Aggregate Indices and Their Corresponding Elementary Indices

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Abstract

"Which index formula at the elementary level, where no expenditure share weights are available, corresponds to a desired aggregate index?" To answer this question, this paper develops a statistical approach. It proposes a theoretical framework which makes it possible to achieve numerical equivalence of an elementary index with the Laspeyres, Paasche or Fisher price index. Depending on the price elasticity, different elementary indices should be applied to different groups of goods in order to approach the desired aggregate index as closely as possible. This is demonstrated empirically in an application using data from German foreign trade statistics.

Keywords: Power Mean, Quadratic Mean, Log-Normal Distribution, Partial Adjustment Model, Price Elasticity, Foreign Trade.

JEL: C43, D12, E31, F14.

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1 Introduction

1.1 Motivation

It is customary in official statistics, although often neglected in theoretical papers, for most price indices to be calculated in two stages. At the first stage, elementary indices are calculated on the basis of prices or their relatives, without having information on quantities or expenditures. At the second stage, the aggregate index is calculated on the basis of the elementary indices from the first stage, using aggregate expenditure share weights.

In general, the question of "What should be measured?" directly yields the optimal index formula at the second stage: for measuring genuine price movements, a Laspeyres price index is used; for deflation purposes, a Paasche price index is preferred; and for the "cost of living", a Fisher price index, among others, is the formula of choice. However, it is less clear which index formula should be used at the first stage, where no expenditure share weights are available. The existing approaches to index numbers including but not restricted to the axiomatic approach are of little guidance in choosing the elementary index corresponding to the characteristics of the index at the second stage.

The point in question is "How can the corresponding elementary index be selected?" The answer to this question is found by the proposition of a statistical approach. A single comprehensive framework, known as "power means", unifies the aggregate and elementary levels. With the aid of this approach, theoretical conditions under which a particular index formula at the elementary level exactly equals the desired aggregate index are identified and empirically approximated.

The remainder of the paper is organised as follows. It continues with a review of a selection of the existing literature on elementary indices. Section 2 introduces basic concepts and approaches in index theory along with a more thorough explanation of the problem at the elementary level. Both the theoretical foundations of power means as well as the application to the Laspeyres, Paasche and Fisher price indices and their corresponding elementary indices are presented in detail in Section 3. The results of an empirical application using data from German foreign trade statistics are to be found in Section 4. The final section concludes.

1.2 Literature Review

After a long period of research into aggregate formulas and an almost equally long policy debate in Europe and the US on whether the Laspeyres or Fisher formula should be used for a consumer price index (cf. Boskin et al., 1996, 1998, and Schultze and Mackie, 2002), the focus of attention has recently moved more to the question of which index formula should be used at the elementary level. Nowadays, the capabilities of modern computers and the increasing coverage of data, first and foremost, through the advent of scanner data, enables statistical offices to calculate more refined price indices even at the elementary level (cf. Silver, 1995, Silver and Webb, 2002, Feenstra and Shapiro, 2003, Diewert, 2004, and Proceedings of the Meetings of the Ottawa Group).

Diewert (2004), and Diewert and Silver (2004, 2008) devote whole chapters in the CPI, PPI and XMPI manuals to elementary indices. They deal with virtually all topics that arise around the calculation of price indices at the elementary level. Theoretical issues, such as the problem of aggregation, are covered as well as practical questions, such as numerical relationships between different elementary indices. They continue by outlining the classical approaches in index theory, i.e. the axiomatic, economic, sampling and stochastic approaches (cf. Subsection 2.3 for a discussion of all four approaches), and discuss the use of scanner data (cf. Subsection 5.2 for an outlook on a prospective study). Currently, there is an active ongoing discussion at Eurostat's Working Group on Harmonisation of Consumer Price Indices – more specifically, in the Task Force on Sampling – on which index formula is to be used at the elementary level (cf. EC, 2001, Section I). The Commission Regulation (EC, 1996, Article 7 in conjunction with Annex II) abandons the use of the Carli index but allows the use of either the Jevons or Dutot index (cf. Subsection 3.1.1 for the definitions of the formulae). More precisely, the Carli index is not prohibited de jure but de facto as it would have to be shown that the results do not differ by more than one-tenth of a percentage point from either the Jevons or Dutot index (cf. the next-but-one paragraph for empirical evidence and Subsection 3.1.2 for the mathematical relation).

Balk (1994) discusses the index formula problem at the elementary level. He questions whether ratios of average prices or an average of price relatives, and which type of average, i.e. arithmetic, geometric or harmonic, should be used.

Turvey (1996) addresses the same problem. He also presents empirical evidence that recalculations of elementary indices with different index formulae give significant changes in aggregate CPIs, annually by more than two percentage points, in Finland, Sweden, Canada and France. The use of unit values (cf. Subsection 3.1.1 for a formal discussion) at the lowest level in a price index is analysed by Balk (1998), which is commonly taken for granted to be an appropriate method of aggregation for prices of homogeneous goods. He tries to answer the questions of the conditions under which a group of goods is sufficiently homogeneous to warrant the use of unit values, and if one needs to restrict the use of unit values to homogenous goods alone. In the context of foreign trade, Silver (2008) criticises the use of aggregate indices which are calculated from unit values at the elementary level in favour of pure price indices. He reveals substantial biases of customs-based unit values: they depend on the structure of quantities and hence, cannot be considered surrogates for survey-based prices.

Szulc (1989) describes the fact that biases at the elementary level are more severe than the pros and cons of the formula at the aggregate level. He finds that if one ignores the particularities of the aggregate index when calculating elementary indices, this might result in surprisingly low differences between different aggregate indices. This is because the indices at the elementary level might not be paying attention to the characteristics of the index formula at the aggregate level, in particular if the same elementary indices are used as building blocks of the aggregate index – no matter which aggregate index should be used. In his 1994 paper he presents numerical evidence for the Canadian CPI that the choice of the elementary index matters the most, particularly in the short term. Dalén (1992, 1995) discusses the impact of the choice of the wrong index formula at the elementary level in the Swedish CPI. Statistics Sweden switched over to the Carli index in January 1990. As soon as April it was replaced by a variant of the geometric index due to the well-known severe upward bias of the Carli index – of more than half a percent in these three months. Using Swedish and Finnish data, he shows in his 1998 paper that the Carli index consistently gives results which are year-on-year two index points and more larger than the Dutot and Jevons indices, while the latter two indices are fairly close to each other. Fenwick (1999) presents evidence that the UK HICP, which is based on the Jevons index at the elementary level, is annually about half a percentage point lower than the national equivalent, the Retail Prices Index, which uses a combination of the Dutot and Carli indices, only because of the different formulae. His main argument for this notable difference is the relative broad item description, leading to aggregation of highly heterogeneous items. Silver and Heravi (2007) show that the difference between the Jevons and Dutot indices is due to different variances in the observed prices at different points in time alone, i.e. these indices will differ if prices exhibit dispersion. From a hedonic regression they derive a heterogeneity-controlled Dutot index and successfully test their approach empirically with scanner data.

2 Aggregate Indices

2.1 First Principles

At the aggregate level, the target of measurement determines the index concept to be used. This is either the cost of goods (COGI) or the cost of living (COLI). In general, the former case leads to Laspeyres (1871) and Paasche (1874) price indices, while the latter results inter alia in the Fisher (1922) price index – other formulas include the Walsh (1901, 1921) and Törnqvist (1936) price indices.

The Laspeyres price index is the arithmetic mean of price relatives with base period expenditure share weights. Here, p_{ib} and q_{ib} denote the price and quantity, respectively, of the i^{th} good at time $b \in \{0, t\}$.

$$P^L = \sum_{i=1}^{n} \frac{p_{it}}{p_{i0}} \frac{p_{i0}q_{i0}}{\sum_{i=1}^{n} p_{i0}q_{i0}} = \frac{\sum_{i=1}^{n} p_{it}q_{i0}}{\sum_{i=1}^{n} p_{i0}q_{i0}} \tag{1}$$

This is the only price index which ensures the principle of pure price comparison (cf. von der Lippe, 2001) over multiple periods by using a fixed basket of goods and which is consistent in aggregation (Subsection 2.2 provides a discussion of this property).

For volume measurement, one would opt for the Laspeyres quantity index Q^L, with $Q^L = V/P^P$, where V is the ratio of expenditures at times t and 0 or the value index and P^P is the Paasche price index. One might call Q^L a (volume) index in constant prices (COPI). The Paasche price index is the harmonic mean of price relatives with current period expenditure share weights.

$$P^P = \left(\sum_{i=1}^{n} \left(\frac{p_{it}}{p_{i0}} \right)^{-1} \frac{p_{it}q_{it}}{\sum_{i=1}^{n} p_{it}q_{it}} \right)^{-1} = \frac{\sum_{i=1}^{n} p_{it}q_{it}}{\sum_{i=1}^{n} p_{i0}q_{it}} \tag{2}$$

This is the only price index leading to volume measures in constant prices which are consistent in aggregation and purely comparable over multiple periods (Laspeyres principle for the quantity index).

The Fisher price index, among others, is a superlative index. It is defined as the geometric mean of the Laspeyres and Paasche price indices.

$$P^F = \sqrt{\frac{\sum_{i=1}^{n} \frac{p_{it}}{p_{i0}} \frac{p_{i0}q_{i0}}{\sum_{i=1}^{n} p_{i0}q_{i0}}}{\sum_{i=1}^{n} \left(\frac{p_{it}}{p_{i0}} \right)^{-1} \frac{p_{it}q_{it}}{\sum_{i=1}^{n} p_{it}q_{it}}}} = \sqrt{P^L P^P} \tag{3}$$

This is the most famous price index approximating the change in the minimum expenditures, which preserve utility at a constant level, owing to changes in (relative) prices (cf. Allen, 1975).

2.2 Two-staged indices

In what follows, the relation between the elementary and aggregate level of two-staged indices is analysed. Firstly, two-staged indices with the same index formula at both levels are described. Secondly, as in the practice of official statistics, different index formulae are applied at the two levels.

