2007-11-30

Current Swedish discussion on housing in the CPI

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Abstract. The paper deals with a current discussion in Sweden on a possible new approach proposed by Professor Anders Klevmarken, to the treatment of owner occupied housing in the Consumer Price Index (CPI). That approach uses a dynamic model in which consumers maximise their utility due to not only current consumption but also future consumption possibilities depending on assets and liabilities. The new approach has subsequently been thoroughly discussed in the Swedish CPI Board, and some questions have been identified that still remain to be fully resolved.

The paper describes some properties and some possible variants of the proposed model, and some main topics in the current discussion. The treatment of mortgage interest may be seen in different ways, potentially leading to notably different outcomes. Particularly, a still open question is if and how the consequences of house price changes on mortgage interest cost should be shown as mortgage interest cost changes in the index.

Call for a new approach

For some years now, the Consumer Price Index Board has been assigned by the Swedish Government (2001) to look into possibilities of improving the Consumer Price Index (CPI), with regard to the calculations of housing costs for owner occupiers. The issue has been previously treated by a Government Commission (SOU 1999:124), in a report from the National Institute of Economic Research (2002) and in an article by Ribe (2004).

Anders Klevmarken, Professor in Econometrics at Uppsala University and member of the Consumer Price Index Board, has now developed a potential new approach for handling owner occupied housing in the CPI. Klevmarken presented the approach to the Ottawa Group Meeting in 2006 (Klevmarken, 2006a), and he has subsequently elaborated further on details and has put forward revised versions (Klevmarken, 2006b, 2007). Discussions are still ongoing and no decisions have been reached yet.

A dynamic model

The model is designed as a theoretical basis for a CPI that includes all consumption, and not only housing. The model can be said to be a further development of the theory for cost-of-living index (cf. ILO et al., 2004, Ch 17-18). Accordingly, the model is based on the assumption that consumers choose their consumption for each time period so as to optimise the relation between utility and cost. The new model also takes into consideration that owner occupied dwellings exist over time, and thus it takes a dynamic or inter-temporal view.

The utility of consumers during a time period (e.g. a certain year or month) in the ideal model is assumed to be a function of basically the following arguments:

(1) Consumption of goods and services other than housing
(2) Housing in rented dwelling
(3) Size and quality of owner occupied dwelling at the start of the period
(4) Size and quality of owner occupied dwelling at the end of the period
(5) Financial assets and liabilities at the end of the period.

Owner occupied housing and financial wealth at the end of the period, mentioned as points (4) and (5) here, give utility for consumers in the form of safe opportunities of future consumption. During the period, consumers can play with the mentioned factors and choose their consumption to receive the maximum utility.

The budget constraint

The maximisation of the consumer's utility is subject to a budget constraint, stating that the income of the consumer must cover her costs during the time period in question. In the model the components of income are:

- Income from labour
- Income from capital such as interest, dividends, gains, etc.
- Withdrawals minus deposits of saved financial capital
- Taking of new loans minus repayment of loans.

The costs to be covered by income are:

- Costs for goods and services other than those for housing (including household costs for electricity, heating oil, water etc).
- Rental costs for dwellings
- Costs for repairs and maintenance of dwellings
- Costs for interest on loans
- New construction, rebuilding or extensions of owner occupied dwellings.

Costs for purchase of used owner occupied dwellings do not need to be included, since these costs are cancelled out with the income received when dwellings are sold. This is clear when the total population is summed up.

Ideal index and practical index calculation

Now an index figure is to be calculated, showing the cost development for consumers due to price changes between two different periods, for example two different years or two different months. In the context of the ideal model just described, the index figure will give the answer to this question:

How much does the consumers' total income after net net financial savings have to change when a price change occurs, in order to enable the consumers to enjoy the same level of utility as before the price change?

In practice a price index is calculated by following price development for a "basket" of goods and services that are consumed by consumers. The index basket includes the components that are on the cost side of the budget constraint. The following simplification is also made in the model:

- *Interest costs are only taken into consideration for loans on owner occupied housing, not for other loans.*

The new approach now implies that the index basket in the CPI would include the following components:

- Other goods and services than housing (including electricity, heating oil, water etc.)
- Rents for rented dwellings
- Repairs and maintenance for housing
- Interest on loans for owner occupied housing
- New construction, rebuilding and extensions of owner occupied dwellings.

Limited changes in methods – sub-indices and weighting data

In practical index calculation, this new approach is similar to that used in the present CPI (see Statistics Sweden, 2001). The presently used approach may be characterised as in the vein of the user cost approach (cf. Diewert, 2004; ILO et al., Ch. 10), although taking account of partial rather than full user cost.

However, the new approach would involve changes in methods as follows:

- The meaning of the component *Interest costs* would partly be changed.
- The present component *Depreciation* would be removed.
- The removed component *Depreciation* would be replaced partly by a new component *New construction,* and partly by the fact that *Repairs* would be given wider coverage.

