Department of Economics
Uppsala University
Uppsala Sweden

A Note on Cost-of-living Indexes, Subsidized Commodities and Income Dependent Prices

by

N. Anders Klevmarken

Abstract
The user cost of publicity provided services is sometimes income dependent. High income earners pay more. A true cost-of-living index which includes income dependent prices is suggested. It is shown that the corresponding Laspeyres index does not only exceed the true index but it exceeds it by more than in the case of no income dependent price.

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1. Introduction: CPI as a compensating price index[1]

A cost-of-living index can be defined as the ratio between the minimum income needed to attain a certain level of utility at prices p_1 and the minimum income needed to attain the same level of utility at prices p_0. A cost-of-living index thus gives the relative income compensation needed to maintain a certain consumption standard when there are price changes. This explains why it is also called a compensating price index. It is of great practical importance as one theoretical foundation of the common consumer price index (CPI).

More or less consistent with its definition and implementation the CPI is used for many purposes. One of its most important uses is in explicit or implicit "contracts" to rule about money compensation for price increases. The CPI is thus frequently used to index pensions and the compensation agreed upon in union contracts, cf. the commonly used cost of living allowances (COLA) in US labor contracts. But the CPI is also in some countries used to index, for instance, student grants, social security compensation for income losses due to sickness, and child benefits and even to index progressive tax schedules to avoid tax increases as a result of inflation only.

In the design of a compensating price index one of the most important issues is to decide which price changes one would like to compensate. In principle the user has to decide for which goods price changes should result in an index change and thus an income compensation. There is neither any theoretical reason to include "all" goods or all goods which in the national accounts are classified as private consumption nor any reason to necessarily compensate for the whole price change. It is entirely a "political" issue. Which price changes does the user want to compensate? It should, however, be noted immediately that even if compensation is only given for price changes of a subset of commodities the prices of *all* commodities which enter the budget set will in general determine the index, i.e. the price level of goods for which compensation is not given will still enter the index formula and thus determine the compensation given for price changes in other goods. This property of a cost-of-living index is discussed more below.

The issue of what to compensate becomes especially important in an economy with goods and services subsidized by the public. In countries like, for instance, the Nordic countries this applies to a large share of total private consumption. Although the method of subsidation varies, food products, public transportation, medical care and medicine, child care and housing are all subsidized. The CPI practice to measure the

[1] This paper was inspired by the discussions in the standing committee for the Swedish CPI (indexnämnden). It is an extension and revision of Klevmarken (1975).

prices of these commodities net or gross of the subsidy varies from country to country and from community to community. In Sweden, for instance, all commodities are currently measured net of subsidies except possibly for housing services. Housing allowances are treated as income transfers. But for the price of medicine this practice is relatively recent, it used to be measured gross of subsidies. If the subsidy is given directly to the consumer or fully forwarded to the consumer by the producer a decrease in subsidy will increase the price net of the subsidy but leave the price gross of the subsidy unchanged. If politicians thus decide to decrease, for instance, the subsidy to public transportation and as a result increase the direct user cost, they will at the same time through the CPI give pensioners, wage earners, families with children and other groups compensation for the consequences of this political decision.

Another example of current interest in Sweden and other countries is the treatment of child care services. They have not yet been included in the Swedish CPI but an inclusion has been considered. Most child care services in Sweden are either provided by the public or by subsidized family cooperatives. In both cases parents only pay about 10 per cent of the total costs in user fees. Consider for the sake of an argument two countries, one which finances public child care entirely through the general budget and one which asks parents to pay a user fee for their children. If the cost of child care increases in both countries all taxpayers have to carry the burden in the first country while in the second only users pay increased fees. In the first country there is no effect on CPI while in the second it increases. Although consumers are worse off in both countries only in the second country pensioners, wage earners and families with children are given compensation because politicians decided to increase the user fees of child care rather than to absorb the increased costs in general tax increases[2]. If this is their intention there is no objection, but it is not obvious why a politically decided increase in user fees should result in a compensation to those who have been asked to pay more as well as to groups who do not use these services at all.

