DEUTSCHE BUNDESBANK
EUROSYSTEM

The CIA (consistency in aggregation) approach

A new economic approach to elementary indices

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* This presentation represents the author's personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank or its staff.

Outline

1. Motivation
2. Test approach
3. Stochastic approach
4. Economic approach
5. Consistency approach
6. Discussion

"Elementary, my dear Watson!" (Sherlock Holmes)

1. Motivation
National Statistician's consultation

− Background: Options for improving the UK's national measure of inflation, the Retail Prices Index (RPI), that have been proposed in **the National Statistician's consultation**.

− Current formulae used in the RPI (ONS, 2012):
  - **Carli: 27% by expenditure weight, 39% by number of items**
  - Dutot: 30% by expenditure weight, 46% by number of items

− Although the scope of the discussion is on **the choice of the index formula at the elementary level**, this choice eventually **depends on the target price index** at the aggregate level.

− RPI is not intended to measure the cost of living (COLI), rather, it is **a cost of goods index (COGI)**.

1. Motivation
Financial Times vs. Royal Statistical Society

- In a recent Financial Times (FT) article, economics editor Chris Giles cast **doubt on the Carli index, i.e. the arithmetic mean**, which is used to calculate the average price of a sub-set of items in the RPI.
- He cited it as **the main cause of the increasing disparity** between RPI and the Consumer Price Index (CPI).
- "**Every year the Carli index** remains part of the RPI calculation, it **imposes a tax of** a little under **£1 bn on society** to give windfall benefits to the holders of index-linked government debt," Giles warned.

- "There is **a fear that the Consumer Price Index underestimates inflation through** the way in which **the geometric mean** is used in its calculation," Jill Leyland, Vice-President of the Royal Statistical Society (RSS), responded in a letter also published by the FT.
- The RSS pointed out that "**CPI also lacks public confidence**."

1. Motivation
Two-staged index calculation

- Practical **consumer price indices are constructed in two stages**:

  1. **a first stage at the lowest level of aggregation** where price information is available but associated expenditure or **quantity information is not available** and
  2. **a second stage of aggregation** where **expenditure information is available at a higher level of aggregation**.

- Paragraph 4 of the 2003 ILO Resolution concerning consumer price indices advises that the CPI should "provide **an average measure of price inflation for the household sector** as a whole, for use as a macro-economic indicator."

- Problem: **The target index is not well defined statistically** (but this topic is part of ONS' research programme).

1. Motivation
Bilateral price indices

- We specify **two accounting periods**, $t \in \{0, 1\}$, for which we have micro price and quantity data for n commodities (bilateral index context).
- Denote the **price and quantity** of commodity $i \in \{1, \ldots, n\}$ in period t by p_i^t and q_i^t, respectively.

- A very simple approach to the determination of a price index over a group of commodities is **the (fixed) basket approach**.
- Define the **Lowe** (1823) **price index**, P_{Lo}, as follows:

- $$P_{Lo} = \frac{\sum_{i=1}^{n} p_i^1 \cdot q_i}{\sum_{i=1}^{n} p_i^0 \cdot q_i}.$$

- There are **two natural choices** for the reference basket:
  - the period 0 commodity vector $\boldsymbol{q^0} = (q_1^0, \ldots, q_n^0)$ or
  - the period 1 commodity vector $\boldsymbol{q^1} = (q_1^1, \ldots, q_n^1)$.

1. Motivation
Laspeyres, Paasche and Fisher

- These two choices lead to
  - the **Laspeyres** (1871) **price index** P_L, if we choose $\boldsymbol{q} = \boldsymbol{q^0}$, and
  - the **Paasche** (1874) **price index** P_P, if we choose $\boldsymbol{q} = \boldsymbol{q^1}$:
- $P_L = \frac{\sum_{i=1}^{n} p_i^1 \cdot q_i^0}{\sum_{i=1}^{n} p_i^0 \cdot q_i^0}, \; P_P = \frac{\sum_{i=1}^{n} p_i^1 \cdot q_i^1}{\sum_{i=1}^{n} p_i^0 \cdot q_i^1}.$

- According to the CPI Manual (ILO et al., 2004), "**the Paasche and Laspeyres price indices are equally plausible**."
- Taking an evenly weighted average of these basket price indices leads to **symmetric averages**.
- **The geometric mean**, which leads to the **Fisher** (1922) **price index**, P_F, is defined as:
- $P_F = \sqrt{P_L \cdot P_P}\ .$

1. Motivation
Keynes' pure theory of money

- In his 1930 *A Treatise on Money* (pp. 95-120), Keynes deals with the theory of **comparisons of purchasing power**.
- Comparisons of purchasing power mean comparisons of the command of money over two collections of commodities which are in some sense "equivalent" to one another, and **not over quantities of utility**.

- Applying **the "method of limits"** establishes that **in any case the measure of the change in the value of money lies between the Laspeyres and Paasche price indices**.

- **The "crossing of formulae"**, to which Fisher has devoted much attention, is, in effect, **an attempt to carry the method of limits somewhat further – further** (in Keynes' opinion) **than is legitimate**.

- We can concoct **all sorts of algebraic function of P_L and P_P**, and **there will not be a penny to choose between them**.

1. Motivation
Test approach

- **Tests**, for example that the formula must treat both positions [time, place or class] in a symmetrical way, **do not prove that any one of the formulae has a leg to stand on**.
- All these **tests are directed to showing**, not that it is correct in itself, but **that it is open to fewer objections than alternative *a priori* formulae**.

