

Retrospective Price Indices and Substitution Bias

by

W. Erwin Diewert
Professor of Economics
University of British Columbia

Marco Huwiler
Senior Investment Strategist
Clariden Leu, Zurich

and

Ulrich Kohli
Chief Economist and Alternate Member of the Governing Board
Swiss National Bank

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Abstract

The consumer price index (CPI) is usually computed as a fixed-weighted Laspeyres price index, with the weights updated at discrete intervals only. It is well known that the Laspeyres functional form entails a substitution bias. One way to reduce it would be to use chained indices, and superlative ones if possible. Unfortunately, the necessary data are often missing. This paper proposes a simple method to retroactively compute the CPI once updated weights become available. The proposed index has the Fisher form. This makes it possible to assess the size of the substitution bias. An application to Swiss data is provided.

Key words: price index, inflation, superlative indices, substitution bias

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Corresponding author: Ulrich Kohli, Swiss National Bank, P.O. Box 2800, CH-8022 Zurich, Switzerland; phone: +41-44-631-3233; e-mail: Ulrich.Kohli@snb.ch.

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1. Introduction

Most countries today still compute their consumer price indices (CPI) as direct (i.e. fixed-weighted) Laspeyres indices.¹ The weights are updated at discrete intervals, often every five or ten years, at which time the old and new series are spliced together. It is well known that, in the consumer context, the Laspeyres functional form tends to overestimate the price level due to substitution bias, i.e. the fact that households tend to reduce (increase) their consumption of those goods that have become relatively dearer (cheaper).² One way to reduce this bias would be to use chained indices and, preferably, superlative ones. The use of chained indices amounts to updating the weights every period. Superlative indices, furthermore, take into account the quantities consumed both before and after the price change. Superlative indices are exact for flexible functional forms, which themselves can provide a second-order approximation to an arbitrary aggregator function.³ By using a quadratic – rather than a linear (e.g. Laspeyres) – approximation, the substitution bias is significantly reduced, or perhaps even eliminated. The difficulty is that to compute chained (whether superlative or not) indices one needs information on quantities and prices for every period. Unfortunately, this information may be missing. Annual data on quantities, in particular, are often not available, and this is why statistical agencies tend to rely on fixed baskets. Sooner or later, however, these baskets will have to be updated, and the question therefore arises at the time when the new information becomes available whether it can be used to assess the importance of the substitution bias, and whether it can be exploited to improve the measure of past price level behavior. Historical price series are often used in economic analysis, and there is no reason why one should limit oneself to the original series if better ones can be computed retroactively. Measurement errors can be an important source of statistical bias in econometric work.

¹ More accurately, statistical agencies compute Lowe indices where the vector of weights refers to a base year which is usually different from the base month for the price relatives. However, Lowe indices basically behave as if they were Laspeyres indices; see Diewert (2004) for material on Lowe indices.

² Another source of measurement bias may be due to changes in quality. We do not address this issue in this paper.

³ See Diewert (1976).

The purpose of this paper is to propose a simple way to use the new information made available at the time of updating in order to get a superlative measure of the price change over the corresponding period. Retroactively computed price indices then make it possible to assess the size of the substitution bias. An application to Swiss data is provided. A comparison between our approach and Hansen's (2007) recent work with Danish data is also presented.

2. A retrospective measure of the price level⁴

The direct Laspeyres price index relies on beginning-of-period fixed weights. Because of changes in relative prices, these weights might progressively become less and less relevant. When dealing with series extending over long periods of time, and when chaining is not possible (perhaps because quantity data are not available for every period), the use of mid-year indices has been advocated in the literature (Diewert, 2004). What we propose here is a simple alternative, namely to take the geometric mean of two Lowe indices, one using the quantities of the first period as reference basket, and the second one using those from the last period. The resulting index has the Fisher form over the entire period.

Let p_i^t and q_i^t denote the price and the quantity of good i at time t , respectively.

Consider the following two runs of fixed-basket indices:

$$(1) \quad 1, \frac{\sum_i p_i^1 q_i^0}{\sum_i p_i^0 q_i^0}, \frac{\sum_i p_i^2 q_i^0}{\sum_i p_i^0 q_i^0}, \dots, \frac{\sum_i p_i^{T-1} q_i^0}{\sum_i p_i^0 q_i^0}, \frac{\sum_i p_i^T q_i^0}{\sum_i p_i^0 q_i^0}$$

$$(2) \quad \frac{\sum_i p_i^0 q_i^T}{\sum_i p_i^T q_i^T}, \frac{\sum_i p_i^1 q_i^T}{\sum_i p_i^T q_i^T}, \frac{\sum_i p_i^2 q_i^T}{\sum_i p_i^T q_i^T}, \dots, \frac{\sum_i p_i^{T-1} q_i^T}{\sum_i p_i^T q_i^T}, 1,$$

where the initial period is denoted by 0, and the terminal one by T . Sequence (1) is simply a run of direct Laspeyres indices. Each element in (1) makes a direct comparison between period t ($t = 1, \dots, T$) and period 0. Each element in sequence

⁴ This section is based on Kohli (2004).

