Retrospective Approximations of Superlative Price Indexes for Years where Expenditure Data is Unavailable

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Outline

- Background
- Superlative and Lloyd-Moulton indexes
- Approximating Superlative indexes
  - Lloyd-Moulton
  - Estimated expenditure shares
  - Quasi Fisher
- Data
- Results
- Conclusion

Background

Statistical agencies may want to inform the public about substitution bias of CPIs by calculating superlative price indexes retrospectively

Expenditure weights often available for distant CPI weight-reference years only

Issue addressed here: retrospective approximations of superlative indexes using a *theoretically-oriented* (Lloyd-Moulton) approach and *statistically-oriented* approaches (linear combinations of expenditure shares in weight-reference years)

Aim: clarify some issues and improve methods applied by several researchers

Superlative and Lloyd-Moulton Indexes

Quadratic Mean (QM) of order *r* price index

$$P_{QM}^{0t}(r) \equiv \left[\frac{\sum_i s_i^0 (p_i^t / p_i^0)^{r/2}}{\sum_i s_i^t (p_i^t / p_i^0)^{-r/2}} \right]^{1/r} = \left\{ \left[\sum_i s_i^0 (p_i^t / p_i^0)^{r/2} \right]^{2/r} \left[\sum_i s_i^t (p_i^t / p_i^0)^{-r/2} \right]^{-2/r} \right\}^{1/2}$$

is superlative. For $r = 2(1-\sigma)$ QM index is the geometric mean of the Lloyd-Moulton (LM) index

$$P_{LM}^{0t}(\sigma) = \left[\sum_i s_i^0 (p_i^t / p_i^0)^{1-\sigma} \right]^{1/(1-\sigma)}$$

and its current weight (CW) counterpart

$$P_{CW}^{0t}(\sigma) = \left[\sum_i s_i^t (p_i^t / p_i^0)^{-(1-\sigma)} \right]^{-1/(1-\sigma)}$$

Superlative and Lloyd-Moulton Indexes (2)

LM index is superlative for $\sigma = \sigma^{0t}$ such that

$$P_{LM}^{0t}(\sigma^{0t}) = P_{CW}^{0t}(\sigma^{0t}) = P_{QM}^{0t}(2(1-\sigma^{0t}))$$

Problem: different superlative index number formulas for different periods

For $\sigma = 0$ QM index becomes Fisher index

$$P_F^{0t} = \left\{ \left[\sum_i s_i^0 (p_i^t / p_i^0) \right] \left[\sum_i s_i^t (p_i^0 / p_i^t) \right]^{-1} \right\}^{1/2} = \left\{ \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \right] \left[\frac{\sum_i p_i^t q_i^t}{\sum_i p_i^0 q_i^t} \right] \right\}^{1/2}$$

Replacing arithmetic averages of price relatives by geometric averages:

$$P_T^{0t} = \left\{ \left[\prod_i (p_i^t / p_i^0)^{s_i^0} \right] \left[\prod_i (p_i^0 / p_i^t)^{s_i^t} \right]^{-1} \right\}^{1/2} = \prod_i (p_i^t / p_i^0)^{(s_i^0 + s_i^t)/2}$$

is Törnqvist index (also QM index for $\sigma \to 1$)

Approximating Superlative Indexes

Two distant benchmark years 0 and *T* for which (CPI) expenditure shares are available

Problem:

approximating superlative indexes for intermediate years $t = 1, \ldots, T-1$

Lloyd-Moulton approach

Assume that σ^{0t} (which makes LM equal to CW) is constant over time:

$\sigma^{0t} \cong \sigma^{0T}$ for $t = 1, \ldots, T-1$

LM index $P_{LM}^{0T}(\sigma^{0T})$ will be numerically close to Fisher or Törnqvist

Estimate $\hat{\sigma}$ such that $P_{LM}^{0T}(\hat{\sigma})$ is equal to Fisher or Törnqvist

