A COMPARISON OF ESTIMATORS FOR ELEMENTARY AGGREGATES OF THE CPI

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1. Introduction

The calculation of the U.S. Consumer Price Index (CPI) involves two distinct levels---the calculation of indexes for elementary aggregates representing 207 strata of items in each of 44 geographical areas, and the aggregation of those indexes across items and areas. The aggregation issues are very familiar to economists from the extensive literature on index number formulas and the economic theory of the cost-of-living index[1].

Research on the estimation of elementary aggregates is more recent, and the literature has developed along two major strands. One line of research was to develop the sampling ideas necessary to estimate a fixed-basket or Laspeyres-type price index for the finite population of all items and outlets using modern survey methods. The 1978 revision of the U.S. CPI implemented an approach, described in more detail below, for drawing a sample of items from outlets with probability proportional to consumer expenditures on each available item.[2] Other countries continue to price a basket consisting of pre-specified items, however, and weighting information is frequently not available. A second literature has appeared recently on estimation of price change for elementary aggregates in that context, examining alternative estimators and particularly the implication of chaining together samples.[3]

Evidence that estimation of elementary aggregates may be a significant problem for the U.S. CPI first appeared in research by Reinsdorf (1993). Reinsdorf compared relative changes in average prices published by the Bureau of Labor Statistic (BLS) with changes in the CPI for closely related strata of items. For example, between January 1980 and January 1995 the average price of bananas increased from $0.319 to $0.503 per pound, an increase of 58 percent. Over the same period the CPI for bananas increased 84 percent. The annualized difference is 1.05 percent per year. Reinsdorf found that for 48 of 52 comparisons between average prices and CPI

[1] ~~Two books that contain~~ excellent surveys of the literature are Pollak (1989) and Diewert and Nakamura (1993).

[2] Early papers on sampling and variance estimation for fixed-basket indexes were Bannerjee (1956) and McCarthy (1961). Leaver and Valliant (1994) describe the implementation in the U.S. CPI and survey recent work on variance estimation and related problems.

[3] See Forsyth (1978), Carruthers, Sellwood and Ward (1980), Forsyth and Fowler (1981), Morgan (1981), Szulc (1983, 1987), Turvey (1989), Dalén (1992), and Diewert (1994).

component series for food and gasoline, the CPI showed a larger increase with average differences of 1.5-2.0 percent per year.

Reinsdorf's (1993) initial hypothesis was that the difference could be explained by "a systematic tendency that is not reflected in the CPI for consumers to shift their retailer patronage patterns in ways that reduce the average prices they pay and hence their cost of living." It soon became clear that the growth of discounters by itself could not explain the full differential between average prices and the CPI food and gasoline components.[4] In early 1993 Reinsdorf suggested that another factor contributing to the average price-CPI differential may be bias resulting from the combination of a ratio index estimator combined with frequent sample replacement.[5] Subsequent research confirmed and quantified the magnitude of this effect, and has already resulted in changes in some BLS procedures.[6] The present paper extends the empirical research comparing the CPI estimator with possible alternatives.

The paper begins with a brief discussion of the population "modified Laspeyres" price index concept that the CPI is currently designed to estimate and discusses some of the problems with the current population concept. Section 3 contains a brief discussion of probability sampling and the estimator used for commodities and services. Section 4 discusses some of the estimator and imputation problems appearing for the shelter components of the CPI. Section 5 introduces two alternative formulas that have been suggested for elementary aggregates. Section 6 presents empirical comparisons of the alternative formulas for elementary aggregates, and Section 7 contains concluding comments and discussion of plans for further research.

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[4] According to Progressive Grocer the market share of economy stores grew from 9.1 percent in 1983 to 14.8 percent in 1991, for a growth in share of roughly 0.7 percent per year. If we assume that economy stores offer a 15 percent discount relative to traditional stores, then the bias from missing consumer switches to economy stores would be roughly 0.15×0.7 percent or 0.1 percent per year, which is substantially less than the 1.5-2.0 percent difference between the CPI and average prices. If the economy stores provide reduced retail services, the quality-adjusted difference may be even smaller.

[5] Reinsdorf (1994). An early version of Reinsdorf's paper was presented at Statistics Canada in April 1993.

[6] See Moulton (1993), Reinsdorf and Moulton (1995), Armknecht, Moulton, and Stewart (1995), and Richardson (1995).

2. Modified Laspeyres Index Concept

The Consumer Price Index has endeavored to maintain a consistent chain of measured price change while samples and market baskets are being updated and replaced. The chaining procedures are intended to avoid having to revise historical series. As mentioned previously, the estimation of the Consumer Price Index consists of two distinct stages, higher level aggregation and elementary aggregate estimation. The higher level aggregation has the following characteristics.

- Uses weights drawn from the Consumer Expenditure Survey,
- Uses a fixed market basket (modified Laspeyres) formula, where 1982-84 is currently the base period,
- Represents a universe, not a sample, of strata of areas and items,
- Updates weights approximately every 10 years.

Elementary aggregate estimation, on the other hand, is characterized by the following:

- weights and probability of selection are drawn from the Point-of-Purchase Survey (POPS) and sales at sample outlets,
- uses a modified Laspeyres formula where the POPS year is the base period,
- estimates using a *sample* of items, outlets, and areas,
- samples are replaced and chained together at five-year intervals (about 20 percent of price sample is replaced each year).

