Impact of the Price-Updating Weights Procedure on the Canadian Consumer Price Index

by

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Abstract

Which index number formula best fits the purpose of a country's consumer price index (CPI) is an intriguing question. Statistics Canada currently uses the Lowe index formula for higher level aggregates of its CPI. With the Lowe Index, to keep the quantities fixed, the expenditure shares need to be price-updated to the price reference period. So far, most empirical research in this area has shown that the Lowe index typically results in a greater substitution bias than the Young index that uses the original expenditure shares. Theoretically, whether the Lowe index is higher than the Young index or vice-versa is ambiguous. The relationship between these two indexes depends on the behaviour of the movements in relative prices and on the sensitivity of quantities to these price movements. Therefore, the findings from other studies on the gap between the Lowe and the Young indexes cannot necessarily be extended to the case of Canada. Moreover, these results can be used to measure the CPI bias by comparing them against a theoretical target index (i.e., Fisher, Walsh or Törnqvist index formulas).

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This paper aims at identifying the impact of price-updated weights on the Canadian CPI. Canadian data from December 1994 to December 2007 were used to compare consumer price indexes compiled using the Lowe index number formula and the Young index number formula. The comparisons between Lowe index and indexes calculated by using other different index number formulas, such as Laspeyres, Paasche, Geometric Young, Fisher, Walsh and Törnqvist, are also presented in order to determine and compare the magnitude of the CPI bias.

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1. Introduction

In the 2007 Ottawa Group Meeting, Carsten B. Hansen presented a paper titled "Recalculation of the Danish CPI 1996-1998". In this paper, the author found that price updating the expenditure weights[2] actually generated higher upward bias in the CPI when compared with using original weights. Greenless and Williams (2009) found similar results using BLS data. Statistics Canada currently uses the Lowe index for the upper

[2] Price-updating weights procedure under investigated in this paper means the procedure used to calculate prices index when the weights have already been determined but the prices reference year and weight reference year are different. The impact of using price-updating weights to determine a new CPI basket on the CPI is not discussed in this paper.

level aggregation. When applying the Lowe index number formula, which is a fixed basket index, the procedure of price-updating weights is required to keep the quantities fixed. Following Hansen's calculation, we would like to investigate the impact of price-updated expenditure weights on the Canadian CPI and to rethink a very fundamental problem—which index number formula best fits the purpose of the Canadian CPI? ***Properly answering these questions is important for us to better understand our CPI and improve the quality of the Canadian CPI.***

In the Consumer Prices Division of Statistics Canada, researchers started investigating this problem by attempting to answer whether the expenditure weights should be price updated to the link month that connects a new CPI basket to the previous basket. At the 2004 Price Measurement Advisory Committee (PMAC) meeting held at Statistics Canada, Ralph Turvey suggested looking at whether price updating earlier baskets better approximates the subsequent expenditure pattern than not price updating the weights. Robin Lowe worked on this project and presented his findings at the 2005 PMAC meeting. Lowe found that the adjusted series, in general, generated higher upward bias than what the unadjusted series did. The price-adjustment procedure probably introduces a further upward bias to the index. In short, Lowe believed that "the unadjusted index is preferable so long as there are no dramatic changes in the periods between the survey data and the link period". Jack Triplett commented on Lowe's findings and pointed out that the problem was really **an effect of using the Lowe index**. He suggested that the preferred solution is to update basket weights quickly and regularly rather than creating a hybrid approach.

Currently, we apply a chain fixed basket concept in our CPI. According to the "Consumer price index manual: theory and practice" (ILO CPI Manual thereafter), the Lowe formula is a good choice for the fixed basket formula. We use the Lowe formula because it offers a simple and convenient way to compile composite price indexes. Due to the lag in obtaining the base year quantity weights, it is impossible to use Laspeyres formula without violating the long-established non-revision policy for the CPI. However, we need to be aware of its inherent limitations. For example, the Lowe index cannot take into

account the effect of the price-induced consumer substitution behaviour; it includes new products in the CPI and reflects changes in the consumer spending patterns with lags and it also has difficulties in fully accounting for quality change. In addition, we know that the Lowe indexes do not belong to the group of "superlative" price indexes[3], which are expected to provide *"fairly close"* approximations to the underlying Cost of Living Indexes (COLI). *"Some kind of superlative index is also likely to be seen as desirable, even when the CPI is not meant to be a cost of living index.*"[4] Thus, it is interesting for us to establish the relationship between Lowe index and a COLI.

The difference between our published CPI and a conditional COLI is called measurement bias in this paper. The main measurement bias includes commodity substitution bias, outlet substitution bias, quality change and new products bias. The bias in the CPI can be a sensitive issue easily questioned by the public media. To face this challenge, we should enhance our research on possible sources of the bias and improve methodologies with the goal of producing a better CPI. Both the 2005 and 2006 Price Management Advisory Committee (PMAC) meetings suggested examining the upward bias in "Laspeyres-type" indexes compared to theoretical target indexes such as Fisher, Walsh or Törnqvist indexes. This comparison can give us some indication of the combined impact of changes in consumers' income and preferences, and substitution effects over the period in question, providing important information for both producers and users of the CPI. Therefore, having the knowledge about the ***magnitude of the bias*** in the fixed-basket price index is valuable for us to correctly interpret the meaning of the CPIs and to control the quality of the CPIs.

Identifying the impact of price-updated expenditure shares on our CPI is essentially evaluating how the Lowe index fits our purpose of the CPI. Hansen's paper shows that price updating of the expenditure weights adds to the upward bias of the CPI. ***Is this also true for the Canadian CPI?*** Robin Lowe only answered whether we should price-update

[3] Superlative index will be discussed in more detail in Section 3.
[4] Refer to ILO CPI Manual 1.13 on page 2.

the expenditure shares when we introduce a new basket[5]. In his calculation, only one basket updating is applied. In this project, we will extend his work to investigate whether we should price update the weights from the weight reference period to the price reference period. We will try to answer how the upward bias accumulates over a certain period. In our calculation, more than one effective CPI basket is used in the comparison through the chaining procedure. Thus, the impact on the CPI is a mixture of price-updating weights and chaining procedure.

This project aims at identifying the impact of price-updated weights on the Canadian CPI. Our strategy of fulfilling this project includes two steps. First, we compare the index series using price-updated weights with those using original weights. Second, we identify the magnitude of the upward bias caused by the procedure of price updating weights by comparing the Lowe index with three superlative indexes: Fisher, Walsh and Törnqvist. Our numerical results show that there is a significant difference between the Fisher index and the other two superlative indexes in a certain circumstance.

The rest of this paper is organized as follows: section 2 discusses the difference between the Lowe index and Young index, and reports the differences between them; section 3 discusses the difference between Lowe index and the three superlative indexes; the last section concludes the paper.

2. Lowe index and Young index

Normally there are three time periods involved in calculating a simple price index: a weight reference period, a price reference period[6], and a current period. The weight reference period is the survey year or years used to determine the CPI basket. The price reference period is the base period for comparing the price changes. If we need to

[5] Currently, Statistics Canada updates CPI basket every four or five years.

[6] In this paper, we assume price reference period and index reference period coincide with each other. Index reference period is the period at which price index is set to 100. Because Young index is not transitive, different price reference period and index reference period might result in different results, thus, we need to clearly define the price reference period and index reference period if they are different.

calculate a ***chained*** index, a link period will be used in the calculation. The link period is the period from which we have information about both the old basket and the new basket. Thus, the index series based on a new basket can be connected to the series based on an old basket through the linking procedure. In Canada, the link period is the month just prior to the introduction of a new CPI basket. For example, we introduced our 2005 basket in May 2007; the link month is April 2007. If the weight reference period is different from the link period or price reference period, a statistical agency has to decide whether or not to price update the weights from the weight reference period to the link month (or to the price reference period). In other words, the statistical agency has the **option** of either fixing physical quantities or fixing expenditure shares of the basket reference year. If it chooses to preserve the quantities, the resulting index is Lowe index; otherwise the resulting index is Young index. Price updating weight procedure is required for fixing the quantities when direct information on quantities is not available. We start our investigation of the impact of price updated expenditure shares on the calculation of CPI by comparing the Lowe index with the Young index. Before we conduct the comparison, we will clarify the purpose of price updating weights first.

