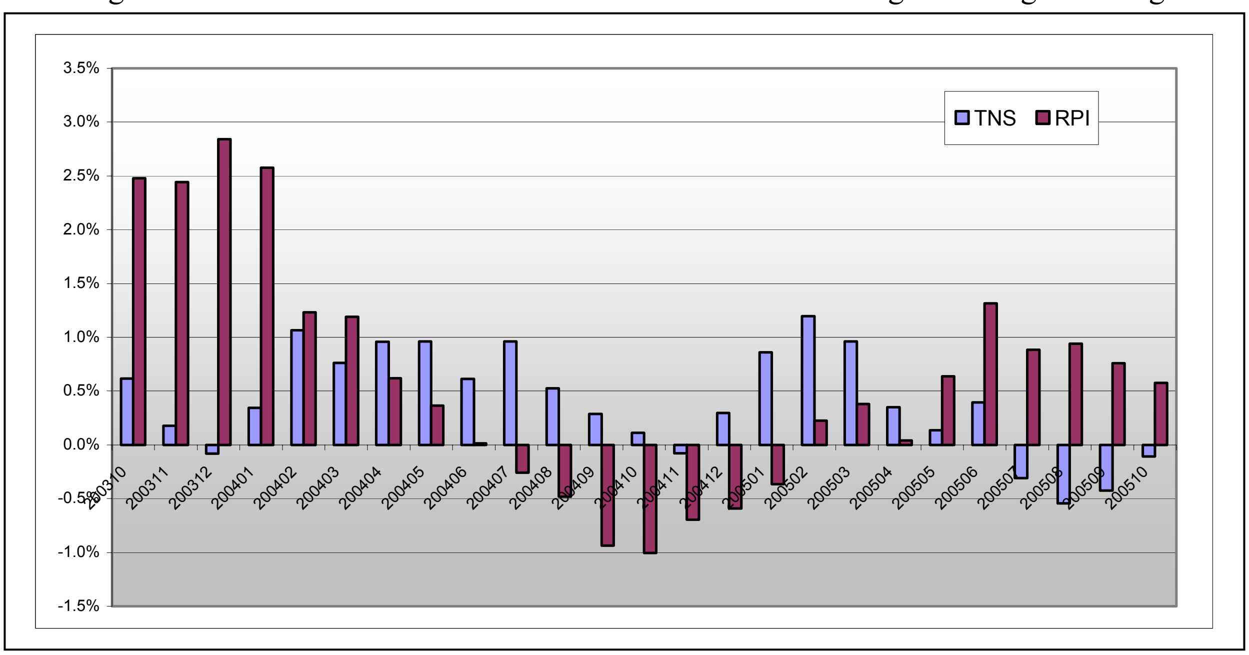


Figure 5: TNS Vs ONS Month on 12 Month Inflation Rate weighted using RPI weights

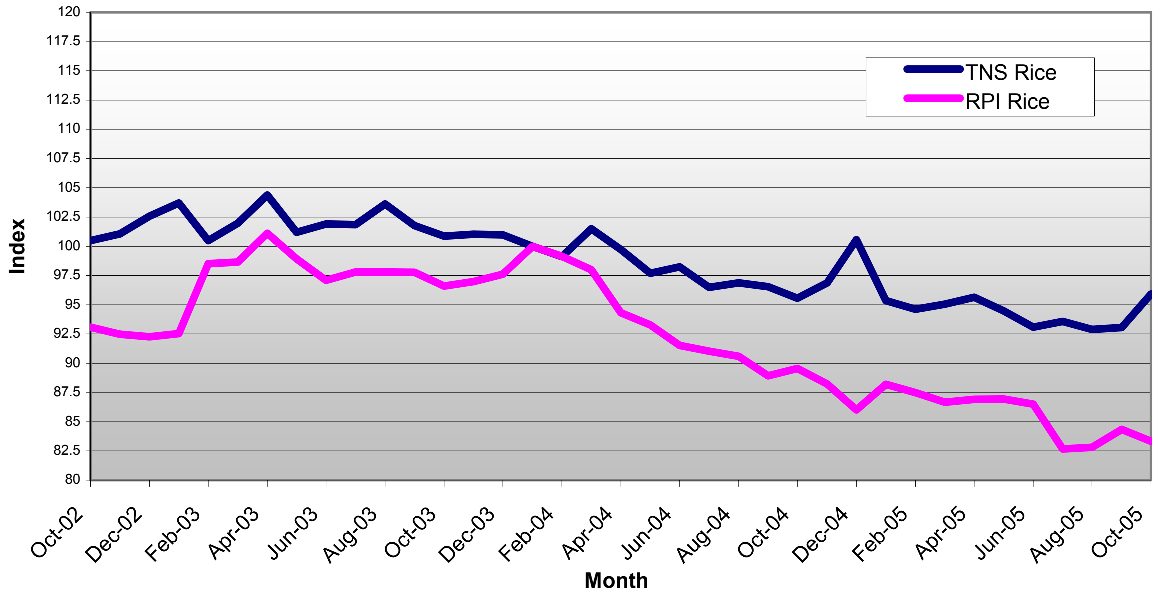


Perhaps more interestingly, the two charts above show that the average overall trend, 0.4% and 0.66% for TNS and RPI respectively conceals significant discrepancies in the headline monthly inflation rate which cannot be explained merely by timing effects.

The immediate question is whether these differences are a true inditement of the way we select items or are caused by the way we have constructed the TNS index (possibly due to selection issues). To examine this we have focussed on two apparently homogenous COICOP+ categories, Oils and Rice, as their charts clearly show a large change at a certain point in time.

4.2.2 Rice

Figure 6: A comparison between the TNS and RPI indices for Rice



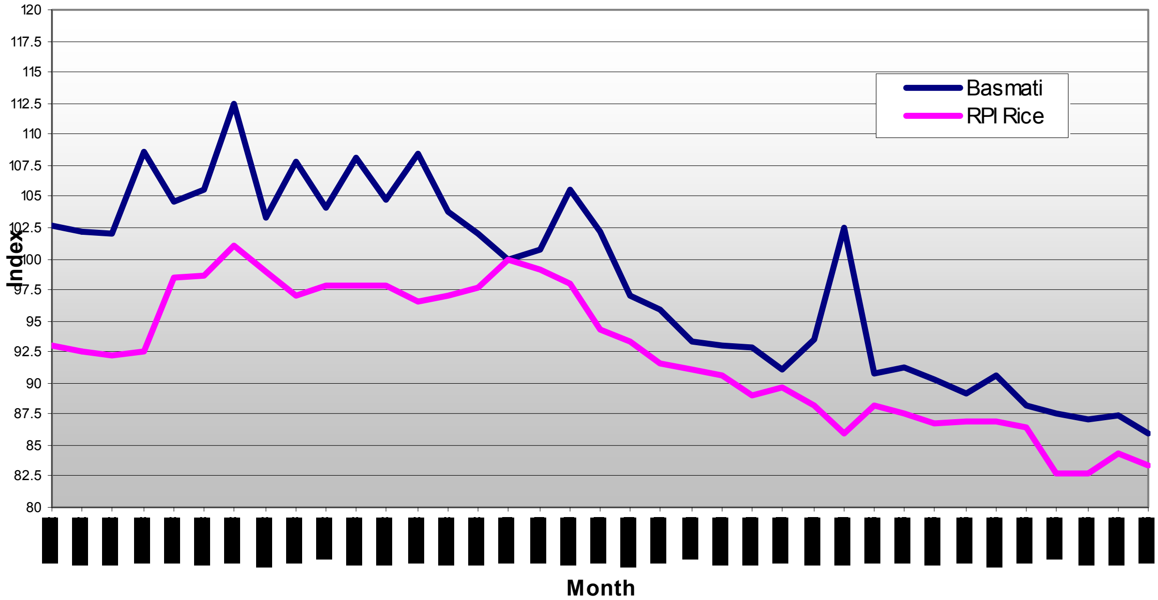
The first interpretation from the chart above is that the rate of decline in the RPI index is much stronger than the TNS dataset from the base period of January 2004. Within the RPI, there is only one item used to represent rice based on the assumption that all rice prices are homogeneous. Long Grain Rice was priced until January 2003, when it was replaced in the index by Basmati Rice. Basmati Rice was chosen to replace Long Grain as research carried out by Mintel[10] indicated that expenditure on Basmati has exceeded that for Long Grain Rice.