Before formally describing the modified Laspeyres index estimated by the CPI, we will first set out the simple Laspeyres index, as usually defined in textbook discussions of price indexes. Items will be indexed by the subscript j, and strata of items and areas (i.e., elementary aggregates) by i. The simple Laspeyres would measure the cost of purchasing in time T the same quantities Q_{ijB} of items that were purchased in base period B. The denominator is the sum of actual base period expenditures E_{ijB}. Denoting prices for the two periods as P_{ijB} and P_{ijT}, the index is

(1) $$I_T^{\text{Lasp}} = \frac{\sum_i \sum_j Q_{ijB} P_{ijT}}{\sum_i \sum_j Q_{ijB} P_{ijB}} = \frac{\sum_i \sum_j E_{ijB} P_{ijT} / P_{ijB}}{\sum_i \sum_j E_{ijB}},$$

The CPI's concept of a *modified* Laspeyres index needs to be defined separately for the two levels of aggregation. At the higher level of aggregation the market basket represents quantities purchased during a base period B which is prior to the pivot period P when the weights are updated. For example, the current market basket represents expenditures drawn from the Consumer Expenditure Survey during 1982-84, but that market basket was not used in index aggregation until the pivot month, December 1986.[7] The modified Laspeyres index aggregation formula can be written as

(2) $$I_T = \frac{\sum_i Q_{iB} I_{iT}}{\sum_i Q_{iB} I_{iP}} \times I_P,$$

where I_{iP} and I_{iT} are the elementary price indexes for pivot month P and period T for the item-area combination i. The Q_{iB}, also known as aggregation weights, do not represent measurable quantities in the ordinary sense. They can be thought of as implicit quantity indexes for the 1982-84 base period, and are calculated from base period expenditures, E_{iB}, divided by a base period index using the formula $Q_{iB} = E_{iB} / I_{iB}$. Thus, formula (2) can be rewritten in terms of actual expenditures and indexes as

(3) $$I_T = \frac{\sum_i E_{iB} I_{iT} / I_{iB}}{\sum_i E_{iB} I_{iP} / I_{iB}} \times I_P.$$

The population concept underlying the estimation of elementary aggregates was designed to appear very similar to (2) and (3), but there are also some important differences. Like (2), the

[7] The pivot month was November 1986 for those strata that are priced bimonthly during odd-numbered months.

modified Laspeyres concept for the elementary aggregates involves a base period, in this case the Point-of-Purchase Survey period for the stratum, A_i, a "link" month L_i (analogous to the pivot month) when the old and new samples are chained together, and prices for the individual items (or "quotes") j within the stratum, P_{ijT}.[8] The population concept for elementary aggregates is thus

$$I_{iT} = \frac{\sum_{\substack{j \in i \\ (\text{during } A_i)}} Q_{ijA_i} P_{ijT}}{\sum_{\substack{j \in i \\ (\text{during } A_i)}} Q_{ijA_i} P_{ijL_i}} \times I_{iL_i}. \tag{4}$$

Several observations can be made at this point about the CPI population concepts in (2) and (4). The modified-Laspeyres concept actually used by the CPI is not the same as the simple Laspeyres index at any level of aggregation. It is therefore incorrect to claim that the CPI, as a Laspeyres index, provides an upper bound to the true cost-of-living index, since the CPI is not a true Laspeyres index.[9] It is somewhat more accurate to say that the CPI measures the change in the cost of a fixed market basket of goods, though even this is not entirely accurate, as can be seen if (4) is substituted for I_{iT} in (2). It is a fixed weight index of many other fixed weight indexes, each with its own base period. It is sometimes said that the CPI is consistent in aggregation (i.e., the index constructed in two or more stages coincides with the value calculated in a single stage). Because of the mixture of various base periods and population definitions, the population CPI clearly depends on the definition of the item-area strata, however, and the value obtained from combining (4) and (2) is different than what would be obtained from applying (2) directly to the individual price quotes. Finally, the restriction in (4) that items exist in the population during the

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[8] The following discussion assumes all of the quotes within the stratum have the same base period. In actual practice, samples are usually replaced simultaneously for a PSU (i.e., a sample urban area), so the 12 index areas with multiple PSUs will have multiple base periods within any given sample. The three largest single-PSU index areas (New York City, Chicago, Los Angeles County), also replace just part of the sample during each sample rotation, and thus will also have multiple base periods within their samples. The CPI treats each sample item at each outlet as a separate quote.

[9] The bounding property of the Laspeyres index relative to the unobservable true cost-of-living index was derived by Konüs (1924), and is also shown in Pollak (1989) and Diewert and Nakamura (1993).

base period constrains the introduction of new items into the sample, though they are now brought into the CPI more rapidly than they were prior to the adoption of periodic sample rotation.[10]

The population concept for the elementary aggregates faces further difficulties when it is implemented using data on expenditures rather than quantities. By analogy to (3), equation (4) would be written in terms of base-period expenditures E_{jA} (suppressing the i subscripts) as

$$(5) \qquad I_{iT} = \frac{\sum_j E_{jA} P_{jT} / P_{jA}}{\sum_j E_{jA} P_{jL} / P_{jA}} \times I_{iL},$$

but now another problem arises. In contrast to (3), in which the index-maker has access to the base-period index I_{iB}, the CPI program generally does not have access to a price history for individual quotes before they are linked into the sample. Therefore the base price, P_{jA}, needs to be imputed somehow. The method used for most CPI items uses the link month price rebased by the index for the stratum, $P^*_{jA} = P_{jL} I_{iA} / I_{iL}$.[11] Applying this imputation to (5), and using a "*" to denote the index with the imputed base price, the population elementary aggregate simplifies to

$$(6) \qquad I^*_{iT} = \frac{\sum_j E_{jA} P_{jT} I_{iL} / P_{jL} I_{iA}}{\sum_j E_{jA} I_{iL} / I_{jA}} \times I_{iL}$$

$$= \frac{\sum_j E_{jA} P_{jT} / P_{jL}}{\sum_j E_{jA}} \times I_{iL},$$

for those index areas that have a only a single base period.

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[10] Some new products can enter the sample between rotations if an old product drops out of the sample and needs to be replaced. See Armknecht, Lane, and Stewart (1994). New construction is brought into the housing sample continuously, however, since the housing sample does not currently undergo periodic sample rotation.

[11] Apparel and food-at-home samples are not used in CPI calculations for several months following the setting of the base price in order to diminish the correlation between imputed base prices and measured price change and the consequent bias in measurement of price change. Apparel samples are held out for eight months, and beginning in January 1995 food-at-home samples are held out for three months. For further discussion see Fixler (1993) and Armknecht, Moulton, and Stewart (1995).

