Unit Value Bias (Indices) Reconsidered

Price- and Unit-Value-Indices in Germany

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*This paper represents the author's personal opinion and does not necessarily reflect the view of the Deutsche Bundesbank or its staff.

1. **Introduction and Motivation**
2. **Unit value index (UVI) and Drobisch's Index (P^{UD})**
3. **Price and unit value indices in German foreign trade statistics** (Tests of hypotheses)
4. **Properties and axioms (uv, UVI, P^{UD})**
5. **Decomposition of the Unit Value Bias**
   (PU^{P}/P^{L} L- and S-effect)
6. **Interpretation of the S-effect in terms of covariances**
   (using a generalized theorem of Bortkiewicz)
7. **Conclusions**

- **Export and Import Price Index Manual (XMPI Man. IMF, 2008)**

- **Unit Value Indices (UVIs) are used in**

  Prices of *trade* (export/import), *land*, *air freight* and certain *services* (consultancy, lawyers etc)

- **Literature (UVIs cannot replace price indices)**

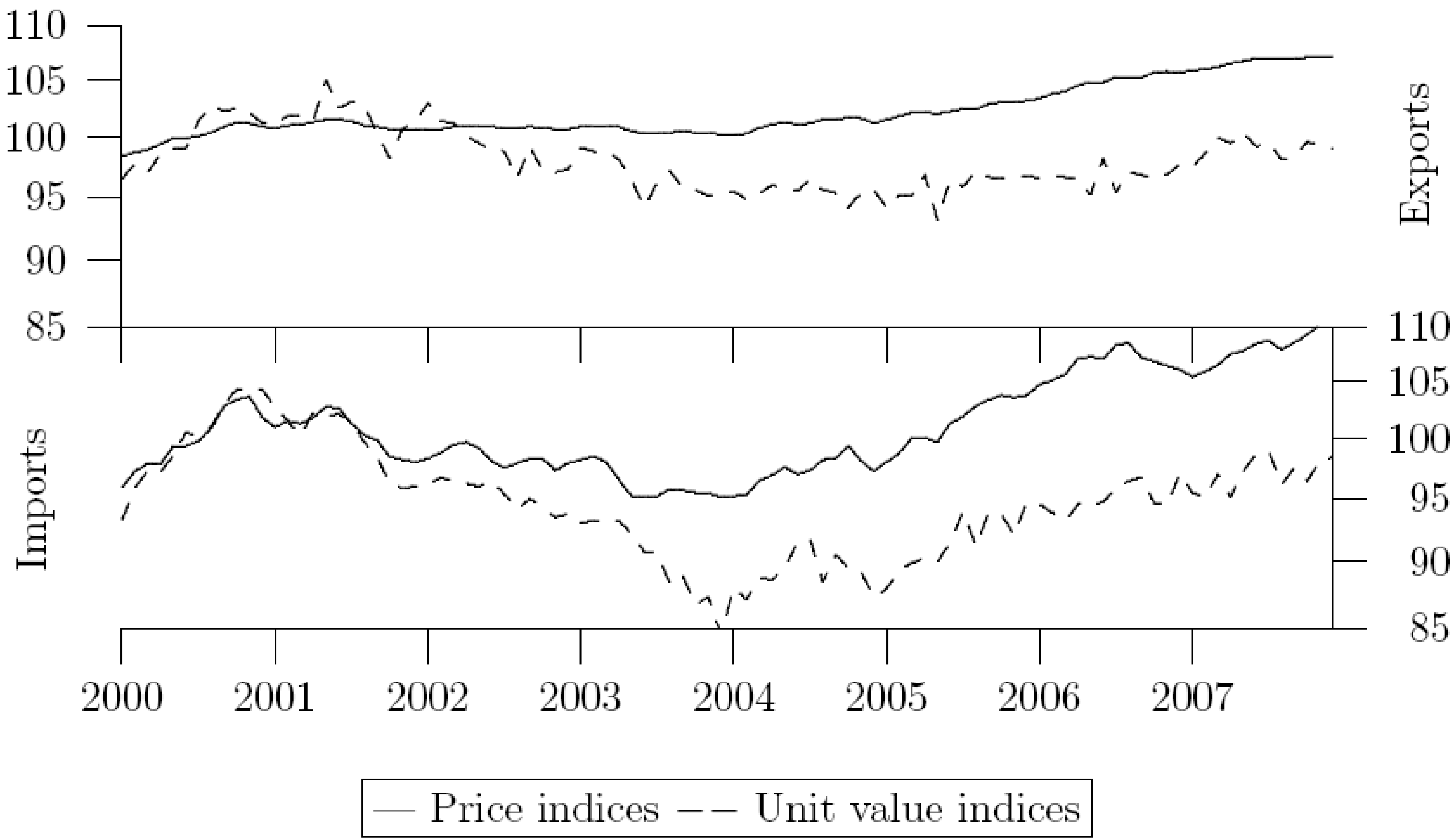
  Balk 1994, 1995 (1998), 2005

  Diewert 1995 (NBER paper), 2004 etc.

