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Recalculations of the Danish CPI 1996 – 2006

Abstract

Walsh, Marshall-Edgeworth, Fisher and Törnqvist price indices are calculated on the basis of the elementary indices included in the Danish CPI. These 'ideal' indices give very similar results and indicate an annual rate of inflation 0,05-0,1 % point below that of the published CPI. Price-updating of the expenditure weights is shown to add to the upward bias of the CPI compared to an ideal index, while the use of original weights as they stand reduces upward bias and gives a better estimate of an ideal index. The CPI is also recalculated as a geometric Young index and as a geometric average of a Young index and a rebased Young index, both of which lies below the ideal indices.

1. Introduction

On the basis of the database applied for the calculation of the Danish CPI it has been possible to establish a complete data set consisting of the elementary aggregate indices and weights that have been used for calculation of the monthly CPI from 1996 to 2006.

The data set makes it possible to perform retrospective calculations and throw some empirical light on three questions:

- What are the "best" indices that can be calculated, when weights referring to the index period are available?
- What is the bias, if any, in the regular CPI, based on expenditure weights of a past period, compared to indices that utilize weights referring to the index period?
- What is the effect of price-updating expenditure weights before they are introduced in the CPI?

All calculations are made bottom-up as in the calculation of the regular monthly CPI in real time, by aggregating the elementary indices, i.e. the indices at the most detailed level, into the overall CPI.

2. The data set

The elementary indices of the CPI

The data set consists of 442 monthly elementary indices all going from January 1996 to December 2006. In this period there have been minor changes in the set of elementary aggregates in the Danish CPI. In order to establish a complete and coherent data set, some elementary indices have either been left out or imputed by the price development of similar ones, and some have been merged to obtain consistent series for the whole period.

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Expenditure weights

From 1996 to 2006 the CPI was calculated using four sets of expenditure data referring to the years 1994, 1996, 1999 and 2003. The corrections made in order to establish the data set concern elementary indices with low weights in the CPI, so that the 442 elementary aggregates included in the calculations account for 98-99 percent of the weighting basis of the CPI.

The recalculated CPI resembles the published CPI

The CPI calculated on the basis of the established data set is almost identical to the published CPI. Out of the 132 monthly indices, the recalculated CPI deviates by a maximum of plus/minus 0,1 % point in 43 months; in the rest of the months the deviation is smaller. With 1996 as 100 both the published and the recalculated CPI equal 123,3 in 2006.

Table 1. The published and the recalculated CPI (1996 = 100)

	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
CPI-pub	102,20	104,09	106,67	109,78	112,37	115,10	117,50	118,87	121,02	123,30
CPI-rec	102,13	104,03	106,64	109,74	112,34	115,09	117,49	118,86	121,01	123,28

CPI-pub: The published CPI rescaled to 1996=100.
CPI-rec: The CPI recalculated on the basis of the established data set.

The similarity of the published and the recalculated CPI makes it possible to draw conclusions about the possible effect in practice of applying different methods for calculation of the CPI. In what follows the recalculated CPI is simply referred to as the CPI.

3. Calculation of ideal indices

Fisher, Walsh and Törnqvist

With the available data set what would be the best measure of the price development from 1996 to 2006? Following the *CPI Manual* [1] the Fisher, Walsh and Törnqvist price indices should be the preferred ones according to both the axiomatic and the economic approach to index number theory:

> "Fisher, Walsh and Törnqvist price indices approximate each other very closely using "normal" time series data. This is a very convenient result since these three index number formulae repeatedly show up as being "best" in all the approaches to index number theory. Hence, this approximation result implies that it normally will not matter which of these indices is chosen as the preferred target index for a consumer price index." (The *CPI Manual*, p. 313)

Marshall-Edgeworth

The CPI has been recalculated using the Walsh, Fisher, Törnqvist and Marshall-Edgeworth formulas. The Marshall-Edgeworth index, while not superlative, can be expected to give results similar to the other three indices.