- It is worth mentioning that **the time reversal test**, which is the main justification of the Fisher, Walsh and Törnqvist price indices, **is meaningful only in interspatial comparisons** (then as the country reversal test).
- **In intertemporal comparisons**, however, **the direction of comparison is *not* arbitrary** (it is not unjustified to prefer a forward movement to moving backwards) (cf. von der Lippe, 2007).

- Moreover, **a two-staged test approach** – and practical consumer price indices are constructed in two stages – **has not been** as **well developed** as the one-staged test approach.

1. Motivation
Elementary indices

- Suppose that there are ***M*** **lowest-level items or specific commodities** in a chosen elementary category.
- Denote the period *t* **price** of item *m* by p_m^t for $t \in \{0, 1\}$ and for items $m \in \{1, \ldots, M\}$.
- The **Dutot** (1738) **elementary price index**, P_D, is equal to the *arithmetic* average of the *M* period 1 prices divided by the *arithmetic* average of the *M* period 0 prices.
- The **Carli** (1764) **elementary price index**, P_C, is equal to the *arithmetic* average of the *M* item price ratios or price relatives, p_m^1/p_m^0.
- The **Jevons** (1865) **elementary price index**, P_J, is equal to the *geometric* average of the *M* item price ratios or price relatives, p_m^1/p_m^0, or the *geometric* average of the *M* period 1 prices divided by the *geometric* average of the *M* period 0 prices.
- $P_D = \frac{\frac{1}{M}\sum_{m=1}^{M} p_m^1}{\frac{1}{M}\sum_{m=1}^{M} p_m^0}$, $P_C = \frac{1}{M}\sum_{m=1}^{M}\frac{p_m^1}{p_m^0}$, $P_J = \sqrt[M]{\prod_{m=1}^{M}\frac{p_m^1}{p_m^0}} = \frac{\sqrt[M]{\prod_{m=1}^{M} p_m^1}}{\sqrt[M]{\prod_{m=1}^{M} p_m^0}}$.

2. Test approach
Axiomatic approach

- **Looking at the mathematical properties of index number formulae** leads to the test or axiomatic approach to index number theory.
- In this approach, **desirable properties for an index number formula are proposed**, and it is then attempted to determine whether any formula is consistent with these properties or tests.

- **It must be decided what tests or properties should be imposed** on the index number.
- Different price statisticians may have different ideas about which tests are important, and **alternative sets of axioms can lead to alternative "best" index number functional forms**.
- This point must be kept in mind since there is **no universal agreement on what the "best" set of "reasonable" axioms is**.
- Hence, the axiomatic approach can lead to **more than one "best" index number formula**.

2. Test approach
Test performance

- The **Dutot index** satisfies all fundamental tests with the important exception of **the commensurability test**, which it fails. If there are heterogeneous items in the elementary aggregate, **this is a rather serious failure** and, hence, price statisticians should be careful in using this index under these conditions.
- The **Carli index** fails **the time reversal test**, and passes the other tests. The failure of the time reversal test is a rather serious matter and so price statisticians should be cautious in using these indices. Note that, however, **not all price statisticians would regard the time reversal test** in the elementary index context **as being a fundamental test** that must be satisfied.
- The **Jevons index** satisfies all the tests but **the test of determinateness as to prices**, i.e. the elementary index is rendered zero by an individual price becoming zero. Thus, when using the Jevons index, **care must be taken to bound the prices away from zero** in order to avoid a meaningless index number value.
- **Hence, no single index formula emerges as being “best” from the viewpoint of this particular axiomatic approach to elementary indices.**

2. Test approach
From principle to practice

- “An economist is someone who sees something **work in practice** and asks whether it would **work in principle**.” (Goldfeld, 1984, J. Money, Credit, Banking)

- What is it **in principle**?
  - bilateral approach
  - one-stage aggregation
  - fixed basket indices
  - constant quality

- And **in practice**?
  - multilateral comparisons
  - two-staged calculation
  - chain method
  - item substitution

2. Test approach
One stage vs. two stages

- **The assertion that the Jevons index appears to be “best” needs to be qualified**: there are many other tests, and price statisticians might hold different opinions regarding the importance of satisfying various sets of tests.

- It can be shown that, for example, **the two-staged Fisher price index** with another index formula at the elementary level **does not satisfy monotonicity in both current and base period prices** (Mehrhoff, 2010).
- This means that **although a price is increasing** in the current period, **the price index does not necessarily increase, too**.
- **Vice versa, the price index does not necessarily decrease either** if a base period price increases.

- **Hence, more attention should be paid to the characteristics of two-staged price indices.**

2. Test approach
Constant quality vs. item substitution

- A CPI should reflect the change in the cost of buying **a fixed basket of goods and services of constant quality**.

- In practice, this represents a challenge to the price statistician as **products can permanently disappear or be replaced with new versions of a different quality** or specification, **and brand new products can also become available**.

- However, **this is not consistent with the idea that outlet prices should be matched to each other in a one-to-one manner** across the two periods.

- Should that be no longer possible due to item substitution, **none of the elementary index formulae will meet the circularity test**. (This test is essentially a strengthening of the *time reversal test*.)
- It illustrates **the use of the chain principle** to construct the overall inflation between periods 0 and 1, **compared to the use of the fixed base principle** to construct an estimate of the overall price change between periods 0 and 1.

2. Test approach
Is the Carli index really "upward biased"?

- The sole argument frequently put forward why the Carli index should be abandoned, is **the claim that it has an "upward bias"** with reference to the time reversal test or circularity test (cf. Diewert, 2012):
- $P_C(\boldsymbol{p}^0, \boldsymbol{p}^1) \cdot P_C(\boldsymbol{p}^1, \boldsymbol{p}^2) = P_C(\boldsymbol{p}^0, \boldsymbol{p}^1) \cdot P_C(\boldsymbol{p}^1, \boldsymbol{p}^0) \geq 1 = P_C(\boldsymbol{p}^0, \boldsymbol{p}^0)$ for $\boldsymbol{p}^2 = \boldsymbol{p}^0$.