Changes would be made in choice of sources of weighting data for some components. It would be possible to discontinue using special calculations that are now used for weights of the components Interest costs and Depreciation. Instead statistics on actual expenditures can be used, such as the survey Household Finances in particular, and the National Accounts.

The changes in methods for the new approach are summarised in the following table.

<table>
<tr><th>Present method</th><th>New method</th></tr>
<tr><td>Component Interest costs

Weighting: Calculated interest on original acquisition prices, before tax deductions (calculation)

Price measurement: Interest rates and acquisition prices. Effects of tax changes are not taken into consideration</td><td>Component Interest costs

Weighting: Observed interest costs for loans for owner occupied housing, after calculated tax deduction (Household Finances)

Price measurement: Interest rates and house prices. Effects of tax changes are not taken into consideration</td></tr>
<tr><td rowspan="2">Component Depreciation

Weighting: Assumed depreciation through wear and tear (calculation)

Price measurement: Prices for material and labour for 'major' repairs (Before 1999: Prices for new houses)</td><td>Component New construction

Weighting: Observed costs (National Accounts)

Price measurement: Prices for (new) houses</td></tr>
<tr><td rowspan="2">Component Repairs (including renovations and extensions)

Weighting: Observed costs (Household Finances, National Accounts)

Price measurement: Prices for material and labour</td></tr>
<tr><td>Component Repairs (minor)

Weighting: Observed costs

Price measurement: Prices for material</td></tr>
</table>

Interest on equity would be excluded

One difference in principles between the new and the present approach is that the new one includes interest costs for loans only. On the other hand, the present approach includes interest costs for the entire purchase price of the house, that is, not only on the loan but also on the paid-up capital.

The new model is based on the choices that consumers make. Interest on equity does not consist of money that the consumer can see, and thus it is not included in the budget constraint. Interest on equity occurs as both a hidden cost and a hidden income, and those two equally large amounts cancel each other out.

Formal expression of the model

The model of Klevmarken (2007) just described verbally may more formally be expressed as follows. The index is to compare periods during each of which the prices are assumed to be unchanged. A basic idea is to model the behaviour of the consumer at a given set of prices, in a setting with an inter-temporal perspective. In each period the consumer chooses her consumption so as to maximise the value of a utility function

$$(1) \qquad U(\mathbf{q}, q_h^0 + \lambda q_m, q_h, q_r, g_A(A), g_M(M))$$

subject to a budget constraint given by an equation,

$$(2) \qquad \begin{aligned} & y + A^0(1 + r_{AA}) + p_h(1-\delta)q_h^0 - M^0 = \\ & \qquad = \mathbf{p}'\mathbf{q} + p_m q_m + p_r q_r + p_h(q_h - (1-\delta)\lambda q_m) + \\ & \qquad\qquad + (r_M - 1)M + (1 - r_A)A \,. \end{aligned}$$

Here $\mathbf{p}$ and $\mathbf{q}$ denote column vectors of prices and consumption volumes, respectively, of other consumer goods and services than housing, but also including owner occupiers' operating consumption for heating, water etc. Further q_h^0 and q_h are the owner occupied housing stock at the beginning an the end of the period considered, respectively; q_m is the volume of maintenance and repair, q_r is the volume of rented dwellings, y is the income from labour etc., and A and M are the financial assets and liabilities, respectively. Also, r_A is interest rate on assets, r_{AA} is rate of assets growth, r_M is interest rate on loans, δ is depreciation rate, and finally λ is a factor by which the lasting value of maintenance and repair is transformed into units of housing stock.

A crucial feature introduced in the latest version of Klevmarken (2007) is that the loan amount M as an argument in the utility function Eq. (1) is transformed so as to pertain to constant prices, by the transformation

$$(3) \qquad g_M(M) = \frac{M}{p_h\, q_h} = \tilde{M} \ .$$

Note that the denominator in Eq. (3) expresses the current value of the dwelling. The transformation of Eq. (3) thus means that the loan amount is expressed as a share of the current dwelling value, or as a debts over assets ratio.

There is a particular motivation for this transformation to be used in the expression Eq. (1) of the utility function. The loan amount entails disutility in two forms: First, the loan amount entails increased future risks, of e.g. not being able to afford a maintained standard of housing in case of interest shocks and the like. Second, the loan requires future interest payments, reducing future consumption possibilities. It appears plausible that those forms of disutility due to the loans primarily depend on the loan amount as a proportion of the collateral.

Denoting the consumer's optimal choices by asterisk, it follows from the budget constraint Eq. (2), after moving some terms between the left-hand and right-hand sides, that the optimal choices satisfy the equation

$$\text{(2A)}\quad \begin{aligned} & y + A^0 r_{AA} + A^* r_A - [(A^* - M^*) - (A^0 - M^0)] \ = \\ & = \ \mathbf{p}'\mathbf{q}^* + p_m q_m^* + p_r q_r^* + p_h (q_h^* - (1-\delta)(q_h^0 + \lambda q_m^*)) + r_M \, p_h q_h^* \tilde{M}^* \ . \end{aligned}$$

The left-hand side of this equation expresses the income net of savings, and the right-hand side measures the cost to be covered by the income.