Since the introduction of Basmati Rice we have seen the index decline rapidly. According to a Mintel Report, Basmati Rice prices have dropped by almost 100% in the past two years as a result of grocery multiples slashing prices on their own brand products. If this is the case, the index could be susceptible to a downward bias unless the other types of rice it is representing are also falling at such a rate.

To investigate this further, we compare the RPI index with a TNS Basmati index, as this provides a like for like comparison.

Figure 7: A like for like comparison between the TNS Basmati index and the RPI Rice Index

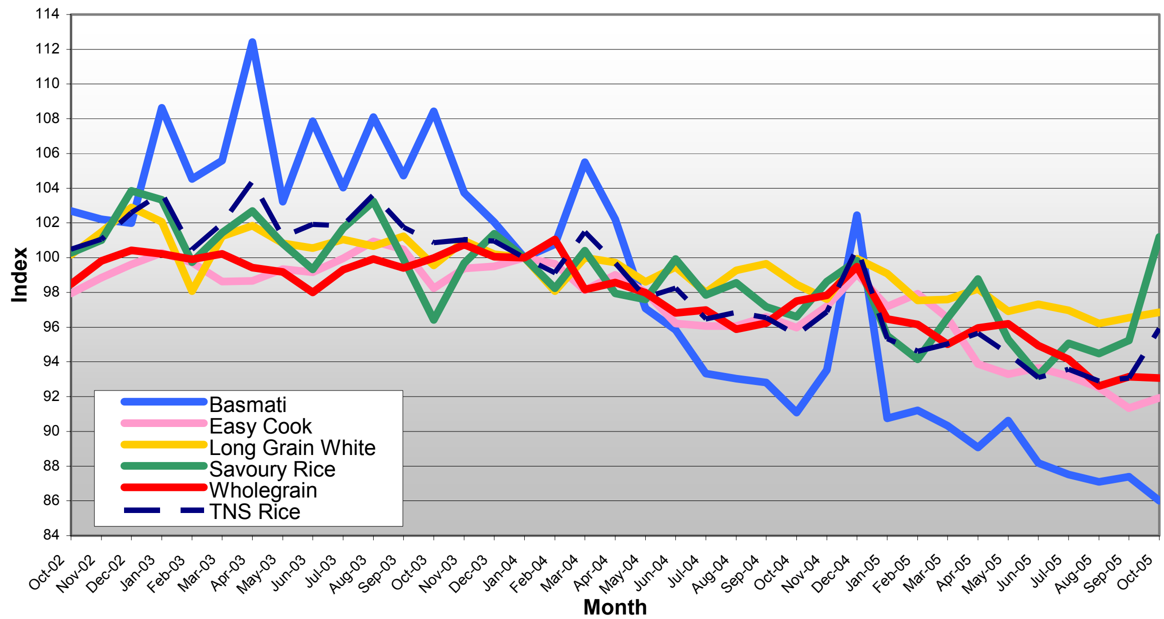
[10] Mintel – A market research and consumer intelligence company that provide reports on consumer expenditure patterns and products.



With the exception of December 2004, which will be further investigated later in this section, the TNS index follows a similar trend to the RPI index. This indicates that the RPI accurately tracks the price of Basmati rice. However, this chart also highlights that we are not accurately tracking the price movements of all types of rice, which Basmati is supposed to represent. The implication is clear- we should also be pricing other types of rice as TNS clearly shows that rice prices do not move in a uniform fashion despite the fact that superficially they represent a homogeneous category of consumption.

In order to establish the different price trends, we extracted data for several rice varieties from the TNS dataset and plotted their movements in the chart below.

Figure 8: An analysis of index trends for separate TNS Rice products



This chart clearly shows that other varieties of rice have smaller fluctuations in price. Basmati is by far the most volatile in terms of price movement, and is not a good indicator of rice prices generally. Further investigations suggest that the observed volatility stems from it being a specialist grain of rice that is mainly imported into the country from North West India and Pakistan and any local weather problems in that area can lead to a sharp change in price.

All other types of rice have shown a steadier decline and a similar trend. The conclusion is clear- it is necessary to price more rice items in order to maintain the representativity of the RPI. This would help to push the RPI index back up towards the TNS index and would provide a more accurate barometer for the rice market than our current Basmati-only index. The inclusion of an extra item would also help to reduce index variability, assuming similar price trends in the future.

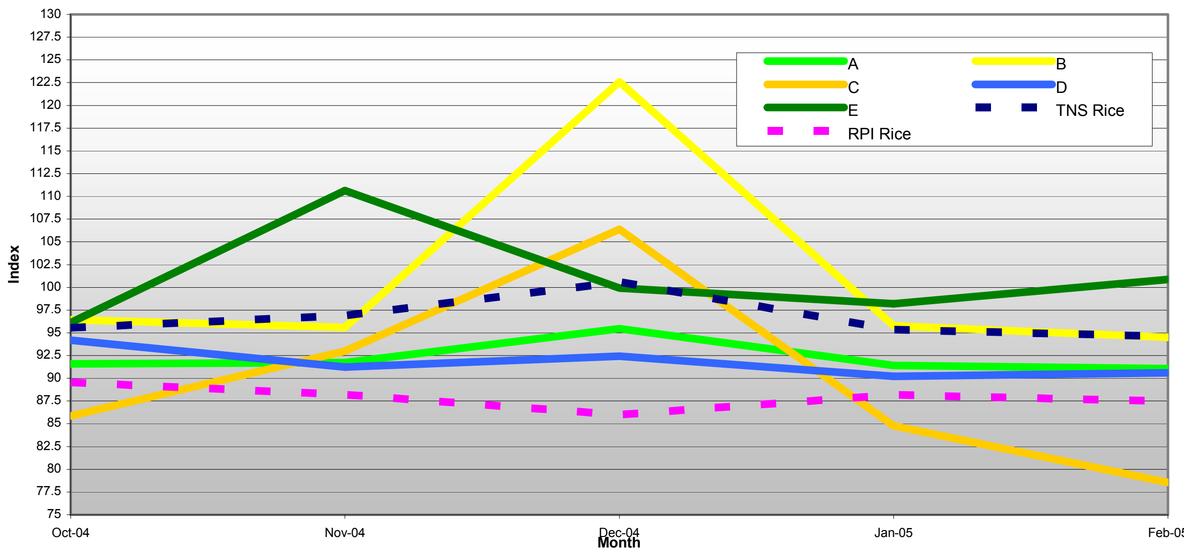
It is instructive to note that the initial reason for selecting Basmati as our representative item was that, out of all varieties, it made up the largest share of expenditure. A look at the number of quotes for each variety of rice in the TNS dataset shows that Basmati makes up only 16% of all rice quotes. Although this does not reflect the relative expenditure, this table combined with the analysis shown earlier demonstrates that there are large sections of the market on which we are not achieving adequate coverage.

TNS Category	TNS Quotes	% of Total Quotes
Basmati	35588	16.41%
Easy Cook	12007	5.54%
Long Grain White	46940	21.65%
Savoury Rice	102586	47.31%
Wholegrain	7926	3.66%
Others	11797	5.44%

As seen in one of the earlier charts there was a significant upturn in prices on Basmati rice in December 2004 based on the TNS index compared with a sharp drop in the RPI figure. We have already expressed reservations about the reliability of TNS data for December. Although the relatively low number of quotes may exacerbate the price increase, there are still almost 4 times as many quotes in the TNS data compared to the RPI so there is still a question mark over the RPI figure.

The story behind the RPI fall is that there was a 20% reduction in 3 regional supermarkets and a drop of 7% in one other. The two supermarkets involved, C and D, show a different pattern in the TNS data, illustrated in the following chart.

Figure 9: Analysis of TNS activity in the major supermarkets between Oct. 2004 and Feb. 2005



If the TNS dataset is to be believed there was a large increase on the price of Basmati, an argument which is supported by the FAO[11], which blamed the rising prices on stocks being hit by floods. This resulted in a limited supply of Basmati from India as well as concerns over future supplies.

The two supermarkets responsible for the price decrease in the RPI both show an increase on the TNS data between November and December 2004. One possible reason for the different results could be the pricing period used to collect prices. In the current RPI system, prices are collected on the middle Tuesday of every month and one day either side, whereas the TNS prices are taken from every day in the month.

The analysis also brings into focus the influence of the regional supermarkets in the current RPI methodology, an influence which, dependent on market circumstances, can lead to misleading

[11] Food and Agriculture Organisation of the United Nations – Information taken from the Food Outlook, December 2004.