- But **this argument is useless in the bilateral index context** where we can compare the two periods under consideration directly, i.e. there is no bias at all:
- $P_C(\boldsymbol{p}^0, \boldsymbol{p}^2) = P_C(\boldsymbol{p}^0, \boldsymbol{p}^0) = 1$ for $\boldsymbol{p}^2 = \boldsymbol{p}^0$.

- In the context of chain indices, **the elementary aggregates only feed into the higher-level indices** in which the elementary price indices – comparing periods t-1 and t (!) – are averaged using a set of pre-determined weights (chain indices are non-aggregable); **the Dutot, Carli and Jevons indices are, thus, not chain-linked**.
- What is more, it apparently fell into oblivion that **the then chain-linked Laspeyres, Paasche, Fisher, Walsh and Törnqvist price indices are subject to chain drift**; i.e. all chain indices are path dependent, which is the opposite of transitivity.

3. Stochastic approach

- The basic idea behind the (unweighted) stochastic approach is that **each price relative**, p_m^1/p_m^0 for $m \in \{1, \ldots, M\}$, **can be regarded as an estimate of a common inflation rate** between periods 0 and 1.

- But the price indices derived from this approach suffer from **a fatal flaw: each price relative** p_m^1/p_m^0 is regarded as being equally important and **is given an equal weight in the index number formulae**.

- The flaw in the argument is **it is assumed that the fluctuations of individual prices round the "mean" are "random"**.

- **There is no general price level**, with individual prices scattered round.

- ***Hence, there is nothing left of the stochastic approach over and above one of the elementary indices already defined.***

4. Economic approach

- The CPI Manual, paragraphs 20.71-20.86, has a section in it which describes **an economic approach to elementary indices**.

- This section has sometimes been used to **justify the use of the Jevons index**, i.e. the geometric mean, **over the use of the Carli index**, i.e. the arithmetic mean, or vice versa **depending on how much substitutability exists** between items within an elementary stratum.

- **This is a misinterpretation of the analysis** that is presented in this section of the Manual.

- **Thus, the economic approach cannot be applied at the elementary level unless price and quantity information are both available.**
- ***Such information is typically not available***, which is exactly the reason elementary indices are used rather than target indices. (Diewert, 2012, "Consumer Price Statistics in the UK")

5. Consistency approach
Consistency in aggregation

- **The consistency in aggregation (CIA) approach** newly developed (Mehrhoff, 2010, Jahr. Nationalökon. Statist.) fills the void of **guiding the choice of the elementary index** (for which weights are not available) **that corresponds to the characteristics of the index at the second stage** (where weights are actually available).
- It contributes to the literature by looking at how **numerical equivalence between an unweighted elementary index and a weighted aggregate index** can be achieved, **independent of the axiomatic properties**.

- *Consistency in aggregation* means that **if an index is calculated stepwise by aggregating lower-level indices** to obtain indices at progressively higher levels of aggregation, **the same overall result** should be obtained **as if the calculation had been made in one step**.

5. Consistency approach
Elementary index bias

- Thus, a relevant, although often neglected, issue in practice is **the numerical relationship between elementary and aggregate indices**.
- This is because if the elementary indices do not reflect the characteristics of the aggregate index, **a two-staged index can lead to a different conclusion** than that reached by the price index calculated directly from the price relatives.

- An elementary index in the CPI is **biased if its expectation differs from its measurement objective**.
- This elementary index bias is **applicable irrespective of which unweighted index is used**.

- **In other words, if the elementary index coincides (in expectation) with the aggregate index, the bias will vanish.**

5. Consistency approach
Cost of goods index

- To reiterate, **we measure the change in the cost of purchasing a fixed basket of goods and services**, and *not* the change in the minimum cost of maintaining a given level of utility or welfare.

- The use of **the Dutot and Carli formulae** at the elementary level of aggregation for *homogeneous* items can be perfectly **consistent with a Laspeyres index concept**.

- **The Laspeyres price index** can be rewritten in an alternative manner as follows:

- $$P_L = \frac{\sum_{m=1}^{M} p_m^1 \cdot q_m^0}{\sum_{m=1}^{M} p_m^0 \cdot q_m^0} = \sum_{m=1}^{M} \frac{p_m^1}{p_m^0} \cdot \frac{p_m^0 \cdot q_m^0}{\sum_{l=1}^{M} p_l^0 \cdot q_l^0} = \sum_{m=1}^{M} \frac{p_m^1}{p_m^0} \cdot s_m^0,$$

- where s_m^0 is the **period 0 expenditure share** on commodity *m*.

5. Consistency approach
A thought experiment

− The first case is where the underlying preferences are **Leontief preferences**, i.e. **consumers prefer not to make any substitutions** in response to changes in relative prices (zero elasticity):

− $q_m^{0} = q_m^{1} = q$ and, hence, $P_L = \frac{\sum_{m=1}^{M} p_m^1 \cdot q}{\sum_{m=1}^{M} p_m^0 \cdot q} = \frac{\frac{1}{M}\sum_{m=1}^{M} p_m^1}{\frac{1}{M}\sum_{m=1}^{M} p_m^0} = P_D$.