Form of the index computation

An ideal index corresponding to the model can now be defined as following the right-hand side of Eq. (2A). In comparing two time periods, the index number is thus taken as the ratio between the values of the right-hand side of Eq. (2A), at the prices in each of the two periods. This index fulfils the aim of answering the question how much the income has to change in order to suffice for a maintained level of utility.

A particular feature may be noted here. In the usual theory of cost-of-living index, the ideal index is equal to the ratio of minimal cost amounts for a given level of utility. Namely, there the numerator and the denominator values result from a utility constrained minimisation that is dual to the budget constrained maximisation of utility. However the ideal index defined here does not have this property. This is so as terms involving the amounts $A*$ and $M*$ determined in the utility maximisation were afterwards moved to the left-hand side of Eq. (2A).

Nevertheless the ideal index may here appear as a sub-index, in line with the theory of Pollack (1975), of an index with the usual property of being equal to a ratio of minimal cost amounts for a given utility level. The index may then be defined as conditional on constant net savings.

The operational index computation uses a fixed basket index as an approximation to the ideal index derived from the model. This index follows the development of the cost for a reference basket consumption $\mathbf{Q}^{\mathrm{B}}$ at the set of prices $\mathbf{P}^t$ of the current period t, and this cost is expressed as

$$\text{(4)}\quad \begin{aligned} \mu^t(\mathbf{Q}^{\mathrm{B}} \mid \mathbf{P}^t) \ = \ & (\mathbf{p}^t)' \, \mathbf{q}^{\mathrm{B}} + p_m^t \, q_m^{\mathrm{B}} + p_r^t q_r^{\mathrm{B}} \ + \\ & + \ p_h^t (q_h^{\mathrm{B}} - (1-\delta)(q_h^{\mathrm{B};0} + \lambda q_m^{\mathrm{B}})) \ + \\ & + \ r_M^t (1 - \tau^t) \ p_h^t q_h^{\mathrm{B}} \ \tilde{M}^{\mathrm{B}} + \tau_h^t \beta^t \, p_h^t q_h^{\mathrm{B}} \ . \end{aligned}$$

In this expression tax effects have also been included. Namely, τ^t is tax rate at which the interest is tax deductible, and τ_h^t and β^t are parameters of real estate tax. Whether or not to include these tax effects in the index computation is a priori a matter of choice, depending on the aim of the index.

The operational index is thus obtained as a ratio between the values of the expression (4) for t taken as the current period and the index reference period, respectively. This ratio is as usual practically computed as a weighted sum of the sub-indices for the components concerned.

Treatment of taxes and loans other than mortgages

As is seen in the expression (4), the index computation may take account of tax effects. First, changes in real estate tax will be shown as price changes in the index, and this is how it is already, and has been for long time, in the Swedish CPI.

However the expression (4) also allows for a relative deduction τ^t on the interest cost, due to possible tax deduction. Such tax effects are disregarded in the present Swedish CPI. The CPI Board has here taken the preliminary position that in the new model, the weight for the interest component should be reduced according to the deduction, but that changes in tax deduction rules for interest should not be shown as price changes in the index. This is also in line with the views of of SOU 1999:124.

As was hinted above, the CPI Board has also taken the preliminary position that the loans considered should be restricted to only mortgages and other house loans. Credit card interests etc. would thus be excluded, as they are now. Although the new model theoretically considers all loans of the household, and not only house loans, nevertheless interest on other loans than house loans are considered to be out of the suitable scope of the present Swedish CPI.

Issue on how to consider loan amount variations

An issue that has been subject to much discussion in the CPI Board concerns the treatment of the variation in the loan amount M in the index computation of the new approach. The question is whether and how the index should reflect interest cost changes due to house prices changes, as the latter affect the loan amounts needed for houses.

Several alternatives have been discussed on what cost to follow in the interest cost computation, mainly these:

Alternative A.	Interest cost on a constant nominal loan amount.
Alternative B.	Interest cost on a constant real loan amount.
Alternative C.	Interest cost on a constant share of the house purchase price, at a constant duration of past house ownership.

By "constant" is here meant constant in the index annual link computation, in the sense of being unchanged between link reference period and comparison period.

The model in its present form, of Klevmarken (2007), corresponds to Alternative B. Namely, the transformation Eq. (3) transforms the loan amount to constant prices. Not using this transformation would yield Alternative A.

