Retrospective Price Indices and Substitution Bias

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1. Introduction

- Most countries today still compute their consumer price indices (CPI) as direct (i.e. fixed-weighted) Laspeyres indices
- The weights are updated at discrete intervals, often every five or ten years, at which time the old and new series are spliced together
- It is well known that, in the consumer context, the Laspeyres functional form tends to overestimate the price level due to a substitution bias

1. Introduction *(continued)*

- Ideally, one should use chained, superlative indices
- However, the necessary data (especially the quantity data) are often not available
- Nonetheless, sooner or later the baskets will need to be updated
- The question therefore arises at the time when the new information becomes available whether it can be used to assess the importance of the substitution bias, and whether it can be exploited to improve the measure of past price level behavior

1. Introduction *(continued)*

- The purpose of this paper is to propose a simple way to use the new information made available at the time of updating in order to get a superlative measure of the price change over the corresponding period
- Retroactively computed price indices then make it possible to assess the size of the substitution bias

2. A retrospective measure of the price level

Consider the following two runs of fixed-basket indices:

$$1,\ \frac{\sum_i p_i^1 q_i^0}{\sum_i p_i^0 q_i^0},\ \frac{\sum_i p_i^2 q_i^0}{\sum_i p_i^0 q_i^0},\ \dots,\ \frac{\sum_i p_i^{T-1} q_i^0}{\sum_i p_i^0 q_i^0},\ \frac{\sum_i p_i^T q_i^0}{\sum_i p_i^0 q_i^0} \qquad (1)$$

$$\frac{\sum_i p_i^0 q_i^T}{\sum_i p_i^T q_i^T},\ \frac{\sum_i p_i^1 q_i^T}{\sum_i p_i^T q_i^T},\ \frac{\sum_i p_i^2 q_i^T}{\sum_i p_i^T q_i^T},\ \dots,\ \frac{\sum_i p_i^{T-1} q_i^T}{\sum_i p_i^T q_i^T},\ 1, \qquad (2)$$

Notation: p_i^t and q_i^t denote the price and the quantity of good i at time t, respectively; the initial period is denoted by 0, and the terminal one by T.

2. A retrospective measure of the price level *(continued)*

Next, normalize run (2) by dividing all its elements by the first one:

$$\frac{\sum_i p_i^0 q_i^T}{\sum_i p_i^T q_i^T}, \quad \frac{\sum_i p_i^1 q_i^T}{\sum_i p_i^T q_i^T}, \quad \frac{\sum_i p_i^2 q_i^T}{\sum_i p_i^T q_i^T}, \quad \ldots, \quad \frac{\sum_i p_i^{T-1} q_i^T}{\sum_i p_i^T q_i^T}, \quad 1, \tag{2}$$

$$1, \quad \frac{\sum_i p_i^1 q_i^T}{\sum_i p_i^0 q_i^T}, \quad \frac{\sum_i p_i^2 q_i^T}{\sum_i p_i^0 q_i^T}, \quad \ldots, \quad \frac{\sum_i p_i^{T-1} q_i^T}{\sum_i p_i^0 q_i^T}, \quad \frac{\sum_i p_i^T q_i^T}{\sum_i p_i^0 q_i^T}. \tag{3}$$

2. A retrospective measure of the price level *(continued)*

Finally, take the geometric means of the corresponding elements of (1) and (3) to get the following sequence of pseudo Fisher indices:

$$1,\ \frac{\sum_i p_i^1 q_i^0}{\sum_i p_i^0 q_i^0},\ \frac{\sum_i p_i^2 q_i^0}{\sum_i p_i^0 q_i^0},\ \ldots,\ \frac{\sum_i p_i^{T-1} q_i^0}{\sum_i p_i^0 q_i^0},\ \frac{\sum_i p_i^T q_i^0}{\sum_i p_i^0 q_i^0} \tag{1}$$

$$1,\ \frac{\sum_i p_i^1 q_i^T}{\sum_i p_i^0 q_i^T},\ \frac{\sum_i p_i^2 q_i^T}{\sum_i p_i^0 q_i^T},\ \ldots,\ \frac{\sum_i p_i^{T-1} q_i^T}{\sum_i p_i^0 q_i^T},\ \frac{\sum_i p_i^T q_i^T}{\sum_i p_i^0 q_i^T}. \tag{3}$$

$$1,\ \sqrt{\frac{\sum_i p_i^1 q_i^0}{\sum_i p_i^0 q_i^0}\frac{\sum_i p_i^1 q_i^T}{\sum_i p_i^0 q_i^T}},\ \ldots,\ \sqrt{\frac{\sum_i p_i^{T-1} q_i^0}{\sum_i p_i^0 q_i^0}\frac{\sum_i p_i^{T-1} q_i^T}{\sum_i p_i^0 q_i^T}},\ \sqrt{\frac{\sum_i p_i^T q_i^0}{\sum_i p_i^0 q_i^0}\frac{\sum_i p_i^T q_i^T}{\sum_i p_i^0 q_i^T}}. \tag{4}$$