The ideal indices are calculated for 1996-1999 and 1999-2003

All these four *ideal* indices rely on weights from the two periods being compared. The indices, therefore, have been calculated for the periods 1996-1999 and 1999-2003, respectively. For 1996-1999 the weights applied are those of 1996 and 1999. For 1999-2003 the weights are those of 1999 and 2003. The definition of the indices and the applied calculation formulas are shown in annex 1.

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[1] ILO/IMF/OECD/UNECE/Eurostat/The World Bank: Consumer price index manual. Theory and practice. International Labour Office, Geneva, 2004.

Annual average index series

In the ideal indices the weights and prices are required to refer to similar periods in time. However, the weights are annual while the elementary indices are monthly, based on monthly recorded prices. To align the weight and price reference periods the annual average of the elementary indices, calculated as a simple arithmetic average of the 12 monthly indices, has been applied.[1] The indices for 1996-1999 are calculated with the elementary indices rescaled to 1996=100, the indices for 1999-2003 are calculated with the elementary indices rescaled to 1999=100. The latter is subsequently chained onto the former. The chaining does not affect the comparison of the series. The indices are shown in Table 2.

Table 2. Ideal indices, 1996 – 2003

	1996	1999	2003	1996-1999	1999-2003	1996-2003
	Index, 1996=100			*Average annual rate of change in %*		
Walsh	100,00	106,39	117,07	2,09	2,42	2,28
M-E	100,00	106,35	117,10	2,07	2,43	2,28
Fisher	100,00	106,34	117,05	2,07	2,43	2,27
Törnqvist	100,00	106,38	117,08	2,08	2,42	2,28

Note: The % changes are calculated on the basis of the un-rounded indices.

The ideal indices give very similar results

The four indices are remarkably similar. The calculations confirm that for all practical purposes the four indices can be expected to give very similar results.

CPI compared with the Walsh index

It is interesting now to compare the CPI with the ideal indices. For the sake of simplicity the Walsh index will be used as reference. The CPI and the Walsh index are shown below in Table 3.

Table 3. The CPI and the Walsh index, 1996 – 2003

	1996	1999	2003	1996-1999	1999-2003	1996-2003
	Index, 1996=100			*Average annual rate of change in %*		
CPI	100,00	106,64	117,49	2,17	2,45	2,33
Walsh	100,00	106,39	117,07	2,09	2,42	2,28
Difference				0,08	0,03	0,05
	Excluding owner-occupied housing					
CPI	100,00	106,54	117,23	2,13	2,42	2,30
Walsh	100,00	106,25	116,75	2,04	2,38	2,24
Difference				0,09	0,04	0,06

Note: The % changes and the differences are calculated on the basis of the un-rounded indices.

At face value, CPI exceeds Walsh by 0,05 % point on the annual rate of change

In the first period from 1996 to 1999 the average annual rate of change of the CPI exceeds that of the Walsh index by 0,08 % point. From 1999 to 2003 there is a difference of only 0,03 % point on the annual rate of change. Taken together for the whole period 1996 to 2003 the CPI overestimates the

[1] The annual indices may instead be calculated as the geometric average of the monthly series. However, for comparison with the CPI, where the annual index is calculated as the arithmetic average of the monthly series, it was chosen to use the arithmetic average. Secondly, using the geometric average instead proved to have only very marginal influence on the indices, and does not influence the relation between the compiled indices.

annual rate of change by 0,05 % point, compared to the Walsh index. The very small difference for 1999-2003 can be explained by some unusual combinations of changes in prices and weights of a few elementary indices.

CPI exceeds Walsh by 0,09 % point if unusual changes are excluded

From 1999 to 2003 car insurance premiums went up by 47 %, financial services by 33 % and gardening by 152 %. At the same time the expenditure weights of these services increased sharply. The weights of the CPI refer to 1994 (for 1999), 1996 (for 2000-2002) and 1999 (for 2003), while in the Walsh index the weights refer to an average of 1999 and 2003. As a result of this the weight of the three indices in the Walsh index is 1,4 % and only around the half in the CPI. Excluding the three indices from both the CPI and the Walsh index, the rate of change of the CPI would exceed that of the Walsh index by 0,10 % point from 1999 to 2003. For the whole period from 1996 to 2003 the annual rate of change of the CPI would then exceed that of the Walsh by 0,09 % point, on average.