In the case of child care services there is an additional complication. In many Swedish municipalities user fees are income dependent. High income families pay more than low income families and there is usually a maximum and a minimum fee. To construct a cost-of-living index one needs to know if all changes in the fee structure should lead to an index change. In principle one could, for instance, choose the position that only common changes in the maximum and minimum levels should lead to a compensation while variations in the span between the two extremes should not. One could argue that political decisions about changes on the degree of subsidation of low income

[2] If the compensating price index was defined such that it gave compensation in before tax income and not in after tax income which is the current CPI practice, consumers in both countries would become compensated, but not necessarily by the same rate.

families should not be counter balanced by a compensation through the CPI. By a similar argument politicians might also choose not to compensate at all for user fee changes. However, if they do choose to give compensation for user fee changes it would seem obvious that a cost-of-living index should only change as a result of changes in the fee *schedules*, but not if user fees increase (decrease) only as a result of an average increase (decrease) in family income.

The issues if and how consumers should be compensated for changes in the user cost of publicity provided goods and services is related to the treatment of direct and indirect taxes in a cost-of-living index. If consumers are compensated for increased user costs caused by decreased subsidies then it would also seem natural to consider how the increased net revenue is used, for instance, if it finances a tax decrease. With the current CPI practice decreased indirect taxes will result in price decreases which in the CPI will counter-balance the increased user costs, while decreased direct taxes will not influence the CPI. In the latter case consumers will get compensation twice, first by the decreased direct taxers and second by the increased CPI.

A cost-of-living index is a function of a reference utility level. It answers questions about the minimum compensation needed to attain a certain level of utility. This implies that the relevant budget set of the consumer will be net of income taxes and transfers, i.e. a cost-of-living index should give the compensation needed taking into account income taxes and transfer payments. This is rarely the case in theory nor in practice. Conventional preference field based index theory usually assumes a linear budget set and no income taxes[3]. In practice CPI is computed without any reference to direct income taxes but it is still used to give various groups in society a compensation *before* tax. This is an unnecessary inconsistency, it for some reason is easier to compensate people before tax than after tax, it is a perfectly well-defined question to ask what compensation *before* tax is needed to attain a certain level of utility. The resulting cost-of-living index will be a function of the income tax and transfer system. In principle one could also design a cost-of-living index which in addition to price changes also compensates for changes in the tax rates of direct taxes, but such an index would presumably be used for other purposes than the current CPI.

The remainder of this paper first includes a more formal definition of a cost-of-living index with and without income taxes, and then follows a discussion of an index with income dependent prices.

[3] One exception is Baye & Black (1986) who discuss at some length cost-of-living indexes in the presence of taxation.

2. Cost-of-living indexes with and without income taxes

Assume the consumer is maximizing a utility function possessing the conventional properties of reflexivity, completeness, transitivity, continuity, nonsatiation and convexity of preferences,[4]

$$u=u(q) \tag{1}$$

subject to the budget constraint

$$p'q=y \tag{2}$$

where q is a vector of quantities, p a vector of prices and y is "income".

The dual problem of this maximization is to find the smallest income needed to reach a given level of utility u for given prices p, and the solution is given by the cost function,

$$e=e(p,u) \tag{3}$$

Following Konüs (1924) a cost-of-living index is defined by,

$$I_{01}=\frac{e(p_1,u)}{e(p_0,u)} \tag{4}$$

where p_0 and p_1 are the reference and comparison vectors of prices respectively.

The well-known Laspeyre approximation is of course,

$$I_{01}^{L}=\frac{(P_1'q_0)}{(P_0'q_0)}; \tag{5}$$

Assume now that consumers have to pay income taxes on their incomes according to a taxsystem,

$$y=Y-T(Y)=\tau(Y) \tag{6}$$

[4] Cf. Deaton & Muellbauer (1980).

where Y is before tax income, y after tax income and T a function which represents the taxsystem. We will assume that T has properties such that there is a unique inverse $\tau^{-1}(y)$.[5]

Assume also that a cost-of-living index is desired which gives the minimum *before* tax income compensation needed to attain a certain level of utility u. As the cost function $e(p,u)$ gives the minimum income after tax, it follows that the desired index is

$$I_{o1} = \frac{\tau^{-1}(e(p_1,u))}{\tau^{-1}(e(p_0,u))} \tag{7}$$

The indices (4) and (7) are equal if the tax system is proportional, while (7) will in general exceed (4) if the tax system is progressive. We can thus conclude that the usual practice to calculate CPI ignoring the income tax system will in general lead to an undercompensation of price increases.