− The second case is when the preferences can be represented by **a Cobb-Douglas function**, i.e. **consumers vary the quantities** in inverse proportion to the changes in relative prices **so that expenditure shares remain constant** (unity elasticity):

− $s_i^{0} = s_i^{1} = M^{-1}$ and, hence, $P_L = \sum_{m=1}^{M} \frac{p_m^1}{p_m^0} \cdot M^{-1} = \frac{1}{M}\sum_{m=1}^{M} \frac{p_m^1}{p_m^0} = P_C$.

5. Consistency approach
Generalised means

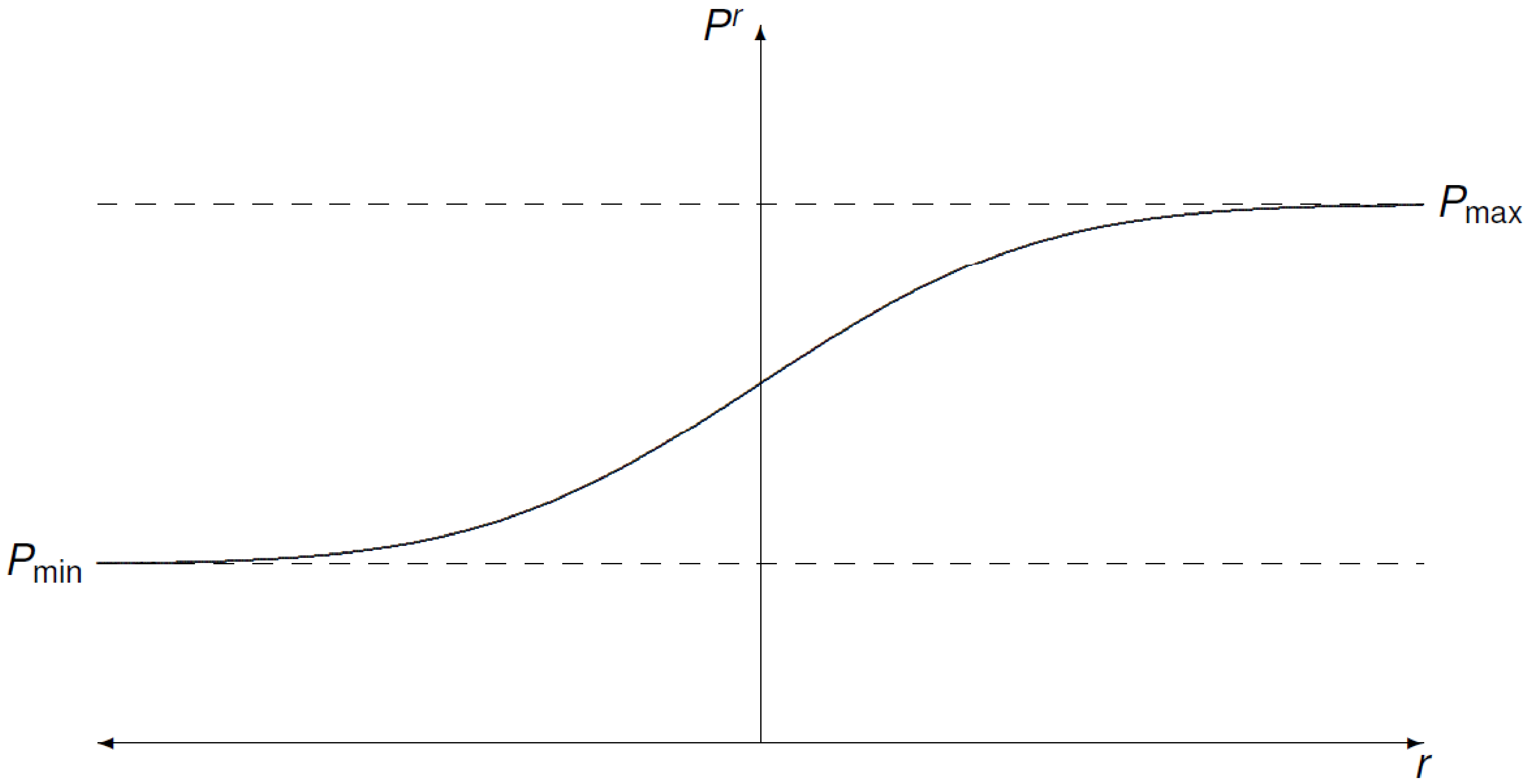
- A single comprehensive framework, known as *generalised means*, **unifies the aggregate and elementary levels**.
- **The generalised mean** of order *r* for the *M* item price ratios or price relatives, p_m^1/p_m^0, is defined as follows:
- $$P^r = \begin{cases} \sqrt[r]{\frac{1}{M}\sum_{m=1}^{M}\left(\frac{p_m^1}{p_m^0}\right)^r} & \text{if } r \neq 0, \\ \sqrt[M]{\prod_{m=1}^{M}\frac{p_m^1}{p_m^0}} & \text{if } r = 0. \end{cases}$$
- The generalised mean represents **a whole class of unweighted elementary indices**, such as the Carli and Jevons indices for $r = 1$ and $r = 0$, respectively.

5. Consistency approach
Numerical equivalence

- Hardy et al. (1934) discuss **the generalised mean** in great detail and prove its properties.
- First, **it covers the whole range between the smallest and largest price relative**, min($\{p_m^1/p_m^0\}$) and max($\{p_m^1/p_m^0\}$), respectively, and **it is a continuous function** in its argument r.
- Moreover, by Schlömilch's inequality, **the generalised mean is strictly monotonic increasing** unless all price relatives are equal.
- The mean value property ensures **the existence of an inverse function**.
- Thus, there exists one and only one r for which **the generalised mean is *numerically* equivalent to an arbitrary aggregate index**:
- $P^r(\mathbf{p}^0, \mathbf{p}^1) = P(\mathbf{p}^0, \mathbf{p}^1, \mathbf{q}^0, \mathbf{q}^1)$.