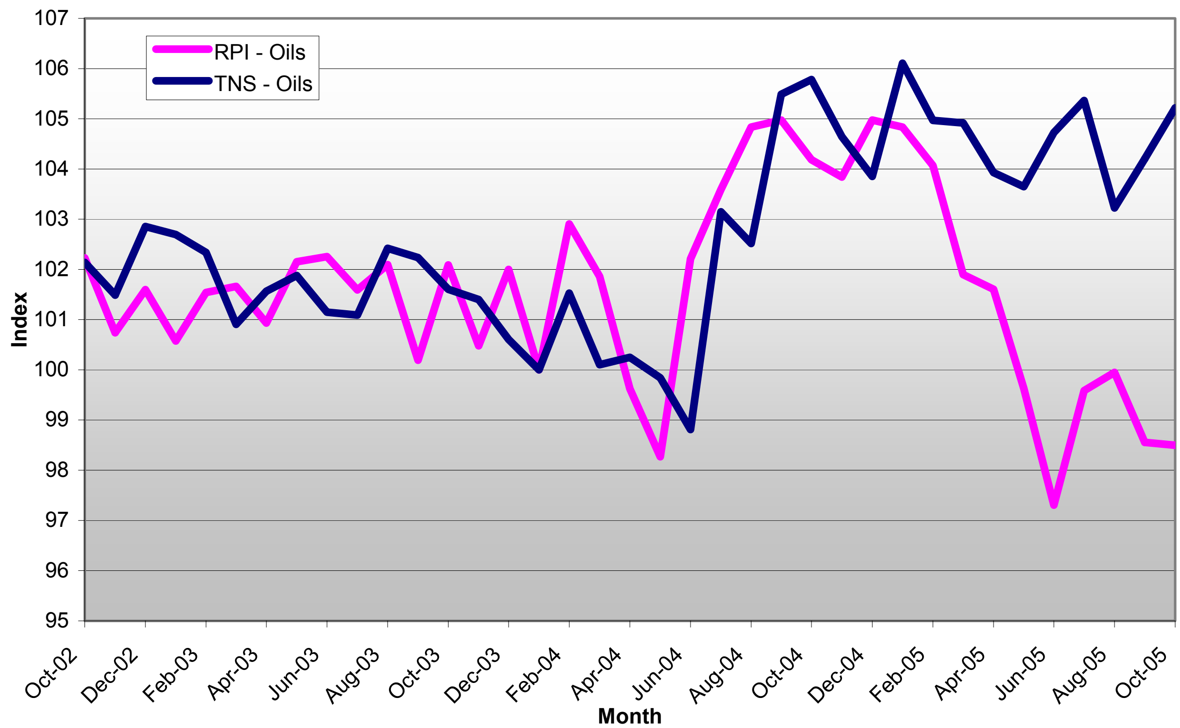
trends. Thus the current method of RPI price collection is to collect one item from one store in each region for all of the large multiple supermarkets and use this to represent the entire region.

This research could also have repercussions for the sample allocation. The analysis of the rice market puts forward a case for increasing the number of quotes collected in each store rather than base an entire region on one quote. Consequently, this analysis confronts us with a number of questions which rhe TNS data should help to answer:

- Is Basmati rice the most suitable indicator to represent rice prices in general due to its volatility?
- Should we be pricing more than one item for rice to counteract the volatile nature of Basmati?
- Does the pricing period need to be extended over the course of the month?
- Is it appropriate to replicate one quote for rice in a supermarket across the whole region?
- Should we collect more quotes within a regional supermarket?

4.2.3 Oils

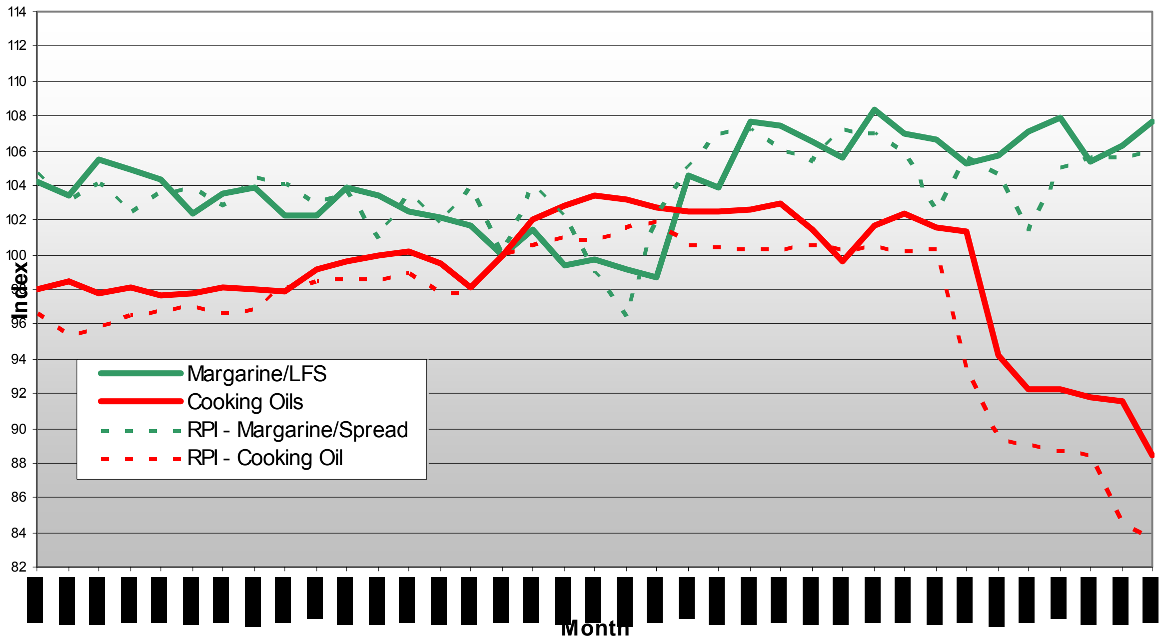
Figure 10: A comparison of TNS and RPI indices for Oil



It can be seen that since the beginning of 2005 there has been a clear divergence in the indices for Oils. This category within the RPI is calculated from two items, Margarine/Low Fat Spread and Cooking Oil.

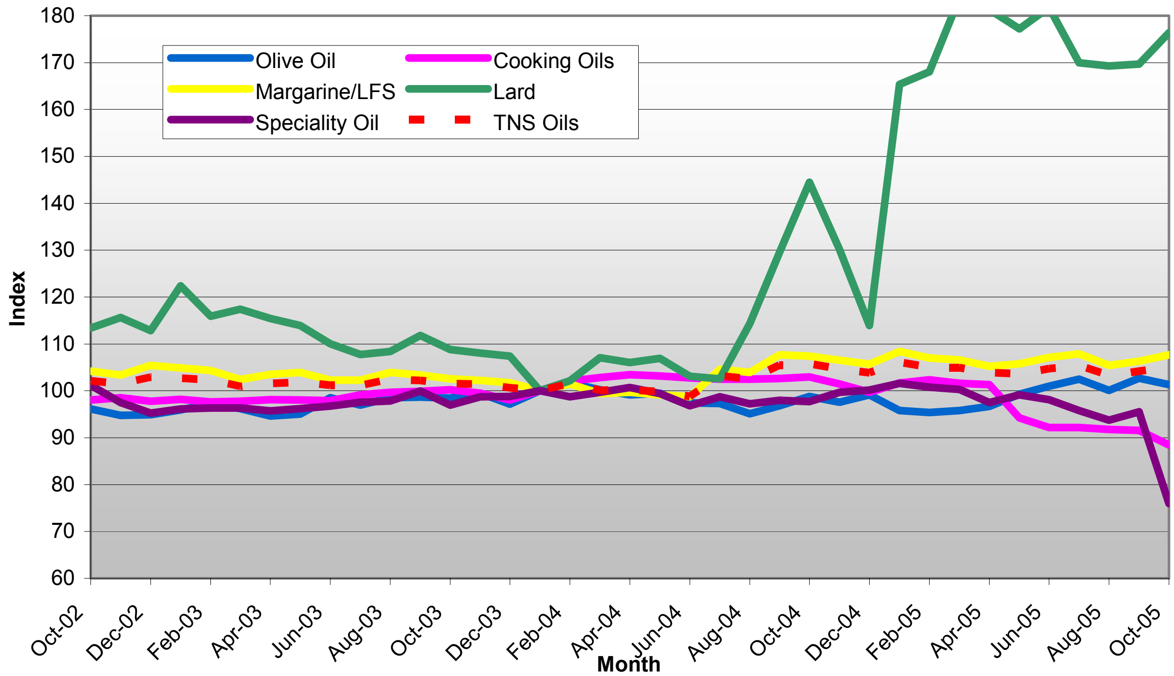
The TNS dataset contains similar product areas and these are shown in the Chart below.