The Laspeyres index corresponding to the index (7) is simply,

$$I^L_{o1} = \frac{\tau^{-1}(p_1' q_0)}{\tau^{-1}(p_0' q_0)}; \tag{8}$$

The indices (7) and (8) are conditional on a given tax system T. If a comparison involves two time points the tax system might have changed and the index user would have to choose either one (or a third alternative).

The additional complexity which follows from the introduction of nonproportional income taxes does not add to the understanding of the subsequent analysis. Any of the indexes suggested below could easily be generalized to an index with income taxes in the same way as the indexes (4) and (5) were generalized to (7) and (8). For this reason income taxes are kept out of the analysis from now on.

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[5] This is a fairly realistic assumption for most income tax systems but may be less realistic for a composite tax and benefit system. For a closer analysis of the properties of a cost-of-living index with income taxes see Baye & Black (1986)

3. A cost-of-living index with subsidized commodities and income dependent prices

Assume now that prices are not exogenously given constants but functions of income y and a vector z.

$$p=p(y,z) \tag{9}$$

z can be thought of as, for instance, the particular schedule of user fees which applies to the consumer.

The budget constraint now becomes

$$p'(y,z)q=y \tag{10}$$

and the corresponding cost functions will have u and z as arguments,

$$e^*=e^*(z,u) \tag{11}$$

The corresponding cost-of-living index is,

$$I_{01}=\frac{e^*(z_1,u)}{e^*(z_0,u)} \tag{12}$$

This index thus compares two user fee schedules and returns the income compensation needed to reach the utility level u when z_0 is replaced by z_1.

Through the cost function the shape of the budget set will thus determine the properties of the cost-of-living index. Just to illustrate with a few simple examples, assume commodity (1) is subsidized by a lump sum $z_{(1)}$ per unit of good 1,

$$p_{(1)}= \bar{p}_{(1)}-z_{(1)} \tag{13}$$

where $\bar{p}_{(1)}$ by definition is the "gross" price $p_{(1)}+z_{(1)}$. The budget restriction now becomes,

$$(\bar{p}_{(1)}-z_{(1)})q_{(1)}+\sum_{\eta=2}^{n} p_{(\eta)}q_{(\eta)}=y \tag{14}$$

and the cost-of-living index

$$I_{01} = \frac{e^{*}(\bar{p}_{(1)1}, z_{(1)1}, p_{(2)1} \cdots p_{(n)1}, u)}{e^{*}(\bar{p}_{(1)0}, z_{(1)0}, p_{(2)0} \cdots p_{(n)0}, u)} \tag{15}$$

Suppose now that politicians do not want to compensate consumers for changes in the lump sum subsidy, but only for price changes which are (directly) unrelated to the subsidy. The index comparison would then have to be made for a certain level of subsidy, say $\bar{z}$. Thus

$$I_{01} = \frac{e^{*}(\bar{p}_{(1)1}, \bar{z}, p_{(2)1}, \ldots, p_{(n)1}, u)}{e^{*}(\bar{p}_{(1)0}, \bar{z}, p_{(2)0}, \ldots, p_{(n)0}, u)} \tag{16}$$

$\bar{z}$ could, for instance, equal $z_{(1)0}$ or $z_{(1)1}$. Although the index (16) is independent of any change in the subsidy it is not independent of its level. The income needed to compensate for any change in $\bar{p}_{(1)}$ or $p_{(2)} \ldots p_{(n)}$ will in general depend on $\bar{z}$.

The implicit model behind the budget set (14) assumes that the consumer´s decisions are based on the price net of subsidies. Suppose now that the subsidy is not immediately given to the consumer but first after the purchase and with a considerable delay. The consumer might then take this into account in his purchase decisions and act as if he payed the following price,

$$p_{(1)} = \bar{p}_{(1)} - z_{(1)}(1+\rho)^{-T} \tag{17}$$

where ρ is a discount rate and T the time span between the purchase and when he obtains the subsidy.[6] If this time span is very long the consumer will "almost" base his decision on $\bar{p}_{(1)}$ and the effect of the subsidy is of course negligible.