On average, the CPI exceeds the Walsh by 0,05 - 0,10 % point on the annual rate of change

The exclusion of such "unusual" changes is questionable, however. On the on hand, they are unusual and not in line with the inverse relation between price and quantity changes often assumed. On the other hand, unusual changes do occur from time to time, and thus should not be excluded. It is therefore difficult to draw any decisive conclusion, but the calculations indicate that the annual rate of change of the CPI exceed that of the CPI by 0,05 – 0.10 %, on average.

Similar results if owner occupied housing is excluded

Rents accounts for 19-20 % of the CPI weighting basis. Out of this 11-12 % are imputed rents for owner occupied housing. Owner occupied housing is included in the Danish CPI by the rental equivalent approach, which means that 19-20 % of the CPI is adjusted by the same price index. To analyse the effect of this the calculations have been carried out excluding owner-occupied housing, leaving only rented dwellings. This index essentially is equal to the Danish Harmonised Index of Consumer prices (HICP). As seen from table 3, excluding owner-occupied housing does not change the results, as the overshooting compared to the Walsh index is still around 0,05 % point on the average annual rate of change; 0,1 % point when car insurance, financial services and gardening are excluded.

4. The effect of price-updating expenditure weights

The Danish CPI is calculated as the expenditure weighted arithmetic average of the elementary aggregate indices:

$$P_{0:t} = \sum w_b^j P_{0:t}^j \tag{1}$$

The CPI is calculated without price-updating the weights – the Young index

$P_{0:t}$ is the overall CPI from period *0* to *t*, $P^j_{0:t}$ are the elementary aggregate indices, and w^j_b the corresponding expenditure weights referring to period *b*. The weights are not price-updated from the weight reference period (*b*) to the price reference period, *0*. Following the CPI Manual, this will be referred to as a Young index.

Chaining of the CPI

When new weights are introduced, the index is chained onto the old index series. From 1996 to 2006 new weights were introduced in 1999, 2003 and 2006, so that the index consists of the following four links:

Table 4. The links in the chained CPI

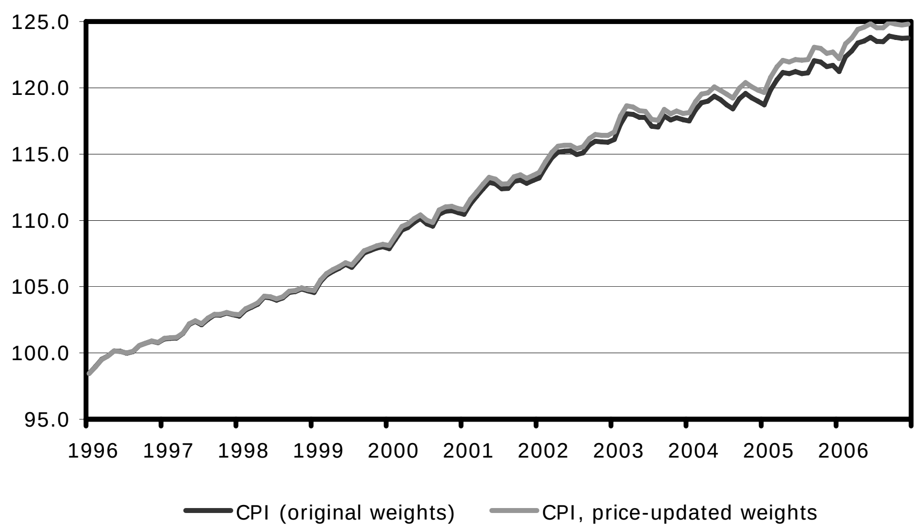
	Index link period	Weight reference period
1	Jan. 1996 – Dec. 1999	1994
2	Dec. 1999 – Dec. 2002	1996
3	Dec. 2002 – Dec. 2005	1999
4	Dec. 2005 – Dec. 2006	2003

Using December as chaining month, CPI in December 2006 with 1996 as index reference period is thus calculated as:

$$(2)\quad P_{96:Dec06} = \sum w_{94}^{j} P_{96:Dec99}^{j} \sum w_{96}^{j} P_{Dec99:Dec02}^{j} \sum w_{99}^{j} P_{Dec02:Dec05}^{j} \sum w_{03}^{j} P_{Dec05:Dec06}^{j}$$

The interesting question is what the CPI would have been if the weights had been price-updated? To answer this, the CPI has been recalculated with price-updated weights. That is, the 1994 weights have been price-updated to the average of 1996; the 1996 weights have been price-updated to December 1999 etc. The results are shown in Figure 1 and Table 5.