Note: extrapolation possible for $t > T$ (real time approximation)

Approximating Superlative Indexes (2)

Assuming constancy of σ is consistent with a CES framework
Elasticity of substitution the same for all pairs of goods

Balk's (2000) two-level (nested) CES approach: elasticity less than 1 at upper aggregation level and greater than 1 at lower level (within strata)

Estimated value depends on actual (upper) aggregation level used

Shapiro and Wilcox (1997):

- BLS data on 9,108 item-area strata
- $\hat{\sigma} = 0.7$: LM approximates Törnqvist

Approximating Superlative Indexes (3)

Using estimated expenditure shares

Approximate unobserved shares in year *t* by moving linear combination, or weighted mean, of shares in benchmark (CPI weight-reference) years 0 and *T*:

$$\hat{s}_i^t = [ts_i^T + (T-t)s_i^0]/T = (t/T)s_i^T + (1-t/T)s_i^0$$

'Natural' approximations of Fisher and Törnqvist indexes:

$$\hat{P}_F^{0t} = \left\{ \left[\sum_i s_i^0 (p_i^t / p_i^0) \right] \left[\sum_i \hat{s}_i^t (p_i^0 / p_i^t) \right]^{-1} \right\}^{1/2} \qquad (\hat{P}_F^{0T} = P_F^{0T})$$

$$\hat{P}_T^{0t} = \prod_i (p_i^t / p_i^0)^{(s_i^0 + \hat{s}_i^t)/2} \qquad (\hat{P}_T^{0T} = P_T^{0T})$$

Approximating Superlative Indexes (4)

Quasi Fisher approach

Re-write 'natural' Törnqvist approximation as

$$\hat{P}_T^{0t} = \left[\prod_i (p_i^t / p_i^0)^{s_i^0} \right]^{1-t/2T} \left[\prod_i (p_i^t / p_i^0)^{s_i^T} \right]^{t/2T}$$

Substituting

$$\prod_i (p_i^t / p_i^0)^{s_i^T} = \prod_i (p_i^t / p_i^T)^{s_i^T} \left[\prod_i (p_i^0 / p_i^T)^{s_i^T} \right]^{-1}$$

yields

$$\hat{P}_T^{0t} = \left[\prod_i (p_i^t / p_i^0)^{s_i^0} \right]^{1-t/2T} \left[\frac{\prod_i (p_i^t / p_i^T)^{s_i^T}}{\prod_i (p_i^0 / p_i^T)^{s_i^T}} \right]^{t/2T}$$

Approximating Superlative Indexes (5)

Replacing geometric averages by arithmetic averages, using $s_i^\tau = p_i^\tau q_i^\tau / \sum_i p_i^\tau q_i^\tau \quad (\tau = 0, T)$ and rearranging yields the *Quasi Fisher* (QF) index:

$$\hat{P}_{QF}^{0t} \equiv \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \right]^{1-t/2T} \left[\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} \right]^{t/2T} = \left[\sum_i s_i^0 (p_i^t / p_i^0) \right]^{1-t/2T} \left[\sum_i s_i^{T*} (p_i^t / p_i^0) \right]^{t/2T}$$

with price backdated shares $s_i^{T*} = (p_i^0 / p_i^T) s_i^T / \sum_i (p_i^0 / p_i^T) s_i^T$

Triplett's (1998) Time-series Generalized Fisher Ideal (TGFI) index:

$$\hat{P}_{TGFI}^{0t} \equiv \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \right]^{1/2} \left[\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} \right]^{1/2} = \left[\sum_i s_i^0 (p_i^t / p_i^0) \right]^{1/2} \left[\sum_i s_i^{T*} (p_i^t / p_i^0) \right]^{1/2}$$

Data

- 444 elementary aggregates from official Danish CPI
- Expenditure shares (CPI weights) for 1996, 1999 and 2003
- Annual price index numbers for 1997-2003 (1996=100)
- Few modifications to cope with changes in commodity classification scheme
- Extreme price increases for some services where expenditure shares rise sharply