2.1 Purpose of price-updating expenditure shares

The CPI basket is meant to represent the consumer expenditure patterns. Frequently updating a CPI basket can ensure the realization of this "objective". However, price updating expenditure share is a different procedure from updating a CPI basket. We are not certain whether price-updating procedure can make our baskets more representative to the spending pattern or not. Regarding to the purpose of price updating expenditure shares, the 2003 Resolution Concerning Consumer Price Index makes the following suggestion:

"Where the weight reference period differs significantly from the price reference period, the weights should be price updated to take account of price changes between the weights reference period and price reference period. Where it is likely that price updated weights

are less representative of the consumption pattern in the price reference period this procedure may be omitted".

This quote actually tells us that price-updating procedure should be used only when it could lead to more representative consumption pattern in the price reference period. In order to implement this recommendation, more detailed information about consumer expenditure pattern should be collected to make the subjective judgement. To our knowledge, we know only New Zealand applies this strategy. It conducts both price updating and quantity updating[7] to keep the CPI weights representative of consumer expenditure pattern in the month when it introduces its new basket.

The ILO CPI Manual also points out:

"It is up to the statistical offices to decide for themselves whether to price-update the expenditure shares or not. If the primary aim is to compile a CPI that measures the price development of an actual fixed basket of goods and services, then the weights should be price-updated. The resulting fixed basket, or Lowe, index will provide a good estimate of the price development if quantities tend to remain constant."[8]

Statistics Canada applies the Lowe index formula in the practice. The purpose of price-updating the expenditure shares in Canada is to keep the quantity fixed, which is required by the Lowe index number formula when the separate information on quantities is not available. The underlying assumption is that consumers would purchase the relative same amount of commodities regardless of the changes in their prices. The magnitude of the bias, which is simplified as the difference between our published CPI and the superlative indexes, generated by using the Lowe index number formula, depends on how well this assumption fits the real world.

[7] For more detail, please refer to Pike(2006).
[8] Refer to 9.101-9.121 of the revised Chapter 9 of the ILO CPI Manual.

2.2 Mathematical difference between Lowe index and Young index

Lowe index is a widely used fixed-basket index. The Laspeyres and Paasche indexes are two special cases of the Lowe index. The physical quantities are fixed for the two comparison periods. Thus, Lowe index reflects pure price changes in the two comparison periods if we could keep constant quality[9]. For the Young index, the "*representative expenditure shares are chosen that pertain to the two periods under consideration*". The overall index is calculated as a "*share-weighted average of the individual price ratios*"[10]. In general, these two index formulas yield different index series. In this sub-section, we show how Lowe index differs mathematically from Young index.

Let $P_{Lo}^{t/0}$ denote Lowe index for the period going from period 0 to period t and $P_{Y}^{t/0}$ denote Young index for the same period. Here t refers to the current period, while 0 refers to the price reference period. Lowe index can be defined as follows:

$$(1)\ P_{Lo}^{t/0} = \frac{\sum_i p_i^t q_i^b}{\sum_i p_i^0 q_i^b} = \sum_i \left(\frac{p_i^t}{p_i^0} \right) w_i^{0b}$$

where w_i^{0b} refers to the price updated weights. 0 is the price reference period and b is the weight reference period. This is a hybrid weight, in which prices and quantities are derived from different periods. Let w_i^b denote the original weights for item i in the weight reference period b. Then, w_i^b and w_i^{0b} are defined as follows:

$$(2)\ w_i^b = \frac{p_i^b q_i^b}{\sum_i p_i^b q_i^b} \text{ and } w_i^{0b} = \frac{p_i^0 q_i^b}{\sum_i p_i^0 q_i^b} = \frac{\frac{p_i^0}{p_i^b} w_i^b}{\sum_i \frac{p_i^0}{p_i^b} w_i^b}.$$

[9] Quality adjustments are not discussed in this paper.
[10] Please refer to ILO CPI Manual 15.54.

Young index can be defined as follows:

$$(3)\quad P_Y^{t/0} = \sum_i \frac{p_i^t}{p_i^0} w_i^b$$

The difference between the Lowe index and the Young index can be calculated as follows:

$$\begin{aligned} P_{Lo}^{t/0} - P_Y^{t/0} &= \sum_i \frac{p_i^t}{p_i^0}(w_i^{0b} - w_i^b) \\ &= \sum_i (\frac{p_i^t}{p_i^0} - P_Y^{t/0})(w_i^{0b} - w_i^b) \qquad \text{using } \sum_i P_Y^{t/0}(w_i^{0b} - w_i^b) = 0 \\ &= \sum_i (\frac{p_i^t}{p_i^0} - P_Y^{t/0})(\frac{w_i^{0b}}{w_i^b} - 1) w_i^b \end{aligned} \quad (4)$$

We can interpret $(\frac{p_i^t}{p_i^0} - P_{t/0}^Y)$ as the difference between the relative price of item i and the weighted average price, which is measured by ($P_Y^{t/0}$), of the all items. The second part of the summation, $(\frac{w_i^{0b}}{w_i^b} - 1)$, captures the difference between the price-updated weight and the original weight. From equation (4), we can see that the differences between the Lowe index and the Young index are determined by both divergence of the price ratio and divergence of the two baskets. Because these two types of divergence do not always work in the same direction and they vary with time, we do not have a certain answer as to whether Lowe index is higher or lower than Young index.

If we decompose the above difference in a slightly different way, we could see the above point more clearly. Let us denote the Young index that uses year b weights but goes from the period b to the period 0 as $P_Y^{0/b}$, which can be written as:

$$(5)\quad P_Y^{0/b} = \sum_i (\frac{p_i^0}{p_i^b}) w_i^b$$

Substituting equation (5) into the price-updated weight defined in equation (2), we have:

$$(6)\quad w_i^{0b} = \frac{\frac{p_i^0}{p_b^i} w_i^b}{\sum_i \frac{p_i^0}{p_b^i} w_i^b} = \left(\frac{p_i^0 / p_i^b}{P_Y^{0/b}} \right) w_i^b$$

Then we can decompose the difference between Lowe index and Young index in the following way:

$$\begin{aligned} P_{Lo}^{t/0} - P_Y^{t/0} &= \sum_i \frac{p_i^t}{p_i^0} (w_i^{0b} - w_i^b) \\ &= \sum_i (\frac{p_i^t}{p_i^0} - P_Y^{t/0})(w_i^{0b} - w_i^b) \qquad \text{using } \sum_i P_Y^{t/0}(w_i^{0b} - w_i^b) = 0 \\ &= \sum_i (\frac{p_i^t}{p_i^0} - P_Y^{t/0})(\frac{p_i^0 / p_i^b}{P_Y^{0/b}} - 1) w_i^b \\ &= \sum_i (\frac{p_i^t}{p_i^0} - P_Y^{t/0})(\frac{p_i^0}{p_i^b} - P_Y^{0/b})(w_i^b / P_Y^{0/b}) \end{aligned} \qquad (7)$$

Equation (7) is a kind of correlation between price relatives above or below their Young means where the first set of deviations goes from period 0 to t and the second set of deviations goes from period b to 0. If the price trends persist all the way from b to 0 and continue on in the same direction from 0 to t, then the above "covariance" will be positive and Lowe index will exceed Young index[11]. Equation 7 indicates that the difference between Lowe index and Young index relies heavily on the price behaviour. The ILO CPI Manual explains this point more intuitively. *"The Lowe index gives more weight to those elementary indices the prices of which have increased by more than average from b*

[11] In this comparison, only one basket is used in the calculation. The price base period is between weight reference period and current period.

to 0 and less weights to those where the prices have increased by less than average. Therefore, if there are long-term trends in the prices, so that prices which have increased relatively from b to 0 continues to do so from 0 to t, and prices which have fallen from b to 0 continues to fall, the Lowe index will exceed the Young index. This indicates a long-run tendency for the Lowe index to exceed the Young index."[12]

Equation (4) or (7) only determines the gap between Lowe index and Young index when we apply only one basket in the calculation. If the Young index and Lowe index are calculated using chaining procedures, i.e., more than one basket will be used in the calculation, we will have a more complicated expression of the difference between the Lowe index and Young index. In the following part of this section, we will derive the difference between Lowe index and Young index when two baskets are applied in the calculation.