Figure 1. The CPI with original and price-updated weights (1996 = 100)



Price-updating the weights increases the annual rate of change by 0,1 % point

From the 1996 to 2006 the CPI increases by 23,28 % using the original weights as they stand, and by 24,29 % using the price-updated weights. The annual rate of change is bigger in all years when using price-updated weights. The average annual rate of change is 2,11 % when weights are not price-updated, and 2,20 % with price-updating. Hence, had the weights been price-updated the annual rate of change of the CPI would, on average, have been 0,1 % point higher than the ones actually published.

Same result if owner occupied housing is excluded

The effect of price-updating the weights is of the same magnitude if owner occupied housing is left out. Excluding owner-occupiers, the average annual rate of change is 2,06 % with the original weights and 2,15 % with the price-updated weights.

The use of original weights provides better estimate of an ideal index

In summary, price-updating the weights increases the upward bias of the CPI compared to an ideal index, and the CPI using original weights provides the better estimate of an ideal index with less upward bias.

Table 5. The CPI with original and price-updated weights

	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
	Annual average, 1996 = 100									
CPI	102.13	104.03	106.64	109.74	112.34	115.09	117.49	118,86	121,01	123,28
CPI-puw	102,18	104,12	106,79	110,03	112,71	115,55	118,01	119,60	121,98	124,29
	Annual rate of change in %									
CPI	2,13	1,87	2,51	2,91	2,37	2,44	2,08	1,17	1,80	1,88
CPI-puw	2,18	1,90	2,56	3,03	2,43	2,52	2,14	1,34	1,99	1,90
Diff.	0,05	0,03	0,06	0,12	0,07	0,08	0,05	0,17	0,19	0,02
	Annual average, 1996 = 100, excl. owner occupied housing									
CPI	102,08	103,95	106,54	109,67	112,21	114,94	117,23	118,35	120,38	122,61
CPI-puw	102,07	103,99	106,65	109,92	112,55	115,39	117,75	119,09	121,40	123,67
	Annual rate of change in %, excl. owner occupied housing									
CPI	2,08	1,83	2,50	2,94	2,32	2,43	1,99	0,95	1,72	1,85
CPI-puw	2,07	1,88	2,56	3,06	2,39	2,52	2,05	1,14	1,94	1,87
Diff.	-0.01	0.05	0.06	0.12	0.07	0.09	0.06	0.19	0.22	0.02

CPI-puw: CPI calculated with price-updated weights.

4.1 Comparing the Lowe and Young indices

2/3 of NSOs price-update weights, 1/3 does not, according to the 2007 CPI Manual Survey

When expenditure weights are available only with a time lag, the statistical office has to decide whether to price-update the weights or not. In the joint UNECE/ILO survey on the *CPI Manual*, as per early September 2007, 32 national statistical offices (NSOs) have replied that they price-update weights for calculation of the regular CPI. 15 NSOs have replied that they use the original weights.

The Lowe index

The main argument in favour of price-updating seems to be that weights and prices should be aligned to the same period in time in order to calculate a Laspeyres price index. However, if the weights are price-updated this corresponds to the calculation of a Lowe index (the *CPI Manual*, chp. 15):

(3)
$$P_{0:t}^{Lo} = \frac{\sum p_t^i q_b^i}{\sum p_0^i q_b^i} = \sum \frac{p_0^i q_b^i}{\sum p_0^i q_b^i} \frac{p_t^i}{p_0^i} = \sum w_{b(0)}^i \frac{p_t^i}{p_0^i},$$
$$w_{b(0)}^i = \frac{w_b^i \left(p_0^i / p_b^i \right)}{\sum w_b^i \left(p_0^i / p_b^i \right)} \ , \quad w_b^i = \frac{p_b^i q_b^i}{\sum p_b^i q_b^i}$$