By contrast, if the full price history were available so that actual base prices could be used, (5) could be written as a weighted average of price relatives:

$$(7) \qquad I_{iT} = \frac{\sum_j C_j P_{jT}/P_{jL}}{\sum_j C_j} \times I_{iL},$$

where $C_j = E_{jA}P_{jL}/P_{jA}$ is the cost in period L of purchasing the same items that were purchased in base-period A. Comparing (6) and (7), we see that the CPI method for imputing base prices (6) results in too little weight being given to quotes with relatively high link-month prices (i.e., if $P_{jL}/P_{jA} > I_{iL}/I_{iA}$ then $E_{jA}/\Sigma E_{jA} < C_{jA}/\Sigma C_{jA}$). Similarly, too much weight is being given to quotes with relatively low link-month prices relative to the preferred population concept in (7). The net effect is an upward bias of the CPI population index using the link-month imputation method (6) relative to the desired index (7). The bias is especially pronounced immediately following sample rotation and for items with volatile prices, as has been documented by Moulton (1993) and Reinsdorf and Moulton (1995).

3. Probability Sampling and Estimation

The selection of samples for price indexes using modern methods of probability sampling was perhaps the most significant single recommendation of the Stigler commission in 1961, and was fully implemented for the CPI beginning with the 1978 revision.[12] The sampling approach required two new tools: the Point-of-Purchase Survey, a household survey which identifies the outlets at which consumers purchase specific categories of items and serves as a sampling frame for outlet selection, and *disaggregation*, a procedure by which the BLS field representative selects a specific sample item for inclusion in the sample and subsequent repricing. The outlet selection is

[12] McCarthy (1961), Layng (1978), Weber and Lambrecht (1979).

for outlet selection, and *disaggregation,* a procedure by which the BLS field representative selects a specific sample item for inclusion in the sample and subsequent repricing. The outlet selection is based on probability proportional to consumer expenditures, and the within-outlet disaggregation relies on probability proportional to item sales.[13]

The probability of inclusion in the sample for any particular outlet or item can be described by an indicator variable N_{jA}, which is a random variable whose only possible values are 0 and 1. Define $N_{jA} = 1$ if quote j is selected for the CPI sample and 0 if it is not.

Suppose the information were available to draw a sample from the population of all items with probability proportional to expenditures, i.e., $P(N_{jB} = 1) = E_{jB} / \Sigma E_{jB}$, (where we are assuming that expenditures are known) and estimate the true Laspeyres index as in equation (4). Then the expected value of the sample index for stratum i, $\hat{I}_{iT}^{\mathrm{Lasp}}$, from (4) would be

$$(8) \qquad E(\hat{I}_{iT}^{Lasp} / I_{iB} | I_{iB}) = \frac{E(\sum N_{jB} W_{jB} P_{jT} / P_{jB})}{\sum E_{jB}} = \frac{\sum P(N_{jB}) W_{jB} P_{jT} / P_{jB}}{\sum E_{jB}}$$

where W_{jB} are weights assigned to each sample quote. For this simple example, the sample weights that would produce an unbiased estimate of the population index are $W_{jB} = \Sigma E_{jB} / n$, where n is the number of quotes included in the sample. The actual sampling methods and weight calculations for the CPI are somewhat more complicated than this (see U.S. Department of Labor [1992] or Swanson [1994]), but the principle is the same—sampling items with probability proportional to base-period expenditures produces indexes that are unbiased measures of expenditure-weighted relative price changes.

Combining the CPI sampling and weighting methods with the usual method for setting base prices implies that CPI sample index is estimating the population concept in (6):

$$(9) \qquad E(\hat{I}_{iT}^{*} / I_{iL} | I_{iL}) = \frac{\sum P(N_{jB} = 1) W_{jB} P_{jT} / P_{jL}}{\sum E_{jB}} = \frac{\sum E_{jB} P_{jT} / P_{jL}}{\sum E_{jB}}.$$

[13] This statement assumes that a respondent at the outlet can provide information on sales of items or categories of items. When information on sales of items is unavailable from the respondent, alternative methods for items selection are used by the field representative, as described in U.S. Department of Labor (1992, p. 188).

The rent and owners' equivalent rent indexes use different sampling and pricing procedures from other commodities and services and face somewhat different estimation and imputation problems. The rent sample is priced at 6-month intervals, and better information is available about quantities consumed (i.e., number of housing units) than expenditures. Therefore an estimator is used that does not require information about base-period prices. The estimator involves calculating price relatives for each area i of the form:

(10) $$Rel_{i,T,T-6} = \frac{\sum W_j P_{jT}}{\sum W_j P_{j,T-6}}$$

where W_j is a weight calculated as the inverse of the probability of selecting sample housing unit j, and P_{jT} and $P_{j,T-6}$ are the rents for periods T and T-6. Before January 1995 the rent estimator also involved an analogously calculated 1-month relative which was based on a retrospective question in which respondents were asked what their rent was one month ago. The two relatives were then combined to calculate the new index using the following composite estimator:

(11) $$\hat{I}_{iT} = 0.65 \times Rel_{i,T,T-1} \times \hat{I}_{T-1} + 0.35 \times Rel_{i,T,T-6} \times \hat{I}_{T-6}$$

Research had shown that the one-month relatives are characterized by substantial respondent underreporting of rent change, and that the composite estimator exhibited highly variable sawtooth behavior over time.[14] Consequently, the estimator was changed in January 1995 to the following formula which involves only the six-month relative:

(12) $$\hat{I}_{iT} = (Rel_{i,T,T-6})^{1/6} \times \hat{I}_{T-1}$$

In the case of owners' equivalent rent (the concept that has been used for measuring homeowner costs in the CPI since 1983), another problem arises in that changes of the owners' implicit rents are not observable and must be imputed. Beginning in 1987, the CPI has matched

[14] Kosary, Branscome, and Sommers (1985), Armknecht, Moulton, and Stewart (1995), and Jacobson (1995).

each owner from a sample to a small set of renters, preferably residing in the same neighborhood (Lane and Sommers [1985]). For each owner j the relative rent change for the n_j matched renters $P_{kT}/P_{k,T-6}$ was then used to impute the change of the owner's implicit rent, m_{jT}. The imputation formula used from 1987-94 was a simple average of rent relatives, i.e., a Carli index formula:

(13) $$m_{jT}/m_{j,T-6} = \frac{1}{n_j} \sum_{k \in \text{Match } j} \frac{P_{kT}}{P_{k,T-6}}.$$

Because of upward bias associated with the Carli formula, discussed in the next section, the imputation formula was changed beginning in January 1995 to the Dutot formula:

(14) $$m_{jT}/m_{j,T-6} = \frac{\sum_k P_{kT}}{\sum_k P_{k,T-6}}.$$

5. Alternative Formulas for Elementary Aggregates

Recent research on alternative formulas for elementary aggregates has largely approached the problem under the assumption that accurate probability samples and weighting information are unavailable, and that use of specification pricing reduces the heterogeneity of the items priced. Thus this research conforms more to price index practices in countries other than the United States.[15]

Statistical agencies principally use one or more of the following three index formulas, which received particular attention in Diewert (1994). The following defines the formulas and briefly discusses their characteristics.