While the Laspeyres and Paasche price indices are consistent in aggregation, the first source of aggregation bias arises from the fact that the Fisher price index is not. This means that the result of a two-staged index calculation does not necessarily coincide with that of a calculation in a single stage. However, as Diewert (1978) shows, superlative indices, such as the Fisher price index, are approximatively consistent in aggregation. Still, the remaining inconsistency can lead to puzzling results. The one-staged index is not necessarily restricted to lie in-between the elementary indices of a two-staged calculation. Even though all elementary indices show decreasing prices, i.e. $P_k^F < 1 \, \forall \, k$ (P_k^F being the Fisher price index for the k^{th} group of goods), the aggregate index can show increasing prices, i.e. $P^F > 1$, and vice versa. Additionally, von der Lippe (2007) proposes the Equality Test and shows that even if all elementary indices are equal, the aggregate index can differ.

Much more severe than this defect of the Fisher price index is the second source of aggregation bias which occurs when statistical offices cannot use a quantity or expenditure-weighted formula at the first stage of the aggregation process. Owing to the unavailability of this information they have to rely on an unweighted index which might not reflect the characteristics of the index formula at the aggregate level. This elementary index bias is equally applicable to the Laspeyres and Paasche price indices as well as to the Fisher price index, no matter which unweighted index is used. A two-staged index with a non-according formula at the elementary level, e.g. $P^{(J)L}$, the Laspeyres price index with Jevons indices as building blocks, can lead to a different conclusion than the true price index. Similarly, as before, one can have decreasing prices with the two-staged index, i.e. $P^{(J)L} < 1$, while the true price index shows increasing prices, i.e. $P^L > 1$, and vice versa. This becomes even worse for the Fisher price index which, in addition, if it is calculated in two stages, can lie outside the bounds of the true Laspeyres and Paasche price indices, i.e. $P^{(J)F} > P^L$ or $P^{(J)F} < P^P$. Both scenarios are due to the fact that the elementary indices may not even be close to the desired target index. Hence, more attention should be paid to the calculation of elementary indices.

2.3 Index Theory

From an index theoretical standpoint, there exist four approaches which offer guidance in the choice of an index formula at both the aggregate and elementary levels. These are the economic, axiomatic, stochastic and sampling approaches and they are described below.

The economic approach gives a microeconomic interpretation to consumer's optimising behaviour. Konüs (1924) develops this approach and derives the cost of living index as the solution to a cost minimisation problem. Moreover, he shows that the upper and lower bounds are, in general, the Laspeyres and Paasche price indices: $P^P \leq P^{COLI} \leq P^L$. A COLI measures the change in the minimum expenditures in order to maintain a given level of utility and hence, substitution between goods is permitted. In practice, these indices are approximated by superlative indices, such as the Fisher price index, as discussed in Diewert (1976). This approach assumes that timely information on quantities or expenditures is available.

The Fisher price index is typically the preferred formula from the viewpoint of the axiomatic approach, too. The axiomatic approach states properties which an index formula should desirably fulfil and checks which axioms are actually fulfilled. Eichhorn (1978), and Diewert (1995) discuss this approach to elementary indices in detail. However, the elementary and aggregate levels are treated individually. In order to fill this gap, an integrated approach for two-staged indices would be desirable. The importance of axioms in general depends heavily on the target of measurement (the possible target indices are introduced in Subsection 2.1) and, to some extent, on personal preferences.

Selvanathan and Prasada Rao (1994) describe a stochastic approach to index numbers in general. In this approach, the price index is the least squares estimator of a weighted regression of price relatives, enabling the calculation of standard errors and confidence intervals. The shortcoming of this approach is that it does not distinguish the fit of the model from the sampling error. The variance of an estimator is rather the expression of the heterogeneity of the price representatives forming the group of goods. One should not take the lowest variance as a measure for determining the most suitable index. Thus, this approach is not designed for judging the adequacy of an index formula. In any case, its main purpose lies in international comparisons.

The sampling approach for elementary indices is presented by Balk (2005). This approach studies elementary indices as sample estimators of unknown population price indices and the required sampling design for unbiasedness. Under an appropriate sampling scheme, both the Dutot and Carli indices can be justified as sample counterparts of the Laspeyres price index. The appropriate sampling scheme in both cases is "probability proportional to size" (PPS) sampling. For the Dutot index to equal the Laspeyres price index, the price representatives should be sampled according to their quantities in the base period. Should the Carli index equal the Laspeyres price index, the appropriate PPS weights are base period expenditures. This approach has the merit of making it possible to achieve numerical equivalence of an elementary index with the desired target index, i.e. $\mathrm{E}(P^D) = P^L$ or $\mathrm{E}(P^C) = P^L$. The demerit is that the informational requirements (quantities or expenditures) are generally not met by statistical offices. If they were met, the desired target index could be calculated directly.

Thus, owing to the aforementioned limitations, none of these four approaches is followed here but a new, fifth approach is proposed. Although a different path is trodden, the goal which is to be achieved is the same as that of the sampling approach: numerical equivalence. The following statistical approach using power means does not rely on PPS but on simple random sampling (SRS), which requires much less additional information and is easier to implement.

3 Corresponding Elementary Indices

3.1 Theoretical Foundations

In order to achieve numerical equivalence between an elementary index and an arbitrary aggregate index, a statistical approach is developed. In Subsection 3.1.1 it is firstly demonstrated that every weighted index can be expressed one-to-one and onto as a "power mean", as long as the former satisfies the strict mean value property. The power mean represents a whole class of unweighted elementary indices, such as the Carli and Jevons indices. However, an analytical derivation of the concrete power mean of a weighted index, aggregate or elementary, is not possible without further assumptions. Hence, secondly the log-normal distribution is introduced in Subsection 3.1.2 and the power means – which correspond to the Laspeyres, Paasche and Fisher price indices, as well as to the Dutot and unit value indices – are related to the distribution's parameters. Although, at that stage, one would be able to numerically calculate elementary indices, corresponding to the desired aggregate ones, the present paper goes one step further and gives an economic interpretation to the parameters through a partial adjustment model in Subsection 3.1.3. Thirdly, the log-normal distribution parameters are related to the price elasticity. Finally, it is shown in the succeeding Subsections 3.2 and 3.3 that the choice of the elementary indices which correspond to the desired aggregate ones can be based on the price elasticity only.

3.1.1 Power Mean

Right at the very beginning, Lemma 1 is needed for the discussion of the problem at the elementary level. Proof for this and all following lemmata and theorems are to be found in the Appendix.

Lemma 1. *The price indices of Laspeyres and Paasche as well as the Fisher price index, Equations (1), (2) and (3), respectively, pass the Mean Value Test of Eichhorn and Voeller (1976):*

$$\min\left(\frac{p_{it}}{p_{i0}}\right) \leq P^* \leq \max\left(\frac{p_{it}}{p_{i0}}\right), \tag{4}$$

where P^* stands for any of the three price indices. This test says that the price index should be greater than or equal to the lowest price relative and less than or equal to the highest one, with equality if and only if all price relatives are equal.

Given this, the problem of choosing the elementary index corresponding to the Laspeyres, Paasche or Fisher price indices becomes solvable. To this end, it is shown that every *weighted* aggregate index can be written as an *unweighted* power mean of price relatives.

Definition 1. Let p_{it}/p_{i0} denote the price relative of the i^{th} good at time t, where $i = 1, 2, \ldots, n$ and $n \geq 2$. Furthermore, all price relatives are assumed to be positive real numbers, $0 < p_{it}/p_{i0} < \infty \, \forall \, i$. Then, their power mean is defined as

$$P^r = \sqrt[r]{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_{it}}{p_{i0}}\right)^r}. \tag{5}$$

By choosing the appropriate powers r, the resulting power means equal some of the most important elementary indices (cf. Table 1). Figure 1 exemplifies the typical shape of the power mean as a function of its argument r. Its analytical properties are stated in Lemma 2.

Lemma 2. *The power mean is a mapping from the affinely extended real numbers* $\mathbb{R} \cup (-\infty, +\infty)$ *on the closed interval* $[P^{min}, P^{max}]$*, or technically speaking*

$$P^r : \mathbb{R} \cup (-\infty, +\infty) \rightarrow [P^{min}, P^{max}]. \tag{6}$$

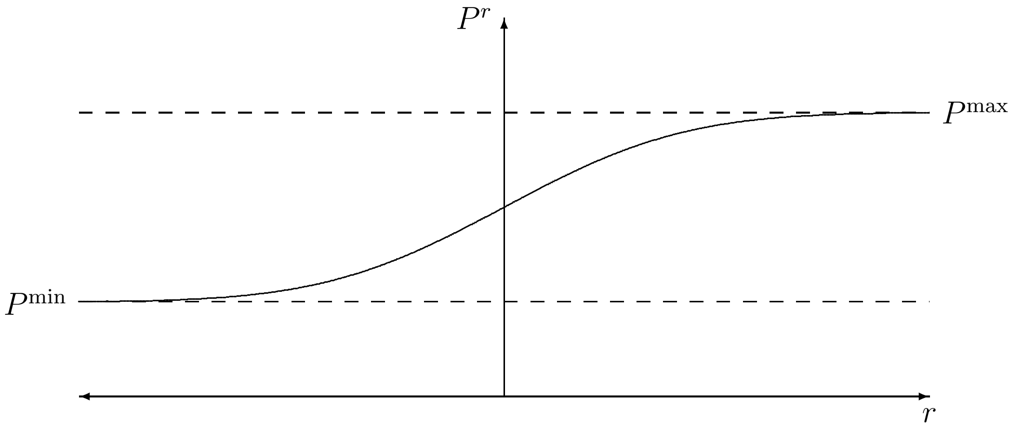

Figure 1: Power Mean of Price Relatives

From these intermediate results, the following theorem is deduced.