The CPI with price-updated weights aims to measure the changing cost of a fixed basket of a past reference year

The Lowe index is a general type of a basket index. It does not require the quantities to refer to any basket in a particular period in time. While not a ‘pure’ Laspeyres index, the calculation of a Lowe index, i.e. the price-updating of weights, can be justified on the basis that it is conceptually clear and provides a measure of a well-defined basket index. It measures, from month to month, the changing costs of buying the same annual basket of a past reference year. The questions that could be raised are, how relevant is it to measure the changing cost of a basket referring to a past period, and how can the Lowe index be expected to estimate an ideal index?

The Young index

If the weights are not price-updated, this corresponds to the calculation of a Young index (the *CPI Manual*, chp. 15):

$$(4) \qquad P^{Yo}_{0:t} = \sum w^i_b \left(\frac{p^i_t}{p^i_0} \right), \qquad w^i_b = \frac{p^i_b q^i_b}{\sum p^i_b q^i_b}$$

The CPI with original weights aims to measure the price development in the index period

The Young index is a *fixed weight index*. It is not a fixed basket index (unless *b* equals *0* or *t*) as it does not measure the changing costs of buying a fixed basket such as Lowe. Focus is that the weights should be as representative as possible of the expenditure shares of the index period from *0* to *t*. Hence, the calculation of a Young index, i.e. the use of original weights, can be justified as an estimate of the average price changes in the index period in which the weights are applied. The period b expenditure shares can be seen as estimates of the average expenditure shares from *0* to *t*, and the Young index as an estimate of an ideal index from *0* to *t*.

The difference between the Young and Lowe indices can be illustrated by subtracting the one from the other:

$$(5) \qquad \begin{aligned} P^{Lo}_{0:t} - P^{Y}_{0:t} &= \sum w^i_{b(0)} \left(p^i_t / p^i_0 \right) - \sum w^i_b \left(p^i_t / p^i_0 \right) \\ &= \sum \left(w^i_{b(0)} - w^i_b \right) \left(p^i_t / p^i_0 \right) \end{aligned}$$

Lowe exceeds Young if there are long-term trends in relative prices

The Lowe index gives more weight to those elementary indices the prices of which have increased by more than average from *b* to *0* and less weights to those where the prices have increased by less than average. Therefore, if there are long-term trends in the prices, so that prices which have increased relatively from *b* to *0* continues to do so from *0* to *t*, and prices which have fallen from *b* to *0* continues to fall, the Lowe index will exceed the Young index. This indicates a long-run tendency for the Lowe index to exceed the Young index. This effect is build into the formulas; it is not related to what may or may not take place in reality in terms of households substituting in response to changing relative prices.

Whether Lowe or Young estimate an ideal index better depends on households' substitution

Whether a Young or Lowe index is the better estimate of an ideal index depends on whether the original (w^i_b) or the price-updated ($w^i_{b(0)}$) weights is the better estimate of the average expenditure shares from *0* to *t*.[1] If the elasticity of substitution at the elementary aggregate level is closer to one, Young is the best estimate. If the elasticity of substitution is closer to zero, Lowe is the best estimate.

Young appears to be the better estimate of an ideal index

Normal consumer behaviour suggests that in general some substitution should be expected, so that the Lowe index will tend to be biased upward compared to an ideal index. In the calculations in this paper Young provides

[1] Note that both Lowe and Young can be expressed in the same form as Walsh and Marshall-Edgeworth, as the expenditure share weighted arithmetic average of the price ratios, the only difference being the weighting component.

a better, less upward biased, estimate of an ideal index, which indicates an elasticity of substitution closer to one than zero. As the Young index allow for some substitution from *b* to *0,* while Lowe does not, it may be argued that the traditional Laspeyres bias to some degree is reduced in the Young index as compared to the Lowe index. Thus, to omit price-updating may be one practical way in which to reduce this type of bias.

Lowe has better axiomatic properties than Young …

The Lowe index satisfies more axioms, or tests, than the Young index (the *CPI Manual*, para. 16.130-134). In particular Lowe satisfies the *time reversal test* and the *circularity test,* which the Young index fails. However, the practical relevance and importance of these tests is not clear.