3. Comparison with Hansen

HANSEN (2007) recently proposed the following Fisher-like price index:

$$P_H^{0:t} = \left[\frac{\sum_i s_i^0 \frac{p_i^t}{p_i^0}}{\sum_i s_i^T \frac{p_i^0}{p_i^t}} \right]^{\frac{1}{2}}, \quad t = 0, 1, \ldots, T, \tag{5}$$

where:

$$s_i^0 = \frac{p_i^0 q_i^0}{\sum_i p_i^0 q_i^0}, \qquad s_i^T = \frac{p_i^T q_i^T}{\sum_i p_i^T q_i^T}.$$

3. Comparison with Hansen *(continued)*:

For comparison purposes, rewrite formulae (1) and (2) in terms of price relatives and expenditure share weights:

$$\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} = \sum_i s_i^0 \frac{p_i^t}{p_i^0}, \quad t = 0, 1, \ldots, T \tag{1'}$$

$$\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^T q_i^T} = \sum_i s_i^T \frac{p_i^t}{p_i^T}, \quad t = 0, 1, \ldots, T. \tag{2'}$$

3. Comparison with Hansen *(continued)*:

Using (2′), it can be seen that the period *t* index in (3) can be written as follows:

$$\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^T q_i^T} = \sum_i s_i^T \frac{p_i^t}{p_i^T}, \qquad t = 0, 1, \ldots, T. \tag{2′}$$

$$\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} = \frac{\sum_i s_i^T \frac{p_i^t}{p_i^T}}{\sum_i s_i^T \frac{p_i^0}{p_i^T}}, \qquad t = 0, 1, \ldots, T. \tag{3′}$$

3. Comparison with Hansen *(continued)*:

Finally, using (1′) and (3′), it can be seen that the period-t pseudo Fisher price index ($P_F^{0:t}$) in the sequence defined by (4) can be written as follows:

$$\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} = \sum_i s_i^0 \frac{p_i^t}{p_i^0}, \quad t = 0, 1, \ldots, T \tag{1′}$$

$$\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} = \frac{\sum_i s_i^T \frac{p_i^t}{p_i^T}}{\sum_i s_i^T \frac{p_i^0}{p_i^T}}, \quad t = 0, 1, \ldots, T. \tag{3′}$$

$$P_F^{0:t} = \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T}\right]^{\frac{1}{2}} = \left[\frac{\sum_i s_i^0 \frac{p_i^t}{p_i^0} \cdot \sum_i s_i^T \frac{p_i^t}{p_i^T}}{\sum_i s_i^T \frac{p_i^0}{p_i^T}}\right]^{\frac{1}{2}}, \quad t = 0, 1, \ldots, T. \tag{4′}$$

3. Comparison with Hansen *(continued)*:

$$P_H^{0:t} = \left[\frac{\sum_i s_i^0 \frac{p_i^t}{p_i^0}}{\sum_i s_i^T \frac{p_i^0}{p_i^t}} \right]^{\frac{1}{2}} , \quad t = 0, 1, \ldots, T , \tag{5}$$

$$P_F^{0:t} = \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} \right]^{\frac{1}{2}} = \left[\frac{\sum_i s_i^0 \frac{p_i^t}{p_i^0} \cdot \sum_i s_i^T \frac{p_i^t}{p_i^T}}{\sum_i s_i^T \frac{p_i^0}{p_i^T}} \right]^{\frac{1}{2}} , \quad t = 0, 1, \ldots, T . \tag{4'}$$

Note that $P_H^{0:T} = P_F^{0:T}$.

3. Comparison with Hansen *(continued)*:

"Laspeyres element":

$$P_{H(L)}^{0:t} = \sum_i s_i^0 \frac{p_i^t}{p_i^0} = P_{F(L)}^{0:t}, \quad t = 0, 1, \ldots, T.$$

"Paasche element":

$$P_{H(P)}^{0:t} = \frac{1}{\sum_i s_i^T \frac{p_i^0}{p_i^t}} \neq \frac{\sum_i s_i^T \frac{p_i^t}{p_i^T}}{\sum_i s_i^T \frac{p_i^0}{p_i^T}} = P_{F(P)}^{0:t}, \quad t = 0, 1, \ldots, T.$$

