Room document at the Ottawa Group Meeting, 2011

Aggregate Properties of Two-Staged Price Indices

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This version: 19 April 2011

Abstract

The present paper contributes to the literature by looking at the empirical evidence of calculations of Laspeyres price indices formed from different elementary indices. For three German price statistics, disaggregate official data are analysed. Generalised means of price relatives are systematically calculated and plugged into the aggregate formula.

The results point to widely different estimates between Laspeyres price indices based on the alternative elementary indices. There is a "price" to be paid at the upper level for suboptimal index formula selection at the lower level. Thus, the need for two-staged price indices to be accurately constructed becomes obvious.

Keywords: Consistency in Aggregation, Laspeyres Price Index, Elementary Indices, Generalised Mean.

JEL: C43, C82, E01, E31.

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1 Motivation

Most price indices in official statistics are calculated in two stages. At the lower level, where no share weights are available, unweighted means of price relatives are taken to form elementary indices. At the upper level, share weights are used to calculate the aggregate index on the basis of the elementary indices. Here, the target of measurement determines the index formula. The European Statistical System aims at tracking genuine price movements and, hence, the Laspeyres price index is chosen. However, it is less clear which index formula should be selected at the lower level.

Thus, a relevant, although often neglected, issue in practice is the numerical relationship between elementary and aggregate indices. This is because if the elementary indices do not reflect the characteristics of the aggregate index, a two-staged index can lead to a different conclusion than that reached by the price index calculated directly from the price relatives. Mehrhoff (2010) demonstrates that every weighted index can be expressed one-to-one as a “generalised mean”. This facilitates the determination of the elementary index that corresponds to the desired aggregate index.

Some authors present empirical evidence for differences, mainly in consumer price indices, due to the choice of the index formula at the lower level. For instance, Dalén (1998) shows using Swedish data that the Carli index consistently gives results which are year-on-year two index points or more larger than the Dutot and Jevons indices, while the latter two indices are fairly close to each other. This is of particular importance since the Commission Regulation (EC, 1996, Article 7 in conjunction with Annex II) de facto abandons the use of the Carli index as it would have to be shown that on average the results do not differ annually by more than one-tenth of a percentage point from either the Dutot or Jevons index.

The present paper contributes to the literature by looking at the empirical evidence of calculations of Laspeyres price indices for three German statistics formed from different elementary indices. The remainder of the paper is organised as follows. Section 2 describes the methodology applied and introduces the data used. Empirical results of the calculations are presented in Section 3. The final section concludes.

2 Background

The Laspeyres price index, among others, is consistent in aggregation. This means that the result of a two-staged index calculation coincides with that of a calculation in a single stage. However, when statistical offices cannot use a share-weighted formula at the lower level of the aggregation process, owing to the unavailability of this information, they have to rely on an unweighted index. This elementary index bias is applicable irrespective of which unweighted index is used. In other words, if the elementary index coincides with the Laspeyres price index, the bias will vanish.

First of all, the generalised mean and the Laspeyres price index are defined. Then, the data and methodology are described.

Definition. Let p_{kjt}/p_{kj0} denote the price relative of the j^{th} good in the k^{th} group of goods at time t, where $j = 1, 2, \dots, n_k$ and $k = 1, 2, \dots, K$. Furthermore, let all price relatives be positive real numbers, $0 < p_{kjt}/p_{kj0} < \infty \, \forall \, j, k$. Then, their generalised mean of order $r \in \mathbb{R}$ for the k^{th} group of goods at time t is defined in Equation (1).

$$P_{kt}^{GM}(r) = \begin{cases} \sqrt[r]{\frac{1}{n_k} \sum_{j=1}^{n_k} \left(\frac{p_{kjt}}{p_{kj0}} \right)^r} & r \neq 0 \\ \sqrt[n_k]{\prod_{j=1}^{n_k} \frac{p_{kjt}}{p_{kj0}}} & r = 0 \end{cases} \tag{1}$$

The generalised mean represents a whole class of unweighted elementary indices, such as the Carli and Jevons indices for $r = 1$ and $r = 0$, respectively. Hardy et al. (1934) discuss the generalised mean in great detail and prove its properties. It covers the whole range between the smallest and largest price relative and it is a continuous function in its argument r. Moreover, by Schlömilch's inequality, the generalised mean is strictly monotonic increasing, i.e. $P^{GM}(r) < P^{GM}(r_0) \, \forall \, r < r_0$, unless all price relatives are equal.