The Carli formula, as discussed in the previous section, is an equally weighted average of price relatives:

[15] See particularly Carruthers, Sellwood and Ward (1980), Morgan (1981), Szulc (1987), Turvey (1989), Dalén (1992), and Diewert (1994).

(15) $$CA_{T,0} = \frac{1}{n}\sum_{j}\frac{P_{jT}}{P_{j0}}.$$

The Dutot formula is a ratio of averages:

(16) $$DU_{T,0} = \frac{\sum_{j} P_{jT}}{\sum_{j} P_{j0}},$$

and the Jevons formula is a geometric mean of price relatives:

(17) $$JE_{T,0} = \prod_{j}(P_{jT}/P_{j0})^{1/n}.$$

The Carli index does not pass the time reversal test, which means that when Carli indexes are chained together and all prices return to their original levels, the chained index will generally not return to its initial value (Dalén [1992] and Diewert [1994]). In particular the index will be biased upward because of the inequality $CA_{T,0}CA_{0,T} \geq 1$. All of the researchers cited earlier have agreed that the Carli formula is definitely not recommended, especially when samples need to be replaced and chained. Reinsdorf and Moulton (1995) concluded that the estimator (including the link-month base price setting method) used by the U.S. CPI for most commodities and services other than shelter (see equations (6) and (9)) has the characteristics of the Carli formula. Also, as previously mentioned, owners' implicit rent used a chained Carli formula from 1987-94.

The Dutot formula satisfies the time reversal and other desirable index properties and is widely used by other countries that price pre-selected specifications (Diewert [1994]). If stratum consists of homogeneous items priced in standard units (e.g., bananas) then sampling the items with probability proportional to expenditures is probably quite close to a sample drawn with probability proportional to quantities. In other words, base-period prices are likely to have been reasonably similar from one outlet to another. In that case, the Dutot index is likely to be close to the conceptual modified Laspeyres index.

Many strata of items within the U.S. CPI, however, are quite heterogeneous. The stratum for televisions includes inexpensive $100 portable sets as well as expensive $3,900 big-screen models. The stratum for musical instruments may include pianos as well as guitars. In such strata of items, the Dutot will be dominated by the high-priced item. For example, suppose a television index sample consists of the two sets just mentioned, and suppose the inexpensive television set increased 40 percent from $100 to $140, but the expensive set remained at $3,900. Then the Dutot index would increase by just 1 percent (i.e., $DU = 4{,}040/4{,}000 = 1.01$). The large price change of the inexpensive item has little effect on the index. Conversely, a 40 percent price increase of the more expensive television set would cause the Dutot index to increase by 39 percent (i.e., $DU = 5{,}560/4{,}000 = 1.39$). It clearly seems undesirable to allow indexes for elementary aggregates to be dominated by the price movements of high priced items.

The Jevons formula also satisfies the time reversal property and other desirable index properties as shown in Diewert (1994). Property T15 of Diewert's paper suggests that the Jevons formula is appropriate if one wants the elementary aggregates to be a function of the relative price changes. Symmetric treatment of relative price changes has intuitive appeal when one is averaging very heterogeneous items within a single stratum. Pollak (1989, 1995), however, has questioned the desirability of requiring indexes to be functions of relative prices. Furthermore, use of the Jevons formula for elementary aggregates combined with the modified Laspeyres approach for higher level aggregation is inherently inconsistent in aggregation and does not permit estimation of a modified Laspeyres population index. Finally, use of the Jevons formula does not permit one to describe the index as measuring the change in price of a fixed basket of goods and services.

Diewert (1994) has suggested other approaches (unit values or Fisher indexes) that would require sample collection of quantity data that are not currently collected in the CPI surveys. Use of supermarket scanner data could make that type of quantity data available for some items. Pollak (1995) has emphasized that the elementary aggregates are highly dependent on commodity definition and classification, and that understanding outlet price differences requires a model of consumer heterogeneity and search. He argues that the economic theory of price indexes needs to be further developed to guide index makers on these problems.

In the following section we simulate price change measurement for the U.S. CPI between June 1992-December 1994 under each of the three elementary aggregate concepts. The "Carli" index will be a simulation of the U.S. CPI as actually calculated. The sample weights and pre-1995 method of setting base prices will be applied to the historical CPI price quotes using the standard CPI estimator, which as shown above is essentially a weighted Carli formula. Higher level aggregation for all three simulated indexes uses the standard modified Laspeyres aggregation. The simulated "Dutot" continues to use the sample weights, but sets all base prices equal to 1. The simulated Jevons calculates the elementary aggregates using a weighted geometric mean.

For shelter, the simulated indexes use the composite estimator that was applied by the CPI during this period. The 1 and 6-month relatives are calculated using the regular CPI estimator and sampling weights under both the "Carli" and "Dutot" simulations. (The estimator for 1 and 6-month relatives for shelter is a weighted ratio of averages, so it is a Dutot-like index formula.) The owners' implicit rent formula, however, is a Carli formula in the Carli simulation (as was used before 1995) and a Dutot formula in the Dutot simulation (as was adopted in January 1995). The Jevons formula uses a geometric mean formula for all of the relative calculations.

6. Empirical results

This section of the paper presents and compares the effects of alternative estimators at the elementary aggregate level in the calculation of consumer price indexes over a 30-month period form June 1992 through December 1994. For these comparisons, a single set of price data was created from the price data that were actually used in calculating the CPI. The data represent price change for approximately 96 percent of the items in the published CPI.[16] This set of data was used in three different price relative calculation programs which differ only by the one month price relative algorithm contained in each.

