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On the RGEKS price index formula for scanner data

M.Ribe
Statistics Sweden, martin.ribe@scb.se

Abstract. The paper discusses some mathematical properties of the RGEKS (or RYGEKS) price index formula recently introduced for use with scanner data, particularly with respect to control of chain drift.

The RGEKS index and the drift problem for scanner data

Ivancic et al. (2011), and de Haan & v. d. Grient (2011), introduce the RGEKS price index formula (or RYGEKS price index, spelled out as Rolling-Year Gini, Eltetö & Köves, Szulc price index). The RGEKS index is meant to meet the problems with chain drift that are noted even for a superlative index formula such as the Fisher index when applied in a very frequently chained form to "scanner data", in the sense of micro-level price and quantity data from retail sales records. The RGEKS index has also recently been studied empirically by Johansen & Nygaard (2011), Krsinich (2011), v. d. Grient & de Haan (2011). For general background reference on Fisher index, GEKS indices etc., cf. further Balk (2008), Diewert (1976), ILO et al. (2004).

Empirical evidence in the mentioned sources, using observed scanner data, suggest that the RGEKS index would be effective in eliminating chain drift. Nevertheless, Ivancic et al. (2011, footnote 19) note that RGEKS does not satisfy transitivity and hence will be potentially subject to chain drift, although they deem that fact unlikely to be a significant problem.

The present paper attempts to further study some properties of the RGEKS index, particularly as related to chain drift attenuation.

The RGEKS index formula

In the notation of de Haan & v. d. Grient (2011, Eq. (18)) the RGEKS index with base period 0 and current period 13 is computed as

$$P^{0,13}_{\text{RGEKS}} = P^{0,12}_{\text{GEKS}} \prod_{t=1}^{13} [P^{12,t} / P^{13,t}]^{1/13}$$

(1)

$$= \prod_{t=0}^{12} [P^{0t} / P^{12,t}]^{1/13} \prod_{t=1}^{13} [P^{12,t} / P^{13,t}]^{1/13} \ .$$

Here $P^{s,t}$ stands for a superlative index, say Fisher index, with base period s and current period t.

The corresponding formula for RGEKS index with base period 0 and current period $T > 13$ should apparently be

$$(2) \qquad P^{0T}_{\text{RGEKS}} = P^{0,12}_{\text{GEKS}} \prod_{t=13}^{T} \prod_{s=t-12}^{t} [P^{t-1,s} / P^{t,s}]^{1/13} .$$

This is apparently how Eq. (19) of de Haan & v. d. Grient (2011), and correspondingly Eq. (8) of Ivancic et al. (2011), should read, with a little more elaborate distinction of the superscripts and in line with Ivancic et al. (2009, Eq. (14)-(18)).

The RGEKS index in re-written form

Proposition 1. *The formula of Eq. (2) for the RGEKS index from base period 0 to current period T > 13 can be identically re-written as*

$$(3) \qquad P^{0T}_{\text{RGEKS}} = P^{0,12}_{\text{GEKS}} \frac{\prod_{t=1}^{13} [P^{12,t}]^{1/13}}{\prod_{t=T-12}^{T} [P^{T,t}]^{1/13}} \prod_{t=13}^{T-1} [P^{t,t+1} / P^{t,t-12}]^{1/13} .$$

Proof. See Annex 1.

The form of Eq. (3) is somewhat simpler than that of Eq. (2), as some redundant repeated occurrences of factors $P^{s,t}$ have been reduced away.

In the case of time-reversible index $P^{s,t}$, such as Fisher index, Eq. (3) can be written a little more intuitively as follows.

Proposition 1A. *If the index $P^{s,t}$ satisfies time-reversibility, the formula of Eq. (2) for the RGEKS index from base period 0 to current period T > 13 can be identically re-written as*

$$(4) \qquad P^{0T}_{\text{RGEKS}} = P^{0,12}_{\text{GEKS}} \cdot \prod_{t=1}^{13} [P^{12,t}]^{1/13} \cdot \prod_{t=13}^{T-1} [P^{t-12,t} \cdot P^{t,t+1}]^{1/13} \cdot \prod_{t=T-12}^{T} [P^{t,T}]^{1/13}$$

The four main factors on the right of Eq. (4) have a natural interpretation as index links. The first factor is a link from period 0 to 12, and the second is a backdating link from period 12 to a geometric mean over periods 0 to 12. The third factor is a geometric mean of chained 13-month multiplicative changes, and the fourth and last factor is an updating link to the current period T from a geometric mean over periods $T-12$ to T.

Eq. (4) may give some alternative impression how the drift attenuation feature of the RGEKS index works. Apparently crucial here is the smoothing by the geometric mean of 13-month changes in the third main factor of Eq. (4).