Definition. The (two-staged) Laspeyres price index, dependent on the order of the generalised mean r, is the arithmetic mean of elementary indices $P_{kt}^{GM}(r)$ with base period share weights $w_{k0} \geq 0$, as defined in Equation (2).

$$P_t^L(r) = \sum_{k=1}^{K} P_{kt}^{GM}(r) w_{k0}, \sum_{k=1}^{K} w_{k0} = 1 \tag{2}$$

Movements of this price index are due to changes in prices alone because the index formula applies fixed (base period) share weights.

Disaggregate official data used in this study stem from three German statistics: the index of import prices, the index of export prices and the index of producer prices for industrial products (domestic sales). The data which are analysed below are the actual prices surveyed at companies by the German Federal Statistical Office and used for the official index calculation. The data set covers the period from January 2005 to November 2010. In addition, base period share weights were supplied, which are derived from foreign trade statistics in the cases of import and export price indices and from turnover and production statistics in the case of the producer price index. This allows the calculation of two-staged Laspeyres price indices.

Assuming log-normal distribution of prices and quantities, Mehrhoff (2010) uses a partial adjustment model to derive the elementary indices that coincide in expectation with the Laspeyres price index in foreign trade. In particular, a generalised mean of order equal to minus the price elasticity of the supply-demand equilibrium yields approximately the same result as the Laspeyres price index. The empirical findings point at the Carli index ($r = 1$) as the corresponding elementary index. Applying the same methodology to producer prices, however, the Jevons index ($r = 0$) performs best at the lower level.

The official publications for all three statistics are based on the Carli index at the lower level. Thus, while the indices of import and export price will be unbiased, the index of producer prices will have a bias. This actual bias and the hypothetical biases of the various formulae at the lower level for all three statistics are quantified below. Specifically, generalised means are systematically calculated and plugged into the aggregate formula.[1] The variation of the order r covers the band from -2 to $+4$ – the lowest and highest estimate, respectively, in the aforementioned empirical study. The basic idea behind this approach is that different elementary indices implicitly weight price relatives differently, although they do not imply an explicit expenditure structure.

3 Results

3.1 Index Levels

For each of the three statistics under consideration, Figure 1 displays the time series of Laspeyres price indices formed from generalised means of orders $+4$ ("maximum"), $+1$ (Carli index), 0 (Jevons index) and -2 ("minimum").

The maximum and minimum are drifting apart at an average annual rate between 0.6% and 1.0%. The difference between Carli and Jevons indices is much lower and the drift rate does not exceed one-sixth of a percent. This is of particular importance for producer price indices since, at the lower level, the official calculation is not performed with the index formula that comes closest to the Laspeyres price index. However, the "price" to be paid for this inaccuracy is rather low. Nonetheless, the upward bias of the Carli index is visible. It should be noted that import and export price indices would have a downward bias if the Jevons index were used. This clearly shows that no statistical one-size-fits-all approach exists but that each subject matter has to be considered separately.