... but how important is this in practice?

Firstly, it is true that when formulated at the level of *individual* prices and quantities as in (3) and (4), Lowe satisfy these tests while Young does not. But is the argument also relevant in practice, when the CPI is calculated by aggregating the elementary indices and where the only difference is the weighting component? *Secondly*, if price-updating is likely to increase upward bias in the CPI compared to an ideal index, how much weight should then be given to the axiomatic properties compared to the importance of providing a better estimate of an ideal index?

The CPI Manual on price-updating of weights

The *CPI Manual* is not very prescriptive on the issue of price-updating weights, but leaves this for the statistical offices to decide. It does mention, though, that automatic price-updating should be undertaken with care: When there is a strong inverse relation between movements of prices and quantities, price-updating may produce perverse results. However, more guidelines or recommendations might be useful, in particular if different practices are likely to influence international comparability.

5. Alternative calculations

The Geometric Young index

The Geometric Young index allows for some substitution

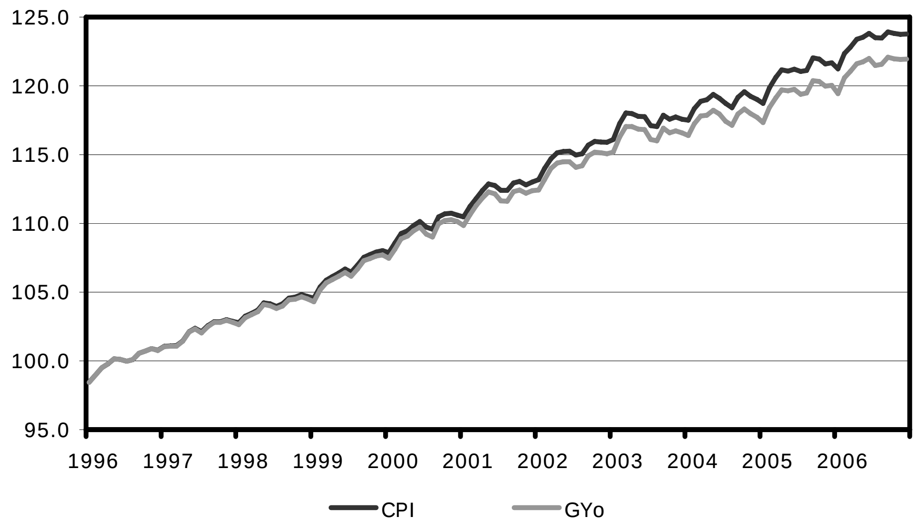
It has been argued that when the CPI is calculated as a Young index with original weights as in equation (1), then, at the elementary aggregate level, an elasticity of substitution of one is assumed from *b* to *0*, while from *0* to *t* no substitution is allowed. To be consistent, therefore, the CPI should be calculated as the expenditure weighted *geometric* average of the elementary aggregate indices, since this would correspond to assume *also* from *0* to *t* an elasticity of substitution of one. To test this, the CPI has been calculated as a *Geometric Young* index:

$$(6) \qquad P_{0:t} = \prod P_{0:t}^{j\ w_b^j}$$

Calculation of the Geometric Young

The Geometric Young index is calculated with the same weights and links as the arithmetic Young index, i.e. the CPI, in equation (2). The weights are not price-updated. Had the weights been price-updated, one could talk of a Geometric Laspeyres index, as in the *CPI Manual* (para. 9.66). The geometric Young index and the CPI are shown in Figure 2.

Figure 2. The Geometric Young index and the CPI



The Geometric Young clearly below the arithmetic Young

Throughout the Geometric Young results in a lower index than the CPI. From 1996 to 2006 the average annual rate of change of the Geometric Young index is 1,96 %, against 2,21 % for the arithmetic Young index.