Figure 11: Comparing TNS and RPI indices for similar product categories



In a similar fashion to what we saw in the Rice index, there is little difference between the TNS and RPI indices for the two items that we price but a more detailed examination of the data reveals a more complex story elsewhere in this category which is not being reflected in the RPI index. In particular, a look at the different types of oils in the more detailed and broader-coverage TNS data shows that there are substantial differences in how prices move in the Oils market.

Figure 12: Analysis of the TNS indices for product categories within Oils



This chart would suggest that Lard prices (not covered in the RPI), which experienced huge price increases in 2004, may be the reason for the TNS index being higher than the RPI[12]. However, as the relative weight of lard in the TNS dataset is minimal compared with other categories, this does not provide the full explanation. Similarly, the price of Olive Oil (also not represented in the RPI) has seen a small upturn in price compared with Cooking Oils and Speciality Oils but that again is not enough to explain the difference in index values.

By analysing the number of quotes for each group we can see that the vast majority of quotes in the TNS data are for Margarine/Low Fat Spreads, approximately 75%, whereas in the RPI it only represents 67% of the market. Similarly, Cooking Oil, which represents 33% of the weight in the RPI, only has 13.8% of the quotes in the TNS.

TNS Group	Quotes	% of Total Quotes
Olive Oil	45317	6.15%
Cooking Oil	101670	13.79%
Lard	25838	3.50%
Margarine/LFS	556599	75.49%
Speciality Oils	7875	1.07%

[12] Prices increased after new EU countries purchased the fat which the UK would usually use for lard and used it for their sausages, resulting in a limited supply to the UK.

Applying these weights to the RPI index would diminish the impact of the falling cooking oil prices and push the index back up towards that of the TNS dataset, in large part due to the increasing importance of Margarine/Low Fat Spread.

Again, we are confronted with a number of questions which need to be answered;

- Are we weighting our items correctly?
- Are the items we are selecting truly representative of other similar items?
- Should we widen our item descriptions to include items such as Olive Oil and even Lard?

4.3 Conclusions and Further thoughts

The initial results of the investigation have certainly provided some intriguing questions, not all of which can be answered at this time. The fact that the apparent contradictions between TNS and ONS price movements seem to disappear when we focus on particular products suggests both that our TNS index calculation procedures are comparable to the ONS indices and that our outlet selection is working well, at least for this segment of the market. However this leaves us with the problem that our item selection appears to leave too many products uncovered and prices for these products may move quite differently from those in our sample on a month to month basis and in some cases can be particularly volatile.

Diversity in variety selection would seem to be important to guarantee the representativity of a sub-index, even with apparently homogeneous items. Whether for the index as a whole this is so important is another issue as the spread of items may iron out the short-term fluctuations, but an index where there is a degree of unrepresentativeness and volatility in the sub-indices not reflected in the market place clearly impedes analysis and an understanding of inflation. There are obvious implications for the basic structure of the RPI sample, in particular the number of quotes collected per item, and the annual update of items. The fact that prices in specific shops can differ widely also suggests that spreading the collection across more outlets may be desirable.

Acknowledgements

This research on sampling issues benefited from a strong team of researchers being in place to take the work forward. Contributions were received both from within Consumer Prices and General Inflation Division and from elsewhere in ONS. Of the joint authors of this paper, acknowledgement is given to Daniel Melser who provided a major contribution to the work on sample optimisation and whose work is reported here. Daniel worked with great enthusiasm in his relatively short time with ONS and thanks should be given to Erwin Diewert for putting Daniel in our direction. On home grown talent, Peter Moran undertook the major share of the work on the relationship between measured prices and prices relating to the total population of consumer purchases and there is no doubt that this will be valuable in informing future decisions. The work continues.

David Fenwick.
May 2006.

Table 1: Optimal Sample Allocation (Average over the years 2002 and 2003).

COICOP Id.	COICOP Description	Avg. CPI Weight (%)	Arithmetic Variance (%)	Geometric Variance (%)	Actual /Current Allocation	Arithmetic Optimal Allocation	Geometric Optimal Allocation	Arithmetic Difference	Geometric Difference
10101	BREAD & CEREALS	1.35	3.46	3.69	2385	2162	2213	-222	-172
10102	MEAT	1.95	4.40	5.23	3577	3236	3366	-341	-210
10103	FISH	0.32	0.64	0.76	558	519	546	-39	-12
10104	MILK, CHEESE & EGGS	1.17	1.68	2.02	1593	1371	1435	-222	-158
10105	OILS & FATS	0.20	0.27	0.31	474	266	278	-208	-196
10106	FRUIT	0.76	4.17	4.75	2042	2026	2079	-16	37
10107	VEGETABLES INCLUDING POTATOES AND OTHER TUBERS	1.34	4.06	4.84	3143	2686	2782	-458	-361
10108	SUGAR, JAM, HONEY, SYRUPS, CHOCOLATE AND CONFECTIO	1.18	0.69	0.81	1358	794	828	-564	-530
10109	FOOD PRODUCTS	0.30	0.50	0.58	916	471	493	-445	-424
10201	COFFEE, TEA, COCOA	0.30	0.58	0.61	522	440	455	-82	-67
10202	MINERAL WATERS, SOFT DRINKS AND JUICES	0.95	2.06	2.25	1717	1408	1454	-309	-263
20101	SPIRITS	0.50	0.37	0.47	639	617	660	-22	21
20102	WINE (INC PERRY)	0.90	1.19	1.48	643	1224	1306	581	664
20103	BEER	0.51	1.13	1.34	451	904	959	453	509
20200	TOBACCO	1.48	0.02	0.02	1030	263	274	-767	-756
30102	GARMENTS	4.47	24.57	25.32	9110	15437	14860	6327	5750
30103	OTHER ARTICLES OF CLOTHING & CLOTHING ACCESSORIES	0.29	0.94	0.91	783	646	622	-138	-162
30104	DRY_CLEANING, REPAIR AND HIRE OF CLOTHING	0.09	0.01	0.02	213	45	48	-167	-165
30200	FOOTWEAR INCLUDING REPAIRS	0.73	2.42	2.55	3475	1994	1969	-1481	-1506
40100	ACTUAL RENTS FOR HOUSING	2.30	3.82	1.79	894	1941	1672	1047	779
40301	PRODUCTS FOR THE REGULAR REPAIR OF THE DWELLING	1.05	0.63	0.75	1194	1022	1059	-172	-135

40302	SERVICES FOR THE REGULAR REPAIR OF THE DWELLING	0.50	0.42	0.21	992	303	296	-688	-696
40503	LIQUID FUELS	0.10	0.05	0.07	103	116	125	13	22
40504	SOLID FUELS	0.10	0.01	0.01	192	45	48	-147	-144
50101	FURNITURE, FURNISHINGS	1.77	12.60	8.69	2855	6030	5278	3175	2423
50200	HOUSEHOLD TEXTILES	0.80	2.66	3.01	1213	2219	2235	1006	1022
50301	MAJOR HOUSEHOLD AND SMALL ELECTRICAL APPLIANCES	0.64	0.63	0.74	1553	916	975	-636	-578
50303	REPAIR OF HOUSEHOLD APPLIANCES	0.05	0.01	0.01	116	26	26	-90	-90
50400	GLASSWARE, TABLEWARE AND HOUSEHOLD UTENSILS	0.63	1.77	1.80	1153	1291	1304	138	151
50500	TOOLS AND EQUIPMENT FOR HOUSE AND GARDEN	0.50	0.39	0.43	1299	504	523	-795	-776
50601	NON-DURABLE HOUSEHOLD GOODS	0.43	0.66	0.76	1019	599	630	-421	-390
50602	DOMESTIC SERVICES AND HOME CARE SERVICES	0.60	0.10	0.14	371	251	261	-120	-110
60101	PHARMACEUTICAL PRODUCTS	0.36	0.32	0.36	317	392	412	75	95
60102	OTHER MEDICAL PRODUCTS & THERAPEUTIC EQUIPMENT	0.33	3.25	0.94	726	510	367	-217	-360
60201	MEDICAL SERVICES AND PARAMEDICAL SERVICES	0.25	0.26	0.35	247	216	229	-31	-18
60202	DENTAL SERVICES	0.08	0.14	0.06	105	58	52	-46	-53
60300	HOSPITAL SERVICES	0.53	0.08	0.09	109	200	208	91	99
70102	MOTOR CYCLES AND BICYCLES	0.14	0.09	0.11	339	152	161	-187	-178
70201	SPARE PARTS & ACCESSORIES	0.43	0.19	0.25	724	318	334	-406	-390
70202	FUELS & LUBRICANTS	2.69	0.10	0.13	1131	766	827	-365	-304
70203	MAINTENANCE & REPAIRS	2.14	0.62	0.85	402	1230	1291	829	889
70204	OTHER SERVICES IN RESPECT OF PERSONAL TRANSPORT EQ	0.58	0.15	0.18	1159	332	341	-827	-818
70302	PASSENGER TRANSPORT BY ROAD	0.13	0.04	0.04	108	66	67	-42	-41
80200	TELEPHONE AND TELEFAX EQUIPMENT AND SERVICES	0.06	0.07	0.09	118	100	107	-18	-11
90101	EQUIP. FOR RECEIVING & REPRODUCING SOUND & PICTURE	0.68	0.82	1.21	1165	1046	1120	-119	-45
90102	PHOTOGRAPHIC AND CINE. EQU. & OPTICAL INSTRUMENTS	0.50	0.45	0.57	345	631	675	286	330

