Lowe and Cobb-Douglas CPIs and their substitution bias

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Lowe

A Lowe price index is defined as

$$P^{Lo}(p^t,p^0;x^b) \equiv p^t \cdot x^b / p^0 \cdot x^b$$

where p^τ ($\tau = 0,t$) is a vector of prices and x^b is a vector of quantities. Typically

$$b \leq 0 < t.$$

Cobb-Douglas

A Cobb-Douglas price index is defined as

$$P^{CD}(p^t,p^0;s^b) \equiv \Pi_n(p_n^t/p_n^0)^{s_n^b}$$

where p^τ ($\tau = 0,t$) is a vector of prices and s^b is a vector of value shares $p_n^b x_n^b / p^b \cdot x^b$. Typically

$$b \leq 0 < t.$$

Lowe and CD compared

Ex. (24) shows that

$$\ln P^{Lo}(p^1,p^0;x^b) – \ln P^{CD}(p^1,p^0;s^b)$$

can be written as covariance between price-update factors over [b,1] and relative price changes over [0,1]. This is likely to be non-negative, especially when [b,0] is short relative to [0,1].

Benchmark Cost-of-Living index

The target Konüs COLI is defined as

$$P^K(p^1,p^0;x^b) \equiv C(p^1,U(x^b)) \,/\, C(p^0,U(x^b))$$

where U(.) is the consumer's utility function and C(.) the dual cost function.

It is assumed that in period b the consumer acts cost-minimizing:

$$C(p^b,U(x^b)) = p^b \cdot x^b \,.$$

Second order approximations (1)

Taylor series around p^b:

$C(p^1,U(x^b)) = p^1 \cdot x^b + e^1$

$C(p^0,U(x^b)) = p^0 \cdot x^b + e^0$

where e^t ($t = 0,1$) are second-order terms which are non-positive.

Bias of Lowe index

The relative substitution bias

$[P^{Lo}(p^1,p^0;x^b) - P^K(p^1,p^0;x^b)] / P^K(p^1,p^0;x^b)$

is given by ex. (29) and likely to be positive.

Second order approximations (2)

An other Taylor series around p^b:

$$\ln C(p^t,U(x^b)) = \ln C(p^b,U(x^b)) +$$

$$\sum_n s_n^b \ln (p_n^t/p_n^b) + \varepsilon^t$$

where ε^t ($t = 0,1$) are second-order terms which are not necessarily non-positive.

Bias of CD index

The relative substitution bias

$\ln P^{CD}(p^1,p^0;s^b) - \ln P^K(p^1,p^0;x^b) = \varepsilon^0 - \varepsilon^1$

is given by ex. (34). *A priori* not much can be said about the sign of this bias.

Comparison

See section 6: On balance it is likely that the relative substitution bias of the CD is less than that of the Lowe index.

The empirical evidence of section 7 is inconclusive.