  von der Lippe 2006 GER
  http://mpra.ub.uni-muenchen.de/5525/1/MPRA _paper_5525.pdf

  Silver (2007), Do Unit Value Export, Import, and Terms of Trade Indices Represent or Misrepresent Price Indices, IMF Working Paper WP/07/121

2000 Jan – 2007 Dec



1. Unit value for the k^th commodity number (CN)

$$\tilde{p}_{k0} = \frac{\sum p_{kj0} q_{kj0}}{\sum q_{kj0}} = \sum_j p_{kj0} \frac{q_{kj0}}{Q_{k0}} = \sum_j p_{kj0} m_{kj0}$$

k = 1, …, K **Unit values are not defined over all CNs**

Examples for CNs

HS (Harmonized System)	Germany (Warenverzeichnis)
19 05 90 Other Bakers' Wares, Communion Wafers, Empty Capsules, Sealing Wafers	**19 05 90 45** Cakes and similar small baker's wares **(8 digits)**
23 09 10 Dog or Cat Food, Put up for Retail Sale	**23 09 10 11 to 23 09 10 90** twelve (!!) CNs for dog or cat food

2. German Unit Value Index (UVI) of exports/imports

the usual Paasche index (unit values instead of prices)

$$PU_{0t}^{P} = \frac{\sum_{k} \tilde{p}_{kt} Q_{kt}}{\sum_{k} \tilde{p}_{k0} Q_{kt}} = \frac{\sum_{k}^{K} \sum_{j}^{m_k} p_{kjt} q_{kjt}}{\sum_{k}^{K} Q_{kt} \left(\sum_{j}^{m_k} \frac{p_{kj0} q_{kj0}}{Q_{k0}} \right)}$$

Aggregation in two stages; $k = 1, \ldots, K$, $j = 1, \ldots, n_K$ commodities in the k^{th} CN; $\Sigma n_k = n$ (all commodities)

3. The Unit value index (UVI) should be kept distinct from Drobisch's index (1871)

$$P_{0t}^{DR} = \frac{\sum_{k} \sum_{j} p_{jkt}\, q_{jkt} \Big/ \sum_{k} \sum_{j} q_{jkt}}{\sum_{k} \sum_{j} p_{jk0}\, q_{jk0} \Big/ \sum_{k} \sum_{j} q_{jk0}} = \frac{\sum_{k} \sum_{j} p_{jkt}\, q_{jkt} \Big/ \sum_{k} Q_{kt}}{\sum_{k} \sum_{j} p_{jk0}\, q_{jk0} \Big/ \sum_{k} Q_{k0}}$$

Drobisch's index

$$P_{0t}^{DR} = \frac{\tilde{p}_t}{\tilde{p}_0} = \frac{V_{0t}}{\tilde{Q}_{0t}}, \quad \tilde{Q}_{0t} = \frac{Q_t}{Q_0}$$

However, Drobisch is better known for $\frac{1}{2}\left(P_{0t}^{L} + P_{0t}^{P}\right)$

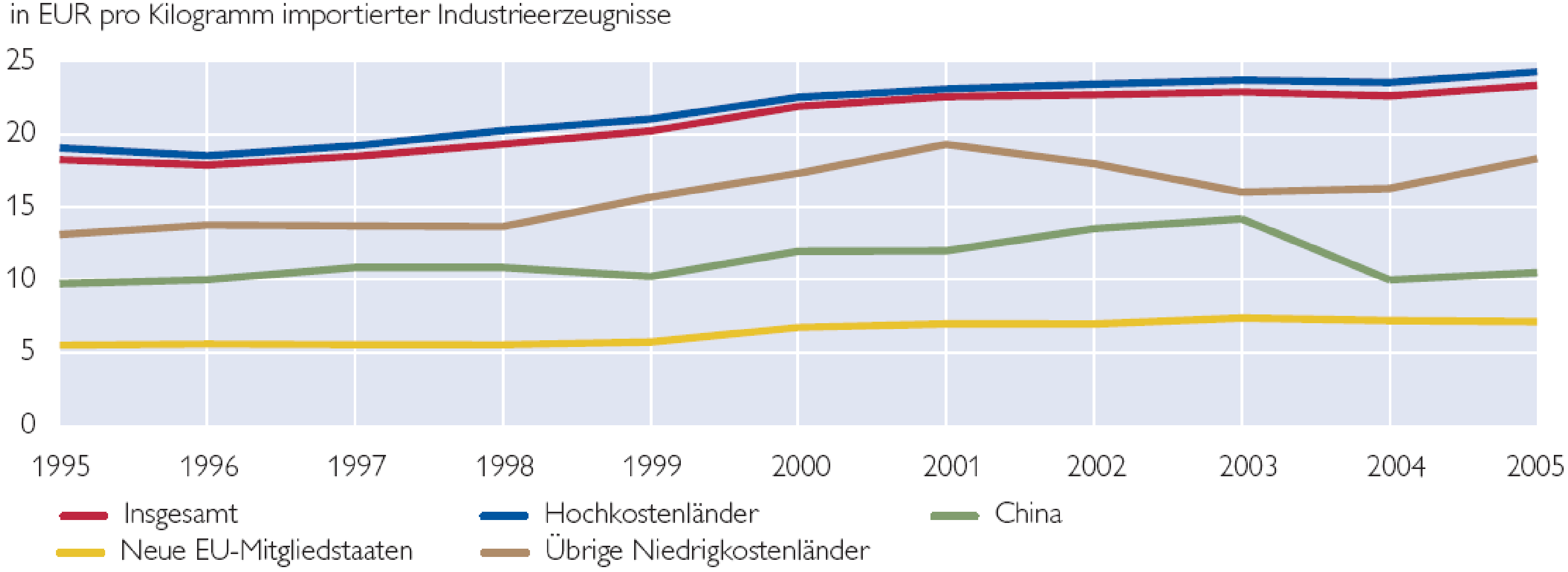
	no information about quantities available	information about **quantities**
the same commodity in different outlets	"normal" usage of the term "low level"	
different goods grouped by a classification		situation of a UVI (Σq needed for unit value)