90102	PHOTOGRAPHIC AND CINE. EQU. & OPTICAL INSTRUMENTS	0.50	0.45	0.57	345	631	675	286	330
90105	REPAIR OF AUDIO-VISUAL PHOTO. DATA PROCESSING EQU.	0.10	0.04	0.04	226	63	64	-163	-162
90301	GAMES TOYS AND HOBBIES	0.92	1.98	1.98	593	1250	1283	656	689
90302	EQUIPMENT FOR SPORT CAMPING & OPEN-AIR RECREATION	0.37	0.77	0.81	398	566	588	169	190
90303	GARDEN PLANTS AND FLOWERS	0.60	0.95	1.30	620	898	925	278	305
90304	PETS AND RELATED PRODUCTS	0.37	0.56	0.46	597	354	349	-243	-248
90401	RECREATIONAL AND SPORTING SERVICES	0.13	0.04	0.05	219	70	73	-149	-147
90402	CULTURAL SERVICES	0.49	3.15	4.08	253	1150	1200	897	947
90503	MISC. PRINTED MATTER, STATIONERY & DRAWING MATERIA	0.64	1.27	0.60	963	437	434	-526	-529
110101	RESTAURANTS & CAFES	9.67	2.13	2.58	8851	5055	5301	-3796	-3551
110102	CANTEENS	0.94	0.45	0.48	870	664	672	-207	-198
120101	HAIRDRESSING SALONS & PERSONAL GROOMING ESTABLISHM	0.75	0.13	0.16	730	363	375	-367	-355
120102	APPLIANCES, ARTICLES & PRODUCTS FOR PERSONAL CARE	1.77	3.49	4.04	1997	2783	2904	786	907
120301	JEWELLERY CLOCKS AND WATCHES	0.61	1.06	1.17	1144	1062	1047	-82	-96
120302	OTHER PERSONAL EFFECTS	0.24	0.42	0.50	916	451	466	-465	-451
120400	SOCIAL PROTECTION	1.10	0.15	0.22	523	423	442	-100	-81
	TOTAL	**55.89**	**100**	**100**	**73,475***	**73,374***	**73,369***	**-101**	**-106**

* These numbers are not the same because of rounding error at the level where allocation is undertaken (i.e. the number of quotes has to be an integer). However, it can be seen that any differences are very small.

ANNEX 1

FUNDAMENTAL PRINCIPLES OF OFFICIAL STATISTICS, ENDORSED BY THE UNITED NATIONS STATISTICAL COMMISSION, 1994.

Principle 1. Official statistics provide an indispensable element in the information system of a society, serving the government, the economy and the public with data about the economic, demographic, social and environmental situation. To this end, official statistics that meet the test of practical utility are to be compiled and made available on an impartial basis by official statistical agencies to honour citizens' entitlements to public information.

Principle 2. To retain trust in official statistics, the statistical agencies need to decide according to strictly professional considerations, including scientific principles and professional ethics, on the methods and procedures for the collection, processing, storage and presentation of statistical data.

Principle 3. To facilitate a correct interpretation of the data, the statistical agencies are to present information according to scientific standards on the sources, methods and procedures of the statistics.

Principle 4. The statistical agencies are entitled to comment on erroneous interpretation and misuse of statistics.

Principle 5. Data for statistical purposes may be drawn from all types of sources, be they statistical surveys or administrative records. Statistical agencies are to choose the sources with regard to quality, timeliness, costs and the burden on respondents.

Principle 6. Individual data collected by statistical agencies for statistical compilation, whether they refer to natural or legal persons, are to be strictly confidential and used exclusively for statistical purposes.

Principle 7. The laws, regulations and measures under which the statistical systems operate are to be made public.

Principle 8. Co-ordination among statistical agencies within countries is essential to achieve consistency in the statistical system.

Principle 9. The use by statistical agencies in each country of statistical concepts, classifications and methods promotes the consistency and efficiency of statistical systems at all official levels.

Principle 10. Bilateral and multilateral co-operation in statistics contributes to the improvement of systems of official statistics in all countries.

ANNEX 2

GOVERNANCE OF THE UNITED KINGDOM STATISTICAL SERVICE: NATIONAL STATISTICS

Responsibilities of Ministers

Minister for National Statistics

The Minister for National Statistics is the Chancellor of the Exchequer. In practice, this function has been delegated to the Economic Secretary to the Treasury.

The Minister for National Statistics appoints the chairman and members of the Statistics Commission, and sets resources for the Commission taking account of the Commission's views.

The Minister will, with the agreement of the Prime Minister and normally following open competition, appoint a National Statistician.

The Minister is responsible for maintenance and development of the co-ordinating structure for National Statistics in the UK, including the Framework for National Statistics. In doing so, the Minister will take account of the international context, including the UN fundamental principles for official statistics.

Departmental Ministers (including the Minister for ONS)

Are accountable for the statistical activities of their department and for the resources allocated to it, but will not normally become involved in the operational aspects of statistical work.

Determine the policy and resources framework within which the statistical work in their Department operates and set performance targets for National Statistics.

Decide the scope of National Statistics and departmental statistical programmes and resources having regard to the comments of the Statistics Commission on the proposed National Statistics programme.

Roles of the National Statistician

The National Statistician has several distinct roles:

UK government's chief adviser on statistical matters

Professionally responsible for those outputs comprising National Statistics - including the duty to maintain and demonstrate their integrity, and to promote coherence and compatibility - and to maintain and publish a Code of Practice to assist this end.

Permanent Secretary, Director and Principal Accounting Officer of ONS, responsible for the operation of the Office in accordance with its aims, objectives and functions, and for its financial management.

Head of the Government Statistical Service, the professional group of statisticians across government, and responsible for promoting the provision across government of co-ordinated, high quality, cost-effective and easily accessible statistics and promoting the integrity and validity of official statistics.

Registrar General of England and Wales, responsible for the registration of births, deaths and marriages in England and Wales and the conduct and publication of censuses of population.

Key roles of the Statistics Commission

To provide independent, reliable and relevant advice on National Statistics to Ministers, and by doing so, to provide an additional safeguard on the quality and integrity of National Statistics. It operates with transparency, and its advice is publicly available.

Assessing user needs, considering and commenting to Ministers on the high-level work programme for National Statistics, drawing on views of users and suppliers; and advising Ministers of widespread concerns about the quality of National Statistics.

Submitting an annual report to the Minister for National Statistics commenting on the annual report of the National Statistician and on the way the Commission fulfilled its remit.

Commenting on the application of the National Statistics Code of Practice and other procedures designed to promote statistical integrity.

Reviewing the need for statistical legislation after two years (by Summer 2002) and report to the Minister for National Statistics.

The Government Statistical Service (GSS) and Office for National Statistics (ONS)

National Statistics are produced by the GSS. The GSS is the professional group of statisticians and their statistical staff who produce statistical outputs and provide statistical advice / service to Ministers and policy officials. It includes departments in devolved administrations (except Northern Ireland).

The GSS is a decentralised organisation. Its largest element is the ONS, which is responsible for a large range of economic, social, demographic, and health statistics. The ONS is also responsible for co-ordinating statistical policy, planning, training and methodology. Many other Government Departments are directly responsible for the production of statistics related to their policy functions.

Devolution

Statistics in Northern Ireland, Scotland and Wales are the direct administrative responsibility of the new devolved administrations. Accordingly the *Framework for National Statistics* was signed by Ministers for each of the three devolved administrations in addition to the Economic Secretary to the Treasury, and it contains specific sections outlining how the Framework will operate in the three administrations.