Data (2)

Direct and chained price index numbers, 1999 and 2003

	Direct indexes			Chained indexes	
	1996=100		1999=100	(1996=100)	
	1999	2003	2003	1999	2003
Laspeyres	106.69	117.90	110.74	106.69	118.15
Paasche	106.00	115.27	109.40	106.00	115.96
Fisher	106.34	116.58	110.07	106.34	117.05
Geo Laspeyres	106.38	116.54	109.96	106.38	116.97
Geo Paasche	106.38	117.15	110.16	106.38	117.20
Törnqvist	106.38	116.85	110.06	106.38	117.08

Results

Chained price index numbers (1996=100)

	1997	1998	1999	2000	2001	2002	2003
Laspeyres	102.11	104.03	106.69	109.88	112.59	115.62	118.15
Paasche	*102.03*	*103.74*	106.00	*108.86*	*111.20*	*113.81*	115.96
Fisher	*102.07*	*103.88*	106.34	*109.37*	*111.90*	*114.71*	117.05
Geo Laspeyres	102.06	103.88	106.38	109.41	111.94	114.71	116.97
Geo Paasche	*102.08*	*103.90*	106.38	*109.40*	*111.97*	*114.83*	117.20
Törnqvist	*102.07*	*103.89*	106.38	*109.41*	*111.96*	*114.77*	117.08
Quasi Fisher	*102.09*	*103.90*	106.34	*109.47*	*112.03*	*114.82*	117.05
TGFI	*102.04*	*103.83*	106.34	*109.29*	*111.83*	*114.68*	117.05
Lloyd-Moulton a)	*102.06*	*103.86*	106.34	*109.39*	*111.95*	*114.75*	117.05
Lloyd-Moulton b)	*102.06*	*103.88*	106.38	*109.43*	*111.99*	*114.79*	117.08

a) Fisher as benchmark; b) Törnqvist as benchmark; approximations in italics

Results (2)