Theorem 1. *If not all price relatives are equal, $\exists\, i \neq j : p_{it}/p_{i0} \neq p_{jt}/p_{j0}$, i.e. the trivial case of perfect homogeneity is neglected, then for any aggregate index P^* that satisfies the mean value property there exists one and only one real r for which the power mean is numerically equivalent,*

$$\exists!\, r \in \mathbb{R} \colon P^r = P^*. \tag{7}$$

Theorem 1 provides the basis for the following derivation of the corresponding elementary indices in the case of the Laspeyres, Paasche or Fisher price indices as desired aggregate indices. An intuitive interpretation of the theorem goes as follows. The aggregate index P^* lies between the smallest and largest price relative, $P^{\min}$ and $P^{\max}$, respectively. The power mean P^r covers the whole range between these two price relatives. Moreover, it is a continuous function and hence, it has to take on the value of the aggregate index at least once. Uniqueness of the power r is secured through the proposition that not all price relatives are equal and therefore, the power mean is a strictly monotonic increasing function in r.

Table 1 depicts some of the most frequently used formulae at the elementary level (cf. Subsection 3.3 for the definitions of quadratic means).

Table 1: Power Means and their Formulae

r	Power Mean	Price Index	Formula
-2	reciprocal quadratic	–	$P^r(-2) = \sqrt{n/\sum_{i=1}^n (p_{i0}/p_{it})^2}$
-1	harmonic	Coggeshall (1887)	$P^h = n/\sum_{i=1}^n (p_{i0}/p_{it})$
$0^\dagger$	geometric	Jevons (1863, 1865)	$P^J = \sqrt[n]{\prod_{i=1}^n (p_{it}/p_{i0})}$
1	arithmetic	Carli (1764)	$P^C = \sum_{i=1}^n (p_{it}/p_{i0})/n$
2	quadratic	–	$P^r(2) = \sqrt{\sum_{i=1}^n (p_{it}/p_{i0})^2/n}$

† The Jevons index is the limit of P^r as r approaches zero.

Another very famous formula at the elementary level is the one of Dutot (1738), the ratio of arithmetic mean prices:

$$P^D = \frac{\frac{1}{n}\sum_{i=1}^n p_{it}}{\frac{1}{n}\sum_{i=1}^n p_{i0}}. \tag{8}$$

Carruthers et al. (1980) show that this index is related to the Jevons index to the second order via $P^J \approx P^D[1 + \mathrm{Var}(p_0^*)/2 - \mathrm{Var}(p_t^*)/2]$, where $\mathrm{Var}(p_0^*)$ and $\mathrm{Var}(p_t^*)$ are the variances of the relative deviations of the prices from their arithmetic mean in the respective periods: $\nu_{ib} = (p_{ib}/\bar{p}_b) - 1$, $b \in \{0, t\}$. Hence, the two indices will closely approximate each other if the variance of the prices remains constant over time. For this reason, the Dutot index is frequently put on a par with the Jevons index.

Drobisch (1871) proposes another index which is of importance at the elementary level. This is the ratio of unit values or the unit value index:

$$P^{UV} = \frac{\sum_{i=1}^n p_{it}q_{it}/\sum_{i=1}^n q_{it}}{\sum_{i=1}^n p_{i0}q_{i0}/\sum_{i=1}^n q_{i0}}. \tag{9}$$

Note that the summation of quantities must be defined and should be economically meaningful. The unit value index is an elementary index in the sense of being a surrogate for a price index (cf. Silver, 2008, and von der Lippe and Mehrhoff, 2009). Using the theorem of von Bortkiewicz (1923), Párniczky (1974) derives criteria under which the unit value index equals the Paasche price index, while Balk (1998) does this for the Fisher price index. They arrive at the following expressions for the ratio of the unit value index to the two indices: $P^{UV}/P^P = 1 + \mathrm{relCov}(p_0, q_t/q_0)$

and $P^{UV}/P^F = \sqrt{[1 + \text{relCov}(p_0, q_t/q_0)][1 + \text{relCov}(p_t, q_t/q_0)]}$, respectively, where $\text{relCov}(X, Y) = \text{Cov}(X, Y)/[\text{E}(X)\text{E}(Y)]$. For the unit value index to equal the Paasche price index, at least one of the following criteria has to hold: a) all base period prices have to be equal, b) all quantity relatives have to be equal, or c) there is no correlation between base period prices and quantity relatives. In the case of the Fisher price index, the situations a) and c) have to hold for current period prices as well. For the reason of Lemma 3, the unit value index is not a price index in the classical meaning.

Lemma 3. *The unit value index in Equation (9) violates the mean value property from Equation (4).*

However, with respect to its importance in both consumer prices and foreign trade, it will be analysed along with power means and the Dutot index in the next subsection.

3.1.2 Log-Normal Distribution

The power r in Subsection 3.1.1 cannot be derived analytically without making any further assumptions. Based on Theorem 2, a closed form solution is provided as to which power corresponds to a given aggregate index as well as to the practically relevant Dutot and unit value indices.

Theorem 2. *Under weak assumptions on the underlying data generating process, which are outlined in the proof (cf. Appendix), prices p_{ib} and quantities q_{ib}, $b \in \{0, t\}$, are jointly log-normally distributed:*

$$\begin{bmatrix} \mathbf{p_i} \\ \mathbf{q_i} \end{bmatrix} \sim \mathcal{LN}\left(\begin{bmatrix} \boldsymbol{\mu}_\mathbf{p} \\ \boldsymbol{\mu}_\mathbf{q} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{\mathbf{p},\mathbf{p}} & \boldsymbol{\Sigma}_{\mathbf{p},\mathbf{q}} \\ \boldsymbol{\Sigma}_{\mathbf{q},\mathbf{p}} & \boldsymbol{\Sigma}_{\mathbf{q},\mathbf{q}} \end{bmatrix} \right). \tag{10}$$

Upon this, an explicit formula is derived by which the power can be computed directly from the log-normal distribution parameters. In Subsection 3.1.3, these distribution parameters will be linked to the price elasticity.

The characteristic run of the log-normal distribution can be inferred from Figure 2.

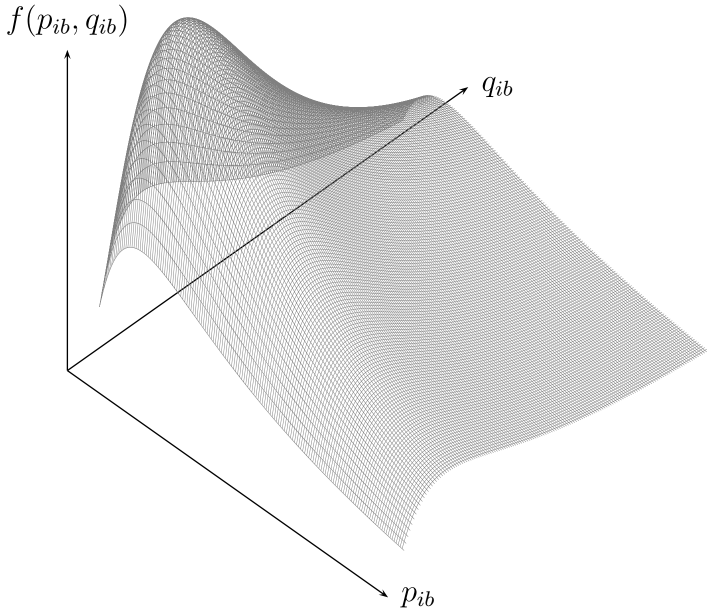

Figure 2: Joint Log-Normal Distribution of Prices and Quantities

The assumption of a quadrivariate log-normal distribution of prices and quantities seems reasonable and predecessors are found in the literature. Moulton (1993), and Dalén (1999) use the log-normal distribution assumption for price relatives, while Silver and Heravi (2007) use it for prices in their own right. Note that the latter assumption is a generalisation of the former one. Log-normal distribution of price relatives is a direct consequence of log-normal distribution of prices. In fact, for power means the results are the same from either of the assumptions. However, it would not be possible to analyse the Dutot and unit value indices without the more general assumption.

The link between the power mean, and the Dutot and unit value indices on the one side, and the log-normal distribution parameters on the other side is built in the following theorem.

Theorem 3. *The power mean in Equation (5) corresponds to the r^{th} root of the r^{th} raw moment of the marginal distribution of price relatives, which is also the log-normal distribution. It follows that*

$$P^r = \exp\left(\mu_{p_t} - \mu_{p_0} + r\frac{\sigma_{p_t}^2 + \sigma_{p_0}^2 - 2\sigma_{p_t,p_0}}{2}\right). \tag{11}$$

The Dutot index in Equation (8) is the ratio of the first raw moments of the marginal distributions of current and base period prices. One finds that

$$P^D = \exp\left(\mu_{p_t} - \mu_{p_0} + \frac{\sigma_{p_t}^2 - \sigma_{p_0}^2}{2}\right). \tag{12}$$

The unit value index in Equation (9) is found to be a ratio of ratios. The ratios, either in the current or base period, are those of the first raw product moment of the marginal distribution of prices and quantities and the first raw moment of the marginal distribution of quantities. This results in

$$P^{UV} = \exp\left(\mu_{p_t} - \mu_{p_0} + \frac{\sigma_{p_t}^2 - \sigma_{p_0}^2 + 2\sigma_{p_t,q_t} - 2\sigma_{p_0,q_0}}{2}\right). \tag{13}$$

From Theorem 3, it can be seen that the Carli index ($r = 1$), unlike the Jevons index ($r \to 0$), is an increasing function of the variance of the price relatives. Hence, a mathematical argument for the upward bias of the Carli index compared with the Jevons index is given through this: the more heterogeneous the goods become at the elementary level, the higher will be the bias.

Theorem 4 establishes the link between the Laspeyres and Paasche price indices and the log-normal distribution parameters (cf. Subsection 3.3 for the solution in the case of the Fisher price index). Moreover, it firstly gives an exact expression for the power mean corresponding to either of the two price indices, and secondly shows to which power mean the Dutot and unit value indices relate.