A (non-legislative) Memorandum of Understanding covers co-operation and co-ordination of matters between the UK Government and the devolved administrations. This document includes a specific Concordat on Statistics. The *Framework for National Statistics* charges the National Statistician with the duty to "work with Heads of Profession in Scotland, Wales and Northern Ireland, to meet users' needs and to promote comparable statistics across the UK - in particular, this will be essential for statistics required to be submitted on a UK basis to the EU or international bodies."

The Legislative Basis for National Statistics

The context for National Statistics is a mixture of legislation, convention and policy. There are 18 statutes, which impact on statistics (e.g. Census Act 1920, Statistics of Trade Act 1947). Statistics legislation in the UK is scattered, incomplete and inadequate for the issues we now face, and does not allow National Statistics to be fully underpinned by a legal framework. The need for Statistics legislation will be the subject of a report to Ministers by the Statistics Commission by June 2002. Increasingly the legislative basis for collecting statistics is governed by European legislation.

Neither the roles of Minister for National Statistics nor that of National Statistician are defined in legislation.

ANNEX 3

BUSINESS EXCELLENCE MODEL

A strategic decision was taken by ONS to adopt the European Foundation for Quality Management's Excellence Model as a practical tool to identify progress made on the path to quality improvement and excellence in business performance, and more specifically to identify gaps and how these might be addressed. It is a self-assessment diagnostic tool that focuses on organisations and general business areas. It looks at performance against five criteria covering what the business area does (the enablers: leadership; people; policy/strategy; partnership/resources; processes) and four criteria on what the business area achieves (the results: people results; customer results; society results; key performance results). Evidence based on feedback from focus groups, questionnaires and personal interviews is used to assess performance across the criteria and a resulting action plan for improvement is introduced. This is then included in the business plan.

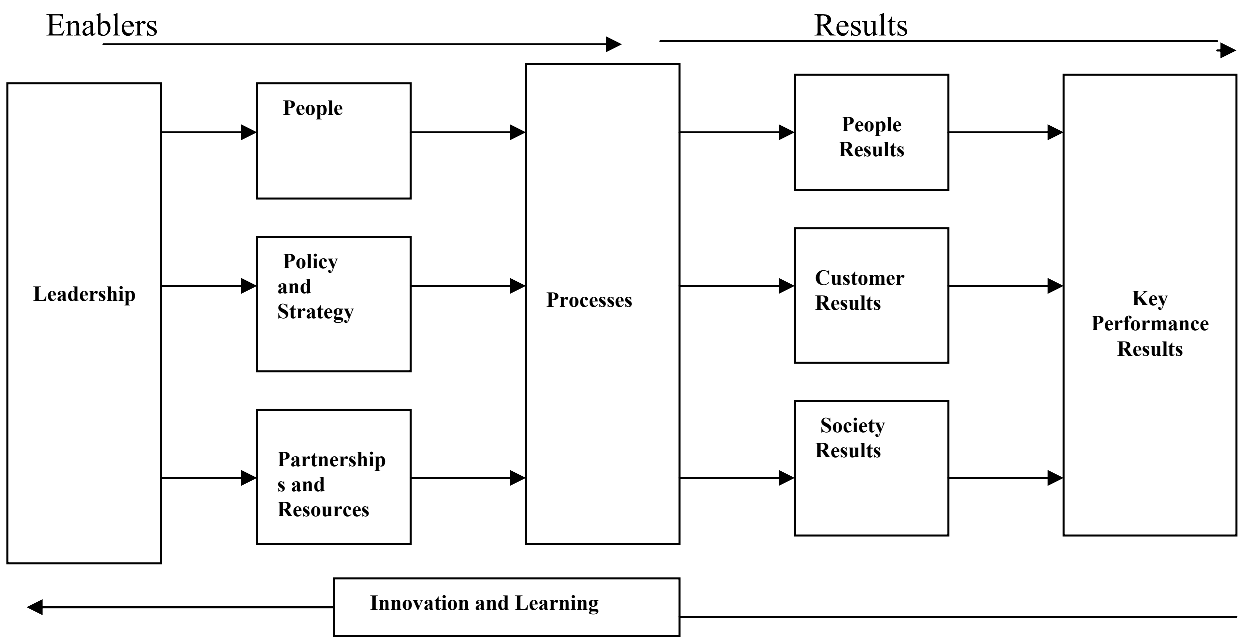
The model provides a useful umbrella for other initiatives and should not be seen as competing with existing quality improvement activities. For instance, it is being used by compilers of the UK Retail Price Index to identify the potential for business improvements in the context of previously achieved Investors in People (IiP) and ISO9002 accreditation.

A general outline of the model is given in Diagram 1. Each box represents a criteria that is linked to the principle underlying the Business Model that "To maintain or increase success as an organisation it is necessary to continually review and improve the way things are done, to strive to achieve better performance as measured by final outputs". The five "enablers" criteria represent what needs to be done to run an organisation. The five enablers describe how an organisation operates. The four "results" criteria are essentially achievements as identified by stakeholders including customers, employees and the community at large. At the heart of the Model is the logic referred to as RADAR, which is used as a matrix to score performance. This consists of:

- **R**esults. This refers to achievements against targets;
- **A**pproach. This covers an integrated set of plans to deliver the results;
- **D**eployment. This refers to systematic implementation in the appropriate business areas identified;
- **A**ssessment. This monitors and analyses the results achieved against objectives.
- **R**eview. This relates to plan, prioritise and implement improvement.

The Results element of RADAR are used to address each of the four "results" criteria and the other elements are used to address the five "enablers" criteria.

Diagram I: The Excellence Model



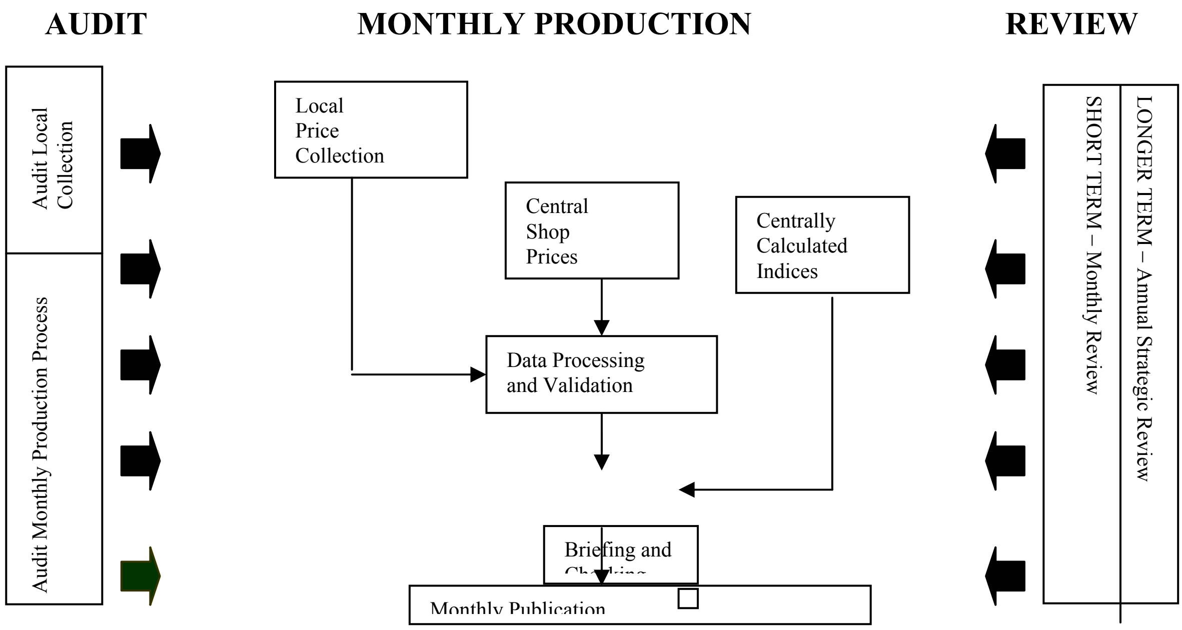
CPI QUALITY MANAGEMENT SYSTEM

Documentation of the Monthly Production Process

A key element of the RPI Quality Management System is that all aspects of the monthly production cycle are not only documented but also quality assured. Contract and Quality Management Branch review each aspect of the monthly and annual production cycle from price collection to index compilation and publication. These audits focus on a particular area of work and:

- check that operational procedures and controls that are carried out in practice comply with documented procedures;
- check that the documented procedures themselves are relevant and effective;
- highlight specific concerns and the action to be taken;
- check that the agreed action has been taken.