When two baskets are applied, we can define Lowe index and Young index in the following ways, respectively:

$$(8)\quad P_{Lo}^{t/0} = P_{Lo-b1}^{t/c} \times P_{Lo-b2}^{c/0} = \left(\sum_i \left(\frac{p_i^t}{p_i^c} \right) w_i^{cb2} \right) \left(\sum_i \left(\frac{p_i^c}{p_i^0} \right) w_i^{0b1} \right)$$

and

$$(9)\quad P_{Y}^{t/0} = P_{Y-b1}^{t/c} \times P_{Y-b2}^{c/0} = \left(\sum_i \left(\frac{p_i^t}{p_i^c} \right) w_i^{b2} \right) \left(\sum_i \left(\frac{p_i^c}{p_i^0} \right) w_i^{b1} \right)$$

where time c is the linking period, and since then the new basket b2 is introduced. From period 0 to period c, b1 is the weight reference period. From period c to period t, b2 is the weight reference period. The second components of equation (8) and equation (9), $\sum_i \left(\frac{p_i^c}{p_i^0} \right) w_i^{0b1}$ and $\sum_i \left(\frac{p_i^c}{p_i^0} \right) w_i^{b1}$, are not affected by the observations of the current period

[12] Refer to the revised Chapter 9 Paragraph 9.115 of the ILO CPI Manual.

t, we can call them as *constant linking factors*. The differences between these two linking factors are divergences between Lowe index and Young index accumulated from the previous period. The original weights and the price-updated weights can be defined as follows:

$$w_i^{b1} = \frac{p_i^{b1} q_i^{b1}}{\sum_i p_i^{b1} q_i^{b1}} \quad \text{and} \quad w_i^{0b1} = \frac{\frac{p_i^0}{p_i^{b1}} w_i^{b1}}{\sum_i \frac{p_i^0}{p_i^{b1}} w_i^{b1}}$$

(10)

$$w_i^{b2} = \frac{p_i^{b2} q_i^{b2}}{\sum_i p_i^{b2} q_i^{b2}} \quad \text{and} \quad w_i^{cb2} = \frac{\frac{p_i^c}{p_i^{b2}} w_i^{b2}}{\sum_i \frac{p_i^c}{p_i^{b2}} w_i^{b2}}$$

The mathematical difference between Lowe index and Young index can be expressed by the following equations:

$$\begin{aligned}
P_{Lo}^{t/0} - P_Y^{t/0} &= P_{Lo-b2}^{t/c} \times P_{Lo-b1}^{c/0} - P_{Y-b2}^{t/c} \times P_{Y-b1}^{c/0} \\
&= P_{Lo-b2}^{t/c} \times P_{Lo-b1}^{c/0} - P_{Y-b2}^{t/c} \times (P_{Lo-b1}^{c/0} - A) \qquad \text{Define } A = P_{Lo-b1}^{c/0} - P_{Y-b1}^{c/0} \\
&= (P_{Lo-b2}^{t/c} - P_{Y-b2}^{t/c}) \times P_{Lo-b1}^{c/0} + P_{Y-b2}^{t/c} \times A \\
&= \left[\sum_i (\frac{p_i^t}{p_i^c} - P_{Y-b2}^{t/c})(\frac{p_i^c}{p_i^{b2}} - P_{Y-b2}^{c/b2})(w_i^{b2} / P_{Y-b2}^{c/b2}) \right] \times P_{Lo-b1}^{c/0} + P_{Y-b2}^{t/c} \times A \\
&= \left[\sum_i (\frac{p_i^t}{p_i^c} - P_{Y-b2}^{t/c})(\frac{p_i^c}{p_i^{b2}} - P_{Y-b2}^{c/b2})(w_i^{b2} / P_{Y-b2}^{c/b2}) \right] \times P_{Lo-b1}^{c/0} + (P_{Lo-b1}^{c/0} - P_{Y-b1}^{c/0}) \times P_{Y-b2}^{t/c}
\end{aligned} \tag{11}$$

The summation part in the brackets of equation (11) indicates the divergence between Lowe index and Young index caused by the most current basket updating, when b2 is introduced in the CPI. The other component of the differences between Lowe and Young index indicates the accumulated divergence between Lowe and Young index from period 0 to link period c. The magnitude of this part is related to the choice of price reference year. From equation (11), we can see that whether Lowe index generates higher upward

bias than Young index is determined by three differences: $(\frac{p_i^t}{p_i^c} - P_{Y-b2}^{t/c})$, $(\frac{p_i^c}{p_i^{b2}} - P_{Y-b2}^{c/b2})$ and $(P_{Lo-b1}^{c/0} - P_{Y-b1}^{c/0})$. If we have the same trends in prices from period b2 to current period, t, the first component of equation (11) will be positive; similarly, if the price trends are the same from price reference period to the link period, $(P_{Lo-b1}^{c/0} - P_{Y-b1}^{c/0})$ will be positive. If we always have the same trend in prices, the divergences between Lowe index and Young index will get bigger and bigger as time moves away from the price reference period. If the trends in price movements are different from period to period, we do not have a certain answer as to which index number formula could yield a better estimate of the superlative indexes.

In the following part of this section, we will use Canadian data from December 1994 to December 2007 to examine the empirical relationship between Lowe index and Young index.

2.3 Empirical results

In this section, we use Canadian data to compare Lowe index and Young index. We have information on expenditures, original weights and price-updated weights of each basic class[13] for five baskets, and corresponding price indexes of each basic class going from Dec 1994 to Dec 2007. The following table lists the weight reference years, starting period and link period for the 5 baskets:

Table 1: Weight reference years with starting period and link Period

[13] Basic classes are the lowest-level aggregates of commodities in Canadian CPI, for which a set of weights is available and consistent with the fixed-basket concept.

Weight reference year	1992	1996	2001 (1)	2001(2)[14]	2005
Starting period	Jan 1995	Jan 1998	Jan 2003	Jul 2004	May 2007
Link period	Dec 1994	Dec 1997	Dec 2002	Jun 2004	Apr 2007

When we calculate our CPI, we use 1995 as the price reference period. Therefore, more than one basket is used for some calculations. The following equation is an example that shows how the chaining procedure works in compiling our CPI:

$$(12)\quad P^{2007-10/1995} = P^{2007-10/2007-04} \times P^{2007-04/2004-06} \times P^{2004-06/2002-12} \times P^{2002-12/1997-12} \times P^{1997-12/1995}$$

The calculated annual Lowe indexes and Young index[15] and their differences are reported in the following table:

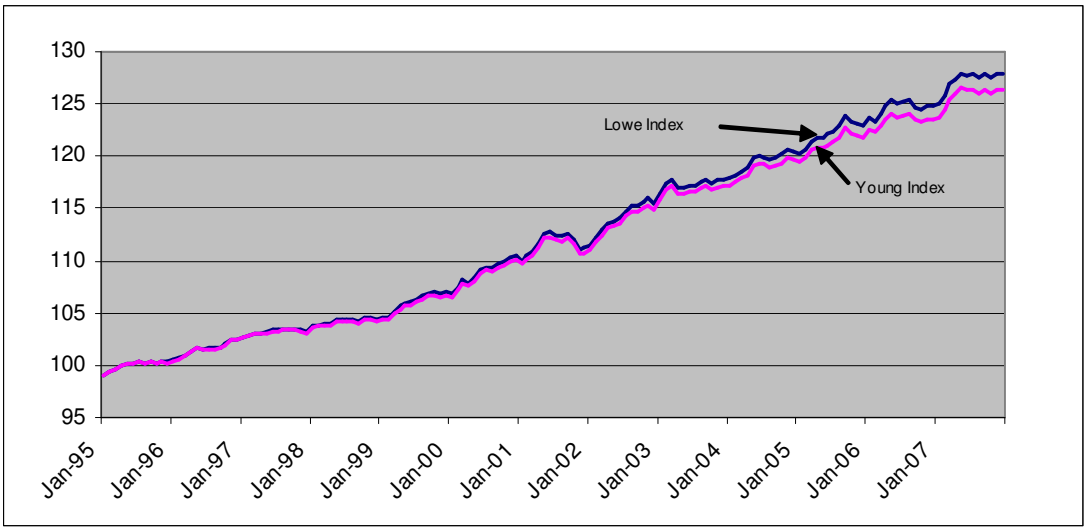
Table 2: Lowe index and Young index (1995=100) for All-items

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Lowe index	101.6	103.2	104.2	106.0	108.9	111.7	114.2	117.4	119.5	122.2	124.6	127.3
Young index	101.5	103.1	104.1	105.8	108.6	111.3	113.7	116.8	118.8	121.2	123.4	125.8
Lowe-Young	0.07	0.11	0.15	0.25	0.34	0.41	0.52	0.59	0.73	0.98	1.25	1.48

We also show these two series of the indexes in the following figure:

Figure 1: Lowe indexes and Young indexes: using monthly data (1995=100)

[14] The mortgage interest rate cost is adjusted for this 2001 basket.
[15] The annual Lowe index and Young index are calculated as simple arithmetic mean of the monthly indexes of each calendar year.