It does not make sense to consider ***absolute*** *unit values ("Euro per kilogram")*

Austrian Import prices rose from ≈ 20 **€ per kilogram** in 1995 to 25 € … in 2005

Grafik 5

Österreichische Importpreisindizes in der Sachgütererzeugung



Quelle: Eurostat (Comext), OeNB.

Glatzer et al "Globalisierung…" http://www.oenb.at/de/img/gewi_2006_3_tcm14-46922.pdf

"Because we use weights as units an increasing import price index could be explained by either rising prices or reduced weights due to quality improvement"

$2^4 = 16$ indices:

type of index (price vs quantity)

Prices (p) vs unit values (uv)

Laspeyres vs Paasche

Export vs import

$$V = \Sigma p_t q_t / \Sigma p_0 q_0 = P^P Q^L = PU^P \; QU^L$$

	Price-indices		Quantity-indices	
	p	**uv**	**p**	**uv**
Laspeyres	P^L	PU^L	Q^L	QU^L
Paasche	P^P	PU^P	Q^P	QU^P

	Price index	Unit value index
Data	**Survey based** (monthly), **sample**; more demanding (weights!)	**Customs based** (by-product), **census**, Intrastat: survey
Formula	**Laspeyres**	**Paasche**
Quality ad-justment	**Yes**	**No** (feasible?)
Prices, aggregates	Prices of specific goods at time of **contracting**	Average value of CNs; time of **crossing border**
New / dis-appearing goods	Included only when a new base period is defined; vanishing goods replaced by *similar* ones **constant selection of goods** *	Immediately included; price quotation of disappearing goods is simply discontinued **variable universe of goods**
Merits	Reflect **pure price** movement (ideally the same products over time)	"**Representativity**" inclusion of *all* products; **data** readily **available**
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CN = commodity numbers

* All price determining characteristics kept constant

Price index (P) Unit value index (U)

Hypothesis	Argument
1. **U < P**, growing **discrepancy**	Laspeyres (P) > Paasche (U) Formula of L. v. Bortkiewicz
2. **Volatility** U > P	U no pure price comparison (U is reflecting changes in product mix [structural changes])
3. **Seasonality** U > P	U no adjustment for seasonally non-availability
4. **U** suffers from **heterogeneity**	Variable vs. constant selection of goods, CN less homogeneous than specific goods
5. **Lead** of P	Prices refer to the earlier moment of contracting (contract-delivery lag; exchange rates)
6. **Smoothing** (due to quality adjustment)	Quality adjustment in P results in smoother series

$p_{10} = p_{1t} = p$
$p_{20} = p_{2t} = \lambda p$
$\mu = m_{2t}/0.5$
$m_{10} = m_{20} = 0.5$

$$\boxed{\Delta = \tilde{p}_{kt} - \tilde{p}_{k0} = \frac{p}{2}(1-\lambda)(1-\mu)}$$

$\lambda > 1$	$\lambda > 1$ and $\mu < 1 \rightarrow \quad \Delta < 0$ less of the more expensive good 2 unit value ***declining***	$\lambda > 1$ and $\mu > 1 \rightarrow \quad \Delta > 0$ **more of the** more **expensive** good 2 unit value ***rising***
$\lambda < 1$	$\lambda < 1$ and $\mu < 1 \rightarrow \Delta > 0$ **less** of the **cheaper** good 2 unit value ***rising***	$\lambda < 1$ and $\mu > 1 \rightarrow \Delta < 0$ more of the cheaper good 2 unit value ***declining***
	$\mu < \mathbf{1}$	$\mu > \mathbf{1}$

"… 'unit value' indices … may therefore be affected by changes in the mix of items as well as by changes in their prices. Unit value indices cannot therefore be expected to provide good measures of average price change over time" (**SNA** 93, § 16.13)