Table 6. Alternative Young indices, annual averages (1996=100)

	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Gyo	102,08	103,89	106,38	109,30	111,71	114,29	116,51	117,67	119,47	121,45
Yo**	102,08	103,89	106,36	109,27	111,67	114,24	116,47	117,61	119,33	121,22
CPI	102,13	104,03	106,64	109,74	112,34	115,09	117,49	118,86	121,01	123,28
Walsh	-	-	106,39	-	-	-	117,07	-	-	-

GYo: Geometric Young index.
Yo**: Geometric mean of a Young index and a rebased Young index.

Table 6a. Average annual rate of change, %

	1996-2003	1996-2006
Gyo	2,21	1,96
Yo**	2,20	1,94
CPI	2,33	2,11
Walsh	2,28	-

GYo: Geometric Young index.
Yo**: Geometric mean of a Young index and a rebased Young index.

The Geometric Young index underestimates an ideal index

From table 6-6a it shows that the Geometric Young also lies below the Walsh index. From 1996-2003 the average annual rate of change of the Geometric Young index is 2,21 %, against 2,28 % for the Walsh index. Thus, the Geometric Young index underestimates the ideal index, and the difference seems to be of more or less same size as the overestimation in the arithmetic Young index. A possible explanation of the downward bias of the Geometric Young index may be that it overestimates the elasticity of substitution at the elementary aggregate level, which then appears to be less than one.

The geometric average of a Young index and a rebased Young index

To overcome the problem of not satisfying the time reversal test the *CPI Manual* (para. 15.60-64) suggests that the CPI be calculated as the geometric average of a Young index and a *rebased* Young index, which will satisfy the time reversal test (and can be calculated in real time). Following the *CPI Manual* it will be denoted by Yo**. The Young index is calculated as in equation (1). The rebased Young index is then given by:

(7) $$P_{0:t}^{\text{YoReb}} = \left[\sum w_b^j \left(P_{0:t}^j \right)^{-1} \right]^{-1}$$

The geometric mean of the Young and the rebased Young index is:

(8) $$P_{0:t}^{\text{Yo**}} = \left(P_{0:t}^{\text{Yo}} \cdot P_{0:t}^{\text{YoReb}} \right)^{1/2}$$

As seen in Table 6-6a Yo** is almost identical to the Geometric Young index[1] and thus also seems biased downward compared to an ideal index.

6. Conclusion

The ideal Walsh Marshall-Edgeworth, Fisher and Törnqvist price indices calculated for 1996-1999 and 1999-2003 give very similar results, so that for any practical reason it will not make any difference which one is chosen.

Taken at their face value the calculations for 1996 to 2003 indicate an annual rate of change of the CPI 0,05 % point above that of an ideal index. If the elementary aggregates for car insurance, financial services and gardening, which showed unusual increases in both expenditure weights and prices, are excluded, the annual rate of change of the CPI exceeds that of an ideal index by 0,09 % point, on average.

When weights are available only with a time lag, the statistical office has to decide whether to price-updated the weights to the price reference period or not. Both practices can be justified, but the target of the resulting indices differs. If weights are price-updated the CPI is a Lowe index, which aims to measure the changing cost of buying the same basket of a past reference year. If the original weights are applied the CPI is a Young index, which aims to measure the average price changes in the index period.

If there are long-term trends in prices a Lowe index will exceed a Young index, whether the households substitute in response to relative price changes or not. This is the case in the calculations from 1996 to 2006, where the annual rate of change of the CPI is 0,1 % point higher, on average, when using price-updated weights instead of the original weights.

[1] This may be a parallel to the similarity of the Jevons index and the Carruthers-Sellwood-Ward-Dalen index, i.e. the geometric average of a Carli index and the harmonic mean of the price ratios.

Whether a Young or Lowe index is the better estimate of an ideal index depends on whether the original or the price-updated weights are the better estimate of the average expenditure shares in the index period. If the elasticity of substitution at the elementary aggregate level is closer to one, the Young index is the better estimate. If the elasticity of substitution is closer to zero, Lowe is the better estimate.

In the calculations in this paper Young provides the better, less upward biased, estimate of an ideal index, compared to a Lowe index. This may indicate an elasticity of substitution closer to one than zero.

The geometric Young index and the geometric mean of a Young index and a rebased Young index give almost identical results. Both indices show price changes below that of the ideal indices. The use of the geometric form to aggregate elementary index thus seems to overestimate the elasticity of substitution.