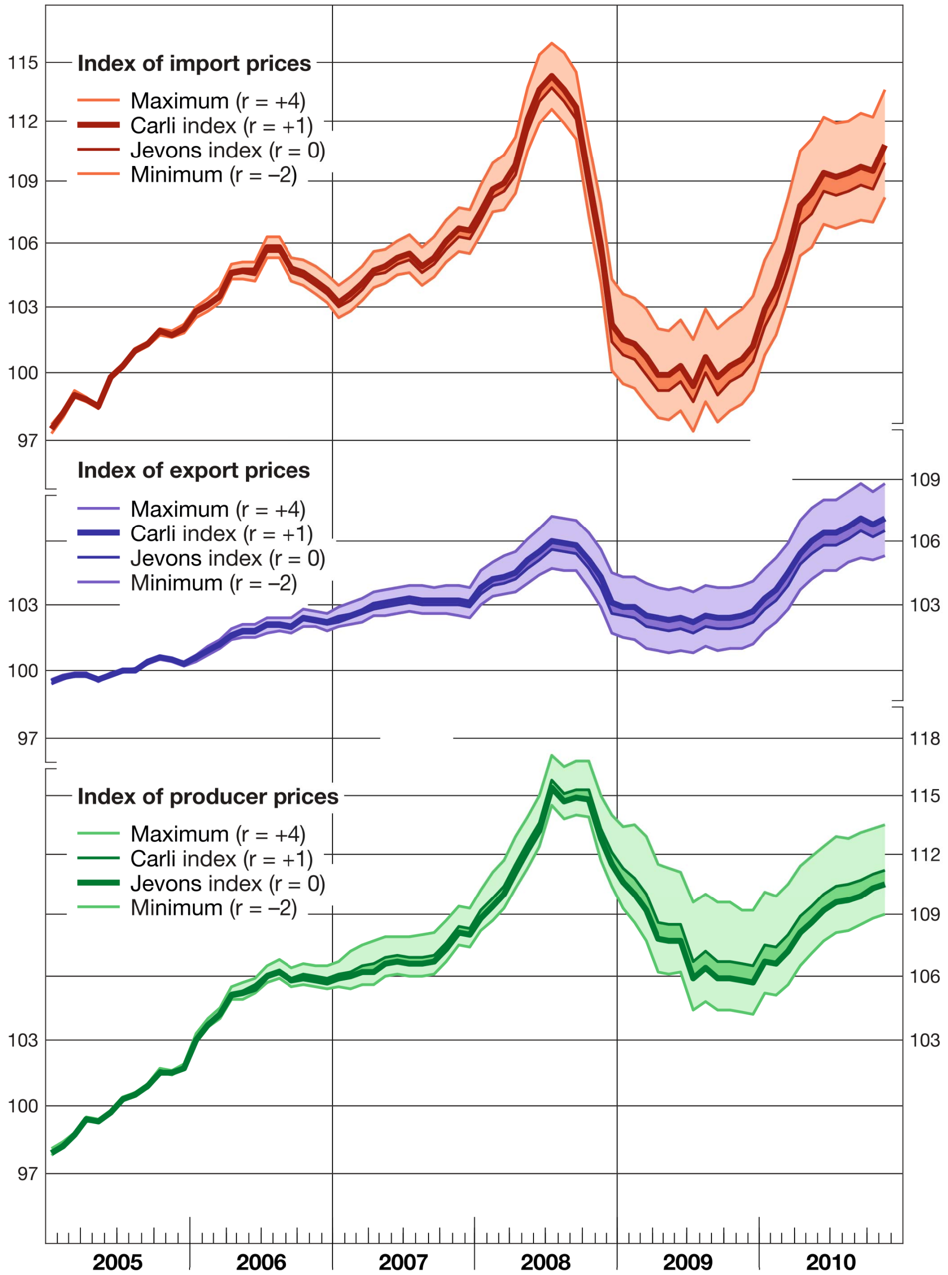
The fact that the indices necessarily move away from each other is also shown analytically below. The relative deviation between any two Laspeyres price indices can be expressed as a sharp mathematical relation. From this it follows that the elementary indices are consistently divergent, which results from increasing variation in the underlying groups of goods over time.

[1] The calculations here match the official methodology exactly except for one minor detail. Some products from the area of energy and water in the producer price index have firm weights in practice, i.e. these groups are not calculated using the Carli index at the lower level but the Laspeyres price index. However, this fact is disregarded hereinafter. Since the following results are derived by comparing price indices among each other rather than to the official figures, this by no means limits their validity.

Figure 1

Laspeyres price indices with alternative elementary indices

2005 = 100, log scale



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If the approach of Diewert (1995) (second-order Taylor series approximations of elementary indices around equal price relatives) is universalised to the generalised mean of order r, the generalised mean is related to the Carli index ($r = 1$) via the expression in Equation (3) below.

$$P_{kt}^{GM}(r) \approx P_{kt}^{GM}(1)\left(1 + \frac{1}{2}(r-1)CV_{kt}^2\right) \tag{3}$$

CV_{kt}^2 is the squared coefficient of variation of the price relatives, i.e. the square of the ratio of their standard deviation to their mean.

Using the transitivity property of the above result, one can easily verify the expression in Equation (4) for the relative deviation between any two indices.

$$\frac{P_{kt}^{GM}(r)}{P_{kt}^{GM}(r_0)} - 1 \approx \frac{\frac{1}{2}(r-r_0)CV_{kt}^2}{1 + \frac{1}{2}(r_0-1)CV_{kt}^2} \tag{4}$$

In particular, a the comparison of the Jevons index ($r = 0$) and the Carli index ($r_0 = 1$) shows that the Jevons index will lie below the Carli index by half the squared coefficient of variation of the price relatives. This implies that, ceteris paribus, the deviation between the two indices will be greater, the larger the heterogeneity of the group of goods becomes.