This example illustrates the dependence of the index on the particular budget set which the consumer is confronted with. In this very simple model ρ represents the market interest rate, but in a more general intertemporal model it would represent the consumer´s time preferences. The budget set will in this way include an element of subjective evaluation on the part of the consumer. More generally, the relevant budget set is the set *perceived* by the consumer, not the set which would hold if the consumer had perfect information and perfect foresight.

[6] In this myopic model with no saving we have to assume that the consumer is able to borrow $z_{(1)}(1+\rho)^{-T}$ at an interest rate of ρ when the purchase is made.

Let's now turn to a simple example with an income dependent price. Assume that

$$p_{(1)}=\bar{p}_{(1)}-\frac{z_{a(1)}}{z_{b(1)}+y} \tag{18}$$

where $\bar{p}_{(1)}\geq 0$, $z_{a(1)}\geq 0$ and $z_{b(1)}>0$ are given parameters. Equation (18) approximates fairly well the user fee schedules for public child care in many Swedish municipalities. Families with zero incomes pay a minimum fee of $\bar{p}_1-z_{a(1)}/z_{b(1)}$. The fee increases with increasing income and reaches a maximum of $\bar{p}_{(1)}$. Figure 1 illustrates the effect of an increase in $\bar{p}_{(1)}$ (or a decrease in $z_{a(1)}$, or an increase in $z_{b(2)}$). Before the price increase the budget line is A_1A_2 and the consumer prefers the commodity bundle $Q(0)$. After the price increase the budget line is B_1A_2 and the consumption bundle $Q(1)$. If $z_{a(1)}$ is zero, i.e. commodity 1 is a regular good, then an income compensation would be needed which shifts the budget line from B_1A_2 to C_1C_2. If, however, $z_{a(1)}>0$ and the subsidy decreases with increasing income, a larger income compensation is needed to reach the same indifference curve. This is illustrated by the line D_1D_2. Because the price of the second commodity is assumed unchanged we can measure the income compensation needed in q_2 units, and we find immediately that the income compensation is larger when there is a subsidy which decreases with increasing income, because $A_2D_2>A_2C_2$.

Suppose now instead that the user fee parameter $z_{a(1)}$ decreases while $\bar{p}_{(1)}$ remains constant and assume that the decrease exactly corresponds to the shift in the budget line from A_1A_2 to B_1A_2. In this case the income compensation is smaller than in the previous case because there is a smaller decrease in the subsidy with increasing income.

An analysis of the implications of a Laspeyres compensation might also be of interest. A conventional Laspeyres compensation would imply a parallel shift of the line C_1C_2 out to the point $Q(0)$. From Figure 1 we immediately obtain the well-known result that a Laspeyres index is greater than or equal to a true cost-of-living index evaluated at the reference period level of utility. In the case of a subsidy the line D_1D_2 would not only be pushed out to Q(0) but its slope would also decrease. Also in this case the result is that the Laspeyres compensation in general will exceed the "true" compensation, but it will in addition exceed it with more than in the normal case with no income dependent subsidy. The last point is illustrated by Figure 2. The line $D_1''D_2''$ is parallel to D_1D_2 and satisfies the point $Q(0)$. Because $OC_3>OD_3$ it follows that

$$\frac{OQ(0)}{OC_3}<\frac{OQ(0)}{OD_3} \tag{19}$$

From the properties of congruent triangles it follows that,

$$\frac{OQ(0)}{OC_3}=\frac{OC'_2}{OC_2} \tag{20}$$

and

$$\frac{OQ(0)}{OD_3}=\frac{OD''_2}{OD_2} \tag{21}$$

Thus

$$\frac{OC'_2}{OC_2}<\frac{OD''_2}{OD_2} \tag{22}$$

Because $OD'_2>OD''_2$ it is also true that

$$\frac{OC'_2}{OC_2}<\frac{OD'_2}{OD_2} \tag{23}$$

and,

$$\frac{OC'_2\ /OA_2}{OC_2\ /OA_2}<\frac{OD'_2\ /OA_2}{OD_2\ /OA_2} \tag{24}$$

where the ratios in the numerators are the two Laspeyres indices and the ratios in the denominators are the true cost-of-living indices.

This proves that in the case of a subsidy which declines with increasing income the Laspeyres index exceeds the true cost-of-living index by more than the Laspeyres index in the case of no income dependent subsidy.