(2), on the other hand, can be interpreted as the inverse of a Paasche price index, making a direct comparison between period t ($t = 0, \dots, T-1$) and period T . Taken as a whole, however, sequence (2) is not a run of Paasche price indices, since in each element the quantities remain those of period T , rather than those of the current period.

Next, we normalize run (2) by dividing all its elements by the first one:

$$(3) \quad 1, \frac{\sum_i p_i^1 q_i^T}{\sum_i p_i^0 q_i^T}, \frac{\sum_i p_i^2 q_i^T}{\sum_i p_i^0 q_i^T}, \dots, \frac{\sum_i p_i^{T-1} q_i^T}{\sum_i p_i^0 q_i^T}, \frac{\sum_i p_i^T q_i^T}{\sum_i p_i^0 q_i^T}.$$

Note that the last element of (3) is a Paasche price index, which makes a direct comparison between prices in period 0 and those in period T . The other elements in (3) are not Paasche indices, however. Rather, they are Lowe indices, using the quantities of period T as weights.

Finally, we take the geometric means of the corresponding elements of (1) and (3) to get the following sequence of pseudo Fisher indices:

$$(4) \quad 1, \sqrt{\frac{\sum_i p_i^1 q_i^0}{\sum_i p_i^0 q_i^0} \frac{\sum_i p_i^1 q_i^T}{\sum_i p_i^0 q_i^T}}, \dots, \sqrt{\frac{\sum_i p_i^{T-1} q_i^0}{\sum_i p_i^0 q_i^0} \frac{\sum_i p_i^{T-1} q_i^T}{\sum_i p_i^0 q_i^T}}, \sqrt{\frac{\sum_i p_i^T q_i^0}{\sum_i p_i^0 q_i^0} \frac{\sum_i p_i^T q_i^T}{\sum_i p_i^0 q_i^T}}.$$

We see two main advantages in using (4) rather than a mid-year index. First, one sees that the last element in (4) has the Fisher (1922) form: it is a direct Fisher index that indicates the price level of period T relative to the price level of period 0. Thus, run (4) will give a superlative measure of the cumulated increase in the price level over the entire period. Although the other elements of (4), strictly speaking, do not have the Fisher form, they can be viewed as elements of a quadratic interpolation. Second, quantity data are often available for the “initial” period only. Yet, baskets do have to be – and indeed are – updated from time to time, so that end-of-period quantities will eventually become available too. Mid-year (i.e. middle of sample) quantities, on the

other hand, might never become available.⁵ We would thus suggest that, in those cases where a Laspeyres price index is being used, a definite price series could be calculated retroactively at the time when a new basket is introduced and splicing has to take place.

3. Comparison with Hansen

Hansen (2007) recently recalculated the Danish CPI, 1996-2006, using a number of different formulae, including one based on the Fisher price index. For period t , the retrospective index Hansen proposed ($P_H^{0:t}$) is as follows:⁶

$$(5) \quad P_H^{0:t} = \left[\frac{\sum_i s_i^0 \frac{p_i^t}{p_i^0}}{\sum_i s_i^T \frac{p_i^0}{p_i^t}} \right]^{\frac{1}{2}}, \quad t = 0, 1, \dots, T$$

where s_i^0 is the base period expenditure share of good i , and s_i^T is the corresponding period T expenditure share:

$$(6) \quad s_i^0 = \frac{p_i^0 q_i^0}{\sum_i p_i^0 q_i^0}$$

$$(7) \quad s_i^T = \frac{p_i^T q_i^T}{\sum_i p_i^T q_i^T}.$$

To see how Hansen's approach differs from ours, it is useful to rewrite formulae (1) and (2) in terms of price relatives and expenditure share weights, rather than in basket form. For the period t price index in each sequence we thus get:

⁵ In fact, if mid-year quantities were available, it would still be advantageous to use the procedure described by (4), but in this case applied to the two half samples separately, i.e. from the first period to the mid-year, and from the mid-year to the last period.

⁶ See Hansen (2007), Annex 1.

$$(8) \quad \frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} = \sum_i s_i^0 \frac{p_i^t}{p_i^0}, \quad t = 0, 1, \dots, T$$

$$(9) \quad \frac{\sum_i p_i^t q_i^T}{\sum_i p_i^T q_i^T} = \sum_i s_i^T \frac{p_i^t}{p_i^T}, \quad t = 0, 1, \dots, T.$$

Using (9), it can be seen that the period t index in (3) can be written as follows:

$$(10) \quad \frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} = \frac{\sum_i s_i^T \frac{p_i^t}{p_i^T}}{\sum_i s_i^T \frac{p_i^0}{p_i^T}}, \quad t = 0, 1, \dots, T.$$

Finally, using (8) and (10), it can be seen that the period t pseudo Fisher price index ($P_F^{0:t}$) in the sequence defined by (4) can be written as follows:

$$(11) \quad P_F^{0:t} = \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} \right]^{\frac{1}{2}} = \left[\frac{\sum_i s_i^0 \frac{p_i^t}{p_i^0} \cdot \sum_i s_i^T \frac{p_i^t}{p_i^T}}{\sum_i s_i^T \frac{p_i^0}{p_i^T}} \right]^{\frac{1}{2}}, \quad t = 0, 1, \dots, T.$$