So the issue is related to the specification of the model for consumer behaviour given by Eq. (1)-(3). To state it in other words, the question has to do with what volume unit the price to be followed is related to. It can be seen that the three alternatives use different factorisations of the interest term in the budget constraint Eq. (2A) and in the operative cost expression Eq. (4), namely:

$$\textit{Alternative } A: \qquad r_M^t(1-\tau^t) \ \times \ M^{\mathrm{B}}$$

$$\textit{Alternative } B: \qquad r_M^t(1-\tau^t)p_h^t \ \times \ q_h^{\mathrm{B}} \ \times \ \frac{M^{\mathrm{B}}}{p_h^t \, q_h^{\mathrm{B}}}$$

$$\textit{Alternative } C: \qquad r_M^t(1-\tau^t)p_h^{t-L} \ \times \ q_h^{\mathrm{B}} \times \ \frac{M^{\mathrm{B}}}{p_h^{t-L} \, q_h^{\mathrm{B}}}$$

These three expressions are equal to each other and equal to the interest term in the cost expression Eq. (4). However the three different factorisations identify different price and volume factors, as follows.

In each of the three above expressions the first factor is the price, and the second factor is the volume, in the sense of usual index formulas. The third factor, occurring for Alternatives B and C, may be seen as a quality adjustment factor. The term L in the expression for Alternative B denotes the duration of past ownership for the present owner. As it stands this form for Alternative C applies to a single household only, and in application on an aggregate (country) level the lagged price p_h^{t-L} is replaced by a moving average.

It may be noted that for Alternative B the mentioned third, quality adjustment, factor is equal to the transformed loan amount $\tilde{M}$ occurring in the utility function Eq. (1), after the transformation of Eq. (3). For Alternative C on the other hand the corresponding factor does not seem to have any sensible role to play in the utility function. This is possibly not an inadequacy, as anyhow the role of $\tilde{M}$ in the utility function is to reflect the disutility of the loan due to future risks and consumption restrictions, and this is not directly related to the current interest cost.

It follows from the stated factorisations that the units for the price in the interest component can be described as follows:

Alternative A -	"Dollars" of interest per "dollar" of loan.
Alternative B -	"Dollars" of interest per house unit with current value covered by loan.
Alternative C -	"Dollars" of interest per house unit with purchase value covered by loan.

In the Swedish application, "dollars" here is of course to be read as SEK (Swedish Kronor; nevertheless, long ago the Swedish currency was "daler", a word of the same origin as "dollar").

In practical computation, the interest cost sub-index is computed as follows in the mentioned alternatives:

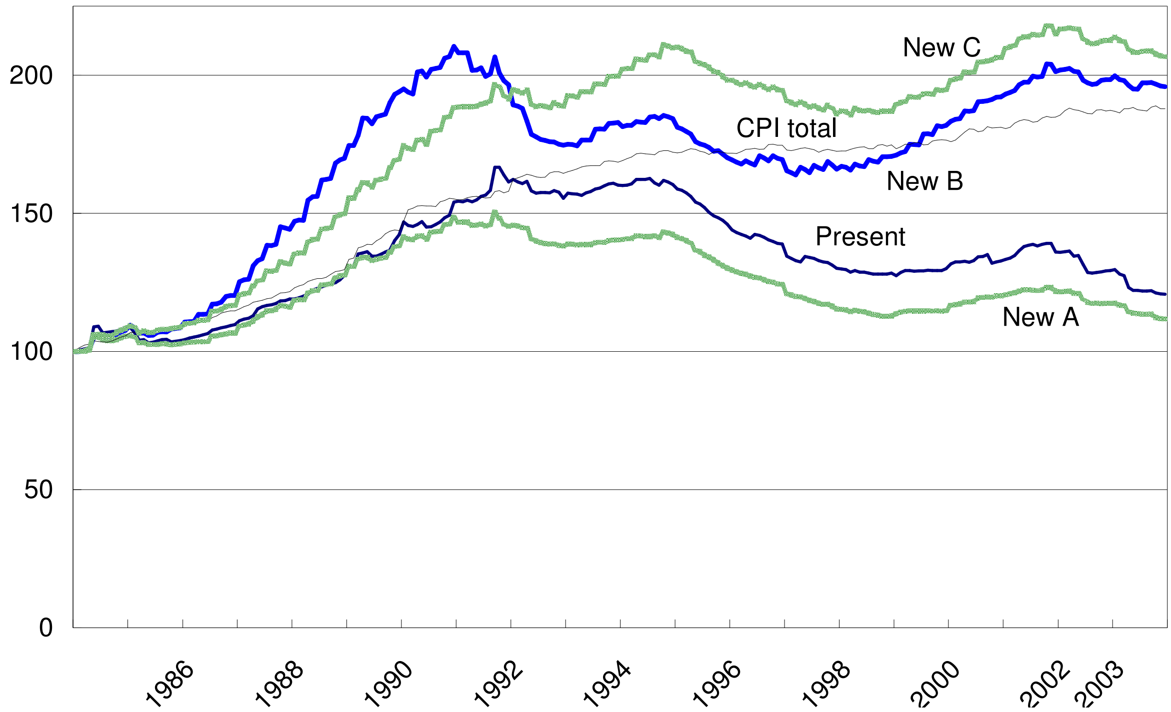
Alternative A.	Interest cost sub-index is taken as an interest rate index.
Alternative B.	Interest cost sub-index is taken as the product of an interest rate index and a house price index.
Alternative C.	Interest cost sub-index is taken as the product of an interest rate index and a long-term moving average of a house price index. The moving average is taken over 25 years or so, to represent typical durations of house ownership.