[16] The remaining items that are not represented by the numbers in the tables are items for which there are exceptional methods of calculating price change for the actual CPI and where it would be inappropriate to use the Dutot or Jevons. In most of these cases accurate weighting and base price information is available for estimating the modified Laspeyres.

First the one month price relative program for the simulated official index (Carli) series uses the same price relative formula as that which is used in the published CPI.[17] The program for creating the Carli series was then modified with the Jevons algorithm and then again with the Dutot algorithm in order to produce the basic component price relatives for each series. The three series of basic component price relatives were used to calculate elementary aggregate price indexes which were then aggregated using the same modified Laspeyres formula as that which is used in aggregating the actual CPI indexes.

Although not as clearly visible in the graph of one month price relatives (attachment 1), the graph of cumulative price relatives for the All Items aggregate (attachment 2) shows an expected pattern of behavior between these series where over time the Carli diverges upward from the Dutot and the Dutot from the Jevons relatives.

As shown in Tables 1 and 2, the differences between the Carli and the experimental indexes are in the same direction for five of the seven major groups. The similarities in the behavior of the experimental indexes may be, in part, attributable to the fact that both are weighted by expenditure, and neither uses imputed base prices.

In the last two of the seven major groups the two experimental indexes seem to behave oppositely with respect to the Carli. This seemingly inconsistent behavior may be due in some instances to the powerful effect on the Jevons price relative of prices near zero and in others, to the fact that high priced items will have a large effect on the Dutot price relative.

Across all of the published areas, both the Jevons and Dutot indexes increased less than did the Carli indexes for Owners' Equivalent Rent. Tables 3 and 4 list the cumulative percent changes for Owners' Equivalent Rent and the differences for each experimental series from the

[17] The program for creating the Carli series is not exactly the same as that which is used in actually producing the CPI. It uses the same price relative formula as the actual CPI production program but handles exceptional and imputed data differently than does the CPI production program. While it is not exactly the same program, the Carli test index program produces nearly the same basic component price relatives as the actual CPI production price relative program. This program was developed by Ken Stewart of the Division of Consumer Prices and Price Indexes.

Table 1. Comparison of percent change for Carli and Dutot from June 1992 to December 1994 for All items and 7 major groups

	Carli	Dutot	Difference
All available items	6.53%	5.60%	0.93%
Food and beverages	6.50%	5.49%	1.02%
Housing	5.59%	4.77%	0.82%
Apparel and upkeep	-0.15%	-2.38%	2.23%
Transportation	6.86%	6.28%	0.58%
Medical care (excludes health insurance)	12.96%	11.01%	1.95%
Entertainment	6.69%	8.22%	-1.53%
Other goods and services	11.35%	9.75%	1.60%

Table 2. Comparison of percent change for Carli and Jevons from June 1992 to December 1994 for All items and 7 major groups.

	Carli	Jevons	Difference
All available items	6.53%	5.32%	1.22%
Food and beverages	6.50%	4.67%	1.84%
Housing	5.59%	4.45%	1.14%
Apparel and upkeep	-0.15%	-2.66%	2.50%
Transportation	6.86%	6.25%	0.61%
Medical care (excludes health insurance)	12.96%	11.35%	1.61%
Entertainment	6.69%	5.37%	1.32%
Other goods and services	11.35%	11.52%	-0.17%

Carli. Though in most cases the differences from the Carli are larger for the Jevons series, the results are mixed.

Bilateral comparisons of the percent changes in detailed indexes calculated with the Carli and the two alternative estimators--Jevons and Dutot--are shown in attachments 3 and 4, respectively. Of the Jevons indexes in attachment 3, the fruits and vegetables aggregate index shows the largest difference from the corresponding Carli index. Women's and girls' apparel shows the second largest difference between Jevons and Carli indexes. This is in keeping with expectations for the behavior of the Carli formula over indexes for groups of items with volatile prices like fruits and vegetables and women's and girls apparel. For most aggregate indexes in attachment 3, with the notable exception of "Fuels and Other Household fuel commodities", the Jevons indexes increased less than did the Carli aggregate indexes.

Table 3. Cumulative percent change from June 1992 to December 1994 by area for Owners' Equivalent Rent - Carli and Jevons

	Carli	Jevons	Difference
Chicago Gary-Lake County, IL-IN-WI	9.86%	8.42%	1.44%
Los Angeles-Anaheim-Riverside, CA	1.63%	0.73%	0.89%
N.Y.-Northern N.J.-Long Island, NY-NJ-CT	8.40%	5.79%	2.62%
Philadelphia-Wilmington-Trenton, PA-NJ-DE-MD	4.55%	3.66%	0.89%
San Francisco-Oakland-San Jose, CA	6.46%	5.33%	1.13%
Baltimore, MD	5.35%	4.66%	0.69%
Cleveland-Akron-Lorain, OH	4.81%	3.66%	1.15%
Miami-Fort Lauderdale, FL	10.80%	9.62%	1.18%
St. Louis-East St. Louis, MO-IL	8.78%	7.98%	0.80%
Washington, DC-MD-VA	3.63%	3.48%	0.15%
Dallas-Fort Worth, TX	5.78%	5.10%	0.68%
Detroit-Ann Arbor, MI	7.96%	6.75%	1.21%
Houston-Galveston-Brazoria, TX	9.23%	8.86%	0.38%
Pittsburgh-Beaver Valley, PA	12.04%	8.53%	3.51%

Table 4. Cumulative percent change from June 1992 to December 1994 by area for Owners' Equivalent Rent - Carli and Dutot