1) UVI mean of uv-ratios

$$\boxed{PU_{0t}^{P} = \sum_{k} \frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} \frac{\tilde{p}_{k0} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}}}$$

2) Ratio of unit values ≠ **mean** of price relatives

$$\boxed{\frac{\tilde{p}_{kt}}{\tilde{p}_{k0}} = \sum_j \frac{p_{kjt}}{p_{kj0}} \left(\frac{p_{kj0} q_{kjt}}{\tilde{p}_{k0} Q_{kt}} \right)}$$

the weights do not add up to unity, but to $\frac{Q_{k0}}{Q_{kt}} \cdot Q_{0t}^{L(k)} = \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^{k}}$

Contribution of k to S-effect

3) Proportionality (identity)

Axioms Drobisch's (price) index and the German UVI (= PU^P)

Axiom		Drobisch*	German PU^P
Proportionality	$U(\mathbf{p}_0, \lambda\mathbf{p}_0, \mathbf{q}_0, \mathbf{q}_t) = \lambda$ (identity = 1)	no	no
Commensurability	$U(\Lambda\mathbf{p}_0, \Lambda\mathbf{p}_t, \Lambda^{-1}\mathbf{q}_0, \Lambda^{-1}\mathbf{q}_t) = U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)$	no	no
Linear homogen.	$U(\mathbf{p}_0, \lambda\mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t) = \lambda\, U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)$	yes	yes
Additivity** (in current period prices)	$U(\mathbf{p}_0, \mathbf{p}^*_t, \mathbf{q}_0, \mathbf{q}_t) = U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t) + U(\mathbf{p}_0, \mathbf{p}^+_t, \mathbf{q}_0, \mathbf{q}_t)$ for $\mathbf{p}^*_t = \mathbf{p}_t + \mathbf{p}^+_t$,	yes	yes
Additivity** (in base period prices)	$[U(\mathbf{p}^*_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)]^{-1} = [U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)]^{-1} + [U(\mathbf{p}^+_0, \mathbf{p}^+_t, \mathbf{q}_0, \mathbf{q}_t)]^{-1}$ for $\mathbf{p}^*_0 = \mathbf{p}_0 + \mathbf{p}^+_0$	yes	yes
Product test	Implicit quantity index of P^{UD} or PU^P	$\Sigma q_t/\Sigma q_0$	QU^L
Time re-versibility	$U(\mathbf{p}_t, \mathbf{p}_0, \mathbf{q}_t, \mathbf{q}_0) = U^{\leftarrow} = [U(\mathbf{p}_0, \mathbf{p}_t, \mathbf{q}_0, \mathbf{q}_t)]^{-1} = [U^{\rightarrow}]^{-1}$	yes	$(PU^{P\leftarrow}) = 1/(PU^{L\rightarrow})$
Transitivity	$U(\mathbf{p}_0, \mathbf{p}_2, \mathbf{q}_0, \mathbf{q}_2) = U(\mathbf{p}_0, \mathbf{p}_1, \mathbf{q}_0, \mathbf{q}_1) \cdot U(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}_1, \mathbf{q}_2)$	yes	no

* Balk1995, Silver 2007, IMF Manual; applies also to subindex $\tilde{p}_{kt}/\tilde{p}_{k0}$

** Inclusive of (strict) monotonicity

Value index $V_{0t} = PU^L_{0t} QU^P_{0t} = PU^P_{0t} QU^L_{0t}$

Bortkiewicz Formula

$$C = \sum_i \left(\frac{p_{it}}{p_{i0}} - P^L_{0t} \right) \left(\frac{q_{it}}{q_{i0}} - Q^L_{0t} \right) \frac{p_{i0} q_{i0}}{\sum p_{i0} q_{i0}}$$

$$= V_{0t} - Q^L_{0t} P^L_{0t} = Q^L_{0t} \left(P^P_{0t} - P^L_{0t} \right)$$

Discrepancy (uv-bias)

$$\boxed{D = \frac{PU^P_{0t}}{P^L_{0t}} = \left(\frac{C}{Q^L_{0t} P^L_{0t}} + 1 \right) \left(\frac{Q^L_{0t}}{QU^L_{0t}} \right) = \frac{P^P_{0t}}{P^L_{0t}} \cdot \frac{PU^P_{0t}}{P^P_{0t}} = L \cdot S}$$

Ladislaus von Bortkiewicz (1923)

$$L = \frac{Q^P_{0t}}{Q^L_{0t}} = \frac{Q^P_{0t}}{S \cdot QU^L_{0t}} = \frac{PU^P_{0t}}{S \cdot P^L_{0t}} \qquad S = \frac{Q^L_{0t}}{QU^L_{0t}} = \frac{Q^P_{0t}}{L \cdot QU^L_{0t}} = \frac{PU^P_{0t}}{L \cdot P^L_{0t}}$$