In both Alternatives B and C the interest cost sub-index would in the very long run tend to follow the development of house prices, given that interest rates then move around a constant level. This may be seen as an indication that in Alternatives B and C, the index may in the long run be able to fulfil its aim, of answering the question on the income development needed for maintaining an unchanged level of utility.

Although Alternatives B and C will thus have largely similar outcomes in the very long run, their outcomes may on the other hand differ vastly in shorter terms. Particularly, an instantaneous house price shock has immediate impact in Alternative B, but in Alternative C it is attenuated off, by the use of a long-term moving average.

Conceptually, both Alternatives B and C can be said to be based on an idea of keeping the quality constant, but they differ in their conceptual approaches to this task. Arguments can be given for both. In a way, the view on quality may in Alternative B perhaps be said to be forward-looking, taking account of future risks and consumption possibilities. On the other hand, that of Alternative C is backward-looking, taking account of conditions in the past with impact on the current cost.

The following diagram shows simulated time-series for the sub-index of capital costs for owner-occupied housing, according to the mentioned alternatives. It is seen that the outcomes differ notably in practice, so the issue really does matter.



An alternative with real interest

At the presentation of Klevmarken (2006a) to the Ottawa Group Meeting in 2006, Professor Erwin Diewert suggested in the discussion the use of real interest rate in the computation of this index. In the operational computation this would mean that in the expression Eq. (4) the nominal interest rate r_M^t would be replaced by a real interest rate $r_M^t - \pi^t$, where an expected future annual inflation rate π^t is deducted. A priori the latter could refer to either the overall inflation or the house inflation.

From the point of view of principles, it is possible to see reasons both for and against the use of real interest rather than nominal interest in the index computation. A reason for using real interest is that the future inflation will in real terms help in repaying the loans. Namely, this is adequately reflected in use of real interest rate, by the deduction of the inflation rate from the nominal interest rate. On the other hand, a reason against using real interest is that the inflation gains on the loans are not realised until the loans are actually repaid. This may largely lie in such a distant future that it is more or less out of consumers' current perspective.

There is also a practical side to be considered. Ideally π^t should be an expected future inflation rate. However such an expected statistic would be problematic to estimate practically, as one would have to rely on either surveyed opinions or time-series forecasting models, and both those alternatives seem problematic; cf. National Institute of Economic Research (2002). Using surveyed opinions would introduce an element of judgement and might open up for possible influence of speculation on the CPI, which would be unallowable. Time-series models for forecasting in turn are dependent on assumptions and cannot be expected to give valid results for more than a rather limited period into the future.

For practical computation of the real interest one would thus probably have to use the current observed inflation rate, as a proxy for the expected future inflation rate, for the term π^t. Potentially such a practice could be defended by an assumption that the inflation rate series is a martingale. However that assumption is at least not evidently justified, and it may also be argued that the long- or medium-term inflation expectations would plausibly be less volatile than the actual inflation.

Considering both the aspects of principle and the practical aspects, the CPI Board has taken the preliminary position that nominal and not real interest is to be used in the index computation.

Impact of using real interest

An attempt can be made to estimate the impact on the index of using real instead of nominal inflation rate, with π^t taken as the current annual overall inflation rate. For feasibility the computation is based on Alternative A mentioned above, that is, without use of the transformation Eq. (3). With the subscript "real" and "nom" used to denote whether real or nominal interest is used for the computation of a statistic, the following relation can be derived from Eq. (4), namely,

$$(5) \quad I^{0;t}_{\text{real}} = \frac{\mu^t_{\text{real}}(\mathbf{Q})}{\mu^0_{\text{real}}(\mathbf{Q})} = \frac{\mu^t_{\text{nom}}(\mathbf{Q}) - \pi^t_{\text{real}} M^0}{\mu^0_{\text{nom}}(\mathbf{Q}) - \pi^0_{\text{real}} M^0}$$

$$\approx \frac{\mu^t_{\text{nom}}(\mathbf{Q})}{\mu^0_{\text{nom}}(\mathbf{Q})} + (\pi^0_{\text{real}} - \pi^t_{\text{real}}) \frac{M^0}{\mu^0_{\text{nom}}(\mathbf{Q})} =$$

$$= I^{0;t}_{\text{nom}} + (\pi^0_{\text{real}} - \pi^t_{\text{real}}) \frac{M^0}{\mu^0_{\text{nom}}(\mathbf{Q})}.$$