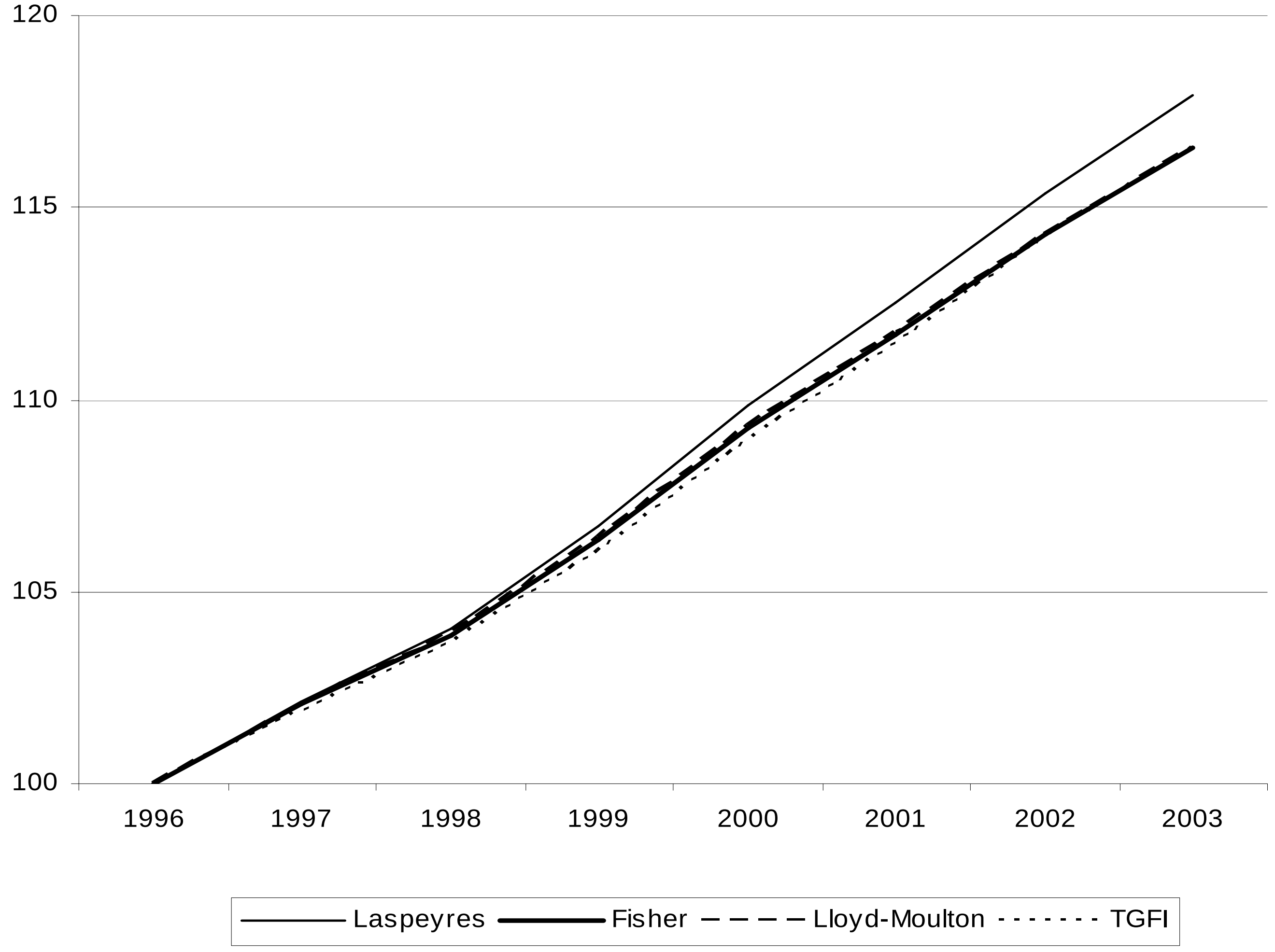
Direct price index numbers (1996=100), excluding observed 1999 shares

	1997	1998	1999	2000	2001	2002	2003
Laspeyres	102.11	104.03	106.69	109.88	112.55	115.39	117.90
Paasche	*102.02*	*103.74*	*105.99*	*108.62*	*110.84*	*113.22*	115.27
Fisher	*102.06*	*103.88*	*106.34*	*109.25*	*111.69*	*114.30*	116.58
Geo Laspeyres	102.06	103.88	106.38	109.29	111.72	114.29	116.54
Geo Paasche	*102.07*	*103.90*	*106.37*	*109.32*	*111.88*	*114.75*	117.15
Törnqvist	*102.06*	*103.89*	*106.37*	*109.30*	*111.80*	*114.52*	116.58
Quasi Fisher	*102.08*	*103.92*	*106.40*	*109.35*	*111.78*	*114.39*	116.58
TGFI	*101.92*	*103.66*	*106.03*	*108.96*	*111.48*	*114.22*	116.58
Lloyd-Moulton a)	*102.07*	*103.89*	*106.39*	*109.31*	*111.74*	*114.32*	116.58
Lloyd-Moulton b)	*102.07*	*103.91*	*106.45*	*109.42*	*111.90*	*114.53*	116.58

a) Fisher as benchmark; b) Törnqvist as benchmark; approximations in italics

Results (3)

Direct price index numbers (1996=100), excluding observed 1999 shares



Conclusions

- All our approximations are numerically similar to Lloyd-Moulton estimates
- Ideally each method should be tested on a data set that enables us to calculate superlative index numbers for intermediate years also
- Lloyd-Moulton approach is grounded in economic theory but statistical agencies might be reluctant to rely on CES assumptions (or the like)
- ‘Natural’ approach is more flexible than Quasi Fisher alternative (e.g., data permitting, estimate important shares directly from available price and quantity data, and estimate remaining shares as linear combinations of benchmark year shares)