The action will often be to amend documentation where it is not clear or to remind staff of the correct procedures but there may be some instances where the procedures themselves need to be changed or where additional staff training is required. Each action is followed up by Contract and Quality Management Branch to ensure the work has been carried out and a re-audit carried out where considered necessary.



ANNEX 5

THE RESEARCH PROGRAMME ON CONSUMER PRICE INDICES

The current research programme was set up in 1998 largely in response to the Boskin Report (Boskin et al, 1996) and its agenda was heavily influenced by Boskin's concentration in identifying and eliminating bias[13]. The initial three year programme focussed on identifying issues and assessing their impact on the RPI. This was followed by more applied work with more emphasis on actually changing the way the index is calculated by improving methods for estimating quality change in new items (hedonics), and for selecting representative models of items in outlets (probability sampling using checkout scanner data).

The programme has also been heavily influenced by the development of the European Harmonised Index of Consumer Prices (known in the UK as the CPI) which the UK Government adopted as its inflation target in December 2003. The Government's requirement for estimates of differences in regional price levels has been another influence and a methodology for calculating these has been developed and results published for 2003 and 2004.

Methodological changes resulting from the research programme are introduced each year for the February index published in March.

Research activities have been guided throughout by the need to:

- ensure unbiased estimates of consumer inflation that measure what they purport to measure;

- ensure measures that are statistically reliable (i.e. a sufficient level of precision is achieved);

- make cost effective use of resources.

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1. Three Year Programme on RPI Methodology. Baxter and Camus, Economic Trends Volume 543, February 1999.

ANNEX 6

THE THEORECTICAL STOCHASTIC MODEL FOR THE PRICE INDEX

The aggregate index I^{0t} can be written as a weighted sum of subindexes.

$$I^{0t} = \sum_{n=1}^{N} w_{nt} I_n^{0t} \qquad (1)$$

Here the weights, w_{nt}, sum to one. Let us further suppose that the subindex is calculated using the Arithmetic method for aggregating the basic price data (i.e. $I_n^{0t} = I_n^{A,0t}$).

$$I_n^{A,0t} = \sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}} \frac{p_m^t}{p_m^0}, \qquad n = 1,...,N \qquad (2)$$

Here p_m^s represents a price for variety m in period $s = t,0$. The basic index is an unweighted arithmetic mean of price relatives. The subscript on the good's price denotes the sampled varieties within a stratum. The overall sample size in the stratum n at time t is M_{nt}. Let us also suppose that the prices for each of the strata (i.e. for each n) have the following stochastic properties.

$$\frac{p_m^t}{p_m^0} = \alpha_{nt} + e_{nmt}, \qquad n = 1,...,N, \; m = 1,...,M_{nt} \qquad (3)$$

$$E(e_{nmt}) = 0, \qquad \forall n = 1,...,N, \; m = 1,...,M_{nt} \qquad (4)$$

$$E(e_{nmt} e_{abt}) = \sigma_{nt}^2, \qquad \forall n = a \text{ and } m = b \qquad (5)$$

$$E(e_{nmt} e_{npt}) = 0, \qquad \forall n \neq a \text{ or } m \neq b \qquad (6)$$

This means that the price relatives are distributed around a mean with the expected value of their deviation from the mean equal to zero. The price relatives share a common variance within each of the n strata and the covariance for each price relative within and between strata is zero.[14] It is straightforward to see that an unbiased estimator of α_{nt} is $I_n^{A,0t}$,

$$\hat{\alpha}_{nt} = I_n^{A,0t} = \sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}} \frac{p_m^t}{p_m^0} \qquad (7)$$

As is well known, the variance term can be unbiasedly estimated by the following estimator.

[14] Note that this stochastic specification embodies very strong assumptions about the relationship between prices. One particular area of concern is with regard to the assumption that the covariance between price relatives within and between strata is zero. This condition is unlikely to be met in practice but more sophisticated models soon become intractable.

$$\hat{\sigma}_{nt}^2 = \frac{1}{(M_{nt}-1)}\sum_{m=1}^{M_{nt}} \left(\frac{p_m^t}{p_m^0} - \hat{\alpha}_{nt}\right)^2 \tag{8}$$

With this framework it is straightforward to show that, as long as some prices are surveyed in each stratum, the index will be unbiased. To see this formally note that,

$$E[I^{0t}] = E\left[\sum_{n=1}^{N} w_{nt} I_n^{A,0t}\right] \tag{9}$$

$$= \sum_{n=1}^{N} w_{nt} E[I_n^{A,0t}] \tag{10}$$

$$= \sum_{n=1}^{N} w_{nt} E\left[\sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}} \frac{p_m^t}{p_m^0}\right] \tag{11}$$

$$= \sum_{n=1}^{N} w_{nt} \sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}} E\left[\frac{p_m^t}{p_m^0}\right] \tag{12}$$

$$= \sum_{n=1}^{N} w_{nt} \alpha_{nt} \tag{13}$$

While the allocation of quotes is not important for bias it is relevant in terms of reducing the variance of the index. To see this note that,

$$Var[I^{0t}] = Var\left[\sum_{n=1}^{N} w_{nt} I_n^{A,0t}\right] \tag{14}$$

$$= \sum_{n=1}^{N} w_{nt} Var[I_n^{A,0t}], \quad \text{by independence} \tag{15}$$

$$= \sum_{n=1}^{N} w_{nt} Var\left[\sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}} \frac{p_m^t}{p_m^0}\right] \tag{16}$$

$$= \sum_{n=1}^{N} w_{nt} \sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}^2} Var\left[\frac{p_m^t}{p_m^0}\right], \text{ by independence} \tag{17}$$

$$= \sum_{n=1}^{N} w_{nt} \frac{\sigma_{nt}^2}{M_{nt}} \tag{18}$$

It can be seen that the variance of the index is inversely proportional to the sample size and as the stratum variance will differ by strata the overall variance of the index can be influenced by how the sample (i.e. the M_n's) are allocated.

ANNEX 7

THE SAMPLE ALLOCATION CRITERIA

Given equation:

$$Var[I^{0t}] = \sum_{n=1}^{N} w_{nt} \frac{\sigma_{nt}^2}{M_{nt}} \quad \text{(see Annex 6)}$$

a natural criterion for allocating the sample amongst the strata is to minimise the variance of the index subject to some cost-of-collection constraint. This can be formalised as the following optimisation problem where c_n denotes the cost of collecting an additional quote for stratum n and C_0 is total available resources for sample collection.

$$\min{}_{M_1, M_2, \ldots, M_N} \qquad Var[I^{0t}] = \sum_{n=1}^{N} w_{nt}^2 \frac{\sigma_{nt}^2}{M_{nt}}$$

$$\text{and} \qquad \sum_{n=1}^{N} c_{nt} M_{nt} = C_t \tag{1}$$

The optimal conditions for this minimisation problem, using the method of Lagrange, are,

$$\frac{\partial L}{\partial M_{nt}} = w_{nt}^2 \frac{\sigma_{nt}^2}{M_{nt}^2} - \lambda c_{nt}, \qquad n = 1, \ldots, N \tag{2}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{n=1}^{N} c_{nt} M_{nt} - C_t \tag{3}$$

With a little manipulation it is possible to write these conditions in a particularly useful way. First, note that by setting (2) to zero we have,

$$M_{nt} = \frac{1}{\sqrt{\lambda}} \frac{w_{nt}\sigma_{nt}}{\sqrt{c_{nt}}}, \qquad n = 1, \ldots, N \tag{4}$$

$$\Rightarrow \sum_{n=1}^{N} M_{nt} = \frac{1}{\sqrt{\lambda}} \sum_{n=1}^{N} \frac{w_{nt}\sigma_{nt}}{\sqrt{c_{nt}}} \tag{5}$$

Combining these two expressions we have,

$$M_{nt} = \frac{\left(\frac{w_{nt}\sigma_{nt}}{\sqrt{c_{nt}}}\right)}{\sum_{n=1}^{N} \left(\frac{w_{nt}\sigma_{nt}}{\sqrt{c_{nt}}}\right)} \left(\sum_{n=1}^{N} M_{nt}\right) \tag{6}$$

The intuition behind this condition is that a larger number of quotes should be collected for those strata with a high weight in the index and/or with a high variance. Note that it is difficult to know exactly the cost of collecting quotes in different strata. In the absence of such information it seems reasonable to assume that the costs are equal across strata $c_1 = c_2 = ... = c_N$. In this case we have the simplified condition,

$$M_{nt} = \frac{w_{nt}\sigma_{nt}}{\sum_{n=1}^{N} w_{nt}\sigma_{nt}} \left(\sum_{n=1}^{N} M_{nt} \right) \qquad (7)$$

ANNEX 8

A VARIANCE FOR A GEOMETRIC MEAN

The use of a geometric mean formula to aggregate prices at the elementary aggregate level can be expressed as follows:

$$I_n^{G,0t} = \prod_{m=1}^{M_{nt}} \left(\frac{p_m^t}{p_m^0} \right)^{\frac{1}{M_{nt}}}, \qquad n = 1, \ldots, N \tag{1}$$

There are potentially a number of different approaches to deriving a variance estimate for the geometric mean index. The approach adopt in this paper is to hypothesise a model of prices and then derive an approximate variance estimate using a Taylor Series approximation.