From the above table and figure, we can see that the Lowe indexes, in general, are higher than the Young indexes for this sample period. The longer the comparison period, the larger are the differences between these two series. These results imply that the prices in the sample period exhibit long-term trends.

The following table shows the annual growth rates for the Lowe and Young indexes:

Table 3: Annual inflation rates measured by Lowe index and Young index and their differences.

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Lowe index	1.57	1.62	0.98	1.74	2.73	2.52	2.27	2.75	1.84	2.23	2.00	2.13
Young index	1.50	1.59	0.94	1.65	2.65	2.46	2.18	2.71	1.73	2.04	1.80	1.96
Difference	0.071	0.036	0.042	0.094	0.077	0.058	0.092	0.042	0.117	0.193	0.206	0.163

In general, we obtain higher annual inflation rates in Lowe index than those in Young index. The size of differences is really small before 2004. Since 2004, we have bigger differences in the inflation rates. The average difference for the later period from 2004 to 2007 is 0.17%.

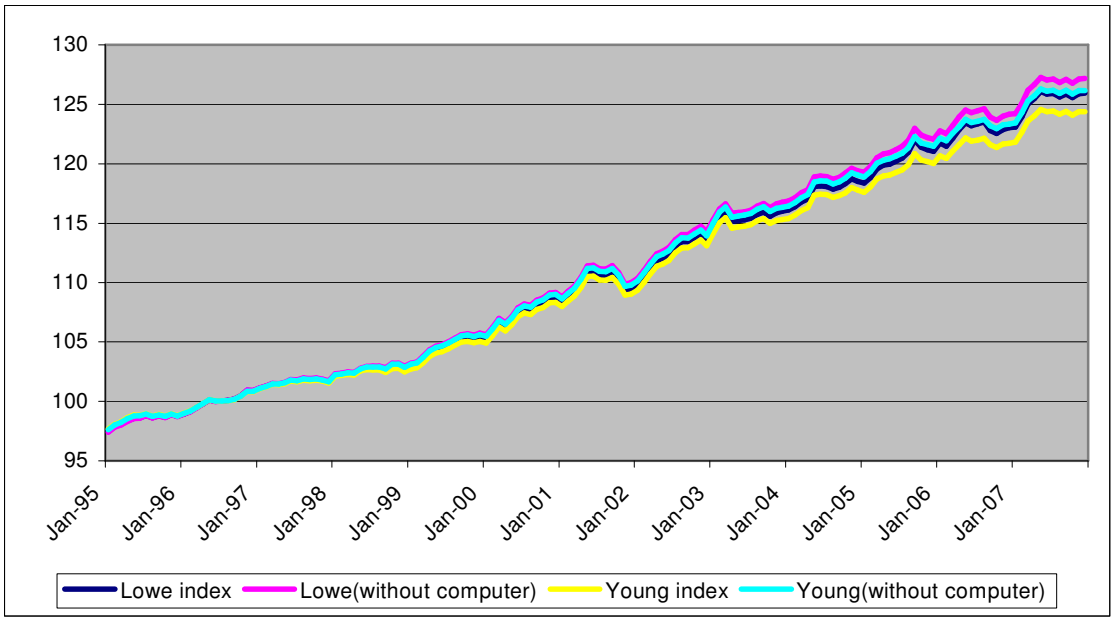
When we did the decomposition of Lowe index and Young index for the most recent period (2007 Dec over 2007 Apr), we find that "Other Fresh Vegetable" contributes most to the negative difference between Lowe index and Young index; while "Mortgage interest cost" and "Computer price index" contribute most to the positive differences between Lowe index and Young index. We use computers as an example to show what the impact of taking "Computers" out of the basket on the divergence between Lowe and Young indexes would be. As consumers, we all benefited from the significant price decline in computers. For the period 1995 to 2007, the price of computer equipment and supplies declines by 89.56%; the relative importance of computer equipment in the All-item CPI is 0.45% in the 1992 CPI basket and 0.84% in the 2005 CPI basket. The following table and figure show the differences between the Lowe index and the Young index with and without computers in the basket:

Table 4: Low index and Young index (with and without computer, 1996=100)

	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Lowe	**101.6**	**102.6**	**104.4**	**107.3**	**110.0**	**112.5**	**115.5**	**117.7**	**120.3**	**122.8**	**125.4**
Young	**101.5**	**102.5**	**104.2**	**106.9**	**109.6**	**111.9**	**115.0**	**117.0**	**119.4**	**121.5**	**123.9**
Difference	**0.086**	**0.144**	**0.242**	**0.328**	**0.398**	**0.507**	**0.567**	**0.712**	**0.953**	**1.219**	**1.443**
Lowe without Computer	**101.7**	**102.8**	**104.8**	**107.7**	**110.5**	**113.0**	**116.2**	**118.5**	**121.3**	**123.8**	**126.6**
Young without Computer	**101.6**	**102.7**	**104.6**	**107.5**	**110.3**	**112.8**	**115.9**	**118.1**	**120.7**	**123.1**	**125.6**

Difference	**0.066**	**0.078**	**0.108**	**0.158**	**0.196**	**0.277**	**0.307**	**0.407**	**0.557**	**0.755**	**0.917**

Figure 2: Lowe indexes and Young indexes (with and without computer price index)



From the above table and figure, we can see that both Lowe index and Young index increase slightly when computers are removed from the calculations. At the same time, removing the Computer price index from the Lowe indexes and Young indexes, the differences between these two indexes are reduced for the period between1996 to 2007.

We also look at the divergence between Lowe indexes and Young indexes for the eight major components of our CPI. The following figures show how these two index series differ for the sample period:

Figure 3: Lowe index and Young index for Food

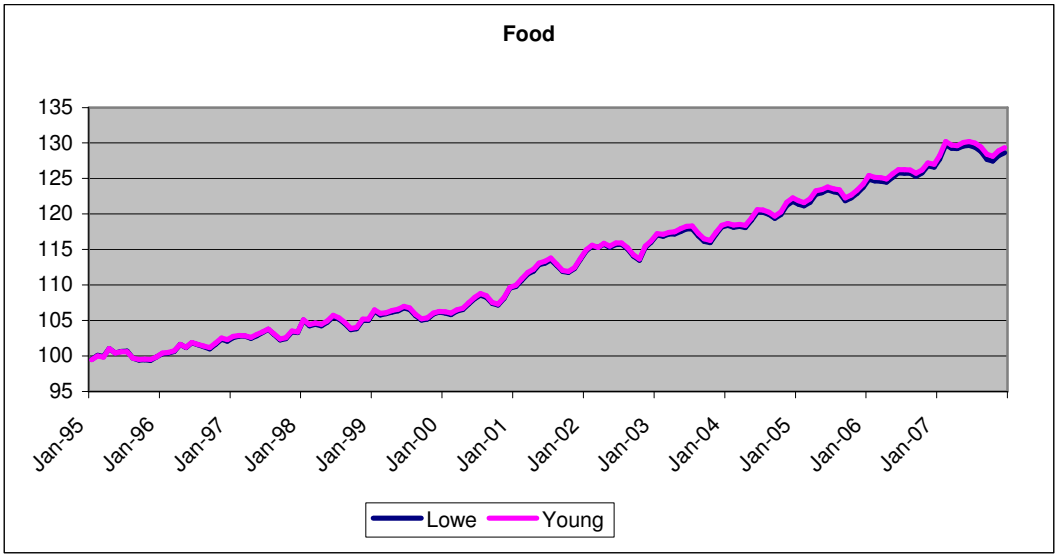


Figure 4: Lowe index and Young index for Household Operations

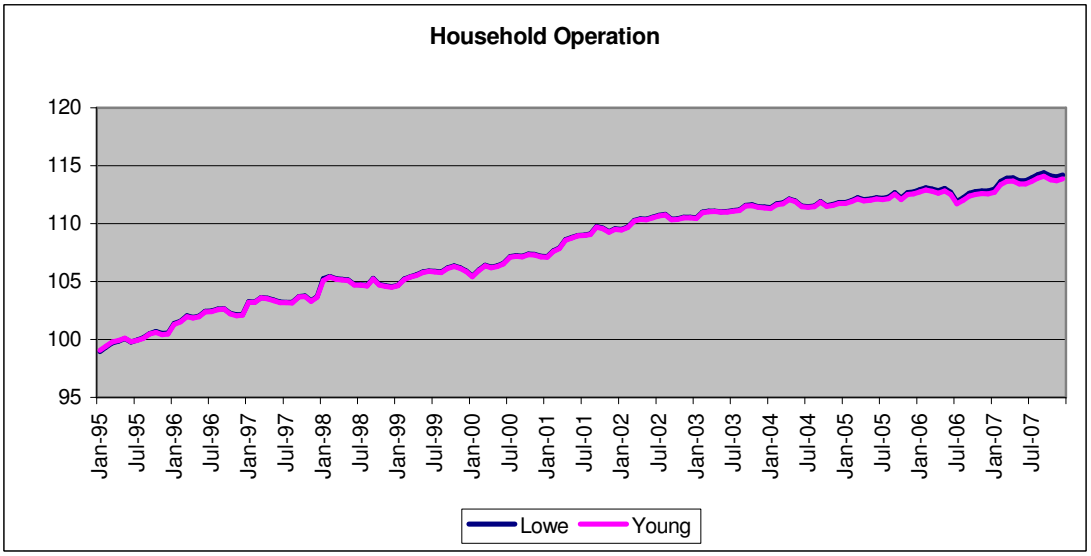


Figure 5: Lowe index and Young index for Shelter:

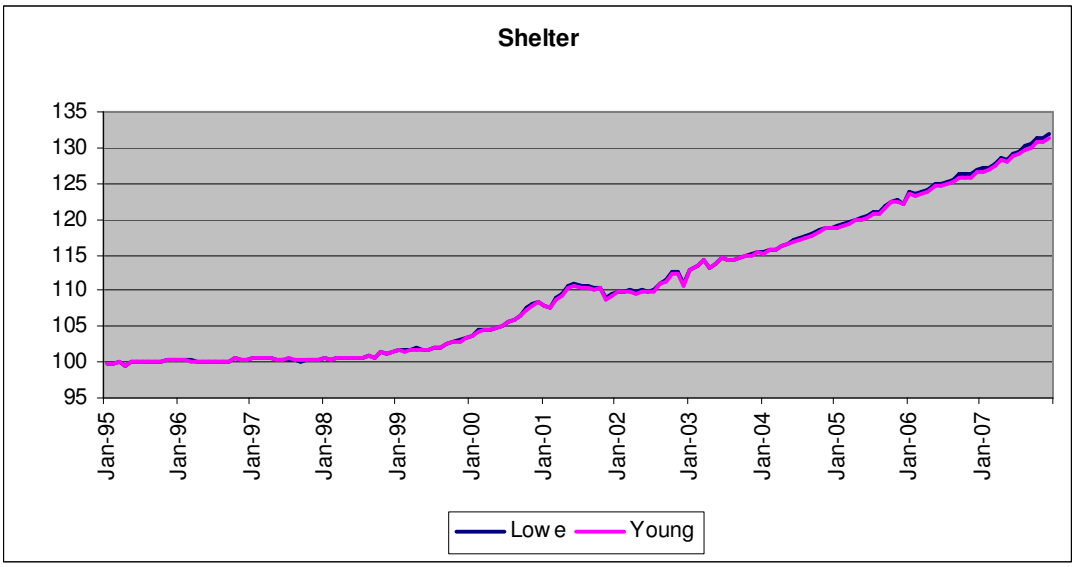


Figure 6: Lowe index and Young index for Clothing and Footwear:

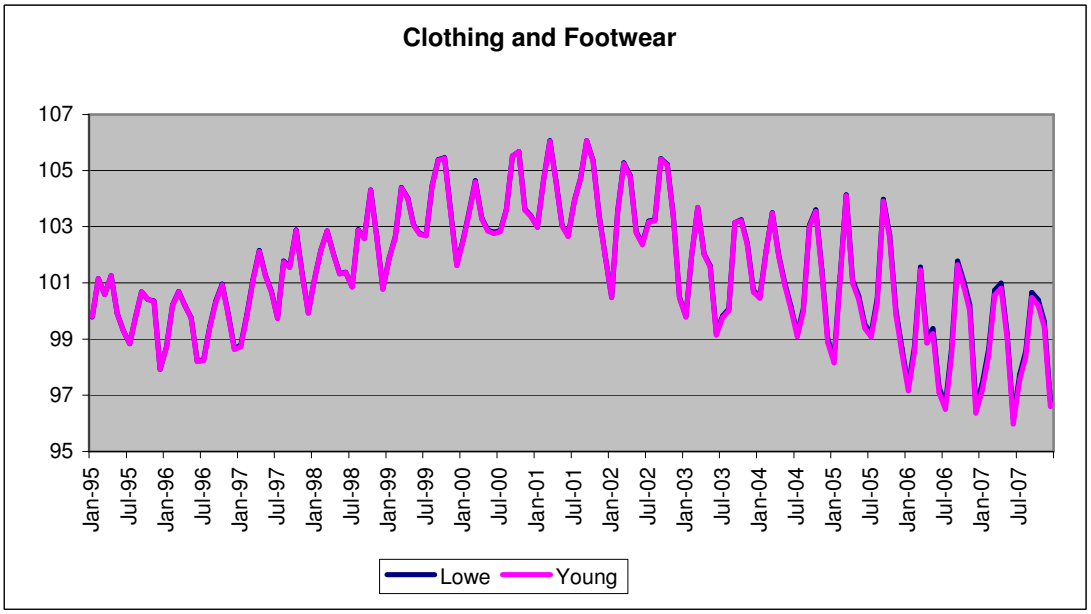


Figure 7: Lowe index and Young index for Transportation

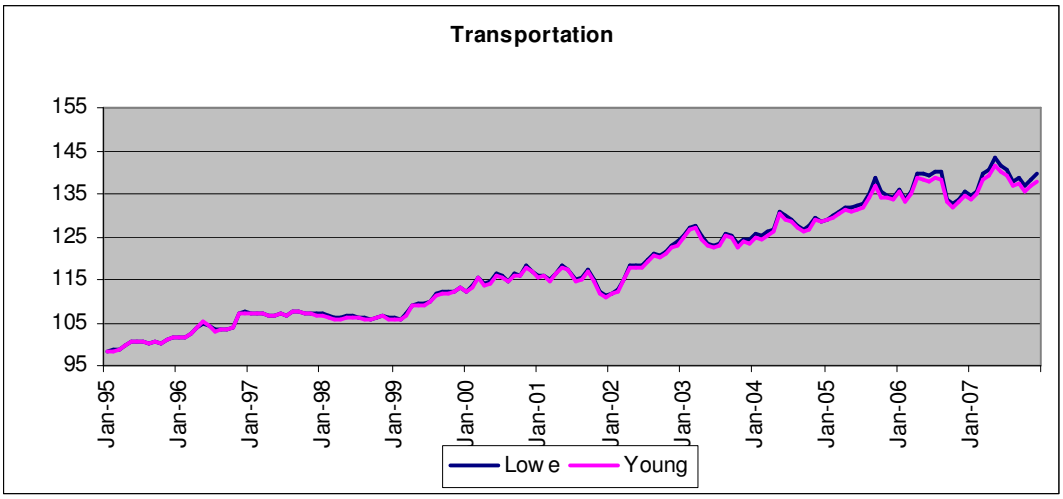


Figure 8: Lowe index and Young index for Recreation

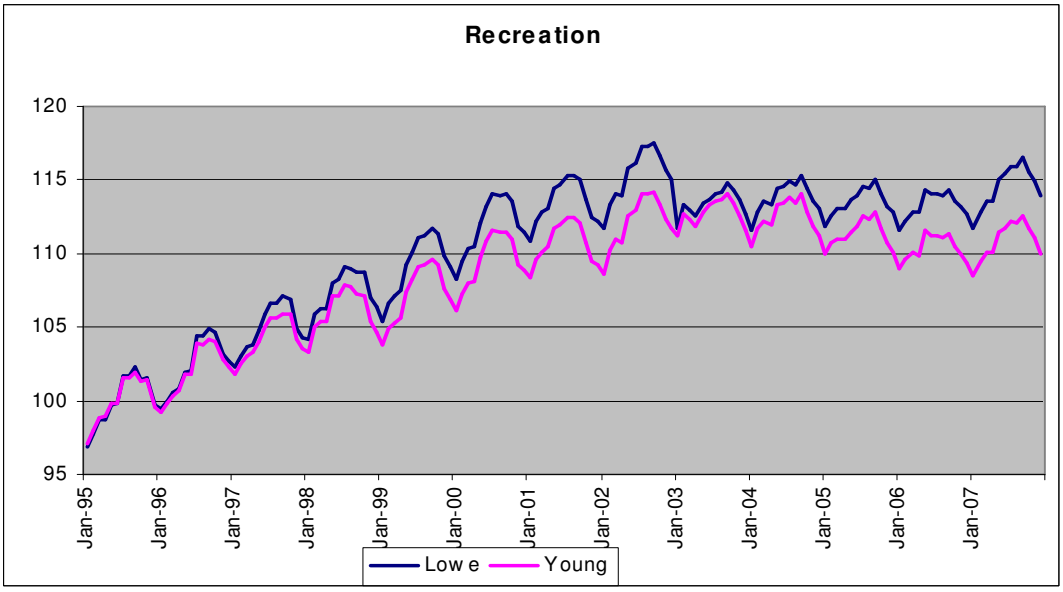


Figure 9: Lowe index and Young index for Health Care

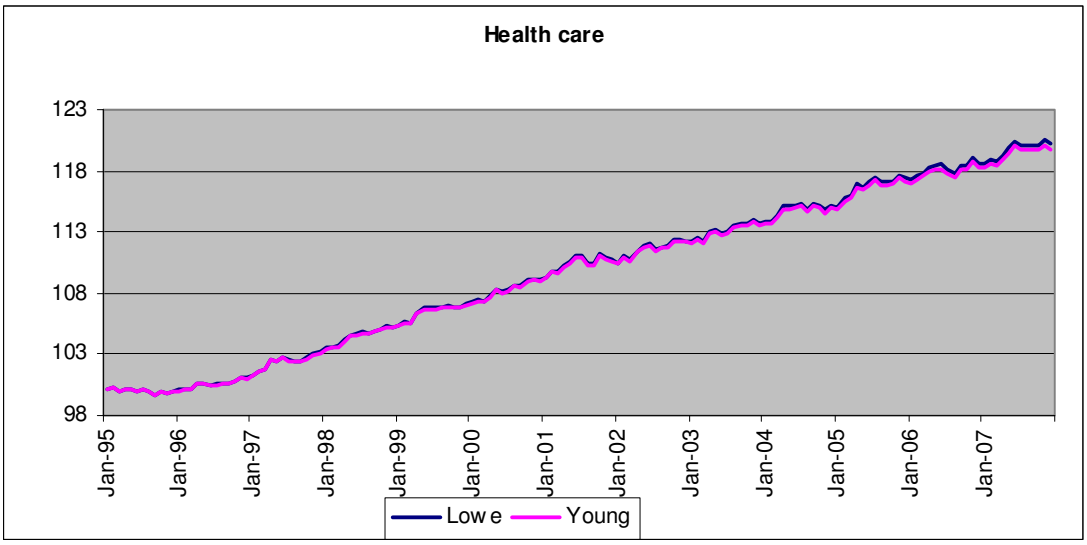
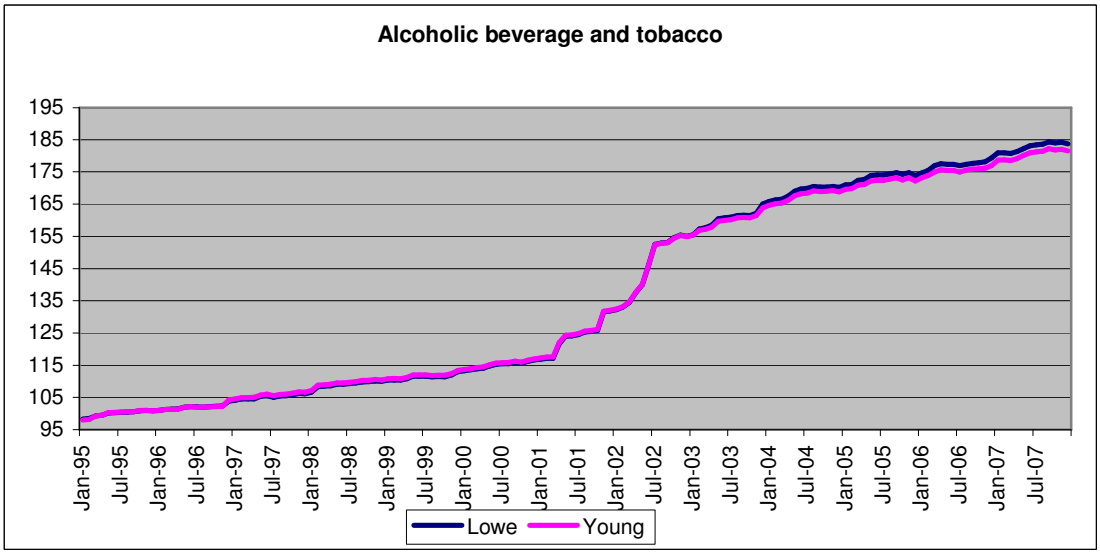


Figure 10: Lowe index and Young index for Alcoholic beverage and tobacco



Comparing Lowe indexes and Young indexes for the eight major components of the CPI, we find that Food is the only component that has higher Young indexes but lower Lowe indexes. For Clothing-and-footwear component the relationship between Lowe index and Young index is mixed for the sample period. The gap between Lowe indexes and Young indexes for Recreation is largest among the eight components. For the other five

components, the Lowe indexes are slightly higher than Young indexes, especially for the latter part of our sample period.

In general, we find that Lowe indexes are higher than Young indexes for the period between 1995 and 2007 using Canadian data. The longer the comparison period is, the bigger the divergences are between these two indexes. This finding indicates the existence of long-term trends in the prices of most items in Canadian CPI basket.

In this section, we compare Lowe index and Young index to identify the impact of price updating weights on Canadian CPI. In the following section, we will conduct comparisons between Lowe index and three superlative price indexes, Fisher, Walsh and Törnqvist indexes, to identify the magnitude of the bias of Lowe index as an estimate of the theoretical target indexes.

3. Lowe index and superlative indexes

Fisher, Walsh and Törnqvist indexes belong to a small class of indexes called "superlative indexes". A characteristic of a superlative index is that it treats the prices and quantities in both periods being compared symmetrically. The index number formulas of these three indexes are flexible and provide second order approximation to each other. In other words, different superlative indexes tend to have similar properties and yield similar results and behave in very similar ways. In addition, they are expected to provide a close approximation to the underlying cost of living index.

A cost of living index (COLI) measures the change in the minimum cost of maintaining a given standard of living. Theoretically speaking, a COLI has some advantages over a fixed-basket index. For example, it allows price-induced substitution of goods and services, and it is not limited to purchase of commodities associated with retail prices. It is, however, extremely difficult to apply in practice.

In this section, we compare Lowe and Young indexes with the three superlative price indexes to identify the magnitude of the bias in Lowe index as an estimate of these indexes. We also try to answer whether Lowe index or Young index generates bigger bias as an estimate of the superlative indexes.

3.1 Data sets

We use Canadian data, including expenditures and prices relatives for all the basic classes for the year of 1992 to 2005. During this sample period, 4 CPI baskets are implemented in the calculation. They are 1992 basket, 1996 basket, 2001 basket (we use 2001 basket after the adjustment on the share of Mortgage Interest Cost Index) and 2005 basket. Because of the introduction of the new baskets, we do not have the same basic classes for the entire comparison period. In order to make our empirical results comparable, we have to obtain consistent basic classes for the whole period. Based on changes from basket to basket, we conduct the following procedures to make the data concordant over time:

1) Leave out some basic classes that do not have any information on them for some periods;
2) Merge some basic classes for some periods;
3) Redefine some basic classes for the whole period under consideration when necessary;
4) Impute price movement for some basic classes from similar ones for some periods.

After conducting the above procedures, we choose 163 basic classes for the sample period. The expenditure share of these 163 basic classes accounts for at least 98.4% of the total expenditure of the original CPI basket. In general, our experimental Lowe indexes are very close to the official CPIs at one-decimal level.