- The basic idea behind this approach is that **different elementary indices implicitly weight price relatives differently**, although they do not imply an explicit expenditure structure.

5. Consistency approach
Typical shape



$$P_{min} = \min\left(\left\{\frac{p_m^1}{p_m^0}\right\}\right),\ P_{max} = \max\left(\left\{\frac{p_m^1}{p_m^0}\right\}\right)$$

5. Consistency approach
Constant elasticity of substitution

- However, **an analytical derivation** of the concrete generalised mean of a weighted aggregate index **is not possible** without further assumptions.

- Hence, both the generalised mean and the target indices are expanded by **a second-order Taylor series approximation** around the point $\ln p_m^t = \ln p^t$ for all $m \in \{1, \ldots, M\}$, $t \in \{0, 1\}$.

- Next, it is usually adequate to assume **a constant elasticity of substitution (CES)** approximation in the context of **approximating changes in a consumer's expenditures** on the *M* commodities under consideration.

- Finally, it is shown that the choice of the elementary indices which correspond to the desired aggregate ones can be based on **the elasticity of substitution** *alone*.

- **Thus, a feasible framework is provided which aids the choice of the corresponding elementary index.**

5. Consistency approach
CES aggregator function

- It is supposed that **the unit cost function** has the following functional form:

- $$c(\boldsymbol{p}) = \begin{cases} \alpha_0 \cdot \left(\sum_{m=1}^{M} \alpha_m \cdot p_m^{1-\sigma} \right)^{1/(1-\sigma)} & \text{if } \sigma \neq 1, \\ \alpha_0 \cdot \prod_{m=1}^{M} p_m^{\alpha_m} & \text{if } \sigma = 1, \end{cases}$$

- where the α_m are non-negative consumer preference parameters with $\sum_{m=1}^{M} \alpha_m = 1$.
- This unit cost function corresponds to **a CES aggregator or utility function**.
- The parameter σ is **the elasticity of substitution**:
  - When $\sigma = 0$, the underlying preferences are **Leontief preferences**.
  - When $\sigma = 1$, the corresponding utility function is **a Cobb-Douglas function**.

5. Consistency approach
Laspeyres and Paasche price indices

- A generalised mean of order *r* **equal to the elasticity of substitution** (σ) yields approximately the same result as **the Laspeyres price index**.
- Hence, if the elasticity of substitution is one (Cobb-Douglas preferences), for example, *r* must equal one and **the Carli index at the elementary level will correspond to the Laspeyres price index as target index**.

- However, if **the Paasche price index** should be replicated, the order of the generalised mean must **equal minus the elasticity of substitution**, in the above example minus one.
- Thus, **the harmonic index gives the same result** and therefore, in this case it should be used at the elementary level.

- **Only if the elasticity of substitution is zero** (Leontief preferences), **the Jevons (Dutot) index corresponds to both the Laspeyres and Paasche price indices** – which in this case coincide.

5. Consistency approach
Fisher price index

- **The Fisher price index** is derived from the Laspeyres and Paasche price indices as their geometric mean.
- Owing to **the symmetry of the generalised means which correspond to the Laspeyres and Paasche price indices**, a quadratic mean corresponds to the Fisher price index, where q must equal two times the elasticity of substitution.

- **A quadratic mean** of price relatives of order q is defined as follows:

- $P^{q} = \sqrt{P^{r=q/2} \cdot P^{r=-q/2}}$.