Theorem 4. *The Laspeyres price index corresponds to the ratio of the first raw product moment of the marginal distribution of current period prices and base period quantities, and the first raw product moment of the marginal distribution of base period prices and quantities. It turns out that*

$$P^L = \exp\left(\mu_{p_t} - \mu_{p_0} + \frac{\sigma_{p_t}^2 - \sigma_{p_0}^2 + 2\sigma_{p_t,q_0} - 2\sigma_{p_0,q_0}}{2}\right). \tag{14}$$

The Paasche price index' correspondence is the same as the one of the Laspeyres price index but with the difference that here there are current period quantities instead of base period ones. It becomes

$$P^P = \exp\left(\mu_{p_t} - \mu_{p_0} + \frac{\sigma^2_{p_t} - \sigma^2_{p_0} + 2\sigma_{p_t,q_t} - 2\sigma_{p_0,q_t}}{2}\right). \tag{15}$$

Equating P^r from Equation (11) with P^L and P^P from Equations (14) and (15), respectively, and solving for r yields after some algebra:

$$P^r = P^L \iff r_L = \frac{\sigma^2_{p_t} - \sigma^2_{p_0} + 2\sigma_{p_t,q_0} - 2\sigma_{p_0,q_0}}{\sigma^2_{p_t} + \sigma^2_{p_0} - 2\sigma_{p_t,p_0}}, \tag{16}$$

$$P^r = P^P \iff r_P = \frac{\sigma^2_{p_t} - \sigma^2_{p_0} + 2\sigma_{p_t,q_t} - 2\sigma_{p_0,q_t}}{\sigma^2_{p_t} + \sigma^2_{p_0} - 2\sigma_{p_t,p_0}}. \tag{17}$$

Finally, the Dutot and unit value indices from Equations (12) and (13), respectively, are related to the power mean as follows:

$$P^r = P^D \iff r_D = \frac{\sigma^2_{p_t} - \sigma^2_{p_0}}{\sigma^2_{p_t} + \sigma^2_{p_0} - 2\sigma_{p_t,p_0}}, \tag{18}$$

$$P^r = P^{UV} \iff r_{UV} = \frac{\sigma^2_{p_t} - \sigma^2_{p_0} + 2\sigma_{p_t,q_t} - 2\sigma_{p_0,q_0}}{\sigma^2_{p_t} + \sigma^2_{p_0} - 2\sigma_{p_t,p_0}}. \tag{19}$$

3.1.3 Partial Adjustment Model

Next, the implied power r of the Laspeyres and Paasche price indices as well as of the Dutot and unit value indices, Equations (16) and (17) in addition to Equations (18) and (19), is connected to the price elasticity derived from a partial adjustment model as in Definition 2.

Definition 2. It is assumed that there exists an equilibrium quantity traded for each good $i = 1, 2, \ldots, n$ and time $b \in \{0, t\}$. This quantity is related to the price of the good, which, in turn, is assumed to be predetermined, and to other, strictly exogenous variables, such as time dummies or a trend. The parameter η_i^q is a panel fixed effect, accounting for unobserved heterogeneity in the data.

$$\ln \overline{q}_{ib} = \alpha + \beta \ln p_{ib} + \mathbf{x_{ib}}\boldsymbol{\delta} + \eta_i^q \tag{20}$$

The adjustment to the equilibrium in Equation (20) is assumed to be both incomplete and erroneous. This is mirrored by the introduction of lagged quantity and an i.i.d. error term. Here, $\beta^* := (1-\rho)\beta$ denotes the effective price elasticity.

$$\begin{aligned} \ln q_{ib} &= (1-\rho)\ln \overline{q}_{ib} + \rho \ln q_{ib-1} + \varepsilon^q_{ib} \\ &= (1-\rho)\alpha + \beta^* \ln p_{ib} + \rho \ln q_{ib-1} + \mathbf{x_{ib}}(1-\rho)\boldsymbol{\delta} + [(1-\rho)\eta^q_i + \varepsilon^q_{ib}] \end{aligned} \tag{21}$$

Prices are assumed to follow a panel AR(1) process:

$$\ln p_{ib} = \gamma_0 + \gamma_1 \ln p_{ib-1} + (\eta^p_i + \varepsilon^p_{ib}). \tag{22}$$

Three remarks have to be made regarding the chosen model. First, the implied cross-price elasticity in Equation (20) is zero. Second, the underlying equilibrium price elasticity β is attenuated by sluggish adjustment of quantities. Third, owing to the problem of identification with observed data on prices and quantities, the estimated effective price elasticity β^* has to be understood as being the one of the supply-demand equilibrium rather than the one of demand. As the focus of this paper is on the effective price elasticity only, it is referred to simply as the price elasticity in what follows.

Using Equations (21) and (22), the covariance matrices can be derived subject to the model parameters. The results are collected in Theorem 5.

Theorem 5. *The covariance matrices* $\mathbf{\Sigma_{p,p}}$ *and* $\mathbf{\Sigma_{p,q}} = \mathbf{\Sigma'_{q,p}}$ *of the log-normal distribution as given in Equation (10) are as follows (the elements of* $\mathbf{\Sigma_{q,q}}$ *do not appear in the calculation of the power* r*):*

$$\mathbf{\Sigma_{p,p}} = \begin{bmatrix} \sigma^2_{p_t} & \sigma_{p_t,p_0} \\ \sigma_{p_t,p_0} & \sigma^2_{p_0} \end{bmatrix} = \sigma^2_p \begin{bmatrix} 1 & \gamma_1^t \\ \gamma_1^t & 1 \end{bmatrix}, \tag{23}$$

$$\mathbf{\Sigma_{p,q}} = \begin{bmatrix} \sigma_{p_t,q_t} & \sigma_{p_t,q_0} \\ \sigma_{p_0,q_t} & \sigma_{p_0,q_0} \end{bmatrix} = \beta^* \sigma^2_p \begin{bmatrix} \frac{1}{1-\rho\gamma_1} & \frac{\gamma_1^t}{1-\rho\gamma_1} \\ \left(\frac{\gamma_1^t - \rho^t}{1-\frac{\rho}{\gamma_1}} + \frac{\rho^t}{1-\rho\gamma_1} \right) & \frac{1}{1-\rho\gamma_1} \end{bmatrix}. \tag{24}$$

In the derivation of Equations (23) and (24) use was made of the weak stationarity assumption, especially of stationarity in covariance.

3.2 Laspeyres and Paasche Price Indices

It turns out that the solution to the problem of corresponding elementary indices depends on the empirical correlation between prices and quantities. In particular, the power r is a function of the price elasticity only. The succeeding theorem summarises the results for the Laspeyres and Paasche price indices (again, cf. Subsection 3.3 for the solution in the case of the Fisher price index) as well as the Dutot and unit value indices.

Theorem 6. *Combining the equations relating the power mean to the log-normal distribution parameters, Equations (16), (17), (18) and (19), with those relating the log-normal distribution parameters to the model coefficients, Equations (23) and (24), gives the final results:*

$$r_L = -\beta^* \frac{1}{1-\rho\gamma_1} \approx -\beta^*, \tag{25}$$

$$r_P \xrightarrow{t\to\infty} \beta^* \frac{1}{1-\rho\gamma_1} \approx \beta^*, \tag{26}$$

$$r_D = 0, \tag{27}$$

$$r_{UV} = 0. \tag{28}$$

From Theorem 6, the general results for the power mean are as follows. A power mean with power r equal to minus the price elasticity ($-\beta^*$) yields approximately the same result as the Laspeyres price index. Hence, if the price elasticity is minus one, for example, the power must equal one and the Carli index (cf. Table 1) at the elementary level will correspond to the Laspeyres price index as target index. This can be seen in the simplest form from the following example: from $q_{i0} = \bar{q}_0/p_{i0}$, where $\bar{q}_0$ is an arbitrary constant, follows $P^L = [\sum_{i=1}^n p_{it}(\bar{q}_0/p_{i0})]/[\sum_{i=1}^n p_{i0}(\bar{q}_0/p_{i0})] = \sum_{i=1}^n (p_{it}/p_{i0})/n = P^C$. However, if the Paasche price index should be replicated, the power of the power mean must equal the price elasticity, in the above example minus one. Thus, the harmonic index gives the same result and therefore, in this case it should be used at the elementary level.

Under the assumption of stationarity in covariance (cf. Subsection 3.1.3), the Dutot and unit value indices both equal the Jevons index. But if price dispersion takes place in reality, violating this assumption, the indices will differ. This is even more the case for the unit value index than for the Dutot index.

3.3 Fisher Price Index

The Fisher price index is derived from the Laspeyres and Paasche price indices as their geometric mean. Owing to the symmetry of the power means which correspond to the Laspeyres and Paasche price indices, a quadratic mean corresponds to the Fisher price index. In Definition 3 the properties of quadratic means in general are presented.

Definition 3. A quadratic mean of price relatives of order q is defined as follows:

$$P^q = \left(\frac{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_{it}}{p_{i0}}\right)^{\frac{q}{2}}}{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_{it}}{p_{i0}}\right)^{-\frac{q}{2}}} \right)^{\frac{1}{q}} \tag{29}$$

The index defined by Equation (29) is symmetric, i.e. $P^q = P^{-q} = P^{|q|}$. Furthermore, it is either increasing or decreasing in $|q|$, depending on the data. Both characteristics can also be seen from Figure 3. Note that a quadratic mean of order q, P^q, should not be mistaken for the quadratic index, $P^r(2)$ (cf. Table 1).

