Measuring and Decomposing Rates of Inflation Derived from Annually Chained Lowe Indices

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Abstract

This note provides an exact decomposition of the inflation rate as derived from annually chained Lowe indices.

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1 Introduction

In a number of European countries the monthly Consumer Price Index (CPI) is calculated as an annually chained Lowe index, whereby December is used as link month. Specifically, the basic building block is a Lowe price index for the current month relative to December of the previous year, the quantities

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being those of some earlier year. Put formally, the basic building block is defined as

$$P(y,m) \equiv \frac{p^{y,m} \cdot q^{y-t}}{p^{y,0} \cdot q^{y-t}}, \tag{1}$$

where y denotes the current year and $m = 1, ..., 12$ a certain month. I will use the convention that $(y, 0) = (y-1, 12)$. The month December thus plays a double role: $P(y, 0) = 1$ as viewed from the year y, while viewed from the year $y-1$ $P(y-1, 12)$ represents the price change relative to December of year $y-2$. The vectors p and q denote N-vectors of prices and quantities respectively, superscripts denote time periods, and the inner product is defined as $p \cdot q \equiv \sum_{n=1}^{N} p_n q_n$. The variable t denotes the time-lag in obtaining quantity data.

The CPI for month m of year y is then calculated as a chained index,

$$P^c(y,m) \equiv P(y,m)P(y-1,12)P(y-2,12)....P(1,12), \tag{2}$$

where $P^c(1, 0) = 1$ has been set for some reference year. For each month, the rate of inflation is usually calculated as the percentage change relative to the corresponding month of the previous year, $P^c(y,m)/P^c(y-1,m) - 1$. Since

$$P^c(y-1,m) \equiv P(y-1,m)P(y-2,12)....P(1,12), \tag{3}$$

it appears that

$$\frac{P^c(y,m)}{P^c(y-1,m)} = \frac{P(y,m)P(y-1,12)}{P(y-1,m)}, \tag{4}$$

which can also be written as

$$\frac{P^c(y,m)}{P^c(y-1,m)} = \frac{p^{y,m} \cdot q^{y-t}}{p^{y,0} \cdot q^{y-t}} \frac{p^{y-1,12} \cdot q^{y-1-t}}{p^{y-1,m} \cdot q^{y-1-t}}. \tag{5}$$

This is the product of two Lowe indices. The first compares prices of the current month (y, m) to the previous December, using quantities of the earlier year $y - t$. The second compares prices of the previous December to the corresponding month $(y - 1, m)$, using quantities of the still earlier year $y - 1 - t$.

The first consequence to notice is that if all the prices of the current month happen to be equal to those of the corresponding month of the previous year, $p^{y,m} = p^{y-1,m}$, then the rate of inflation will not necessarily be equal to zero.

More generally, if $p^{y,m} = \lambda p^{y-1,m}$ for some $\lambda > 0$, then the rate of inflation will not necessarily be equal to $\lambda - 1$.

2 Decomposing the inflation rate

The question addressed in this note is whether it is possible to decompose the inflation rate into contributions of the individual commodities and/or contributions of the current and the previous year. Thus, consider the first factor at the righthand side of expression (5) and use the logaritmic mean[1] and its linear homogeneity (that is, property (3)) to obtain the following decomposition

$$\begin{aligned}
&\ln\left(\frac{p^{y,m}\cdot q^{y-t}}{p^{y,0}\cdot q^{y-t}}\right)\\
&= \frac{p^{y,m}\cdot q^{y-t} - p^{y,0}\cdot q^{y-t}}{L(p^{y,m}\cdot q^{y-t}, p^{y,0}\cdot q^{y-t})}\\
&= \frac{1}{L(P(y,m),1)}\frac{(p^{y,m} - p^{y,0})\cdot q^{y-t}}{p^{y,0}\cdot q^{y-t}}\\
&= \frac{1}{L(P(y,m),1)}\sum_{n=1}^{N}\frac{p_n^{y,0}q_n^{y-t}}{p^{y,0}\cdot q^{y-t}}\frac{p_n^{y,m} - p_n^{y,0}}{p_n^{y,0}}\\
&= \sum_{n=1}^{N} w_n^{y,m}\frac{p_n^{y,m} - p_n^{y,0}}{p_n^{y,0}},
\end{aligned} \tag{6}$$

where the weights $w_n^{y,m}$ are defined as

$$w_n^{y,m} \equiv \frac{1}{L(P(y,m),1)}\frac{p_n^{y,0}q_n^{y-t}}{p^{y,0}\cdot q^{y-t}} \; (n = 1, ..., N). \tag{7}$$