If, in addition, the definition of the Laspeyres price index is used, it is even possible to derive such a formula at the upper level as in the following Equation (5).

$$\begin{aligned} \frac{P_t^L(r)}{P_t^L(r_0)} - 1 &= \sum_{k=1}^{K}\left(\frac{P_{kt}^{GM}(r)}{P_{kt}^{GM}(r_0)} - 1\right)\frac{P_{kt}^{GM}(r_0)w_{k0}}{\sum_{k=1}^{K} P_{kt}^{GM}(r_0)w_{k0}} \\ &\approx \frac{\frac{1}{2}(r-r_0)\sum_{k=1}^{K} CV_{kt}^2 \frac{P_{kt}^{GM}(1)w_{k0}}{\sum_{k=1}^{K} P_{kt}^{GM}(1)w_{k0}}}{1 + \frac{1}{2}(r_0-1)\sum_{k=1}^{K} CV_{kt}^2 \frac{P_{kt}^{GM}(1)w_{k0}}{\sum_{k=1}^{K} P_{kt}^{GM}(1)w_{k0}}} \end{aligned} \tag{5}$$

Turning again to the Jevons-Carli example, the Laspeyres price index with Jevons indices at the lower level will lie below that with Carli indices at the lower level by half the weighted mean of the squared coefficients of variations. The (implicit) weights are the individual relative contributions of each group of good to the aggregate Laspeyres price index with elementary Carli indices.

3.2 Growth Rates

The Table presents summary statistics of the deviations of 59 monthly year-on-year growth rates between Laspeyres price indices based on the appropriate formula and those based on the various alternative elementary indices. To reiterate, in the case of import and export price indices, the Carli index is the reference at the lower level; in the case of the producer price index, the Jevons index is the reference.

The location parameters mean and median carry the sign that was to be expected given the results of the above Taylor approximations. In particular, if the order of the generalised mean is lower than that of the reference index, the parameters turn out to be negative – and vice versa. Dispersion parameters are a function of the absolute difference between r and r_0. The further the elementary index is from the reference index, the greater is the variation of deviations in terms of standard deviation / root mean square error and minimum / maximum, respectively. As regards the sign of the growth rates, almost no problems exist in the identification of turning points.

For import and export price indices, three-quarters of the mean square error is due to differences in the means, while the variances between the annual growth rates of Laspeyres price indices based on the various elementary indices are virtually identical. By contrast, one-seventh of the mean square error of the producer price index is due to differences in the variance and only half of the MSE can be explained with recourse to differences in the mean. Skewness behaves in a similar manner to the location parameters. Excess kurtosis is positive on average for foreign trade price indices, while it is negative throughout for the index of producer prices. It is noteworthy that the discrepancies between import and export price indices on the one hand and the producer price index on the other hand do not depend on the choice of the reference index.

For the famous Jevons-Carli example, Figure 2 illustrates the distribution of deviations between year-on-year growth rates of Laspeyres price indices formed from these two elementary indices.

The figure demonstrates the empirical differences between the two most used formulae, the choice of which is also subject to discussion. Skewness and excess kurtosis of deviations of the indices of import prices and producer prices are close to

Table: Summary statistics of deviations of growth rates of Laspeyres price indices with different elementary indices