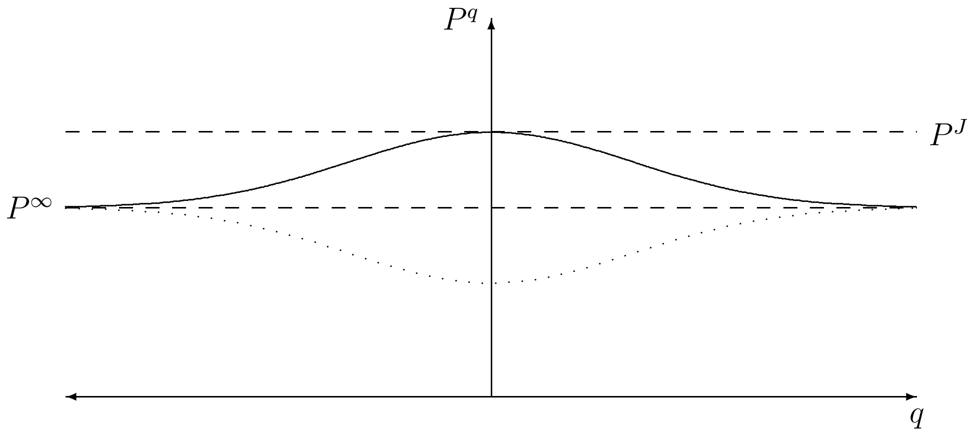

Figure 3: Quadratic Mean of Price Relatives

Dalén (1992), and Diewert (1995) show via a Taylor series expansion that all quadratic means approximate each other to the second order. However, as Hill (2006) demonstrates, the limit of P^q if q diverges is $P^{\infty} = \sqrt{P^{\min} P^{\max}}$. He concludes that quadratic means are not necessarily numerically similar.

For $q \to 0$ the quadratic mean becomes the Jevons index. For $q = 1$ a hybrid index results, which was first described by Balk (2005) and independently devised by Mehrhoff (2007) as a linear approximation to the Jevons index by crossing the implicit quantities of the Carli and harmonic indices, which explains the name. Implicit quantities are derived by equating the Carli index to the Laspeyres price index and the harmonic index to the Paasche price index; these are the inverses of base and current period prices, respectively (cf. Subsection 3.2). Lastly, one arrives at the CSWD index (Carruthers, Sellwood and Ward, 1980, and Dalén, 1992) for $q = 2$, which is the geometric mean of the Carli and harmonic indices. Table 2 contrasts these indices.

Table 2: Quadratic Means and their Formulae

q	Quadratic Mean	Formula
$0^{\dagger}$	Jevons	$P^J = \sqrt[n]{\prod_{i=1}^{n} (p_{it}/p_{i0})}$
1	Hybrid	$P^H = \sum_{i=1}^{n} \sqrt{(p_{it}/p_{i0})} / \sum_{i=1}^{n} \sqrt{(p_{i0}/p_{it})}$
2	CSWD	$P^{CSWD} = \sqrt{\sum_{i=1}^{n} (p_{it}/p_{i0})} / \sqrt{\sum_{i=1}^{n} (p_{i0}/p_{it})}$
3	cubic	$P^q(3) = \sqrt[3]{\sum_{i=1}^{n} \sqrt{(p_{it}/p_{i0})^3}} / \sqrt[3]{\sum_{i=1}^{n} \sqrt{(p_{i0}/p_{it})^3}}$
4	quartic	$P^q(4) = \sqrt[4]{\sum_{i=1}^{n} (p_{it}/p_{i0})^2} / \sqrt[4]{\sum_{i=1}^{n} (p_{i0}/p_{it})^2}$

† The Jevons index is the limit of P^q as q approaches zero.

Applying the preceding definitions gives the final result which is stated in Theorem 7.

Theorem 7. *A quadratic mean of order two times the absolute price elasticity corresponds to the Fisher price index:*

$$P^F \approx \sqrt{\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_{it}}{p_{i0}}\right)^{-\beta^*}\right)^{-\frac{1}{\beta^*}} \left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_{it}}{p_{i0}}\right)^{\beta^*}\right)^{\frac{1}{\beta^*}}} = P^q(2|\beta^*|). \qquad (30)$$

The approximate equality in Equation (30) follows from Equations (25) and (26) in conjunction with Equation (5).

4 Findings in Foreign Trade Statistics

4.1 Data Description

For the purpose of illustrating the methodology outlined here, an application to scanner data for homogeneous goods would be suited best because information on both prices and quantities at the elementary level would be necessary. Unfortunately, scanner data are not available for the German CPI. Hence, as an empirical application, data from German foreign trade statistics are analysed as an alternative. The source of these data is the German Federal Statistical Office. At the time of frontier crossing, movements of goods in special trade are to be reported for statistical purposes; with member states of the European Union in the Intrastat system, and with non-member states via the customs' Single Administrative Document (EC, 2006). Declarations are to be made according to the Commodity Classification for Foreign Trade Statistics and consist inter alia of the goods' values and quantities, the latter generally in terms of the weights. Based on these declarations, albeit not derived from homogeneous goods, unit values are calculated at the elementary level as $\tilde{p}_{ib} = (\sum_{i=1}^{n} p_{ib}q_{ib})/(\sum_{i=1}^{n} q_{ib})$, $b \in \{0, t\}$, which, in turn, form the basis for the succeeding analysis.

Owing to the nature of collected data, their structure is repeated cross-sections rather than a panel. Repeated cross-sections arise by independent cross-sectional surveys at consecutive points in time. Unlike in price statistics, it is not ensured in foreign trade statistics that the same goods are observed over time. The coverage of the universe of goods is time-varying and it is not possible to establish a one-to-one correspondence between goods over time. In this case, Deaton (1985) suggests estimation to be performed on a pseudo panel. This is averaging the data within a cohort, where a cohort is a group of goods sharing common characteristics and every good belongs to one group and one group only which is the same over time. Here, unique transactions are aggregated at the lowest level available, that is their reporting level: the eight-digit code of the Commodity Classification. These lower level aggregates are the individual observations which are nested at the four-digit code level to form an upper level aggregate.

The data set covers 1,264 pseudo panels (nests) consisting of 12,948 groups of goods (cohorts), for exports as well as for imports, and a total of 1,839,384 observations over the period January 2000 to December 2007. Only goods measured in kilograms – these are about three-quarters of all goods – are included in the analysis. The data, unit values in €1,000 per 100 kg (hereafter "prices") and weights in 100 kg (hereafter "quantities"), are transformed into their natural logarithms. Although the goods at the elementary level are not homogeneous, they are treated as if they were for the following analysis.

4.2 Regression Results

As weak stationarity of prices and quantities is assumed in the derivation of corresponding elementary indices (cf. Subsection 3.1.3), panel unit root tests are performed prior to estimation in order to test the validity of this assumption. In particular, these are the test of Levin, Lin and Chu (LLC, 2002), Breitung (2000), Im, Pesaran and Shin (IPS, 2003), augmented Dickey and Fuller (ADF, 1979), and Phillips and Perron (PP, 1988). The first two assume a common unit root process under the null hypothesis and no unit root under the alternative. The last three, by contrast, test the null hypothesis of an individual unit root process against the alternative of some cross-sections without a unit root. The latter two tests for panel data are derived as a combination of their time series variants using the results of Fisher (1925). Included in the test specification are individual effects and individual linear trends. Lag lengths, if necessary, are selected automatically based on the Schwarz information criterion; if applicable, the spectral estimator's bandwidth is selected according to Newey and West (1994) using the Bartlett kernel.

As can be seen from Table 3, the tests show stationarity of both prices and quantities for almost all panels in exports as well as in imports. Throughout, quantities perform better than prices, and exports and imports do equally well. That not all of them are stationary is largely due to non-unity power of the tests. Thus, the issue of (co-)integration can safely be ignored for the remainder of the analysis.

Table 3: Percentages of Stationary Panels at the 5% Significance Level

	Exports		Imports	
Test	Prices	Quantities	Prices	Quantities
LLC	89.81%	94.82%	90.46%	93.07%
Breitung	84.47%	90.88%	85.66%	92.26%
IPS	93.34%	97.70%	92.50%	97.72%
ADF	93.51%	97.78%	93.07%	98.04%
PP	96.87%	98.72%	96.88%	99.36%

The price elasticity β^* is estimated in the framework of the log-linear partial adjustment model given in Equation (21) by means of dynamic panel data one-step system GMM (Arellano and Bover, 1995, and Blundell and Bond, 1998). Neither time dummies nor a deterministic trend are included. Prices are assumed to be predetermined and are instrumented accordingly. The instrument set is collapsed in order to reduce the instrument count.

The overall results are fairly robust to different specifications of the model (inclusion of dummies or a trend), choice of instruments (limited lag depth) and estimation methods (fixed effects or difference GMM). Thus, only results which are derived from the above set-up are reported.

After adjusting for outliers, 1,246 panels in exports and 1,249 in imports remain. The distribution of the price elasticity in exports and imports can be gathered from Figure 4. The histograms show positive excess kurtosis, or leptokurtosis, for exports as well as for imports. Compared with the associated normal distribution, the peak around the mean is more pronounced, i.e. there is a higher probability of values near the mean, and the tails are fatter, i.e. there is a higher probability of extreme values. However, the distributions look both quite unimodal and symmetric. The distribution of imports lies slightly more to the right than the one of exports.

As a goodness-of-fit measure, a Pseudo-$R^2 = \text{Corr}^2(\ln q_{ib}, \ln \hat{q}_{ib})$ is used. This is the squared coefficient of correlation between observed and fitted values with the obvious interpretation of explained variance of a regression of observed on fitted values and an intercept.