Given the price structure (18) it is easy to calculate an explicit expression for a Laspeyre index. It is,

$$I_{01}^{L}=\frac{z_{b(1)1}+\bar{p}_{(1)1}q_{(1)0}+p_{(2)1}q_{(2)0}\pm\sqrt{(z_{b(1)1}-\bar{p}_{(1)1}q_{(1)0}-p_{(2)1}q_{(2)0}+4z_{a(1)1}q_{(1)0}}}{z_{b(1)0}+\bar{p}_{(1)0}q_{(1)0}+p_{(2)0}q_{(2)0}\pm\sqrt{(z_{b(1)0}-\bar{p}_{(1)0}q_{(1)0}-p_{(2)0}q_{(2)0}+4z_{a(1)0}q_{(1)0}}} \tag{25}$$

where + or - is choosen as appropriate. This index gives the compensation needed for a change in any of the price parameters $\bar{p}_{(1)}$, $z_{a(1)}$ and $z_{b(1)}$. If one, for instance, would prefer an index which only compensates for a change in $\bar{p}_{(1)}$ then $z_{a(1)}$ and $z_{b(1)}$ would have to remain constant. In a Laspeyre index it might be natural to fix these two parameters at their reference values, i.e. at $z_{a(1)0}$ and $z_{b(1)0}$, but this is at the index user´s discretion.

4. Conclusions

A cost-of-living index is a compensating price index which gives the minimum income change needed to attain a certain level of utility (standard of living) when a certain price change has occurred. The compensating property makes it necessary to design a cost-of-living index with certain applications in mind. It is the user who must decide if it is desirable to give compensation for changes in the prices of all goods or only of a subset of goods. In principle one might choose to include or exclude any good from the price comparison of an index. There is no theoretical reason to include any particular good. This issue becomes particularly relevant when goods and services subsidized by the public are included in the price comparison. Should changes in the degree of subsidizing be allowed to influence the index?

When a decision has been reached on this issue the construction of a cost-of-living index follows from the preference field of the consumer and the budget set which confronts her, or more generally by our model of the consumer´s decision process. Even if the price comparison is limited to a certain number of goods the entire budget set will determine the size of the compensation derived from a cost-of-living index, and in the case of subsidized commodities, the whole user fee structure will determine the index number even if changes in the degree of subsidizing is not allowed to influence the index. In these cases one has to condition on the prices of goods which are not part of the price comparison and on the degree of subsidation respectively.

Nonlinear budget sets were introduced by means of income dependent user fees. It was shown that if user fees increase with increasing income the corresponding cost-of-living index will in general exceed an index for a situation with user fees unrelated to income.

The Laspeyre compensation is at least as large as the true compensation. It was also demonstrated that the excess compensation given by a Laspeyre index when user fees are an increasing function of income exceeds the excess compensation when there are no income dependent user fees.

The index formula needed in practice when user fees depend on income will depend on the properties of the functional relation. By way of example this note has demonstrated that it is possible to find applicable approximation to a true cost-of-living index. The design issues which need to be solved before an index can be implemented empirically are not much different from those of an ordinary CPI. One difference is though that data on the whole fee structure are needed not only on a single price or on what people actually pay.

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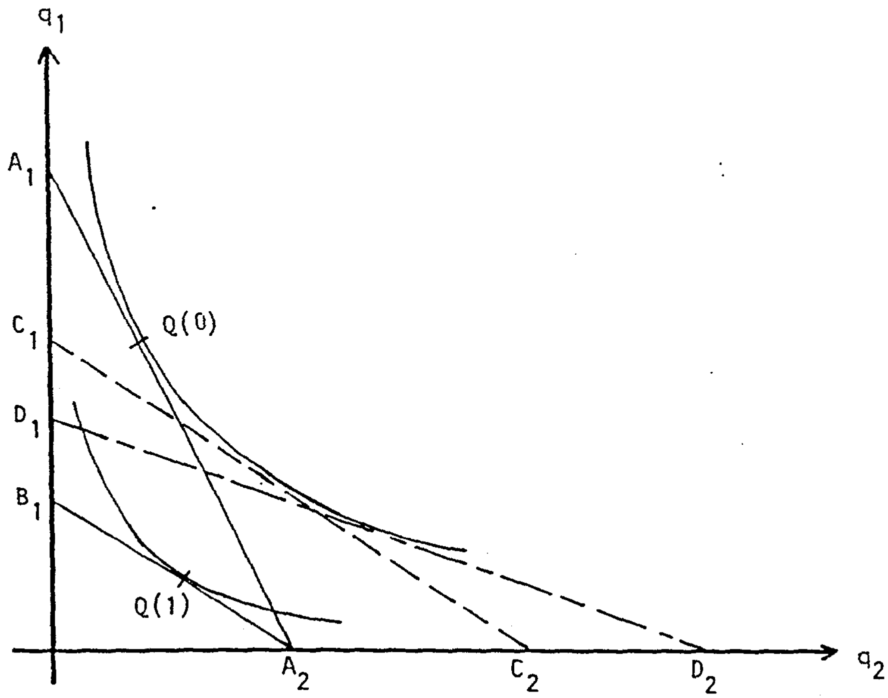