Similarly, decomposing the second factor delivers

$$\ln\left(\frac{p^{y-1,12}\cdot q^{y-1-t}}{p^{y-1,m}\cdot q^{y-1-t}}\right)$$

[1]The logarithmic mean is, for two positive real numbers a and b, defined by $L(a,b) \equiv (a-b)/\ln(a/b)$ when $a \neq b$, and $L(a,a) \equiv a$. It has the following properties: (1) $\min(a,b) \leq L(a,b) \leq \max(a,b)$; (2) $L(a,b)$ is continuous; (3) $L(\lambda a, \lambda b) = \lambda L(a,b)$ $(\lambda > 0)$; (4) $L(a,b) = L(b,a)$; (5) $(ab)^{1/2} \leq L(a,b) \leq (a+b)/2$; (6) $L(a,1)$ is concave. This tool was used by Balk (2004) to obtain decompositions of the Fisher index.

$$
\begin{aligned}
&= \frac{p^{y-1,12} \cdot q^{y-1-t} - p^{y-1,m} \cdot q^{y-1-t}}{L(p^{y-1,12} \cdot q^{y-1-t}, p^{y-1,m} \cdot q^{y-1-t})} \\
&= \frac{1}{L(P(y-1,12)/P(y-1,m),1)} \frac{(p^{y-1,12} - p^{y-1,m}) \cdot q^{y-1-t}}{p^{y-1,m} \cdot q^{y-1-t}} \\
&= \frac{1}{L(P(y-1,12)/P(y-1,m),1)} \sum_{n=1}^{N} \frac{p_n^{y-1,m} q_n^{y-1-t}}{p^{y-1,m} \cdot q^{y-1-t}} \frac{p_n^{y-1,12} - p_n^{y-1,m}}{p_n^{y-1,m}} \\
&= \sum_{n=1}^{N} w_n^{y-1,m} \frac{p_n^{y-1,12} - p_n^{y-1,m}}{p_n^{y-1,m}}, \qquad (8)
\end{aligned}
$$

where the weights $w_n^{y-1,m}$ are defined as

$$
w_n^{y-1,m} \equiv \frac{1}{L(P(y-1,12)/P(y-1,m),1)} \frac{p_n^{y-1,m} q_n^{y-1-t}}{p^{y-1,m} \cdot q^{y-1-t}} \ (n = 1, ..., N). \qquad (9)
$$

Combining the two factors delivers

$$
\begin{aligned}
&\ln \left(\frac{P^c(y,m)}{P^c(y-1,m)} \right) \qquad (10) \\
&= \sum_{n=1}^{N} w_n^{y,m} \frac{p_n^{y,m} - p_n^{y,0}}{p_n^{y,0}} + \sum_{n=1}^{N} w_n^{y-1,m} \frac{p_n^{y-1,12} - p_n^{y-1,m}}{p_n^{y-1,m}} \\
&= \sum_{n=1}^{N} \left(w_n^{y,m} \frac{p_n^{y,m} - p_n^{y,0}}{p_n^{y,0}} + w_n^{y-1,m} \frac{p_n^{y-1,12} - p_n^{y-1,m}}{p_n^{y-1,m}} \right).
\end{aligned}
$$

Since for any positive real number $a \neq 1$, $\ln a = (a-1)/L(a,1)$, the lefthand side of the foregoing expression is a simple transformation of the inflation rate. In fact, for $a \approx 1$, the usual approximation is $\ln a \approx a - 1$. The last line of expression (10) then provides a decomposition of the inflation rate into contributions of the individual commodities, divided with respect to current year and previous year. The second line of expression (10) provides a decomposition of the inflation rate into parts due to the current year and the previous year respectively. Both decompositions are exact.

3 Two closing remarks

1. The solution of the (draft) *HICP Manual* Annex 1 is, using our notation,[2]

$$\begin{aligned}&\frac{P^c(y,m)}{P^c(y-1,m)} - 1 \\ &= \frac{P(y-1,12)}{P(y-1,m)}\left(P(y,m)-1\right) + \left(\frac{P(y-1,12)}{P(y-1,m)} - 1\right).\end{aligned} \tag{11}$$

Notice that $P(y,m)-1$ as well as $P(y-1,12)/P(y-1,m)-1$ can simply be decomposed commoditywise.

The first factor at the righthand side of expression (11) is then interpreted as this year's contribution to the inflation, and the second factor as last year's contribution. One should notice, however, that the first factor actually conflates last year's contribution in ratio form, $P(y-1,12)/P(y-1,m)$, with this year's contribution in difference form, $P(y,m)-P(y,0)=P(y,m)-1$. Thus the interpretation of the first factor seems difficult to maintain.

2. It is interesting to observe that the decomposition method proposed in Section 2 can easily be adapted to situations where the annual inflation rate involves more factors than the two contained in expression (5). Also, the extension to other indices than those of Lowe should be straightforward.

References

[1] Balk, B. M., 2004, "Decompositions of Fisher Indexes", *Economics Letters* 82, 107-113.

[2] This solution coincides with the UK practice of computing contributions of components to annual changes in the all items RPI, as detailed in Section 8.9 of the *Consumer Price Indices Technical Manual 2005.*