	Carli	Dutot	Difference
Chicago Gary-Lake County, IL-IN-WI	9.86%	8.32%	1.54%
Los Angeles-Anaheim-Riverside, CA	1.63%	0.04%	1.58%
N.Y.-Northern N.J.-Long Island, NY-NJ-CT	8.40%	6.79%	1.61%
Philadelphia-Wilmington-Trenton, PA-NJ-DE-MD	4.55%	3.93%	0.62%
San Francisco-Oakland-San Jose, CA	6.46%	5.76%	0.71%
Baltimore, MD	5.35%	4.52%	0.83%
Cleveland-Akron-Lorain, OH	4.81%	4.26%	0.55%
Miami-Fort Lauderdale, FL	10.80%	9.98%	0.82%
St. Louis-East St. Louis, MO-IL	8.78%	8.25%	0.53%
Washington, DC-MD-VA	3.63%	3.53%	0.10%
Dallas-Fort Worth, TX	5.78%	5.21%	0.58%
Detroit-Ann Arbor, MI	7.96%	7.23%	0.73%
Houston-Galveston-Brazoria, TX	9.23%	7.78%	1.45%
Pittsburgh-Beaver Valley, PA	12.04%	10.18%	1.86%

For the Dutot indexes in attachment 4, the second largest difference from the corresponding Carli aggregate was for fruits and vegetables while the largest difference was for women's and girls' apparel. Because of the effect of the Dutot weighting scheme on indexes for items with volatile prices, these differences were expected to be positive and larger for aggregates across items with volatile prices. Overall, the differences between the Carli and Dutot indexes, while less than those between Jevons and Carli indexes, still show the Dutot indexes increasing at a substantially lesser rate than the Carli.[18]

7. Conclusions

The estimators used by BLS for most commodities and services and owners' equivalent rent have tended to share the undesirable properties of the Carli index, particularly overstatement of inflation after new sample quotes are linked into the index. The Dutot and Jevons formulas have been suggested as alternatives that do not share these undesirable linking properties, though neither formula represents a true modified Laspeyres index except under special circumstances. BLS has already taken actions to alleviate some of the functional form problems, as described by Armknecht, Moulton, and Stewart (1995), but many subindexes still exhibit Carli-like properties.

To address the remaining problems, two options might be considered. One would be to change the population concept from the fixed basket of the modified Laspeyres approach, and adopt an alternative target such as the Jevons. As pointed out earlier, the fixed-basket concept is already tenuous under the current system of sample rotations and mixing of base periods at different levels of aggregation. Another option would be to develop alternative methods of imputing base prices. The authors are currently conducting research on new methods for estimating base prices, for which, in contrast with the current methods, the errors in estimating would not be correlated with the price levels and movements of items that are entering the sample. In particular, sample average prices during the base period for items possessing specified characteristics might be used as imputed base prices for new items having the same characteristics.

[18] Marshall Reinsdorf has suggested that the Dutot index may have shown a higher growth rate than the geometric mean because it may be more affected by small sample bias than the geometric mean. In small samples with heterogeneous items, such as the television set example given in the previous section, the Dutot formula reduces the effective sample size to n=1, by giving most of the weight to a single quote. Richardson (1995) has also investigated small sample bias.