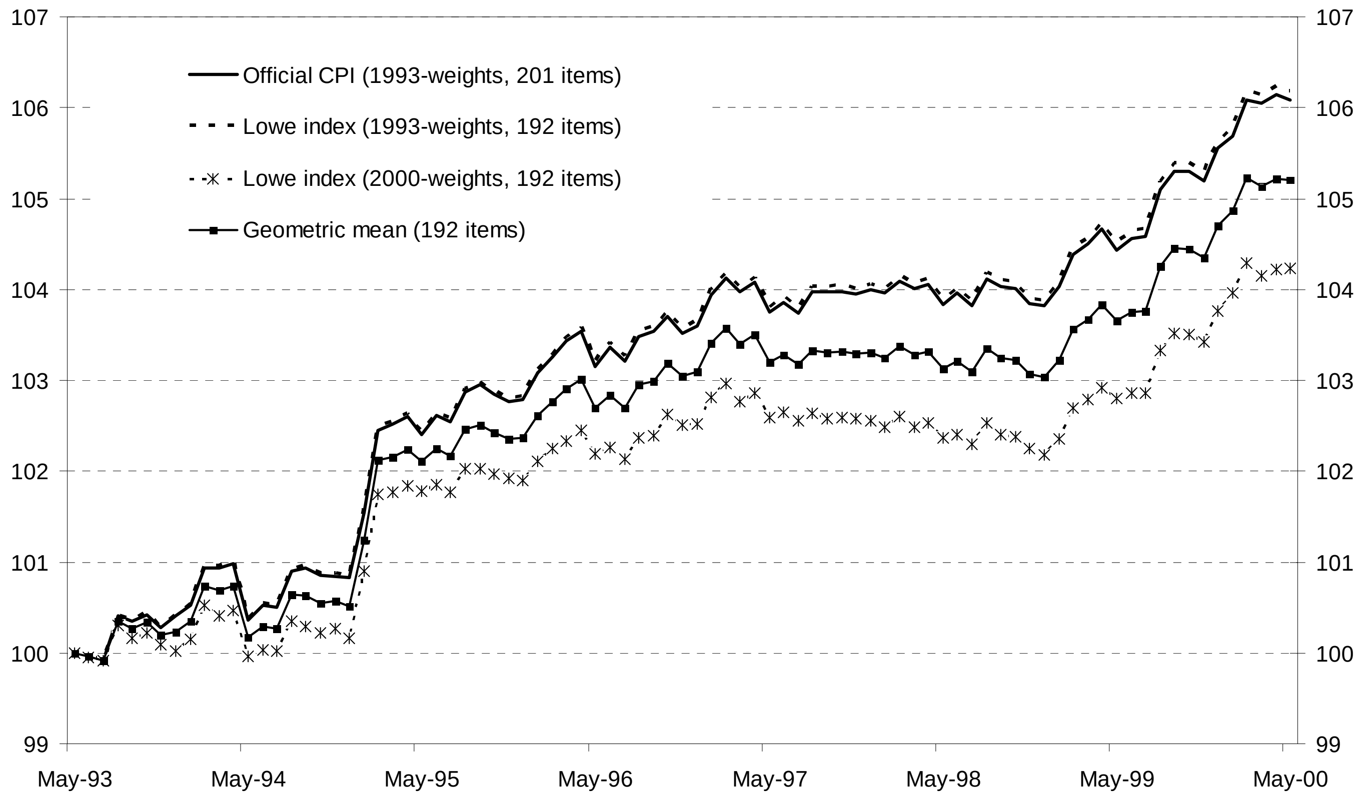
$P_{H(P)}^{0:t}$ can be interpreted as a harmonic period-*T* weighted Young index. It does not have the Paasche form, though.

4. Application to Swiss data

- The application is for the Swiss CPI data for the period 1993 to 2000
- At the time the Swiss CPI was computed as a direct Laspeyres quantity index, with fixed weights using May 1993 as the reference period
- The weights were eventually revised in 2000
- The number of categories common to both surveys amounts to 192 out of 201
- In value terms, these categories represented over 99% of the CPI

Figure 1

Alternative consumer price indices, Switzerland, 1993-2000



5. Conclusion

- Between 1993 and 2000 the Swiss price level (in terms of CPI) increased by about 5.2%, rather than the 6.1% that the official data suggest
- In terms of yearly averages, this implies an inflation rate of about 0.73%, rather than 0.85%
- This suggests a yearly substitution bias of 0.13% on average
- This estimate gives considerable empirical support to BRACHINGER, SCHIPS and STIER (1999) who contended that the (upper level) substitution bias probably does not exceed 0.15 percentage points per year
- The approach that we have used for the CPI could obviously also be applied to other indices, including quantity indices