With disregard of the annual chaining and with $t-1$ denoting the time 12 months before t, the following approximate equation is obtained,

$$\pi^t_{\text{real}} \approx \pi^t_{\text{nom}} + (\pi^{t-1}_{\text{real}} - \pi^t_{\text{real}}) \frac{M^0}{\mu^0_{\text{nom}}(\mathbf{Q})},$$

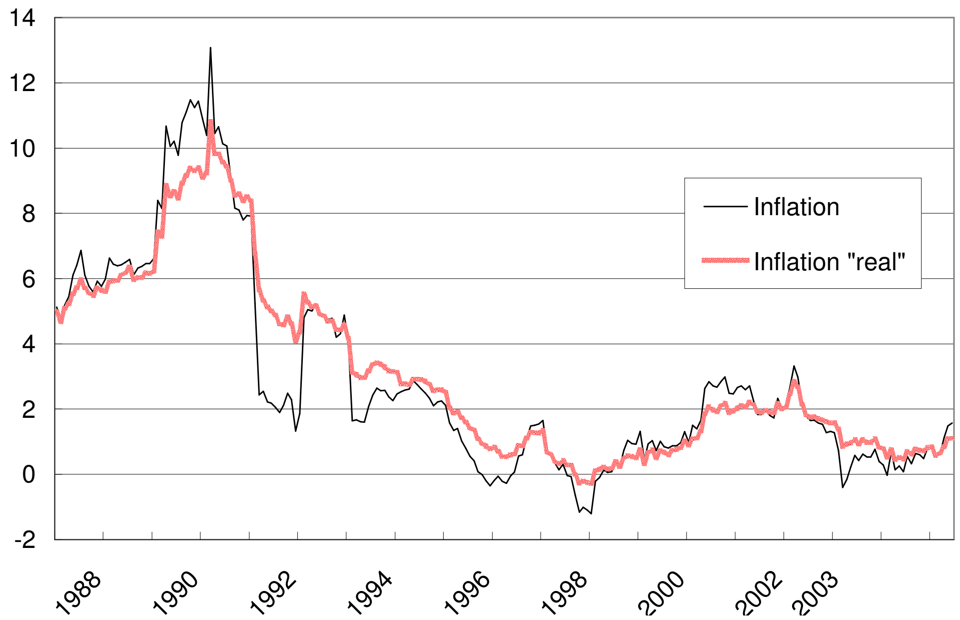
and this equation is solved by

$$(6) \qquad \pi^t_{\text{real}} \approx \frac{\pi^t_{\text{nom}} + \pi^{t-1}_{\text{real}} \dfrac{M^0}{\mu^0_{\text{nom}}(\mathbf{Q})}}{1 + \dfrac{M^0}{\mu^0_{\text{nom}}(\mathbf{Q})}} \, .$$

This relation can be used for a recursive re-computation of the usual inflation rate series π^t_{nom} into one based on real interest, that is π^t_{real} . The values of the ratio $M^0 / \mu^0_{\text{nom}}(\mathbf{Q})$ needed here can be obtained from official statistics, as shown in the following table:

Year	Consumtion MSEK	Mortgages MSEK	Relation %	House prices 1981=100
1985	418 000	266 000	64	109
1987	459 000	287 000	63	130
1989	508 000	309 000	61	181
1991	707 000	423 000	60	217
1993	744 000	473 000	64	175
1995	805 000	467 000	58	184
1997	864 000	469 000	54	198
1999	926 000	527 000	57	237
2000	935 000	529 000	57	263
2002	1 030 000	644 000	63	302
2003	1 076 000	674 000	63	322
2004	1 103 000	774 000	70	353

The result is shown in the following diagram.



Note that that the diagram shows the overall inflation, covering the entire consumption. It is seen that using real instead of nominal interest may have a quite dramatic impact on the inflation rate, up to some two percentage points, as is seen in the diagram. Such a huge impact might be hard to explain to users of the index.

Mathematical properties of the utility function

In the standard theory of the cost-of-living index, the utility function is mathematically a concave function. Often it is for convenience also assumed to correspond to homothetic preferences, which means that it is a function of a linearly homogeneous function, or practically equivalently that the level hyper-surfaces are homothetic images of each other. In the dynamic model of Klevmarken (2007), the utility function Eq. (1) is still concave in the "q-arguments", but the properties with respect to the arguments A and M deserve some elaboration. Consider he utility function given in simplified notation as

(1A) $$\tilde{U}(\mathbf{q}, q_h^0 + \lambda q_m, q_h, q_r, A, M) \ .$$

Three of the arguments here affect the utility in the current period depending on future consumption, which in turn is affected by the consumer's resources and choices in the current period. These three arguments are