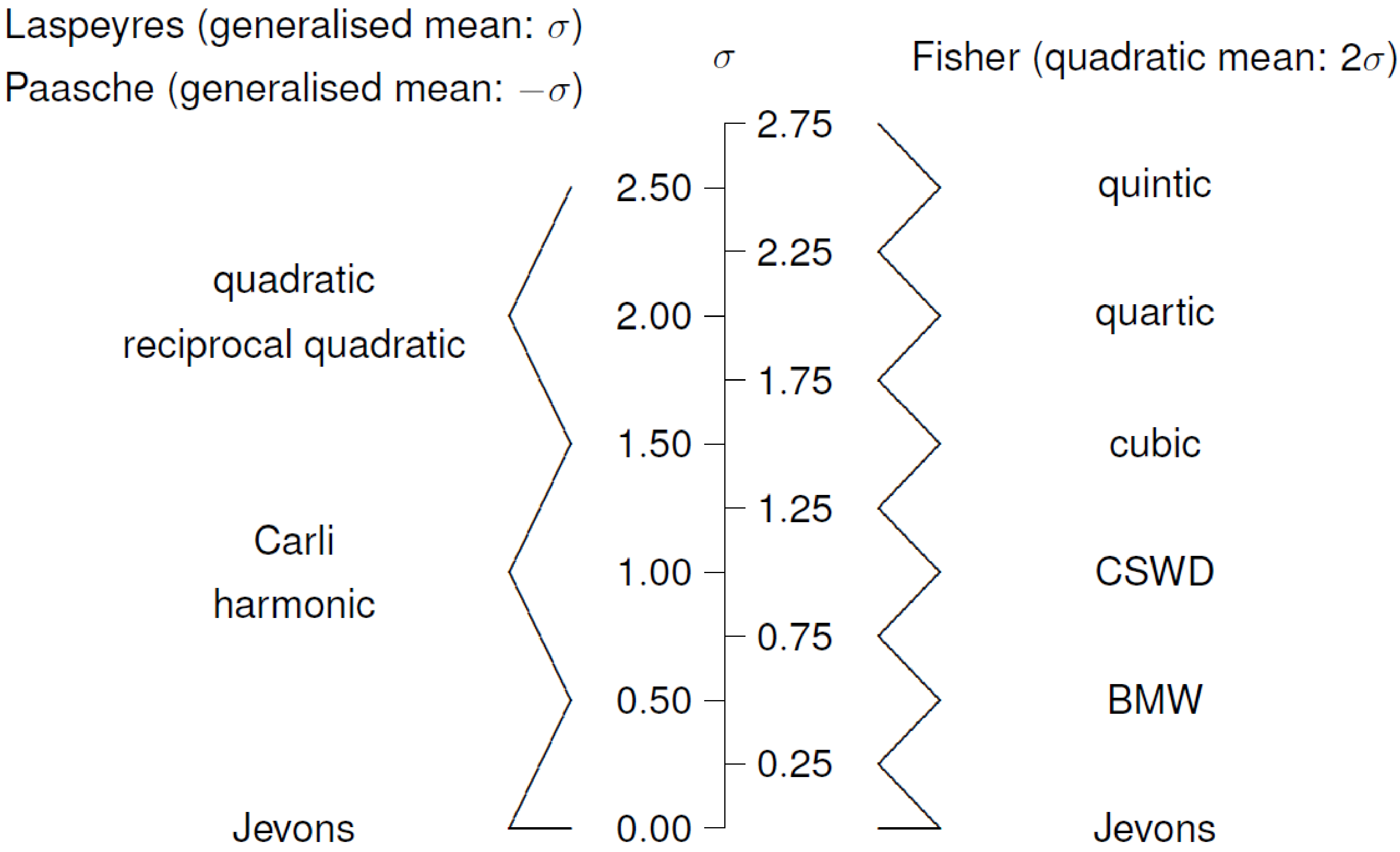
- **The index is symmetric**, i.e. $P^q = P^{-q}$. Furthermore, **it is either increasing or decreasing** in $|q|$, depending on the data.
- Note that **a quadratic mean** of order q, P^q, **should not be mistaken for the quadratic index**, $P^{r=2}$.

5. Consistency approach
Quadratic means

- Dalén (1992), and Diewert (1995) show via a Taylor series expansion that **all quadratic means approximate each other** to the second order.
- However, as Hill (2006) demonstrates, the limit of P^q if q diverges is $\sqrt{P_{\min} \cdot P_{\max}}$; he concludes that **quadratic means are not necessarily numerically similar**.

- For $\sigma = 0$ ($q = 0$) **the quadratic mean** becomes **the Jevons index**.
- For $\sigma = .5$ ($q = 1$) an index results, which was first described by Balk (2005, 2008) as the unweighted Walsh price index and independently devised by Mehrhoff (2007, pp. 45-46) as **a linear approximation to the Jevons (CSWD) index**; hence, this index number formula is referred to as **the Balk-Mehrhoff-Walsh index**, or, for short, “BMW”.
- Lastly, one arrives at **the CSWD index** (Carruthers, Sellwood and Ward, 1980, and Dalén, 1992) for $\sigma = 1$ ($q = 2$), which is **the geometric mean of the Carli and harmonic indices**.