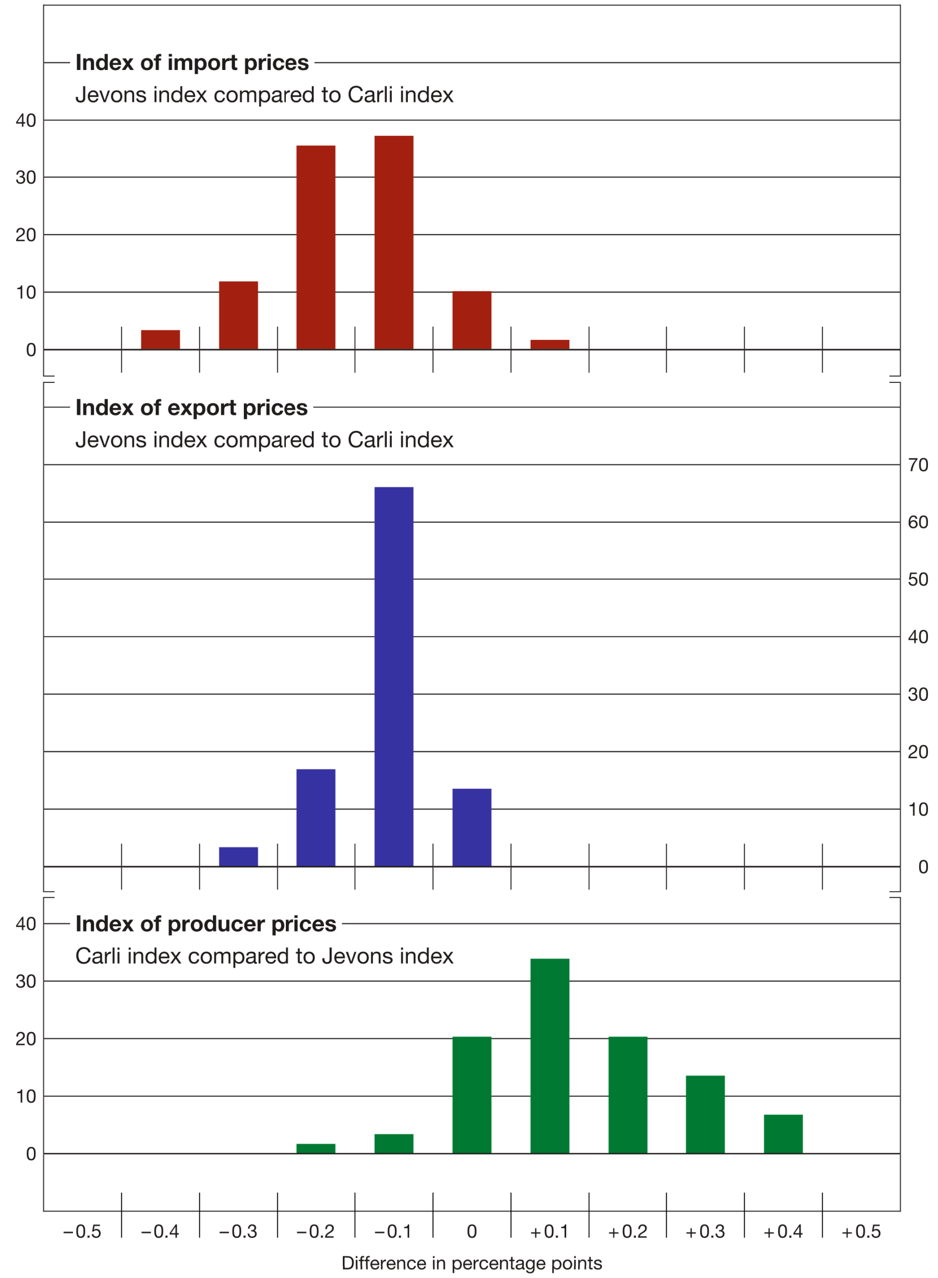
		Import price index ($r_0 = +1$)						Export price index ($r_0 = +1$)						Producer price index ($r_0 = 0$)					
Statistic†	$r =$	−2	−1	0	+2	+3	+4	−2	−1	0	+2	+3	+4	−2	−1	+1	+2	+3	+4
Mean	(pp)	−0.4	−0.3	−0.2	0.2	0.3	0.5	−0.3	−0.2	−0.1	0.1	0.2	0.3	−0.3	−0.1	0.1	0.3	0.4	0.6
SD	(pp)	0.2	0.1	0.1	0.1	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.1	0.1	0.3	0.5	0.7
RMSE	(pp)	0.5	0.3	0.2	0.2	0.4	0.5	0.4	0.3	0.1	0.1	0.2	0.3	0.4	0.2	0.2	0.4	0.6	0.9
MSE Mean	(%)	79	82	71	71	80	82	75	74	74	59	71	73	62	55	52	50	46	43
MSE Var	(%)	1	2	1	3	1	1	0	0	0	0	0	0	11	8	13	16	17	16
MSE Cov	(%)	19	16	28	26	18	17	25	26	26	41	29	27	27	37	35	34	37	41
Minimum	(pp)	−1.0	−0.7	−0.4	−0.1	0.0	0.0	−0.7	−0.5	−0.3	−0.2	−0.1	−0.2	−0.8	−0.4	−0.2	−0.3	−0.5	−0.8
Median	(pp)	−0.4	−0.3	−0.2	0.1	0.3	0.4	−0.3	−0.2	−0.1	0.1	0.2	0.3	−0.3	−0.1	0.1	0.2	0.4	0.5
Maximum	(pp)	0.1	0.0	0.1	0.4	0.7	1.0	0.0	0.0	0.0	0.3	0.6	0.7	0.1	0.1	0.4	0.8	1.3	1.8
Skewness		−0.4	−0.5	−0.1	0.3	0.4	0.6	−0.8	−0.3	−0.6	−0.2	0.3	0.0	−0.4	−0.1	0.1	0.2	0.2	0.1
Excess		0.4	0.1	0.3	−0.1	−0.2	0.2	−0.2	−0.4	1.3	2.1	1.4	0.2	−0.2	−0.4	−0.1	−0.5	−0.3	−0.3
Sign	(%)	97	97	100	100	100	97	100	100	100	100	100	100	98	100	100	98	98	97