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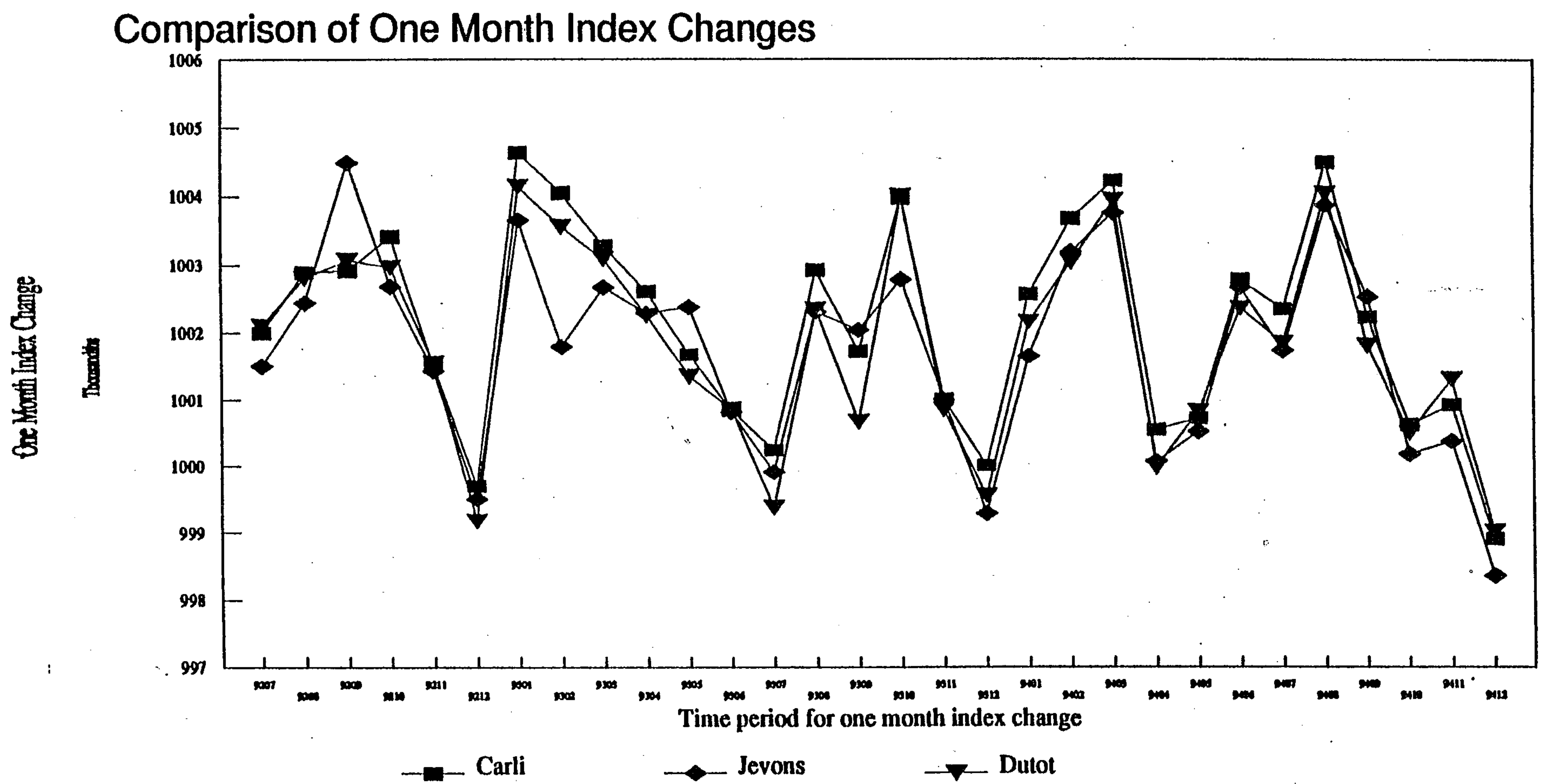
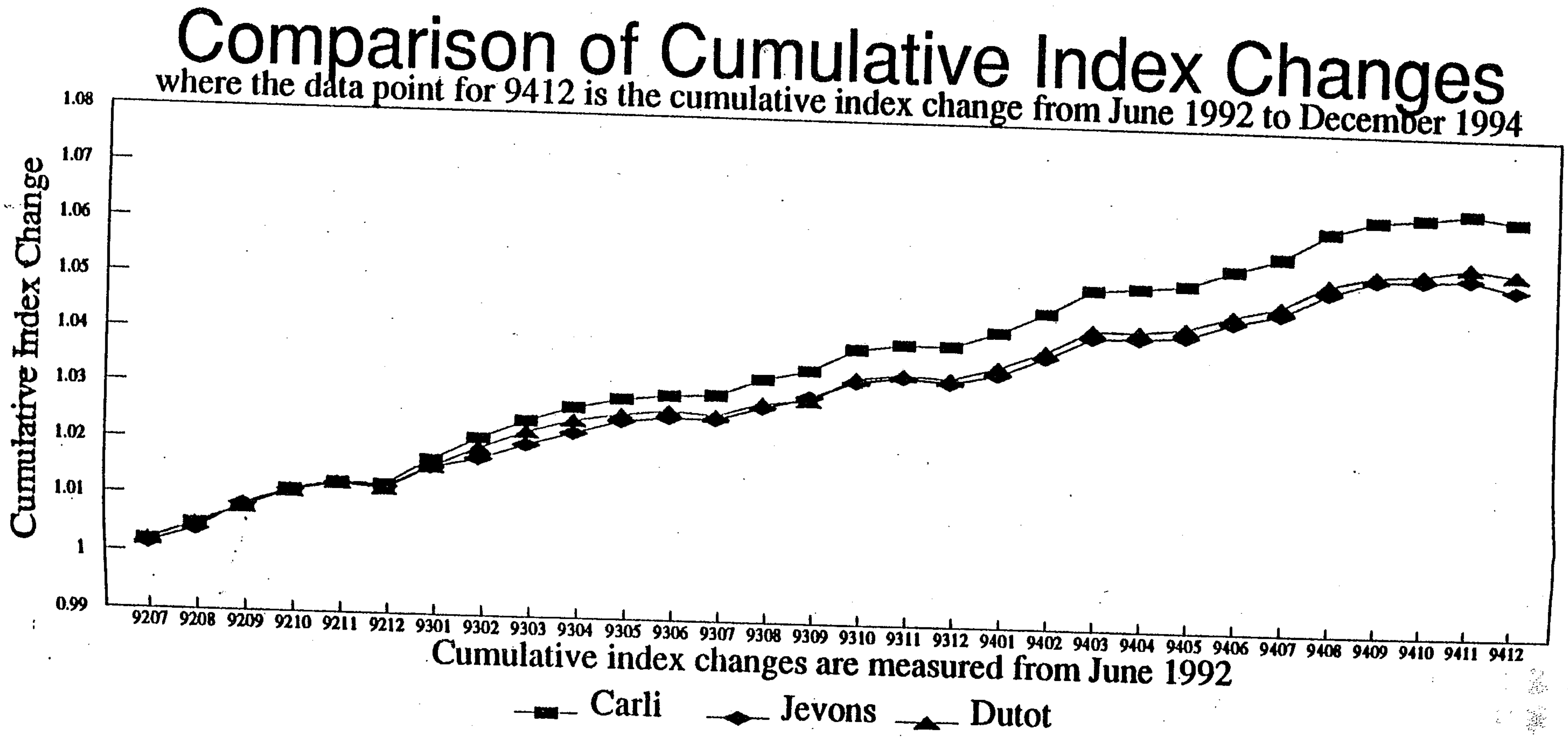
Attachment 1.



Attachment 2.



Attachment 3.
Cumulative percent change from June 1992 to December 1994 for Carli and Jevons Indexes

	Carli	Jevons	Difference
All available items	6.53%	5.32%	1.22%
Food and beverages	6.50%	4.67%	1.84%
Food	6.88%	4.87%	2.00%
Food at home	8.21%	5.30%	2.91%
Cereals and bakery products	8.26%	5.75%	2.51%
Meat, poultry, fish and eggs	4.83%	3.31%	1.52%
Dairy products	2.94%	1.86%	1.08%
Fruits and vegetables	18.10%	9.66%	8.44%
Fresh fruits and vegetables	28.18%	16.23%	11.94%
Processed fruits and vegetables	-0.33%	-2.36%	2.03%
Other food at home	7.54%	5.95%	1.59%
Sugar and sweets	0.94%	-0.17%	1.11%
Fats and oils	3.10%	3.12%	-0.02%
Nonalcoholic beverages	14.63%	13.26%	1.36%
Other prepared food	5.75%	3.44%	2.31%
Food away from home	4.59%	4.13%	0.45%
Alcoholic beverages	2.97%	2.69%	0.28%
Housing	5.59%	4.45%	1.14%
Shelter	7.12%	5.52%	1.60%
Owners' Equivalent Rent	8.20%	6.75%	1.45%
Maintenance and repairs	3.77%	5.90%	-2.12%
Fuels and other utilities	2.57%	3.33%	-0.76%
Fuels	-0.18%	0.74%	-0.91%
Fuel oil and other household fuel commodities	-1.78%	-1.62%	-0.16%
Gas (piped) and electricity (energy services)	-0.01%	0.98%	-0.99%
Other utilities and public services	6.05%	6.62%	-0.57%
Household furnishings and operation	2.01%	0.62%	1.39%
Housefurnishings	1.27%	-0.21%	1.48%
Housekeeping supplies	2.67%	0.54%	2.13%
Housekeeping services	4.40%	4.47%	-0.07%
Apparel and upkeep	-0.15%	-2.66%	2.50%
Apparel commodities	-0.72%	-3.45%	2.73%
Men's and boys' apparel	-1.07%	-1.76%	0.69%
Women's and girls' apparel	-1.50%	-5.08%	3.58%
Infants' and toddlers' apparel	3.29%	0.21%	3.08%
Footwear	-1.42%	-2.79%	1.37%
Other apparel commodities	3.06%	-2.85%	5.92%
Apparel services	5.27%	4.91%	0.36%
Transportation	6.86%	6.25%	0.61%
Private transportation	6.13%	5.62%	0.52%
New vehicles	8.57%	7.71%	0.86%