S > 1

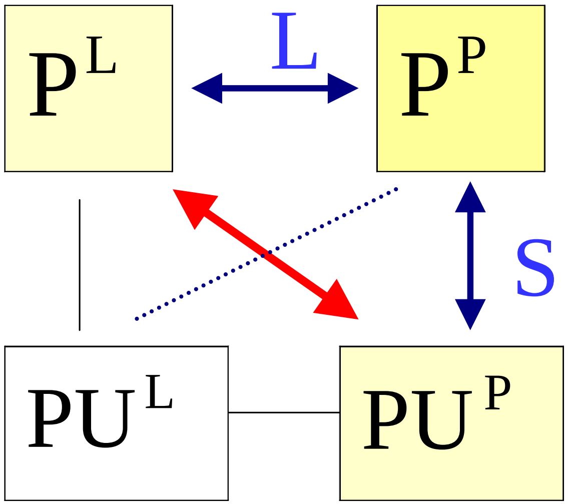
L < 1 | L > 1

II opposite direction
D indeterminate

Quadrant I same
Direction D > 1

III same direction
D < 1

IV opposite direction
D indeterminate

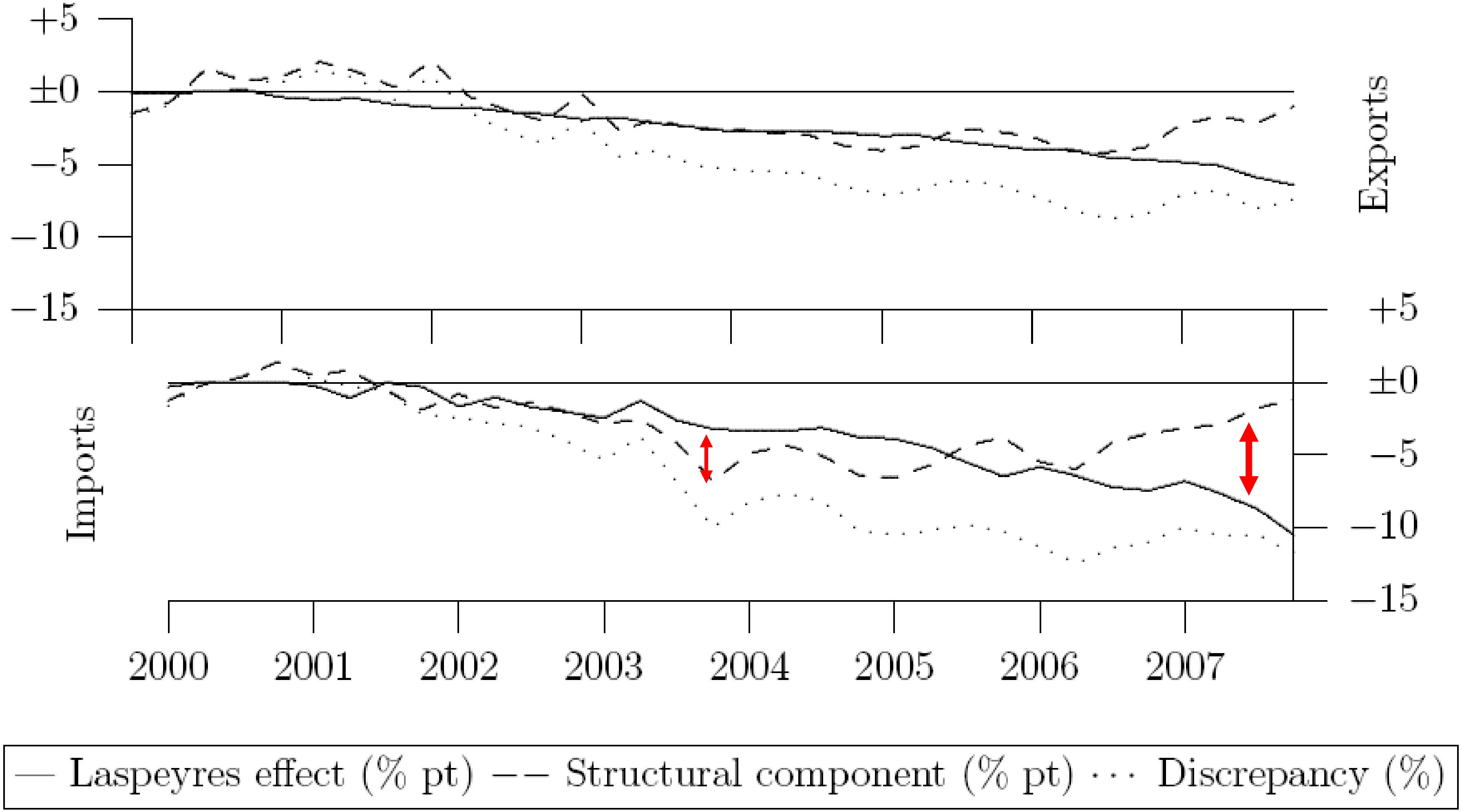


In I and III we can combine two inequalities

	S < 1	**S = 1**	**S > 1**
L < 1	$PU^P < P^P < P^L$	$PU^P < P^L$	indefinite
L = 1	$PU^P < P^L = P^P$	$PU^P = P^P = P^L$	$PU^P > P^L = P^P$
L > 1	indefinite	$PU^P > P^L$	$PU^P > P^P > P^L$