5. Consistency approach
Corresponding elementary indices

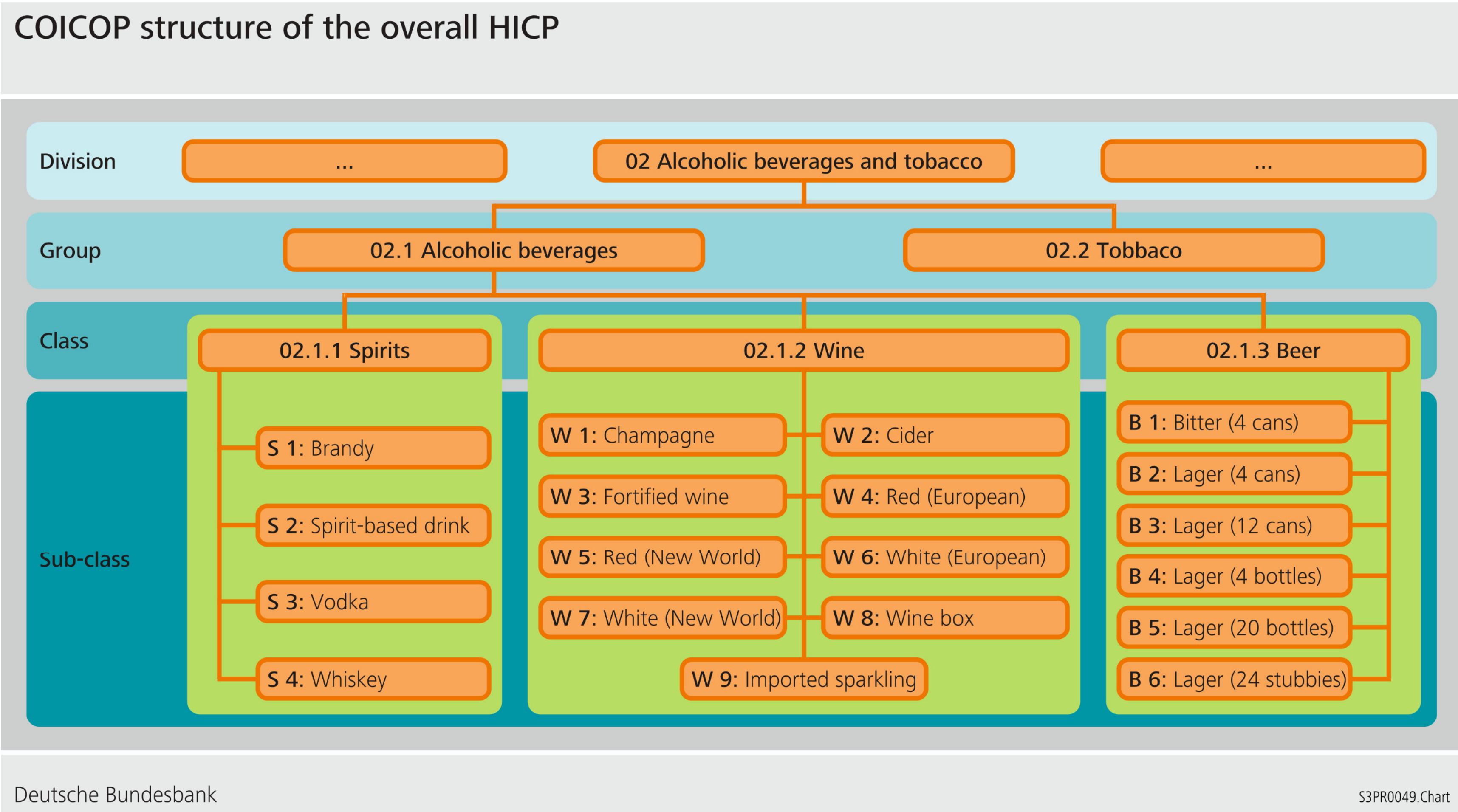


5. Consistency approach
Empirical results

- As an empirical application, **detailed expenditure data** from Kantar Worldpanel for elementary aggregates within the COICOP group of **alcoholic beverages in the UK** are analysed.
- The data cover the period **from January 2003 to December 2011**; the data set consists of transaction level data, which records inter alia purchase price and quantity, and includes **192,948 observations** after outlier identification.

- **The elasticity of substitution** is estimated in the framework of a log-linear model by means of **ordinary least squares**. (Note that the consumer preference parameters are removed via differencing products common to adjacent months and, thus, there is no need for application of seemingly unrelated regression.)
- As a robustness check to the CES model based results, **the generalised mean** which minimises relative bias and root mean squared relative error to the desired aggregate index is found directly by **numerical optimisation techniques**. (Rather than at the aggregate transaction level, like the econometric method, this analysis, however, is performed one level above – at the elementary level.)