New cars	8.04%	7.42%	0.62%
Used cars (excluded)	-----	-----	-----
Motor fuel ,Motor Oil, Coolant, etc.	-2.46%	-2.37%	-0.08%
Maintenance and repairs	7.50%	7.05%	0.45%
Other private transportation	4.17%	3.88%	0.30%
Other private transportation commodities	-2%	-1%	-0.75%
Other private transportation services	12.64%	11.41%	1.23%
Automobile insurance	12.82%	11.53%	1.29%
Automobile finance charges (excluded)	-----	-----	
Automobile fees	10.44%	7.63%	2.81%
Public transportation	13.91%	12.39%	1.52%
Medical care (excludes health insurance)	12.96%	11.35%	1.61%
Medical care commodities	7.87%	7.03%	0.84%
Medical care services(excludes health ins)	14.19%	12.40%	1.79%
Professional medical services	11.63%	10.41%	1.22%
Hospital and related services (- h.ins)	18.39%	15.65%	2.73%
Entertainment	6.69%	5.37%	1.32%
Entertainment commodities	4.19%	2.64%	1.55%
Reading materials	8.55%	8.42%	0.13%
Sporting goods and equipment	1.05%	-1.96%	3.00%
Toys, hobbies, and other entertainment	1.50%	-0.80%	2.31%
Entertainment services	8.65%	7.50%	1.14%
Other goods and services	11.35%	11.52%	-0.17%

Attachment 4.

Cumulative percent change from June 1992 to December 1994 for Carli and Dutot Indexes

	Carli	Dutot	Difference
All available items	6.53%	5.60%	0.93%
Food and beverages	6.50%	5.49%	1.02%
Food	6.88%	5.77%	1.10%
Food at home	8.21%	6.52%	1.69%
Cereals and bakery products	8.26%	7.18%	1.08%
Meat, poultry, fish and eggs	4.83%	3.04%	1.79%
Dairy products	2.94%	1.92%	1.02%
Fruits and vegetables	18.10%	15.10%	3.00%
Fresh fruits and vegetables	28.18%	24.58%	3.60%
Processed fruits and vegetables	-0.33%	-2.24%	1.91%
Other food at home	7.54%	6.26%	1.27%
Sugar and sweets	0.94%	1.82%	-0.88%
Fats and oils	3.10%	1.44%	1.66%
Nonalcoholic beverages	14.63%	11.06%	3.57%
Other prepared food	5.75%	5.51%	0.24%
Food away from home	4.59%	4.48%	0.10%
Alcoholic beverages	2.97%	2.77%	0.20%
Housing	5.59%	4.77%	0.82%
Shelter	7.12%	6.12%	0.99%
Owners' Equivalent Rent	8.20%	6.92%	1.29%
Maintenance and repairs	3.77%	3.57%	0.21%
Fuels and other utilities	2.57%	2.25%	0.32%
Fuels	-0.18%	-0.04%	-0.13%
Fuel oil and other household fuel commodities	-1.78%	-1.46%	-0.31%
Gas (piped) and electricity (energy services)	-0.01%	0.11%	-0.12%
Other utilities and public services	6.05%	5.15%	0.89%
Household furnishings and operation	2.01%	1.38%	0.63%
Housefurnishings	1.27%	0.67%	0.60%
Housekeeping supplies	2.67%	1.32%	1.35%
Housekeeping services	4.40%	4.66%	-0.26%
Apparel and upkeep	-0.15%	-2.38%	2.23%
Apparel commodities	-0.72%	-3.33%	2.61%
Men's and boys' apparel	-1.07%	-1.66%	0.60%
Women's and girls' apparel	-1.50%	-6.24%	4.75%
Infants' and toddlers' apparel	3.29%	1.46%	1.83%
Footwear	-1.42%	-2.02%	0.60%
Other apparel commodities	3.06%	1.44%	1.63%
Apparel services	5.27%	6.74%	-1.47%
Transportation	6.86%	6.28%	0.58%
Private transportation	6.13%	5.59%	0.54%
New vehicles	8.57%	8.05%	0.52%

New cars	8.04%	7.42%	0.62%
Used cars (excluded)	-----	-----	-----
Motor fuel ,Motor Oil, Coolant, etc.	-2.46%	-2.37%	-0.08%
Maintenance and repairs	7.50%	7.44%	0.06%
Other private transportation	4.17%	3.51%	0.66%
Other private transportation commodities	-2%	-2%	-0.03%
Other private transportation services	12.64%	10.61%	2.03%
Automobile insurance	12.82%	10.70%	2.12%
Automobile finance charges (excluded)	-----	-----	
Automobile fees	10.44%	10.82%	-0.38%
Public transportation	13.91%	13.00%	0.91%
Medical care (excludes health ins.)	12.96%	11.01%	1.95%
Medical care commodities	7.87%	7.64%	0.23%
Medical care services(excludes health ins)	14.19%	11.82%	2.37%
Professional medical services	11.63%	8.88%	2.76%
Hospital and related services (- h.ins)	18.39%	16.65%	1.74%
Entertainment	6.69%	8.22%	-1.53%
Entertainment commodities	4.19%	4.14%	0.05%
Reading materials	8.55%	6.97%	1.58%
Sporting goods and equipment	1.05%	1.96%	-0.91%
Toys, hobbies, and other entertainment	1.50%	2.44%	-0.94%
Entertainment services	8.65%	11.41%	-2.77%
Other goods and services	11.35%	9.75%	1.60%