Different practice in countries may influence the measured rate of inflation and hence also international comparability of the CPIs, and further theoretical and empirical research should be encouraged.

ANNEX 1. Calculation formulas for ideal price indices

	Definition	1996-1999 calculation	1999-2003 calculation
Walsh	$P_{0:t}^{W} = \frac{\sum p_t^i \sqrt{q_0^i \cdot q_t^i}}{\sum p_0^i \sqrt{q_0^i \cdot q_t^i}} = \sum w_W^i \cdot \left(\frac{p_t^i}{p_0^i} \right)$, $w_W^i = \frac{\sqrt{\left(w_0^i \cdot w_t^i \right) / \left(p_t^i / p_0^i \right)}}{\sum \sqrt{\left(w_0^i \cdot w_t^i \right) / \left(p_t^i / p_0^i \right)}}, w_t^i = \frac{p_t^i q_t^i}{\sum p_t^i q_t^i}$	$P_{96:99}^{W} = \sum w_W^j \cdot P_{96:99}^j,$ $w_W^i = \frac{\sqrt{\left(w_{96}^j \cdot w_{99}^j \right) / P_{96:99}^j}}{\sum \sqrt{\left(w_{96}^j \cdot w_{99}^j \right) / P_{96:99}^j}}$	$P_{99:03}^{W} = \sum w_W^j \cdot P_{99:03}^j,$ $w_W^i = \frac{\sqrt{\left(w_{99}^j \cdot w_{03}^j \right) / P_{99:03}^j}}{\sum \sqrt{\left(w_{99}^j \cdot w_{03}^j \right) / P_{99:03}^j}}$
Marshal-Edgeworth	$P_{0:t}^{ME} = \frac{\sum p_t^i \left(q_0^i + q_t^i \right) \ /2}{\sum p_0^i \left(q_0^i + q_t^i \right) \ /2} = \sum w_{ME}^i \left(\frac{p_t^i}{p_0^i} \right)$ $\mathrm{w}_{ME}^i = \frac{v_0^i + \left(v_t^i / \left(p_t^i / p_0^i \right) \right)}{\sum \left(v_0^i + \left(v_t^i / \left(p_t^i / p_0^i \right) \right) \right)}, v_t^i = p_t^i q_t^i$	$P_{96:99}^{ME} = \sum w_{ME}^j \cdot P_{96:99}^j,$ $w_{ME}^j = \frac{v_{96}^j + \left(v_{99}^j / P_{96:99}^j \right)}{\sum v_{96}^j + \left(v_{99}^j / P_{96:99}^j \right)}$	$P_{99:03}^{ME} = \sum w_{ME}^j \cdot P_{99:03}^j,$ $w_{ME}^j = \frac{v_{99}^j + \left(v_{03}^j / P_{99:03}^j \right)}{\sum v_{99}^j + \left(v_{03}^j / P_{99:03}^j \right)}$
Fisher	$P_{0:t}^{F} = \left(\frac{\sum p_t^i q_0^i}{\sum p_0^i q_0^i} \frac{\sum p_t^i q_t^i}{\sum p_0^i q_t^i} \right)^{1/2}$	$P_{96:99}^{F} = \left[\left(\sum w_{96}^j P_{96:99}^j \right) \left(\sum w_{99}^J \left(P_{96:99}^j \right)^{-1} \right)^{-1} \right]^{1/2}$	$P_{99:03}^{F} = \left[\left(\sum w_{99}^j P_{99:03}^j \right) \left(\sum w_{03}^J \left(P_{99:03}^j \right)^{-1} \right)^{-1} \right]^{1/2}$
Törnqvist	$P_{0:t}^{T} = \prod \left(\frac{p_t^i}{p_0^i} \right)^{\left(w_0^i + w_t^i \right) / 2}$	$P_{96:99}^{T} = \prod \left(P_{96:99}^j \right)^{\left(w_{96}^j + w_{99}^j \right) / 2}$	$P_{99:03}^{T} = \prod \left(P_{99:03}^j \right)^{\left(w_{99}^j + w_{03}^j \right) / 2}$