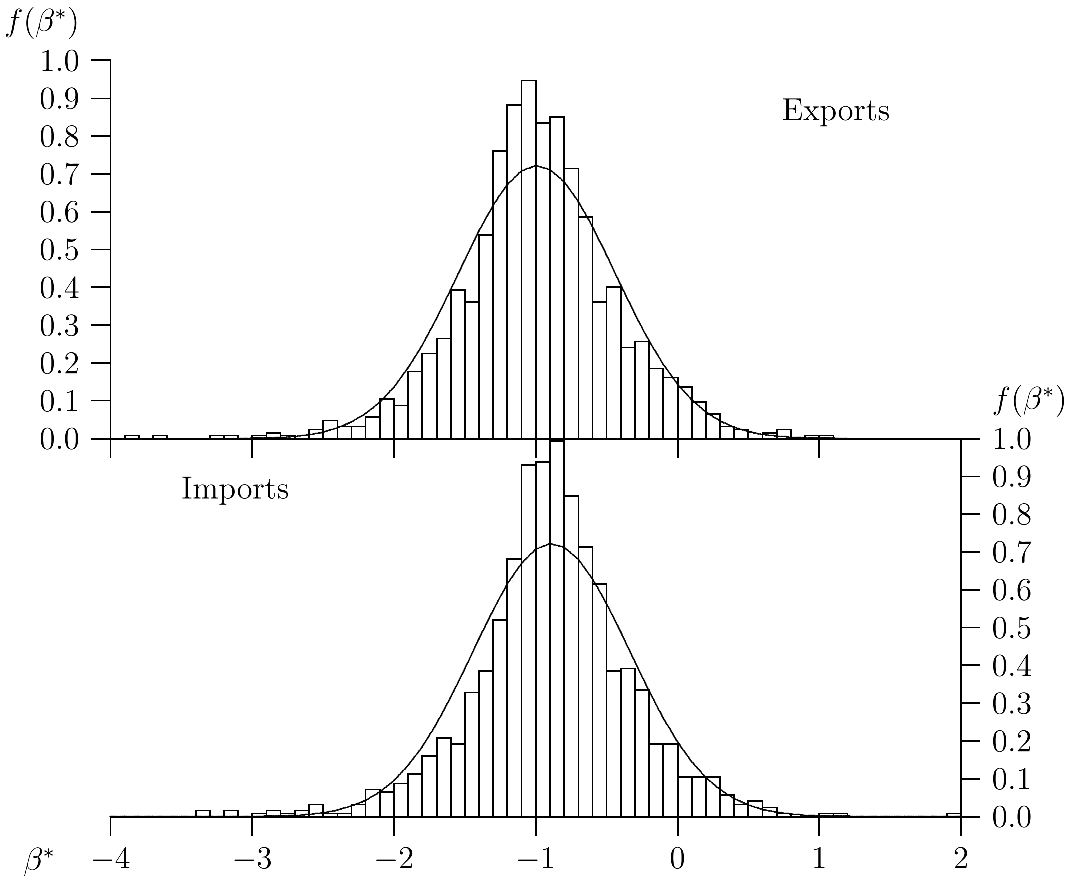

Figure 4: Density Histogram (Bin Width = 0.1) and Normal Density Plot of β^*

The most important descriptive summary statistics are collected in Table 4. The average price elasticity (β^*) for exports is -0.99, ranging from -3.9 to 1.1. Adjustment to the equilibrium is strong with the adjustment parameter $(1 - \rho)$ being 0.80 on average, lying in the range from 0.0 (no adjustment) to 1.6 (over-adjustment). The goodness-of-fit measure is high at 0.51 on average, covering the whole range from 0.0 to 1.0. The results for imports are almost the same with the notable difference of the average price elasticity, which is -0.89, i.e. a significant 0.1 point lower than for exports. Adjustment and goodness-of-fit are as strong as for exports. Yet results for imports are less stable than those for exports. This is due to higher heterogeneity of observed data owing to the large number of different countries from which German companies import goods.

Table 4: Descriptive Summary Statistics of the Partial Adjustment Model

	Exports			Imports		
Statistic	β^*	$1-\rho$	Pseudo-R^2	β^*	$1-\rho$	Pseudo-R^2
Mean	-0.9911	0.8014	0.5060	-0.8877	0.8071	0.5116
Variance	0.3055	0.0466	0.0775	0.3055	0.0425	0.0748
Minimum	-3.8923	-0.0157	0.0001	-3.3727	0.0491	0.0000
Maximum	1.0826	1.6344	0.9961	1.9547	1.4127	0.9850

Persistence of the process of prices given in Equation (22) is relatively low; on average, the autoregressive parameter γ_1 is 0.17 for exports and 0.19 for imports, thus rendering the simplification of Theorem 6 valid.

More important than the regression results itself are their implications for price statistics. These are summarised in Tables 5 and 6, respectively, in terms of the proportions to which each of the elementary indices corresponds to the Laspeyres, Paasche and Fisher price indices for panels with at least two groups of goods.

For the Laspeyres price index as the desired aggregate index, 70% of the panels in exports and 72% in imports imply the use of the Carli index at the elementary level. This means that if one wants to calculate a Laspeyres price index at the aggregate level, the Carli index will yield approximately the same result at the elementary level in these panels (as it is shown in an example in Subsection 3.2). Regarding trade values, these figures reduce to 62% and 66%, respectively. The Jevons index performs best at the first stage in 14% of the panels in exports and 17% in imports with much higher shares with respect to trade value, i.e. 29% and 28%, respectively. In 15% of the panels in exports and 10% in imports, the quadratic index is desirable at the lower level of aggregation; trade value shares here are 7% and 5%, respectively. Shares missing to 100% reflect other indices.

Table 5: Elementary Indices Corresponding to a Laspeyres Price Index

		Panels		Trade Values	
r	Price Index	Exports	Imports	Exports	Imports
0	Jevons	14%	17%	29%	28%
1	Carli	70%	72%	62%	66%
2	quadratic	15%	10%	7%	5%

If the Paasche price index is taken as the desired aggregate index, the corresponding power means are inverted: instead of the Carli index, the harmonic index, and, instead of the quadratic index, the reciprocal quadratic index have to be used. As mentioned before, if the Jevons index corresponds to the Laspeyres price index, it does so for the Paasche price index, too.

If the desired aggregate index is chosen to be the Fisher price index, the results are as follows. The use of the Jevons index is suggested by 6% of the panels in exports and 7% in imports; with respect to trade values these shares increase to 20% and 17%, respectively. The hybrid index is found to be superior in 21% of the panels in exports (trade values: 19%) and 28% (25%) in imports. 46% of the panels in exports (48%) and 44% in imports (43%) favour the CSWD index. A quadratic mean of cubic order should be used for 21% of the panels in exports (9%) and 15% in imports (12%). For a quadratic mean of quartic order the figures are 6% (3%) and 4% (2%), respectively. Again, quintic and higher orders make up shares missing to 100%.

Table 6: Elementary Indices Corresponding to a Fisher Price Index

		Panels		Trade Values	
q	Price Index	Exports	Imports	Exports	Imports
0	Jevons	6%	7%	20%	17%
1	Hybrid	21%	28%	19%	25%
2	CSWD	46%	44%	48%	43%
3	cubic	21%	15%	9%	12%
4	quartic	6%	4%	3%	2%

All in all, different elementary indices should be applied to each panel in order to approach the Laspeyres, Paasche or Fisher price index as closely as possible.

4.3 A Case Study

In Subsection 4.2 neither the Dutot nor the unit value index could be analysed. The presumption that these indices will differ from the Jevons index due to price dispersion (as discussed in Subsection 3.2) can only be tested with sufficient data. The intention of the following case study is firstly to discuss the empirical behaviour of these two indices, and secondly to test the results from Subsection 4.2 for their robustness. Exports of passenger cars are chosen as an example. With an export value of more than €100 billion in 2007, the more than five million cars exported make up more than 10% of trade value of all exported goods. The panel 8703 (four-digit code of the Commodity Classification: motor cars and other motor vehicles principally designed for the transport of persons, including station-wagons and racing cars) consists of 21 groups of goods and 1,895 observations of trade values and quantities in terms of both weight and number (the average 2007 car weights about 1.5 tonnes). The data set ends in 2007 and hence, is not affected by the recent financial crisis which has hit car makers hard around the world.

Both prices and quantities with respect to weight as well as number pass all of the five panel unit root tests of Subsection 4.2 at any conventional significance level. The partial adjustment model is robust to the specification of quantities as either weight or number as the results in Table 7 indicate. Irrespective of the definition of quantities, the results are virtually the same. The price elasticity for exports is close to zero and insignificant. The adjustment to the equilibrium, at 81%, is as strong as the Pseudo-R^2 is high at 95%. Hence, the Jevons index, corresponding to a price elasticity of zero, seems appropriate for all three, the Laspeyres, Paasche and Fisher, price indices.

Table 7: Partial Adjustment Model for Passenger Cars

	Weight			Number		
Statistic	β^*	$1-\rho$	Pseudo-R^2	β^*	$1-\rho$	Pseudo-R^2
Parameter	0.0469	0.8078	0.9572	−0.0277	0.8111	0.9525
Standard Error	0.0303	0.0317	—	0.0290	0.0317	—

Explanations for the counterintuitive result of equal car sales irrespective of prices are threefold, two of which are technical and one is economic. First, the price elasticity derived from the partial adjustment model is the effective one, i.e. lagged adjustment to the equilibrium lowers the absolute value of the price elasticity. Second, the estimated price elasticity should not be mistaken for the one of demand owing to the problem of identification (as explained in Subsection 3.1.3). Third, car makers might be wanting to hold sales stable by compensating for exchange rate fluctuations, and might thus be willing to accept short-term reductions in their profits.

After balancing the panel, 15 groups of goods remain for robustness testing of the regression results. Given the strongly balanced feature of this new panel, time series of the desired aggregate indices can be directly calculated. The power mean P^r, Equation (5), which minimises the root mean squared error (RMSE) to the desired aggregate index P^*, that is either the Laspeyres or Paasche price index, Equations (1) and (2), respectively, is found by non-linear least squares:

$$r_{\min} = \arg\min_{r \in \mathbb{R}} \sqrt{\frac{1}{t} \sum_{b=1}^{t} (P_b^* - P_b^r)^2}. \tag{31}$$

Analogously to Equation (31), one finds the quadratic mean P^q, Equation (29), which minimises the root mean squared error to the Fisher price index, Equation (3).

In Table 8 the outcomes of the partial adjustment model and non-linear optimisation are compared, along with the corresponding power means of the Dutot and unit value indices. The findings do not change qualitatively. In fact, the deviation from the symmetry proposition is insignificant and it turns out that the regression results coincide with the direct calculation of the power mean. The use of the Jevons index is justified. Note that the linear regression is based on a panel data set of 1,817 observations, while the non-linear direct calculation is based on a time series of 95 observations, which makes the latter more prone to erratic behaviour.