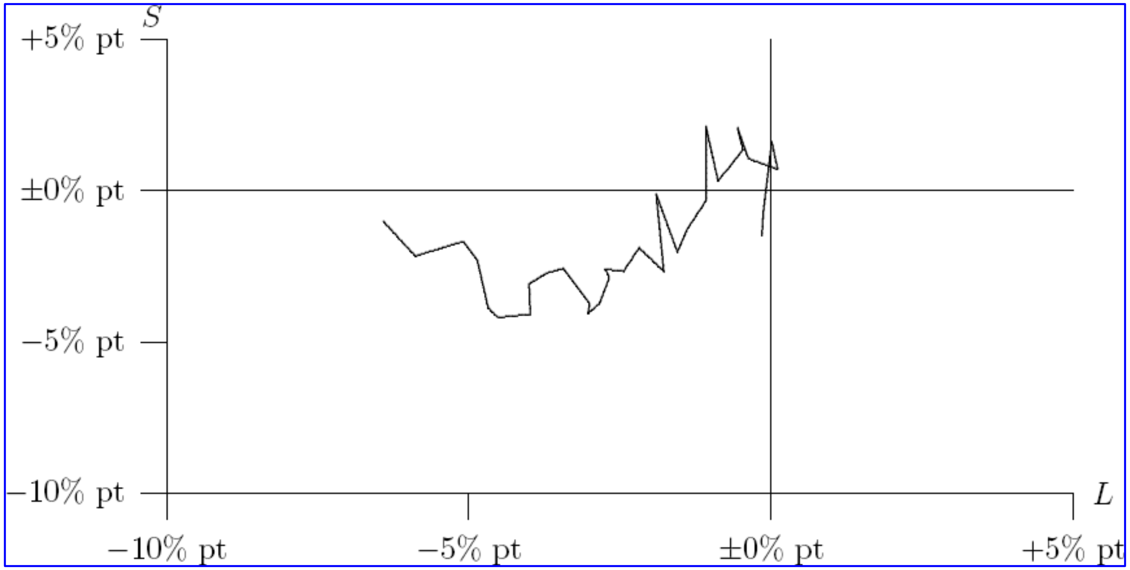
Deflator X and M respectively taken for P^P

S and L independent ?

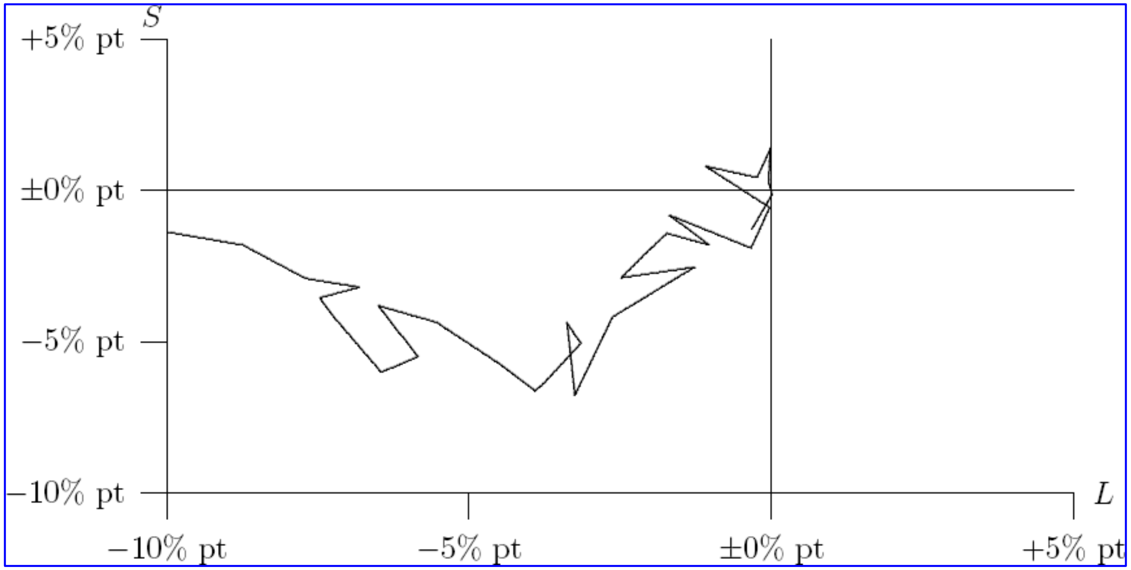


Normal reaction:
L and S negative
more likely in the case
of imports

exports



imports



Interpretation L-Effect: **contributions to the covariance** (Szulc)

$$R = \frac{P_P - P_L}{P_L} = \sum_i \left[\left(\frac{p_i^1 / p_i^0 - P_L}{P_L} \right) \cdot \left(\frac{q_i^1 / q_i^0 - Q_L}{Q_L} \right) \cdot \left(\frac{p_i^0 q_i^0}{\sum p_i^0 q_i^0} \right) \right]$$