In the following section, we report the main empirical results of the comparison.

3.2 Main empirical results

In order to compare Lowe index with three superlative indexes, we compute consumer price indexes for three different periods, 1992-1996, 1992-2001 and 1992-2005, using different index number formulas, including Laspeyres, Paasche, Fisher, Walsh and Törnqvist. For example, let $P_F^{1996/1992}$ denote Fisher index for the period 1996 over 1992, we can calculate it using the following formula:

$$(12)\quad P_F^{1996/1992} = \left(\frac{\sum_{i=1}^{N} p_i^{1996} q_i^{1992}}{\sum_{i=1}^{N} p_i^{1992} q_i^{1992}} \right)^{1/2} \left(\frac{\sum_{i=1}^{N} p_i^{1996} q_i^{1996}}{\sum_{i=1}^{N} p_i^{1992} q_i^{1996}} \right)^{1/2} = \left(\sum_{i=1}^{N} \frac{p_i^{1996}}{p_i^{1992}} w_i^{1992} \right)^{1/2} \left(\sum_{i=1}^{N} \left(\frac{p_i^{1996}}{p_i^{1992}} \right)^{-1} w_i^{1996} \right)^{-1/2}$$

where w_i^{1992} and w_i^{1996} are expenditure share for item i at year of 1992 and 1996, respectively. They can be defined in the following way:

$$(13)\quad w_i^{1992} = \frac{p_i^{1992} \times q_i^{1992}}{\sum_{i=1}^{N} p_i^{1992} \times q_i^{1992}} \quad \text{and} \quad w_i^{1996} = \frac{p_i^{1996} \times q_i^{1996}}{\sum_{i=1}^{N} p_i^{1996} \times q_i^{1996}}$$

In the following table, we show consumer price indexes calculated using different index number formulas[16]:

Table 5: Comparisons among different price indexes (1992=100)

[16] The numerical results of Lowe index in Table 5 are obtained from CANSIM directly.

	Indexes (1992=100)			Annual Growth Rate (%)			Differences in Growth Rate		
	92-96	92-2001	92-2005	92-96	92-2001	92-2005	92-96	92-2001	92-2005
Laspeyres	106.4	117.3	129.4	1.55	1.78	2.00			
Paasche	106.0	113.9	118.4	1.46	1.45	1.31			
Fisher	106.2	115.6	123.8	1.51	1.62	1.66	-0.06	0.08	0.22
	Chained[17]	**115.6**	**125.0**		**1.62**	**1.73**			
Walsh	106.2	116.2	126.3	1.51	1.68	1.81	-0.07	0.02	0.06
	Chained	**115.8**	**125.3**						
Törnqvist	106.2	116.2	126.8	1.50	1.69	1.84	-0.06	0.02	0.03
	Chained	**115.8**	**125.3**						
Lowe	105.9	116.4	127.3	1.44	1.70	1.87			

Examining Table 5, we find that Lowe index is higher than three superlative indexes except for the period of 1992-1996. For the period of 1992 to 2005, the official CPI indicates that Canadian CPI has increased by 27.3%, while Fisher index indicates a 23.84% increase in consumer prices; Walsh index indicates a 26.35% increase and Törnqvist index indicates a 26.78% increase in the price level. These numerical results imply a yearly average difference of 0.216% between the official CPI and Fisher index, of 0.06% between the official CPI and Walsh index, and of 0.033% between the official CPI and Törnqvist index. One thing we have to point out here is that the official CPI is not really comparable with the superlative indexes in this setting. Official CPI[18] is compiled using the real CPI basket, while the superlative indexes only use 163 basic classes of the CPI basket.

[17] Chained Fisher, Walsh and Törnqvist indexes are reported in bold and blue in this table.
[18] We do not have enough information to compile the experimental Lowe index for this comparison period.

One interesting finding is that our Fisher index (fixed base) for the period of 1992-2005 is much lower than the other two superlative indexes, Walsh and Törnqvist. This finding is inconsistent with the theory that the three superlative indexes should behave in a similar manner. Although, using chaining procedure, we can reduce the differences among the three superlative indexes. We are still interested in exploring possible sources of this divergence. Taking a further look at the above table, we believe that a relatively big gap between Laspeyres and Paasche is the direct reason causing Fisher index to behave differently from the other two superlative indexes. Then, we go further to investigate which item is the biggest contributor to the gap between Laspeyres and Paasche index. A decomposition of the difference between Paasche and Laspeyres indicates that "Computer Equipment and Supplies" (thereafter computer) contributes most to the gap between Laspeyres and Paasche. Thus we remove computer from our basic classes and calculate consumer price indexes using the same index number formulas. The following is another table showing different indexes:

Table 6: Indexes without Computer (1992-2005)

	1992-1996	1996-2001	2001-2005	1992-2001	1992-2005
Laspeyres	106.5	110.7	109.6	117.7	130.0
Paasche	106.3	109.8	108.4	116.9	126.8
Fisher	106.4	110.3	109.0	117.3	**128.4**
Walsh	106.4	110.2	109.0	117.3	**128.5**
Törnqvist	106.3	110.2	109.0	117.3	**128.6**

From the above table, we find that the three superlative indexes are closely approximated to each other after we remove computer from the calculation. It is a little bit surprising to see such a big impact of one single item, which accounts for only a small share (less than 1 percent) of the total CPI basket, on the aggregate level of the CPI.

To avoid the impact of possible measurement errors in the computer prior to 1996 on our calculation, we conduct similar calculation for the period 1996 to 2005. In addition, all the indexes are compiled using same basic classes. We also compile Geometric Young indexes[19] in the following tables. Table 7 reports indexes including computer while Table 8 reports indexes without computer in the calculation:

Table 7: Different Indexes (with computer, 1996-2005)

	Indexes			Difference		
	1996-2001 (1996=100)	2001-2005 (2001=100)	1996-2005 (1996=100)	Lowe (1996-2005)	Young (1996-2005)	GYoung (1996-2005)
Laspeyres	110.0	109.0	121.4			
Paasche	107.8	107.3	112.9			
Fisher	108.9	108.2	117.1	3.26	2.31	0.36
	Chained		**117.8**	**2.55**	**1.59**	**-0.35**
Walsh	109.0	108.2	118.1	2.24	1.28	-0.66
	Chained		**118.0**	**2.35**	**1.39**	**-0.55**
Törnqvist	109.1	108.2	118.5	1.87	0.92	-1.03
	Chained		**118.1**	**2.25**	**1.29**	**-0.65**
Lowe	110.0	109.4	120.3		-0.95	-2.90
Young	109.6	108.8	119.4	0.95		-1.94
GYoung	108.7	108.0	117.4	2.90	1.94	

Table 8: Different Indexes (without computer, 1996-2005)

[19] The annual Geometric Young index is calculated as simple geometric mean of the monthly indexes of each calendar year.

	Indexes			Difference		
	1996-2001 (1996=100)	**2001-2005 (2001=100)**	**1996-2005 (1996=100)**	**Lowe (1996-2005)**	**Young (1996-2005)**	**GYoung (1996-2005)**
Laspeyres	110.7	109.6	122.3			
Paasche	109.8	108.4	118.8			
Fisher	110.3	109.0	120.6	0.74	0.18	-1.17
	Chained		**120.2**	**1.14**	**0.59**	**-0.77**
Walsh	110.2	109.0	120.5	0.77	0.22	-1.14
	Chained		**120.1**	**1.19**	**0.63**	**-0.73**
Törnqvist	110.2	109.0	120.5	0.76	0.21	-1.15
	Chained		**120.1**	**1.18**	**0.62**	**-0.73**
Lowe	110.5	109.8	121.3		-0.56	-1.91
Young	110.3	109.5	120.7	0.56		1.35
GYoung	109.7	108.8	119.4	1.91	1.35	

Note that in Table 7 and Table 8, Lowe, Young and Geometric Young indexes are chained indexes, while the superlative indexes are compiled using fixed bases. If we also calculate chained-superlative indexes, the divergences between Lowe index and superlative indexes will be ***increased***. For example, the chained Törnqvist index (with computer) for the period going from 1996 to 2005 is 118.1, and the resulted difference between Lowe index and the Törnqvist index is 2.35, which is larger than 1.87. There is only one exception, the chained Fisher index (with computer) for the period going from 1996 to 2005, which is caused by the significant price decline in the computer.