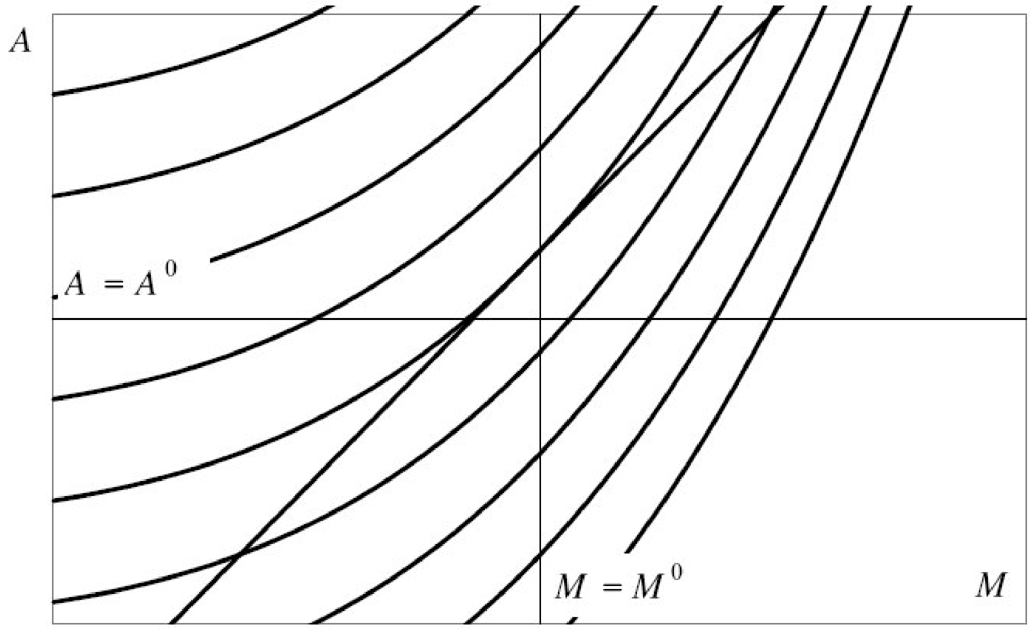
- The dwelling volume q_h , enabling future housing.
- The assets A , enabling future spending on consumer products.
- The loan amount M, entailing financial risks for the future, and future interest payments and instalments which restrict future spending on consumer products.

Plausibly the following conclusions should hold:

- $\tilde{U}$ *is an increasing concave function in each of its arguments except* M – as an increment in any of these variables yields positive and decreasing marginal utility. (That this holds also for A follows from its truth for the "q-variables" in the future consumption.)
- $\tilde{U}$ *is a decreasing concave function in the argument* M – as an increment in M adds to the disutility of future risks and future restrictions of consumption, and furthermore a larger M tends to push down future consumption possibilities to regions with worse scarcity and a higher gradient of the utility function.

Suppose now that the consumer has chosen optimal values of all arguments in the utility function, so that the utility function attains the value that is maximal attainable value under the budget constraint. Then keep the "q-variables" at their values so chosen but allow A and M to vary freely again, so that the utility thus restricted becomes a function of A and M only. The level curves (indifference curves) of this function of A and M

may look as in the following diagram, where also a line corresponding to the budget constraint is included.



Compared to the usual picture in the theory of the cost-of-living index, this picture has some unusual features. First, the indifference curves are increasing (and convex) rather than decreasing (and convex). Second, the budget line has positive slope, with inclination angle close to 45 degrees, rather than negative slope. This is quite natural and due to the fact that larger M gives less utility. Note that the consumer has the possibility of e.g. moving downwards-left along the budget line, by using bank assets to repay loans.

Necessary conditions for non-triviality

Just as in the usual theory, the optimal choice lies in a point where the budget line is tangent to an indifference curve. This also holds in the A-M-plane shown in the diagram. However, potentially situations may occur where the budget line is not a tangent to any indifference curve.

For instance, imagine for the moment a case where the utility function is such that all indifference curves are straight lines. That case has two possible sub-cases. First, if the budget line happens to coincide with an indifference curve, then there is no unique optimal choice, hut a whole range of choices that are all optimal. Second, and more likely, the budget line may not coincide with any of the straight indifference lines, and then the optimal choice must be on the boundary of the domain of feasible choices. This means that the optimal choice would be either to increase the loans as much as any lender would agree to, or to do just the opposite, to repay as much as ever possible, to the last available penny.

Although such "bang-bang" type situations may of course occur, they would apparently often not describe a realistic optimisation setting for the consumer. To be generally realistic the model must be able to describe not only such trivially extreme situations, but also situations which involve a non-trivial trade-off, between utility of liquidity and disutility of debt. The latter kind of situations is such that the budget line is a tangent to an indifference curve in the mentioned optimal *A*-*M*-plane. To make this possible, the indifference curves must be strictly convex and have a derivative with sufficiently wide range.

In the usual theory of the cost-of-living index, homothetic preferences are often assumed to hold. This assumption apparently is largely made for mathematical convenience, as its realism can generally be questioned (it disregards "Engel's law"). However in the present setting preferences homothetic in all variables are unfeasible also mathematically, as potentially this would necessarily imply boundary optimum ("bang-bang"). Namely, if the budget line in the *A*-*M*-plane happens to go through the origin, then a level curve to which it is tangent would intersect some homothetic image of itself, and then this image cannot be a level curve.