R a "centred" covariance $\frac{s_{XY}}{\overline{X} \cdot \overline{Y}}$ $\qquad L = R + 1$

A. **Chaffe**, M. **Lequain**, G. **O'Donnell**, Assessing the Reliability of the CPI Basket Update in Canada Using the Bortkiewicz Decomposition, Statistics Canada, September 2007

No L-effect ($L = 1$) if

1. all p^1/p^0 equal (P_L) or $= 1$
2. all $q^1/q^0 = Q_L$ or $= 1$
3. covariance $= 0$

No S-effect ($S = Q^L/QU^L = 1$) if

1. no CNs, only individual goods (or: each $n_k = 1$, perfectly homogeneous CNs)
2. all q^1/q^0 equal (or $= 1$) **3.** all $m_{kjt} = m_{kj0}$ $\forall j, k$ **4.** all prices in 0 equal $p_{kj0} = \tilde{p}_{k0}$

prices in t are irrelevant

1. $$S = \frac{Q_{0t}^{L}}{QU_{0t}^{L}} = \sum_k \frac{Q_{0t}^{L(k)}}{\tilde{Q}_{0t}^{k}} \cdot \frac{\tilde{p}_{k0} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}}$$

2. Generalized theorem of Bortkiewicz

for two linear indices X_t and X_0

$$X_t = \frac{\sum x_t y_t}{\sum x_0 y_t} \qquad X_0 = \frac{\sum x_t y_0}{\sum x_0 y_0}$$

$$\frac{X_t}{X_0} = 1 + \frac{s_{xy}}{\overline{X} \cdot \overline{Y}}$$

$$w_0 = x_0 y_0 \Big/ \sum x_0 y_0$$

$$\sum \frac{x_t}{x_0} w_0 = \overline{X} = X_0$$

$$s_{xy} = \sum \left(\frac{x_t}{x_0} - \overline{X} \right) \left(\frac{y_t}{y_0} - \overline{Y} \right) w_0 = \frac{\sum x_t y_t}{\sum x_0 y_0} - \overline{X} \cdot \overline{Y}$$

The "usual" theorem (page 15) is a special case $\rightarrow$

Theorem for the L-effect

$$\frac{X_t}{X_0} = 1 + \frac{s_{xy}}{\overline{X} \cdot \overline{Y}}$$

$x_0 = p_0$	$y_0 = q_0$	$X_t = P^P$	$C = \sum_i \left(\frac{p_{it}}{p_{i0}} - P^L_{0t} \right) \left(\frac{q_{it}}{q_{i0}} - Q^L_{0t} \right) \frac{p_{i0} q_{i0}}{\sum p_{i0} q_{i0}}$
$x_t = p_t$	$y_t = q_t$	$X_0 = P^L$	

1. for S

$$S = Q^L_{0t} / QU^L_{0t}$$

$x_0 = q_0$	$y_0 = 1$	$X_t = Q^{L(k)}_{0t}$	$\sum \left(\frac{q_{kjt}}{q_{kj0}} - \tilde{Q}^k_{0t} \right) \left(p_{kj0} - \tilde{p}_{k0} \right) \frac{q_{kj0}}{\sum q_{kj0}}$
$x_t = q_t$	$y_t = p_0$	$X_0 = \tilde{Q}^k_{0t}$	

2. for 1/S

$x_0 = q_0$	$y_0 = p_0$	$X_t = \tilde{Q}^k_{0t}$	$\sum \left(\frac{q_{kjt}}{q_{kj0}} - Q^{L(k)}_{0t} \right) \left(\frac{1}{p_{kj0}} - \frac{1}{\tilde{p}_{k0}} \right) \frac{p_{kj0} q_{kj0}}{\sum p_{kj0} q_{kj0}}$
$x_t = q_t$	$y_t = 1$	$X_0 = Q^{L(k)}_{0t}$	

6. Two commodities example with both, S and L effect (example of page 12)

$p_{10} = p_{1t} = p$
$p_{20} = p_{2t} = \lambda p$
$\mu = m_{2t}/0.5$
$m_{10} = m_{20} = 0.5$

$\pi = p_{1t}/p_{10}$
$p_{2t}/p_{20} = \eta\pi$

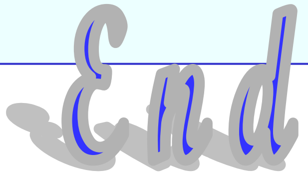
$\lambda > 1$	$\Delta < 0 \rightarrow S < 1$	$\Delta > 0 \rightarrow S > 1$
$\lambda < 1$	$\Delta > 0 \rightarrow S > 1$	$\Delta < 0 \rightarrow S < 1$
	$\mu < 1$	**$\mu > 1$**