Table 8: Partial Adjustment Model Compared to Non-Linear Optimisation

Statistic	β^*	r_L	r_P	q	r_D	r_{UV}
Expectation	—	$-\beta^*$	β^*	$2\lvert\beta^*\rvert$	0	0
Parameter	0.0469	0.3000	−0.4224	0.0000	0.2508	0.9920
Standard Error	0.0303	0.1399	0.1453	†	0.0428	0.1528
Pseudo-R^2	0.9572	0.1957	0.1437	0.1548	0.9328	0.2754
RMSE	—	0.0404	0.0388	0.0389	—	—

† Standard error of q is not stable with respect to different initial values.

That the implied power means of the Dutot and unit value indices are significantly off their expectations can be explained with recourse to Equations (18) and (19). While scanner data in a CPI may be well-behaved in terms of their covariance stability, things are different in an export or import price index (cf.

Subsections 1.2 and 3.1.1 for empirical and theoretical evidence, respectively). Unlike scanner data, the basis of index calculation is not a panel but rather repeated cross-sections with time-varying coverage of the universe of goods, i.e. new goods are introduced while others disappear. Thus, the relative broad item description in foreign trade is likely to cause heterogeneity to increase over time (cf. Subsection 3.2 for a discussion of this issue). Neither the variance of prices nor the concurrent covariance between prices and quantities is stable over time. While the variance is increasing, the covariance is decreasing, which explains the gap between the indices and to their expectations.

However, when allowing for non-power means as the Dutot and unit value indices, the findings change slightly. For all three target indices, the Dutot index has a lower RMSE than the Jevons index, which shows a lower RMSE than the unit value index. This is depicted in Table 9, which compares the RMSEs of the respective elementary indices as estimators of the desired aggregate ones. Nonetheless, the Dutot and Jevons indices are numerically very close.

Table 9: RMSEs of Elementary Indices to Desired Aggregate Ones

Estimator	Laspeyres (r_L)	Paasche (r_P)	Fisher (q)
β^*	0.0418	0.0413	0.0389
$r_{\min}$ / $q_{\min}$	0.0404	0.0388	0.0389
Jevons Index	0.0415	0.0408	0.0389
Dutot Index	0.0338	0.0387	0.0338
Unit Value Index	0.0418	0.0650	0.0531

In Figure 5 the time series of the Dutot, Jevons and unit value indices as estimators of the Fisher price index are drawn on the semi-logarithmic scale with base month January 2000 = 100. Both the Dutot and Jevons indices are similar to the Fisher price index and to each other. This was to be expected from the regression results as well as non-linear optimisation (cf. Table 8). Owing to missing expenditure share weights, these approximations to the Fisher price index are the closest that one can get. The unit value index is much more volatile (cf. Table 9) and lies well above the Fisher price index, although it is fairly close at the beginning of the time series. This was to be expected as well given the aforementioned time-varying variances of prices and covariances between concurrent prices and quantities.

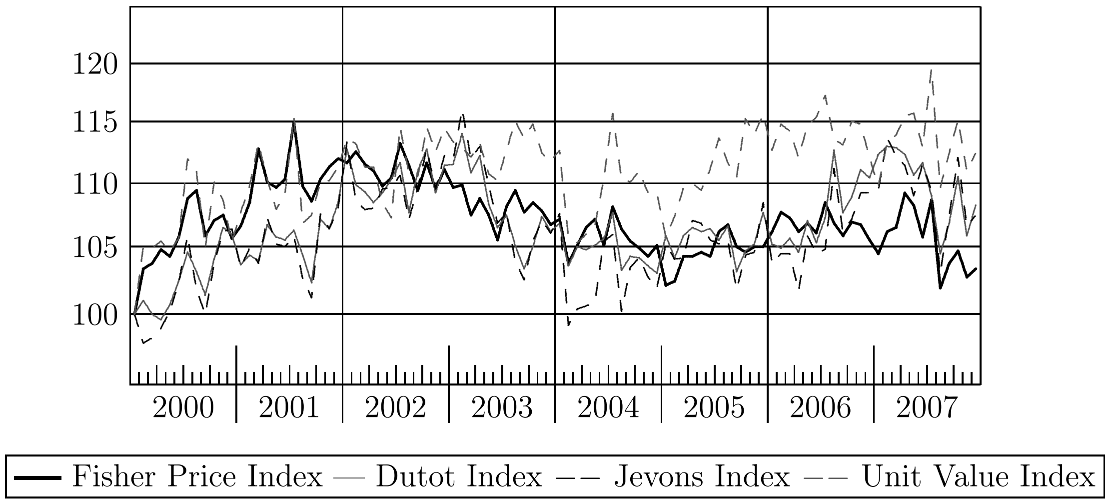

Figure 5: Elementary Indices as Estimators of the Fisher Price Index

5 Conclusion

5.1 Summary

This paper addresses the problem of index calculation at the elementary level, where no expenditure share weights are available. The question of "Which index formula at the elementary level corresponds to the characteristics of the index at the aggregate level?" is dealt with. A statistical approach is proposed which makes it theoretically possible to achieve numerical equivalence of an elementary index with the desired aggregate index – in this instance, the Laspeyres, Paasche or Fisher price index. Based on "power means" and the assumption of joint log-normal distribution of prices and quantities, it is shown that the solution depends on the price elasticity only, which is derived from a partial adjustment model. Thus, a feasible framework is provided which aids the choice of the corresponding elementary index. The results are graphically produced in Figure 6.

From an empirical application to German foreign trade statistics, it can be seen that the choice of the elementary index does matter (cf. Figure 5). The choice itself depends on the characteristics of prices and quantities. Therefore, depending on the price elasticity, different elementary indices should be applied

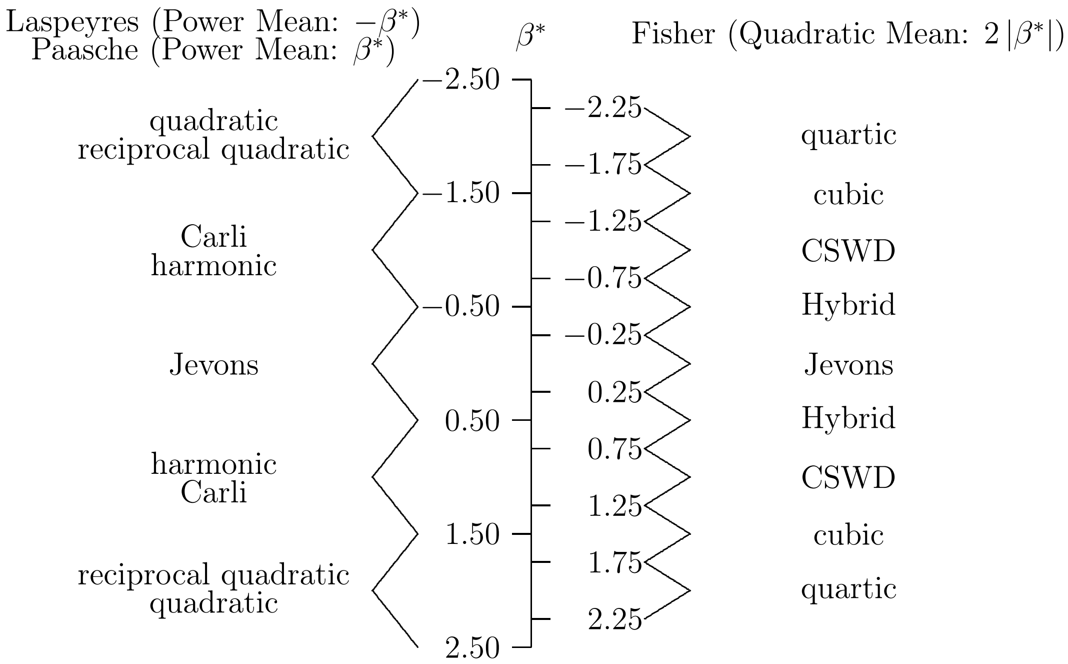

Figure 6: Overview of Corresponding Elementary Indices

to each group of goods in order to approach the Laspeyres, Paasche or Fisher price index as closely as possible. While not relying on axiomatic considerations, this paper finds notable empirical differences between different elementary indices and aggregate indices formed from them. Furthermore, the results indicate that a range of elementary indices should be applied in the calculation of price indices. This is in line with the findings of other authors (cf. the review of the empirical literature in Subsection 1.2). In particular, the Carli index performs remarkably well at the elementary level of a Laspeyres price index, corresponding to a price elasticity of minus one. Sometimes, it is argued that the Carli index is upward biased. However, this holds only in comparison with the Jevons index. Yet, the comparison in question is not with another elementary index but with the desired aggregate index. So, it may be the case that the Carli index is unbiased or even downward biased compared with the Laspeyres price index (cf. Subsection 3.1.2 for the discussion of the Carli index' upward bias).

5.2 Outlook

Two possible applications of the approach outlined in this paper arise immediately after a decision has been taken on which aggregate index is desired. Firstly, index calculation can be rendered more precisely if different elementary indices are applied to each group of goods, reflecting their specific price elasticities. At least for prominent groups of goods with high expenditure shares, studies on the price elasticity should be available. This will drive down biases of official price indices. In fact, the desired aggregate index can be approximated by using appropriate elementary indices. Secondly, for different purposes, different elementary indices should be calculated. This means that if the Carli index is applied as the single formula at the elementary level of a Laspeyres price index, implying a price elasticity of minus one, for the same data, the harmonic index must be used at the elementary level of a Paasche price index. Still, this is in contrast to the current practice as regards foreign trade in Germany, where the Carli index is used at the elementary level in both price statistics and volume measurement in national accounts. The former task is achieved via the Laspeyres price index, while the latter results in an implicit deflator in the form of the Paasche price index.

An application of this approach to scanner data in a CPI would be worthwhile. Scanner data in its most familiar form are collected at the checkouts of retail stores by the scanning of bar codes. Thus, they provide a census of all transactions rather than a sample. Furthermore, they are collected continuously and provide simultaneous information on both prices and quantities, unlike discrete surveys of prices alone. Lastly, qualitative information may be linked to scanner data, allowing for hedonic adjustment. The foreign trade application of this paper and the prospective study of scanner data are different subject matters. In foreign trade statistics, the data are intermediately aggregated and unit values are used, which are neither seasonally nor quality adjusted, rather than observed purchase prices. Disaggregate scanner data allow the calculation of unbiased price indices and hence, a more thorough analysis based on them might change the results.