It is noteworthy that $P_H^{0:T} = P_F^{0:T}$. That is, when $t = T$, both (5) and (11) simplify to the usual Fisher ideal price index between periods 0 and T . For values of t between 0 and T , however, the two formulae are likely to give different results. Note that the numerator in the square bracket of (5) is identical to the first term in the numerator of the square bracket of (11). This is the *Laspeyres element* of the Fisher formula. The difference between (5) and (11) is due to the *Paasche element*. As it can be seen from (10), the Paasche element in our formulation is indeed a true Paasche price index that makes a comparison between period- t and period- T prices, normalized to ensure that $P_F^{0:0} = 1$. The component that plays the role of the Paasche element in Hansen's formula ($P_{H(P)}^{0:t}$), on the other hand, is as follows:

$$(12) \quad P_{H(P)}^{0:t} = \frac{1}{\sum_i s_i^T \frac{P_i^0}{P_i^t}}, \quad t = 0, 1, \dots, T.$$

$P_{H(P)}^{0:t}$ can be interpreted as a harmonic period- T weighted Young index. It does not have the Paasche form, however. It is therefore debatable whether $P_H^{0:t}$ can be viewed as a legitimate member of the Fisher family for $0 < t < T$.

4. Application to Swiss data

There are no direct measures of the substitution bias available for Switzerland, although some informed estimates do exist. Thus, Brachinger, Schips and Stier (1999) argue that the (upper level) substitution bias probably does not exceed 0.15 percentage points per year. They base their opinion on the findings of the Boskin Report, and on the fact that the Swiss weights are updated more frequently and put in place with less delay than in the United States.

The procedure that we propose makes it possible to get an independent estimate of the substitution bias for Switzerland. The application is for the Swiss CPI data for the period 1993 to 2000. Until recently the Swiss CPI was computed as a direct Laspeyres quantity index. The weights were often not revised for considerable periods of time. Thus, from 1993 to 2000 the index was computed with fixed weights, using May 1993 as the reference period. The weights were eventually revised in 2000. Of course, because of the introduction of new goods and the dropping out of old ones, the coverage was not exactly the same in the 1993 and 2000 surveys. It turns out, however, that the number of categories common to both surveys amounted to 192 out of 201. In value terms, these categories represented over 99% of the CPI.

We show in Figure 1 the path of the official CPI series for Switzerland. It is calculated as a run of direct Laspeyres quantity indices, using May 1993 as the reference period. According to this series, the price level increased by 6.09% between 1993 and 2000. Next, we calculated the same series, but only retaining those 192 goods that were common in both the 1993 and the 2000 surveys. It can be seen that

this series is almost identical to the official one: the price increase over the seven year period now approximates 6.18%. The yearly average difference between the two series thus amounts to little more than 0.01%.

We next computed the run of Lowe indices, as shown by (3), using the May 2000 weights. Over the entire period the price increase is now estimated to be 4.23%, i.e. two percentage points less than the Laspeyres measure. Remember that the final element of sequence (3) is a true, direct Paasche index. It was therefore to be expected that it would be less than the last element of the run of direct Laspeyres indices (1). It is also noteworthy that series (3) lies throughout the sample underneath the series given by (1) as it ought to.

We finally compute the geometric mean of the two series just reported. It is also shown in the graph. As expected, it is situated between series (1) and (3). The direct Fisher index (the last element of sequence (4)) indicates that the Swiss price level has increased by 5.20% over the seven year period. This is nearly a percentage point (about 15%) less than what the official series suggests. This measure can be viewed as a superlative measure, and indeed, in our opinion, this is the best measure of the increase in the Swiss price level between 1993 and 2000 that can be reconstituted today on the basis of the available information.

Our results also make it possible to quantify the annual substitution bias. Comparing the Fisher index with the 192-item Laspeyres index, we get a total bias of 0.93% over seven years. This implies a yearly substitution bias of 0.13% on average. This estimate gives considerable empirical support to Brachinger, Schips and Stier's (1999) assertion.

5. Conclusion

With the benefit of hindsight, one can affirm that the Swiss price level (in terms of CPI) has increased by about 5.2% between 1993 and 2000, rather than the 6.1% that the official data suggest. In terms of yearly averages, this implies an inflation rate of about 0.73%, rather than 0.85%. Given that the CPI is used for many indexing

purposes, a difference of close to one percentage point (or nearly one sixth in relative terms) over a seven year period is not trivial.

Needless to say, the approach that we have used for the CPI could also be applied to other indices, including quantity indices. Economic analysts and researchers often work with long time series. Given that measurement errors can be an important source of statistical biases, econometricians might find it advantageous to retroactively compute pseudo Fisher indices on the model that is proposed here.

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Figure 1
Laspeyres CPI, 1993- and 2000-based Lowe Indices, and Pseudo Fisher Index
Switzerland, 1993-2000