S-effect	**L-effect**	$\pi = \eta = 1$
$S = \frac{Q_{0t}^L}{\tilde{Q}_{0t}} = 1 + \frac{(1-\lambda)(1-\mu)}{1+\lambda} = 1 + \frac{\Delta}{\tilde{p}_0}$	$P_{0t}^L = \frac{\pi(1+\eta\lambda)}{1+\lambda}$	$= 1$
$s_{xy}^{(1)} = \sum_j \left(\frac{q_{jt}}{q_{j0}} - \tilde{Q}_{0t} \right)(p_{j0} - \tilde{p}_0) \frac{q_{j0}}{\sum q_{j0}} = \tilde{Q}_{0t}\Delta$	$P_{0t}^P = \frac{\pi(2-\mu+\eta\lambda\mu)}{2-\mu+\lambda\mu}$	$= 1$
$s_{xy}^{(2)} = \frac{2\tilde{Q}_{0t}(\lambda-1)(1-\mu)}{p(1+\lambda)^2} = -\frac{\Delta}{(\tilde{p}_0)^2}$	$L = \frac{P_{0t}^P}{P_{0t}^L} == \frac{2-\mu+\eta\lambda\mu}{1+\eta\lambda} \cdot \frac{1+\lambda}{2-\mu+\lambda\mu}$	

	if $\pi = \eta = 1$
$\Delta^* = \tilde{p}_t - \tilde{p}_0 = \frac{p}{2}\left[\pi\left(2-\mu\left(1-\eta\lambda\right)\right)-\left(1+\lambda\right))\right]$	$\Delta^* = \Delta$
$C = s_{xy}^{(L)} = \frac{2\tilde{Q}_{0t}\lambda(1-\eta)(1-\mu)}{\left(1+\lambda\right)^2}$	$C = 0$
$\Delta^* = \tilde{p}_t - \tilde{p}_0 = \pi \frac{s_{xy}^1}{\tilde{Q}_{0t}} + \frac{s_{xy}^L (1+\lambda)^2}{2\tilde{Q}_{0t}} + \pi\left(1-\lambda\eta\right)-\left(1-\lambda\right)$	

7. What remains to be done

- Analysis of the time series of UVIs and PIs on various levels of disaggregation, cointegration and Granger-Causality
- Microeconomic interpretation of S-effect (in terms of utility maximizing behaviour)



No structural change **between** CNs (that is $Q_{k0} = Q_{kt}$) yields

$$V_{0t} = PU_{0t}^{P} = PU_{0t}^{L} \text{ and } QU_{0t}^{L} = QU_{0t}^{P} = 1$$

This is, however, not sufficient for the S-effect to vanish

$$S = Q_{0t}^{L} \neq 1$$

No mean value property of PUP

$$PU^{P} = \sum_k \sum_j \frac{p_{kjt}}{p_{kj0}} \left(\frac{p_{kj0} q_{kjt}}{\sum \sum p_{kj0} q_{kjt}^{*}} \right)$$

$$P^{P} = \sum_k \sum_j \frac{p_{kjt}}{p_{kj0}} \left(\frac{p_{kj0} q_{kjt}}{\sum \sum p_{kj0} q_{kjt}} \right)$$

$$q_{kjt}^{*} = q_{k_{j0}} \frac{Q_{kt}}{Q_{k0}}$$

a fictitious quantity in t

The same applies to Laspeyres

$$PU^{L} = \sum_k \sum_j \frac{p_{kjt}}{p_{kj0}} \frac{p_{kj0} \left(q_{kjt} \dfrac{Q_{k0}}{Q_{kt}} \right)}{\sum \sum p_{kj0} q_{k_{j0}}}$$

The relation $S = PU^P/P^P$ instead of $S = Q^L/QU^L$ is not interesting

$$PU^P = \frac{\sum_k \tilde{p}_{kt} Q_{kt}}{\sum_k \tilde{p}_{k0} Q_{kt}} = \sum_k P^P_{(k)} \frac{Q_{kt} \sum_j p_{kj0} m_{kjt}}{\sum_k Q_{kt} \sum_j p_{kj0} m_{kj0}}$$

$$P^P = \frac{\sum_k \sum_j p_{kjt}\, q_{kjt}}{\sum_k \sum_j p_{kj0}\, q_{kjt}} = \sum_k P^P_{(k)} \frac{Q_{kt} \sum_j p_{kj0}\, m_{kjt}}{\sum_k Q_{kt} \sum_j p_{kj0}\, m_{kjt}}$$

Sum of weights!

UVI in XMPI Manual

§ 2.14
Drobisch's formula

$$P_U = \left(\frac{\sum_m p_m^1 q_m^1}{\sum_m q_m^1} \right) \Bigg/ \left(\frac{\sum_n p_n^0 q_n^0}{\sum_n q_n^0} \right)$$