Appendix: Proof of Lemmata and Theorems

Proof of Lemma 1. The Laspeyres and Paasche price indices are basket indices, i.e. they are a weighted means of price relatives, either arithmetic with base period expenditure share weights or harmonic with current period expenditure share weights. Either way, the weights $\omega_i > 0$ sum up to one, $\sum_{i=1}^{n} \omega_i = 1$ and the proposition follows. This holds as well for the Fisher price index as it is the geometric mean of the Laspeyres and Paasche price indices. □

Proof of Lemma 2. That the limits towards $r \to \pm\infty$ are the maximum and minimum, respectively, can be shown by straightforward algebraic manipulations. The geometric index as the limit towards $r \to 0$ is found via a Taylor series expansion. From this the proposition follows. □

Proof of Theorem 1. From Lemma 1, it follows that the aggregate index P^* lies between the smallest and largest price relative, $P^{\min}$ and $P^{\max}$, respectively. To reiterate, the exclusion of the trivial case of perfect homogeneity ensures that the mean value property is fulfilled in its strict from.

$$P^{\min} < P^* < P^{\max}$$

That the power mean P^r is continuous on its whole domain follows from Lemma 2. Moreover, it covers the whole range between the smallest and largest price relative as its co-domain. Over and above that, the strict mean value property leads to r being real, $r \in \mathbb{R}$.

$$P^{\min} < P^r < P^{\max}$$

In addition, from the intermediate value theorem (under a continuous function the image of a connected space is connected), it follows that the power mean takes on all values of its co-domain, of which the aggregate index is an element, at least once. Hence, the image equals the co-domain.

$$P^* \in (P^{\min}, P^{\max}) \leftarrow \mathbb{R} : P^r$$

Eventually, the uniqueness of the power r is secured through the proposition that not all price relatives are equal, and with Jensen's inequality it can be shown that the power mean is a strictly monotonic increasing function.

$$P^s > P^r \, \forall \, s > r$$

From this, it follows that the power mean is bijective and therefore an inverse function exists. □

Proof of Lemma 3. If one writes the unit value index in its price relatives form, $\sum_{i=1}^{n}(p_{it}/p_{i0})\omega_i$, the assigned weights $\omega_i = (p_{i0}q_{it}/\sum_{i=1}^{n} q_{it})/(\sum_{i=1}^{n} p_{i0}q_{i0}/\sum_{i=1}^{n} q_{i0})$ do not necessarily sum up to unity. This contradiction proves the proposition. □

Proof of Theorem 2. The processes of prices and quantities are assumed to have both started in the infinite past.

$$p_{i,t} = p_{i,-\infty} \cdot \ldots \cdot \frac{p_{i,0}}{p_{i,-1}} \cdot \frac{p_{i,1}}{p_{i,0}} \cdot \frac{p_{i,2}}{p_{i,1}} \cdot \ldots \cdot \frac{p_{i,t-1}}{p_{i,t-2}} \cdot \frac{p_{i,t}}{p_{i,t-1}}$$
$$q_{i,t} = q_{i,-\infty} \cdot \ldots \cdot \frac{q_{i,0}}{q_{i,-1}} \cdot \frac{q_{i,1}}{q_{i,0}} \cdot \frac{q_{i,2}}{q_{i,1}} \cdot \ldots \cdot \frac{q_{i,t-1}}{q_{i,t-2}} \cdot \frac{q_{i,t}}{q_{i,t-1}}$$

However, the period-to-period changes are not independently distributed. The sequences are assumed to satisfy a mixing condition, which implies ergodicity; hence, a central limit theorem under weak dependence becomes applicable. Thus, it follows that prices and quantities are marginally log-normally distributed. Having proven marginal log-normal distribution, it follows that they are also jointly log-normally distributed by imposing a functional relationship between prices and quantities and autoregressive relationships within them. □

Proof of Theorem 3. The expectation of a log-normally distributed random variable is given by $\exp(\mu + \sigma^2/2)$. After taking natural logarithms it applies that $a \ln X \pm b \ln Y \sim \mathcal{N}(a\mu_X \pm b\mu_Y,\, a^2\sigma_X^2 + b^2\sigma_Y^2 \pm 2ab\sigma_{X,Y})$. Using this and the definitions of the power mean, and the Dutot and unit value indices one finds the following results.

$$P^r = \sqrt[r]{\mathrm{E}\left(\frac{p_{it}^r}{p_{i0}^r}\right)} = \exp\left[\frac{1}{r}\left(r(\mu_{p_t} - \mu_{p_0}) + r^2\frac{\sigma_{p_t}^2 + \sigma_{p_0}^2 - 2\sigma_{p_t,p_0}}{2}\right)\right]$$

$$P^D = \frac{\mathrm{E}(p_{it})}{\mathrm{E}(p_{i0})} = \frac{\exp\left(\mu_{p_t} + \frac{\sigma_{p_t}^2}{2}\right)}{\exp\left(\mu_{p_0} + \frac{\sigma_{p_0}^2}{2}\right)}$$

$$P^{UV} = \frac{\mathrm{E}(p_{it}q_{it})/\mathrm{E}(q_{it})}{\mathrm{E}(p_{i0}q_{i0})/\mathrm{E}(q_{i0})} = \frac{\exp\left(\mu_{p_t} + \mu_{q_t} + \frac{\sigma_{p_t}^2 + \sigma_{q_t}^2 + 2\sigma_{p_t,q_t}}{2}\right) / \exp\left(\mu_{q_t} + \frac{\sigma_{q_t}^2}{2}\right)}{\exp\left(\mu_{p_0} + \mu_{q_0} + \frac{\sigma_{p_0}^2 + \sigma_{q_0}^2 + 2\sigma_{p_0,q_0}}{2}\right) / \exp\left(\mu_{q_0} + \frac{\sigma_{q_0}^2}{2}\right)}$$

By reducing the terms, the proposition follows. □

Proof of Theorem 4. Using the definitions of the Laspeyres and Paasche price indices, the expectations are as follows.

$$P^L = \frac{\mathrm{E}(p_{it}q_{i0})}{\mathrm{E}(p_{i0}q_{i0})} = \frac{\exp\left(\mu_{p_t} + \mu_{q_0} + \frac{\sigma_{p_t}^2 + \sigma_{q_0}^2 + 2\sigma_{p_t,q_0}}{2}\right)}{\exp\left(\mu_{p_0} + \mu_{q_0} + \frac{\sigma_{p_0}^2 + \sigma_{q_0}^2 + 2\sigma_{p_0,q_0}}{2}\right)}$$

$$P^P = \frac{\mathrm{E}(p_{it}q_{it})}{\mathrm{E}(p_{i0}q_{it})} = \frac{\exp\left(\mu_{p_t} + \mu_{q_t} + \frac{\sigma_{p_t}^2 + \sigma_{q_t}^2 + 2\sigma_{p_t,q_t}}{2}\right)}{\exp\left(\mu_{p_0} + \mu_{q_t} + \frac{\sigma_{p_0}^2 + \sigma_{q_t}^2 + 2\sigma_{p_0,q_t}}{2}\right)}$$

The proposition follows by reducing the terms. The corresponding power means are found by solving the equations for r. □

Proof of Theorem 5. Stationarity in covariance of the processes, i.e. $0 \leq \rho < 1$ and $0 \leq \gamma_1 < 1$, imply that the covariance between any two observations depends only on the lag between them. For the covariance of logarithmic prices, it follows that it is an exponentially decreasing function.

$$\sigma_{p_\kappa,p_\ell} = \gamma_1^{|\kappa-\ell|}\sigma_p^2$$

Using the lag operator and inverting the lag polynom in the function of logarithmic quantities, it can be written as follows.

$$\ln q_{ib} = \alpha + \beta^* \sum_{\tau=0}^{\infty} \rho^\tau \ln p_{ib-\tau} + \left(\sum_{\tau=0}^{\infty} \rho^\tau \mathbf{x_{ib-\boldsymbol{\tau}}} \right) (1-\rho)\boldsymbol{\delta} + \left(\eta_i^q + \sum_{\tau=0}^{\infty} \rho^\tau \varepsilon_{ib-\tau}^q \right)$$

Taking the expectation and subtracting it on both sides yields the following expression.

$$\ln q_{ib} - \mu_q = \beta^* \sum_{\tau=0}^{\infty} \rho^\tau (\ln p_{ib-\tau} - \mu_p) + \sum_{\tau=0}^{\infty} \rho^\tau \varepsilon_{ib-\tau}^q$$

Multiplying this expression with $\ln p_{i\xi} - \mu_p$ and taking the expectation results in the desired covariances.

$$\sigma_{p_\xi, q_b} = \beta^* \sum_{\tau=0}^{\infty} \rho^\tau \sigma_{p_\xi, p_{b-\tau}} = \beta^* \sigma_p^2 \sum_{\tau=0}^{\infty} \rho^\tau \gamma_1^{|\xi-(b-\tau)|}$$

Substituting the appropriate expressions for ξ and b, either 0 or t, the proposition follows by applying the formula for the sum of a geometric series. □

Proof of Theorem 6. Substituting the respective expressions into the equations directly yields the stated results. Under the stationarity in covariance assumption, the difference of (co-)variances at different points in time vanishes and approaches zero. For the powers corresponding to the Laspeyres and Paasche price indices, r_L and r_P, respectively, it is assumed that the product of the autoregressive parameters is sufficiently small to be negligible, i.e. the sluggishness of adjustment of quantities or the persistence of the process of prices is low: $\rho\gamma_1 \to 0$. The power corresponding to the Paasche price index is derived under the additional assumptions of sufficiently large t in order for the serial correlation to converge to zero: $\rho^t \to 0$ and $\gamma_1^t \to 0$. □

Proof of Theorem 7. The proposition follows directly by reducing the terms. □

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