† The first three rows display the means, standard deviations (SD) and root mean square errors (RMSE) in percentage points (pp) of the empirical differences between the year-on-year growth rates of $P_t^L(r)$ and $P_t^L(r_0)$. In the subsequent three rows, the MSE is decomposed into the proportions that are due to differences in the means and differences in the variances as well as the (unsystematic) covariance component. Minimum, median and maximum are given in the next three-row block. In the first two of the last three rows, skewness and excess (kurtosis) are to be found. The final row shows the ratio of equal signs in the growth rates.

Figure 2

Distribution of deviations of year-on-year growth rates of Laspeyres price indices

Relative frequency in %



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zero. Yet, deviations of the index of export prices are left-skewed and leptokurtic. This means that the left tail is longer compared to a symmetric distribution and that the peak around the mean is more acute than that of a normal distribution.

Combining the results from the table and the figure, one can draw conclusions on the properties of the underlying data. High positive excess kurtosis, large absolute skewness and virtually no variance component in the decomposition of the MSE indicate that the groups of goods of the export price index have the lowest coefficients of variation. Next in line are the groups of goods of the import price index. The groups of goods of the producer price index have by far the highest coefficients of variation as excess kurtosis is negative and a considerable part of the MSE is due to differences in the variances.

4 Summary

The conclusions of this paper are two-fold. The results point to widely different estimates between Laspeyres price indices based on the alternative elementary indices. This is due to the fact that the elementary indices may not even be close to the desired target index. The main argument for the notable differences found is a relative broad item description, leading to aggregation of highly heterogeneous items, meaning that the choice of the elementary index is significant. The price indices will differ if prices exhibit dispersion, i.e. if the observed price relatives have increasing variances over time (cf. Silver and Heravi, 2007). Conversely, the more homogeneous the groups of goods are, i.e. the lower the coefficients of variation are, the lower the variability of the aggregate indices on the certain order of the generalised mean will be.

Finally, on the practically more relevant issue of biases in the index calculation: although the producer price index inaccurately uses the Carli index at the lower level, rather than the Jevons index, which would be the a priori correct choice, the resulting distortion is in fact negligible. This result stems from a rather low heterogeneity of the groups of goods and, as such, accounts for the high quality of German price statistics. Thus, the importance of the lower level and the elementary index cannot be emphasised enough.

References

European Commission (1996), "Commission Regulation (EC) No 1749/96," *Official Journal of the European Communities*, L 229, 3-10.

Dalén, J. (1998), "Studies on the Comparability of Consumer Price Indices," *International Statistical Review*, 66, 83-113.

Diewert, W.E. (1995), "Axiomatic and Economic Approaches to Elementary Price Indexes," *NBER Working Paper*, 5104.

Hardy, G.H., Littlewood, J.E., and Pólya, G. (1934), *Inequalities*, Cambridge, United Kingdom: Cambridge University Press.

Mehrhoff, J. (2010), "Aggregate Indices and Their Corresponding Elementary Indices," *Journal of Economics and Statistics*, 230, 709-725.

Silver, M., and Heravi, S. (2007), "Why Elementary Price Index Number Formulas Differ: Evidence on Price Dispersion," *Journal of Econometrics*, 140, 874-883.