Figure 1. Income compensation when the price of one good is an increasing function of income

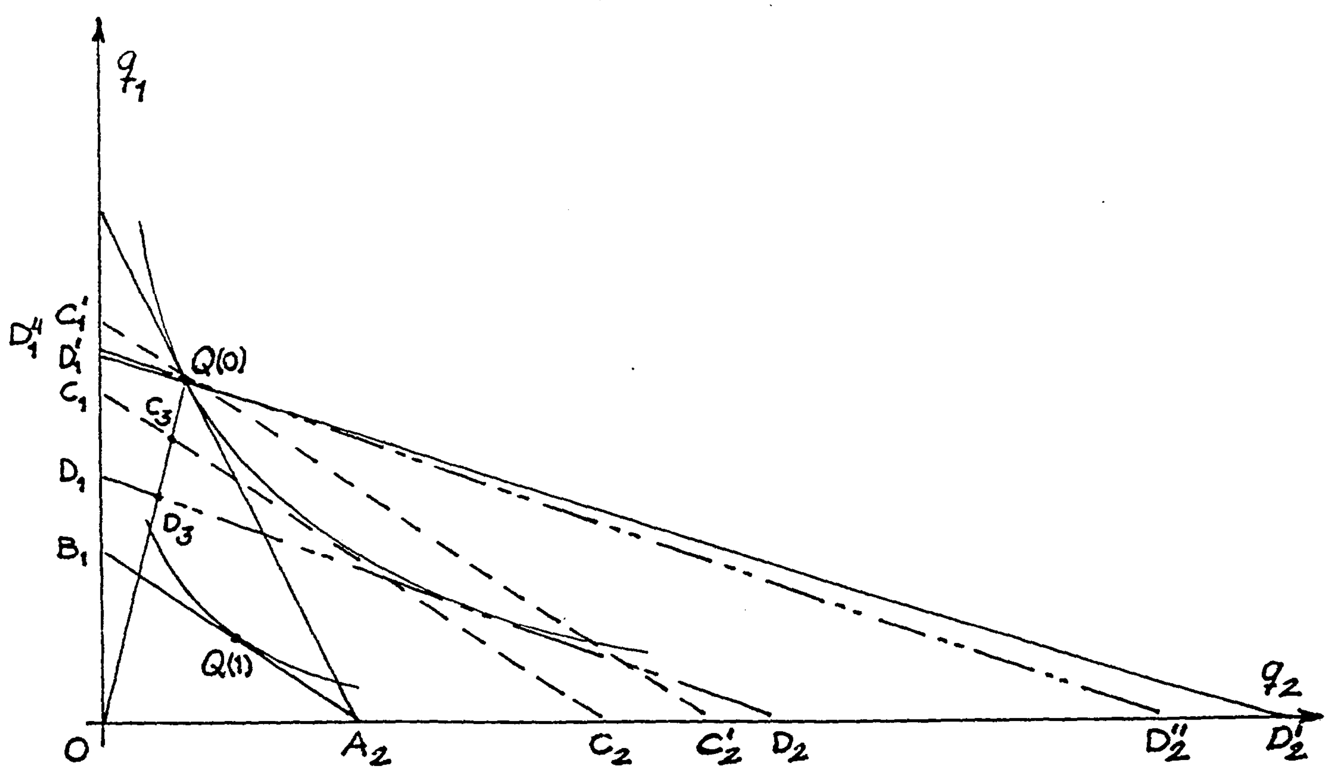

Figure 2. Laspeyres compensation with and without an income dependent price