For the period of 1996-2005, Fisher index does not behave so differently from the other two superlative indexes as it does in the previous case; however it still differs obviously from the other two superlative indexes. Removing computer from the calculation makes Fisher closer to the other two superlative indexes.

Examining the results, we find that the Lowe indexes yield bigger bias than the Young indexes as the estimates of the superlative indexes. Geometric Young indexes are lower than the three superlative indexes in most cases. In addition, we find that removing computer from the calculation reduces the size of the bias, as well as the gap between the Lowe index and the Young index.

Table 9 and Table 10 show the average annual growth rates in the CPI with computer and without computer in the calculations.

Table 9: Average Annual Growth Rates (with computer)

	Average annual growth rate			Difference		
	1996-2001	2001-2005	1996-2005	Lowe (1996-2005)	Young (1996-2005)	GYoung (1996-2005)
Laspeyres	1.93	2.19	2.17			
Paasche	1.51	1.77	1.36			
Fisher	1.72	1.98	1.77	0.31	0.22	0.04
Walsh	1.74	1.99	1.86	0.21	0.12	-0.06
Törnqvist	1.76	1.99	1.90	0.18	0.09	-0.10
Lowe	1.92	2.28	2.08		-0.09	-0.28
Young	1.84	2.13	1.99	0.09		-0.19
GYoung	1.68	1.95	1.80	0.28	0.19	

Table 10: Average Annual Growth Rates (without computer)

	Average annual growth rate			Difference		
	1996-2001	2001-2005	1996-2005	Lowe (1996-2005)	Young (1996-2005)	GYoung (1996-2005)
Laspeyres	2.06	2.31	2.27			

Paasche	1.89	2.04	1.93			
Fisher	1.97	2.17	2.10	0.07	0.02	-0.11
Walsh	1.97	2.17	2.10	0.07	0.02	-0.11
Törnqvist	1.97	2.17	2.10	0.07	0.02	-0.11
Lowe	2.01	2.37	2.17		-0.05	-0.18
Young	1.97	2.28	2.12	0.05		-0.13
GYoung	1.87	2.14	1.99	0.18	0.13	

From the above two tables, we can see that both Lowe index and Young index yield a higher inflation rate than the superlative indexes; however, Geometric Young indexes yield lower inflation rates than the superlative indexes in most of the comparison period. For the period of 1996-2005, with computer in the calculation, the average yearly difference between Lowe index and Fisher index is **0.31%;** between Lowe index and Walsh index is **0.212%;** between Lowe index and Törnqvist index is **0.17%.** The average yearly differences between Young index and the three superlative indexes are relatively smaller in size compared with the difference between Lowe index and the superlative indexes. The yearly biases between Young index and three superlative indexes, Fisher, Walsh and Törnqvist indexes, are **0.221%, 0.122% and 0.087%** respectively. Again, because of the significant difference between the Fisher and the other two superlative indexes, we obtain different yearly average bias in our CPI.

For the same sample period, excluding computer in the calculation, the average yearly bias generated by Lowe index as an estimate of the three superlative indexes is dramatically reduced. Compared with Fisher, Walsh and Törnqvist indexes, Lowe index yields a yearly bias of **0.07%, 0.073% and 0.072%** on average, respectively. Young index yields even lower bias than Lowe index as an estimate of the three superlative indexes. Therefore, we can see that including computer or excluding computer in the calculation makes significant differences in our results. Computer equipment and supplies account for only a small portion of the Canadian CPI basket, but the changes in their prices are significant in our sample period. Our empirical results show that if the changes in both quantity and price of one item are very big, it would have a significant impact on

the overall price indexes, even if it only accounts for a small portion of the basket. This also implies that we need to be more careful when we make decisions on excluding some items from our CPI basket.

We also compare CPIs excluding eight of the most volatile components (called as Quasi Core CPI in this study) identified by the Bank of Canada with the superlative indexes. This is similar to comparing the Core CPI with the superlative indexes, except that we do not exclude the effect of changes in indirect taxes. The main results are reported in the following tables:

Table 11: Quasi Core CPIs and the Other Price Indexes (with Computer, 1996-2005)

	Indexes			Difference (1996-2005)		
	1996-2001	2001-2005	1996-2005	Lowe	Young	GYoung
Laspeyres	108.0	107.1	116.7			
Paasche	106.1	105.6	109.1			
Fisher	107.1	106.4	112.8	4.08	3.24	1.81
Walsh	107.2	106.5	114.1	2.85	2.01	0.59
Törnqvist	107.3	106.5	114.5	2.44	1.60	0.17
Lowe	108.3	108.0	116.9		-0.84	-2.27
Young	107.9	107.5	116.1	0.84		-1.43
GYoung	107.3	106.9	114.7	2.27	1.43	

Table 12: Average Annual Growth Rates and their Differences (Quasi Core CPI, 1996-2005, with Computer)

	Average annual growth rate			Difference (1996-2005)		
	1996-2001	2001-2005	1996-2005	Lowe	Young	GYoung

Laspeyres	1.55	1.74	1.73			
Paasche	1.20	1.38	0.98			
Fisher	1.37	1.56	1.35	0.40	0.32	0.18
Walsh	1.41	1.58	1.47	0.28	0.20	0.06
Törnqvist	1.42	1.58	1.51	0.24	0.16	0.02
Lowe	1.60	1.94	1.75		-0.08	-0.22
Young	1.53	1.81	1.67	0.08		-0.14
GYoung	1.41	1.68	1.53	0.22	0.14	

Comparing Table 7 with Table 11, and Table 9 with Table 12, we find that the biases in the Quasi Core CPI caused by Lowe Index number formula as the estimates of superlative indexes are slightly higher than those in the All-items CPI. For example, the yearly average difference in All-item CPI calculated using Lowe index number formula is **0.31** percent higher than that computed using Fisher index, while the yearly average growth rate in Quasi Core CPI calculated using Lowe index is **0.4** percent higher than that computed using Fisher. The Quasi Core CPI compiled using Geometric Young index formula is slightly higher than those based on the three superlative indexes. We also notice that the gap between Fisher index and the other two superlative indexes are also enlarged when we exclude the 8 of the most volatile components from the computation of the prices indexes. One possible reason to explain this divergence is that the relative importance of the computer equipment in the CPI basket increases after the exclusion.

4. Conclusion

Mathematically speaking there is no fixed relationship between the Lowe index and the Young index. The empirical results of which index number formula yields a bigger upward bias depend on the price movements during the examined period. Canadian data for the period of 1996-2007 shows that the procedure of price-updating expenditure shares adds higher upward bias to the superlative indexes than using original expenditure

shares. Lowe indexes also yield higher average annual growth rates of the prices than Young indexes and the three superlative indexes for the period of 1996-2005.

Even though our calculation shows that Young index is a better estimate of a superlative index than the Lowe index for the period under consideration, we do not suggest switching to the use of Young index. Lowe index is a fixed-basket index that reflects the pure price change for the comparison period, while Young index is a fixed-weight index that reflects weighted price change. If we plan to switch to Young index, we have to change the fundamental concept used in the Canadian CPI, and make changes to our system to accommodate the different data requirements. In addition, the Young index has its own problems. For example, it does not satisfy some important axiom tests, such as the time reversal test and transitivity test. Non-transitivity of the Young index might cause some practical problems in the index calculation. Instead of switching to the Young index, we could give a serious thought to the Geometric Young index. As the CPI pointed out, it is "*likely to be less subject than their arithmetic counterparts to the kinds of index number biases*".

Our empirical results also imply that the procedure of price-updating weights cannot make our CPI basket more representative of the consumer expenditure pattern[20]. In order to keep our basket representative of the spending pattern of consumers, we should update our basket more frequently and regularly.

Both the Lowe index and the Young index would generate a certain amount of upward bias in the CPI as an estimate of a superlative index. It is useful for CPI compilers to have knowledge about the scale of the bias. Because the size of the bias depends on the period under review, we should be calculating the retrospective superlative indexes regularly as suggested by the ILO's "Resolution Concerning Consumer Price Indexes" once the required data are available.

[20] There is some misunderstanding about price-updating procedure. It is believed that it could make a CPI basket more representative of the consumers' current expenditure pattern.

Our numerical results indicate that the Fisher ideal index behaves differently from the Walsh and Törnqvist indexes in some circumstances. Thus there is also an issue for us to choose an appropriate comparison target when we try to identify the upward bias in our CPI.

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