Numerical stress test

Attempting to somewhat stress test the drift attenuation capacity of the RGEKS index, consider a simple scenario of just two products. The two products are supposed to show a repetitive pattern of prices and quantities with a cycle of 4 periods, as in the following table.

Period	p_1	p_2	q_1	q_2
...				
$t-4$	100	100	10	10
$t-3$	30	100	100	10
$t-2$	30	100	20	10
$t-1$	100	100	2	10
t	100	100	10	10
...				

Over a full cycle from period $t-4$ to period t the prices and quantities return to the same state. To be non-drifting a price index should then also return to the same value. Chained Fisher index comes far from that, as it falls by 29 percent. As expected the RGEKS index performs much better and falls by just 2.6 percent (for $t > 20$). But still this is not negligible, and is it good enough?

Conclusion

The observations above seem to indicate that although undeniably present the drift attenuation property of the RGEKS is not quite complete. It may have to be further studied theoretically, to settle whether it can be considered effective quite enough to be trusted under all potential practical circumstances for scanner data. At present it may not seem quite clear whether the reported encouraging experience with RGEKS could be safely generalised to all plausible potential future possible conditions, including e.g. other patterns and levels of inflation. As is indicated by the graphs in Fig. 1 of de Haan & v. d. Grient (2011), scanner data are extraordinary data with respect to volatility, and this may considerably stress the drift attenuation capacity of the RGEKS index.

Disclaimer

Views expressed in this paper are those or the author solely.

References

B.M. Balk (2008), Price and Quantity Indices, Cambridge: Cambridge University Press.

W.E. Diewert (1976), Exact and Superlative Index Numbers, Journal of Econometrics, 4, 115–145; reprinted in Essays in Index Number Theory, Vol. 1 (eds. W.E. Diewert and A.O. Nakamura), Amsterdam: North-Holland, 1993, pp. 223–252.

ILO/IMF/OECD/UNECE/Eurostat/The World Bank (2004), Consumer Price Index Manual: Theory and Practice, Geneva: International Labour Office, http://www.ilo.org/public/english/bureau/stat/guides/cpi/index.htm

L. Ivancic, W.E. Diewert, K.J. Fox (2009), Scanner data, time aggregation and the construction of price indexes, , Ottawa Group Meeting in Neuchâtel [Preprint of Ivancic et al. (2011)].

L. Ivancic, W.E. Diewert, K.J. Fox (2011), Scanner data, time aggregation and the construction of price indexes, Journal of Econometrics 161, 24–35.

I. Johansen & R. Nygaard (2011), Dealing with bias in the Norwegian superlative price index of food and non-alcoholic beverages, Ottawa Group Meeting in Wellington, http://www.stats.govt.nz/ottawa-group-2011/agenda.aspx

J. de Haan & H. A. v. d. Grient (2011), Eliminating chain drift in price indexes based on scanner data, Journal of Econometrics 161, 36–46.

F. Krsinich (2011), Price Indexes from Scanner Data: A comparison of different methods, Ottawa Group Meeting in Wellington, http://www.stats.govt.nz/ottawa-group-2011/agenda.aspx

H. A. v. d. Grient & J. de Haan (2011), Scanner Data Price Indexes: The "Dutch Method" versus Rolling Year GEKS, Ottawa Group Meeting in Wellington, http://www.stats.govt.nz/ottawa-group-2011/agenda.aspx

Annex 1

Proof of Proposition 1.

(A1)
$$\begin{aligned}
P_{\text{RGEKS}}^{0T} &= P_{\text{GEKS}}^{0,12} \prod_{t=13}^{T} \frac{\prod_{s=t-12}^{t} [P^{t-1,s}]^{1/13}}{\prod_{s=t-12}^{t} [P^{t,s}]^{1/13}} \\
&= P_{\text{GEKS}}^{0,12} \frac{\prod_{t=1}^{13} [P^{12,t}]^{1/13}}{\prod_{t=T-12}^{T} [P^{T,t}]^{1/13}} \prod_{t=13}^{T-1} \frac{\prod_{s=t-11}^{t+1} [P^{t,s}]^{1/13}}{\prod_{s=t-12}^{t} [P^{t,s}]^{1/13}} \\
&= P_{\text{GEKS}}^{0,12} \frac{\prod_{t=1}^{13} [P^{12,t}]^{1/13}}{\prod_{t=T-12}^{T} [P^{T,t}]^{1/13}} \prod_{t=13}^{T-1} \frac{[P^{t,t+1}]^{1/13}}{[P^{t,t-12}]^{1/13}}
\end{aligned}$$