Suppose that prices are generated by the following model. Note that this is the logarithmic equivalent of the model outlined above.

$$\log\left(\frac{p_m^t}{p_m^0} \right) = \beta_{nt} + \varepsilon_{nmt}, \qquad n = 1, \ldots, N, \; m = 1, \ldots, M_{nt} \tag{2}$$

$$E(\varepsilon_{nmt}) = 0, \qquad \forall n = 1, \ldots, N, \; m = 1, \ldots, M_{nt} \tag{3}$$

$$E(\varepsilon_{nmt}\varepsilon_{abt}) = \phi_{nt}^2, \qquad \forall n = a \text{ and } m = b \tag{4}$$

$$E(\varepsilon_{nmt}\varepsilon_{npt}) = 0, \qquad \forall n \neq a \text{ or } m \neq b \tag{5}$$

It can readily be shown that the least squares (and Normal Maximum Likelihood) estimator for β_n^{0t} is the log of the geometric mean index. This is clearly unbiased.

$$\hat{\beta}_{nt} = \frac{1}{M_{nt}} \sum_{m=1}^{M_{nt}} \log\left(\frac{p_m^t}{p_m^0} \right) \tag{6}$$

Similarly, an unbiased estimator for the variance is,

$$\hat{\phi}_{nt}^2 = \frac{1}{(M_{nt} - 1)} \sum_{m=1}^{M_{nt}} \left(\log\left(\frac{p_m^t}{p_m^0} \right) - \hat{\beta}_{nt} \right)^2 \tag{7}$$

In this case it can be seen that,

$$Var[\hat{\beta}_{nt}] \qquad = Var\left[\frac{1}{M_{nt}} \sum_{m=1}^{M_{nt}} \log\left(\frac{p_m^t}{p_m^0} \right) \right] \tag{8}$$

$$= \sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}^2} Var\left[\log\left(\frac{p_m^t}{p_m^0} \right) \right] \tag{9}$$

$$= \frac{\phi_{nt}^2}{M_{nt}} \tag{10}$$

However, we want a variance estimate for the exponent of $\hat{\beta}_n^{0t}$ as this is equal to the geometric mean index. It is straightforward to show that, by taking a first order Taylor Series approximation to $\exp(\hat{\beta}_n^{0t})$, we can derive an approximate estimate of the variance. The result is shown below and derived in the Appendix.

$$Var[\hat{\beta}_{nt}] \approx \frac{\phi_{nt}^2}{M_{nt}} \exp(2\beta_{nt}) \tag{11}$$

This is the expression that we use in our empirical analysis with population parameters replaced by estimates. It gives an overall index variance estimate of the form,

$$Var[I^{0t}] = \sum_{n=1}^{N} w_{nt}^2 \frac{\phi_{nt}^2}{M_{nt}} \exp(2\beta_{nt}) \tag{12}$$

The use of this expression for the variance leads to a slightly different form for the optimal allocation formula. With common relative costs of collection (i.e. $c_1 = c_2 = ... = c_N$) and replacing population parameters with estimates we have,

$$M_{nt} = \frac{w_{nt}\phi_{nt} \exp(\hat{\beta}_{nt})}{\sum_{n=1}^{N} w_{nt}\phi_{nt} \exp(\hat{\beta}_{nt})} \left(\sum_{n=1}^{N} M_{nt} \right) \tag{13}$$

The only difference between this approach and that for the arithmetic mean of price relatives is the inclusion of a level effect for the change in the index. With our theoretical approach outlined we now turn to the empirical implementation of this method.

APPENDIX

Deriving an Approximation to the Variance of the Geometric Mean Index

Taking a first order approximation to the transformation of the parameter β_n^{0t} around the expected value we have:

$$f(\hat{\beta}_{nt}) \approx f(E(\hat{\beta}_{nt})) + \frac{\partial f(E(\hat{\beta}_{nt}))}{\partial \hat{\beta}_{nt}}(\hat{\beta}_{nt} - E(\hat{\beta}_{nt})) \qquad (1)$$

Calculating the variance for this linear approximation it can be seen that,

$$Var(f(\hat{\beta}_{nt})) \approx \left[\frac{\partial f(E[\hat{\beta}_{nt}])}{\partial \hat{\beta}_{nt}}\right]^2 Var(\hat{\beta}_{nt}) \qquad (2)$$

For the exponential transformation the variance has the form,

$$Var(\exp(\hat{\beta}_{nt})) \approx \exp(2E[\hat{\beta}_{nt}])Var(\hat{\beta}_{nt}) \qquad (3)$$

This gives the result in the text where, $E[\hat{\beta}_{nt}] = \beta_{nt}$.

APPLYING THE VARIANCE ESTIMATION TO ADJACENT MONTHS (MONTHLY PRICE CHANGES)

We can calculate a monthly price index by doing the following.

$$I^{t-1t} = \frac{I^{0t}}{I^{0t-1}} \tag{1}$$

$$= \frac{\sum_{n=1}^{N} w_{nt} I_n^{A,0t}}{\sum_{n=1}^{N} w_{nt} I_n^{A,0t-1}} \tag{2}$$

$$= \frac{\sum_{n=1}^{N} w_{nt} \left(\sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}} \frac{p_m^t}{p_m^0} \right)}{\sum_{n=1}^{N} w_{nt} \left(\sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}} \frac{p_m^{t-1}}{p_m^0} \right)} \tag{3}$$

$$= \sum_{n=1}^{N} \sum_{m=1}^{M_{nt}} \frac{\left(\frac{w_{nt}}{M_{nt}} \frac{p_m^{t-1}}{p_m^0} \right)}{\left(\sum_{n=1}^{N} \sum_{m=1}^{M_{nt}} \frac{w_{nt}}{M_{nt}} \frac{p_m^{t-1}}{p_m^0} \right)} \left(\frac{p_m^t}{p_m^{t-1}} \right) \tag{4}$$

This can perhaps more intuitively be written as,

$$I^{t-1t} = \sum_{n=1}^{N} w_{nt} \left(\frac{I_n^{A,0t-1}}{\sum_{n=1}^{N} w_{nt} I_n^{A,0t-1}} \right) \sum_{m=1}^{M_{nt}} \frac{1}{M_{nt}} \left(\frac{\left(\frac{p_m^{t-1}}{p_m^0} \right)}{I_n^{A,0t-1}} \right) \left(\frac{p_m^t}{p_m^{t-1}} \right) \tag{5}$$

From (5) it can be seen that we need to update the weights by the change in relative prices of the stratum and variety.

The equivalent expression for the geometric mean elementary aggregator is simpler) and is shown below.

$$I^{t-1t} = \sum_{n=1}^{N} w_{nt} \left(\frac{I_n^{G,0t-1}}{\sum_{n=1}^{N} w_{nt} I_n^{G,0t-1}} \right) I_n^{G,t-1t} \tag{6}$$

$$= \sum_{n=1}^{N} w_{nt} \left(\frac{I_n^{G,0t-1}}{\sum_{n=1}^{N} w_{nt} I_n^{G,0t-1}} \right) \prod_{m=1}^{M_{nt}} \left(\frac{p_m^t}{p_m^{t-1}} \right)^{\frac{1}{M_{nt}}} \qquad (7)$$

These formulae can be used in practice. In the empirical application (7) has been used directly in computing the results however a slightly modified version of (5) was used in order (see (8) below) to ease the computational burden associated with the geometric mean. We used the expression below.

$$I^{t-1t} = \sum_{n=1}^{N} w_{nt} \left(\frac{I_n^{A,0t-1}}{\sum_{n=1}^{N} w_{nt} I_n^{A,0t-1}} \right) I_n^{A,t-1t} \qquad (8)$$

The only difference from (5) is that we haven't updated the weights in the elementary index calculation.