Corollaries. It can now be concluded that in order to express non-trivial consumer trade-offs concerning A and M, the utility function must have these properties:

- $\tilde{U}$ *must depend on both* A *and* M.
- $\tilde{U}$ *cannot depend on* A *and* M *only by way of one single linear combination, such as* $A - M$.
- $\tilde{U}$ *cannot correspond to preferences that are homothetic in all their arguments.*

Possible derivation from a multi-period-model (?)

Klevmarken (2007) deliberately refrains from formulating a model where the consumer optimises her utility with regard to prospects over an indefinite future, as such a model involves specifications that are redundant to the index computation. Without objecting to this view one might still perhaps take it as an exercise to try to derive the model of Klevmarken (2007) from a multi-period model, in the spirit of, e.g., Adda & Cooper (2003), Li & Löfgren (2004), Reis (2005), Weitzmann (2003).

Such a multi-period model would be more dependent on specific assumptions than that of Klevmarken (2007). It could though possibly be of value in exemplifying what could possibly be hidden in the utility function, and then perhaps serve as a test of potential consistency problems. The following is just a sketch of how such a model might be constructed.

With simplified notations analogous to those used above, the following form of utility function may be postulated:

$$(7) \qquad V(\{\mathbf{Q}^{(s)}\}_{s\geq t}) \;=\; \sum_{s=t}^{\infty} c^{s-t}\, W(\mathbf{Q}^{(s)}) \qquad (c<1)$$

This utility function reflects the utility in the current period t due to both the consumption in that period and the anticipated consumption in all coming periods in an indefinite future. The consumer is assumed to maximise this utility function subject to budget constraints,

$$(8) \qquad \begin{aligned} y^{(s)} + dA^{(s-1)} - M^{(s-1)} \;&=\; \mathbf{P'}^{(s)}\,\mathbf{Q}^{(s)} + d'A^{(s)} - d''M^{(s)} && (s \geq t) \\ dA^{(s-1)} \;&=\; \mathbf{P'}^{(s)}\,\mathbf{J}\mathbf{Q}^{(s)} + d'A^{(s)} && (s \geq t+1)\;. \end{aligned}$$

Here $\mathbf{J}$ denotes a diagonal matrix with entries ≤ 1, with the non-zero entries marking product categories with purchases to be financed from savings, such as capital goods.

From this model it is possible to derive a utility function where the anticipated future consumption is built in. This will be a utility function directly corresponding to the utility function Eq. (1) although in simplified notation,

$$(9) \qquad U(\mathbf{Q}^{(t)}, A^{(t)}, M^{(t)})$$

Like Eq. (1) this utility function depends on the consumer's current choices only, not her future choices. The future choices are eliminated from the current period's considerations, by the assumption that they will be made optimally when their time comes. This utility function could be constructed by use of customary Lagrange's multipliers, from the multi-period model defined by Eq. (7) and (8) as

$$(10) \qquad \begin{aligned} U(\mathbf{Q}^{(t)}, A^{(t)}, M^{(t)}) \;=\;& W(\mathbf{Q}^{(t)}) + \sum_{s=t+1}^{\infty} c^{s-t}\, W(\mathbf{Q}^{*(s)}) + \\ &+ \lambda^{*}\cdot(\mathbf{P'}^{(t+1)}\,\mathbf{Q}^{*(t+1)} + d'A^{*(t+1)} - d''M^{*(t+1)} - y^{(t+1)} - dA^{(t)} + M^{(t)}) + \\ &+ \mu^{*}\cdot(\mathbf{P'}^{(t+1)}\,\mathbf{J}\mathbf{Q}^{*(t+1)} + d'A^{*(t+1)} - dA^{(t)})\,, \end{aligned}$$

where asterisk denotes optimal choices made by the consumer in periods $t+1,\;t+2,\;...$ on the condition of the period t choices specified by the arguments $\mathbf{Q}^{(t)}, A^{(t)}, M^{(t)}$. It is not necessary to go further into mathematical tools for the optimising over the future involved here, as it suffices to ensure that U can be sensibly defined.

It may be noted that the utility function depends on future prices, as these affect the consumer's future choices denoted by asterisk in Eq. (10). In this context the future prices are to be taken as expected prices, which would have to be determined by some model.

Also, in the above multi-period consideration, the transformation by Eq. (3) of the loan amount to constant prices was for simplicity not explicitly considered.

The multi-period consideration presented in this section is thus not more than a first sketch, to illustrate some conceptual considerations that would have to be dealt with in an attempt to embed the dynamic model in a multi-period model. To be used in e.g. sensible numerical simulations the multi-period model would have to be elaborated on further.

Disclaimer

The views expressed in this paper are those of the author solely.

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