5. Consistency approach
COICOP structure for alcoholic beverages

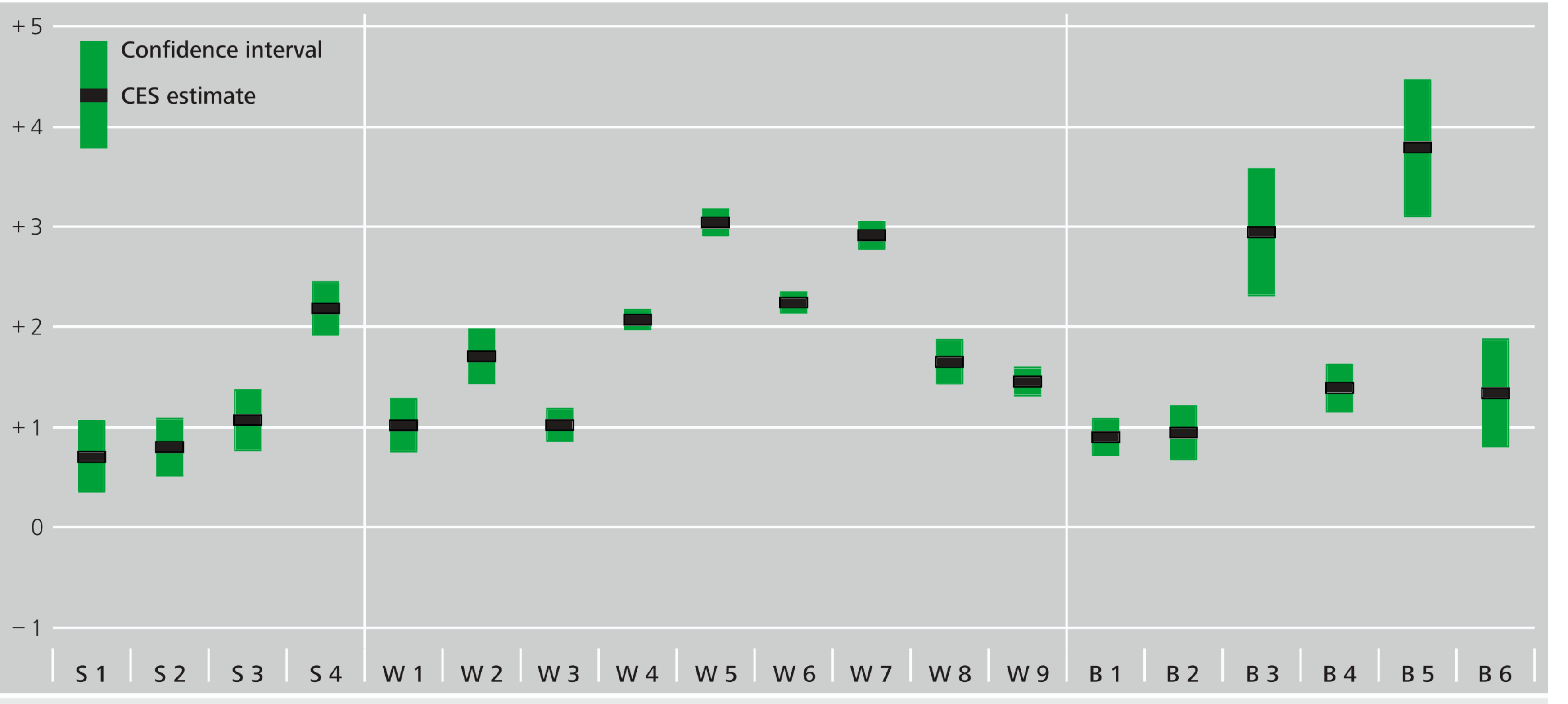


5. Consistency approach
Findings on substitution behaviour

- The median elasticity of substitution is 1.5, **ranging from .7 to 3.8**.
- All estimates are **statistically significantly greater than zero**; for 8 out of 19 sub-classes **the difference to iso-elasticity is insignificant**, while for the remaining 11 sub-classes substitution is found to even exceed unity elasticity.
- **In spirits, consumers are more willing to substitute between different types of whiskey (S 4) than is the case for brandy or vodka (S 1 and S 2).**
- **For both red and white wines, substitution is more pronounced for the New World (W 5 and W 7) than for European wines (W 4 and W 6).**
- **Also, the elasticity of substitution tends to be higher for 12 cans and 20 bottles of lager (B 3 and B 5), respectively, than for 4 packs (B 2 and B 4).**
- These results are **consistent with the findings of Elliott and O'Neill (2012)**.
- Furthermore, comparing the CES regression results with the direct calculation of the generalised means, **the outcomes do not change qualitatively**.
- In particular, **the Carli index performs remarkably well** at the elementary level of a Laspeyres price index, questioning the argument of its "upward bias" – in fact, **it is the Jevons index that has a downward bias**.

5. Consistency approach
CES estimation results

CES estimates with 95% confidence interval



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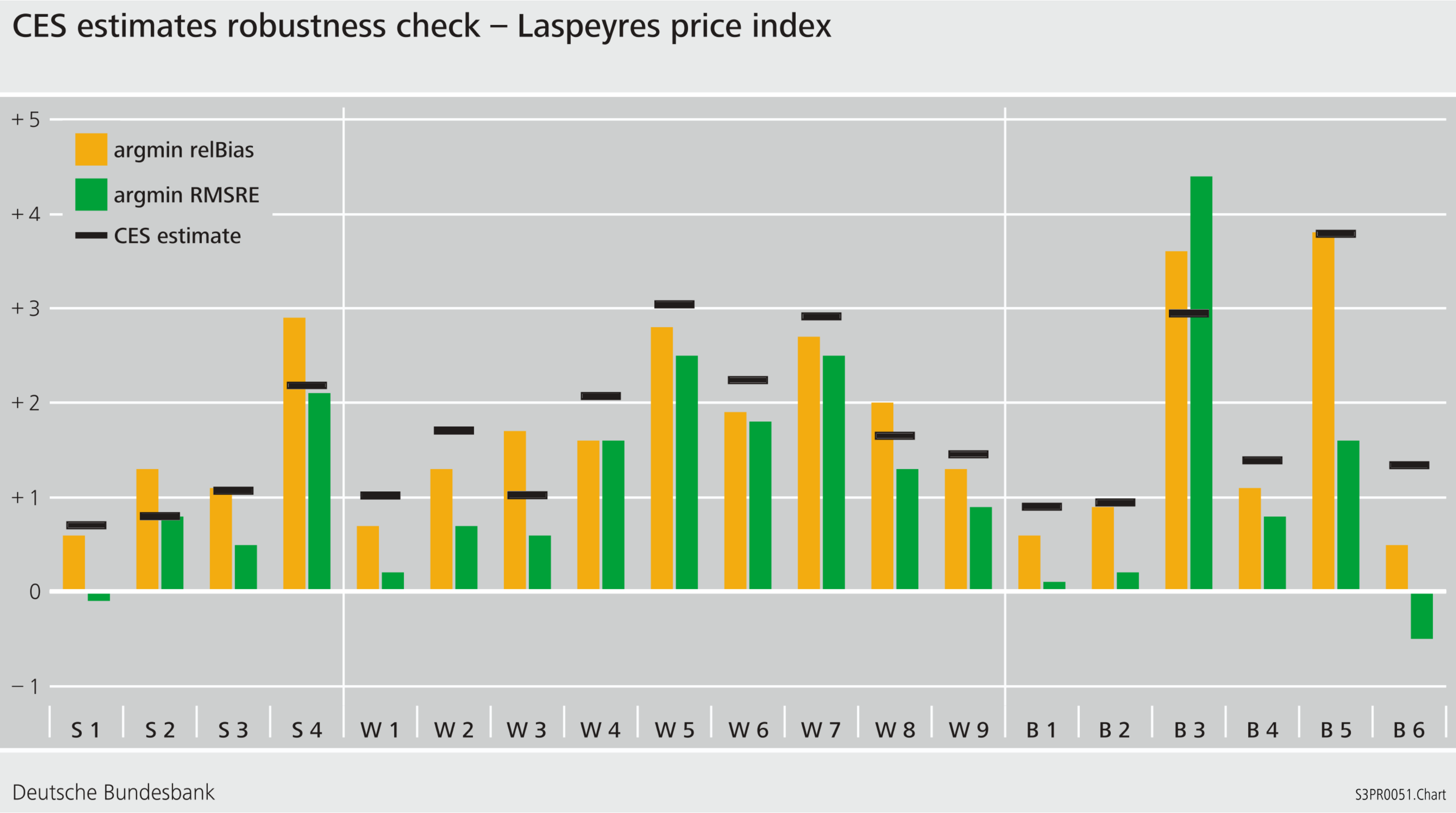
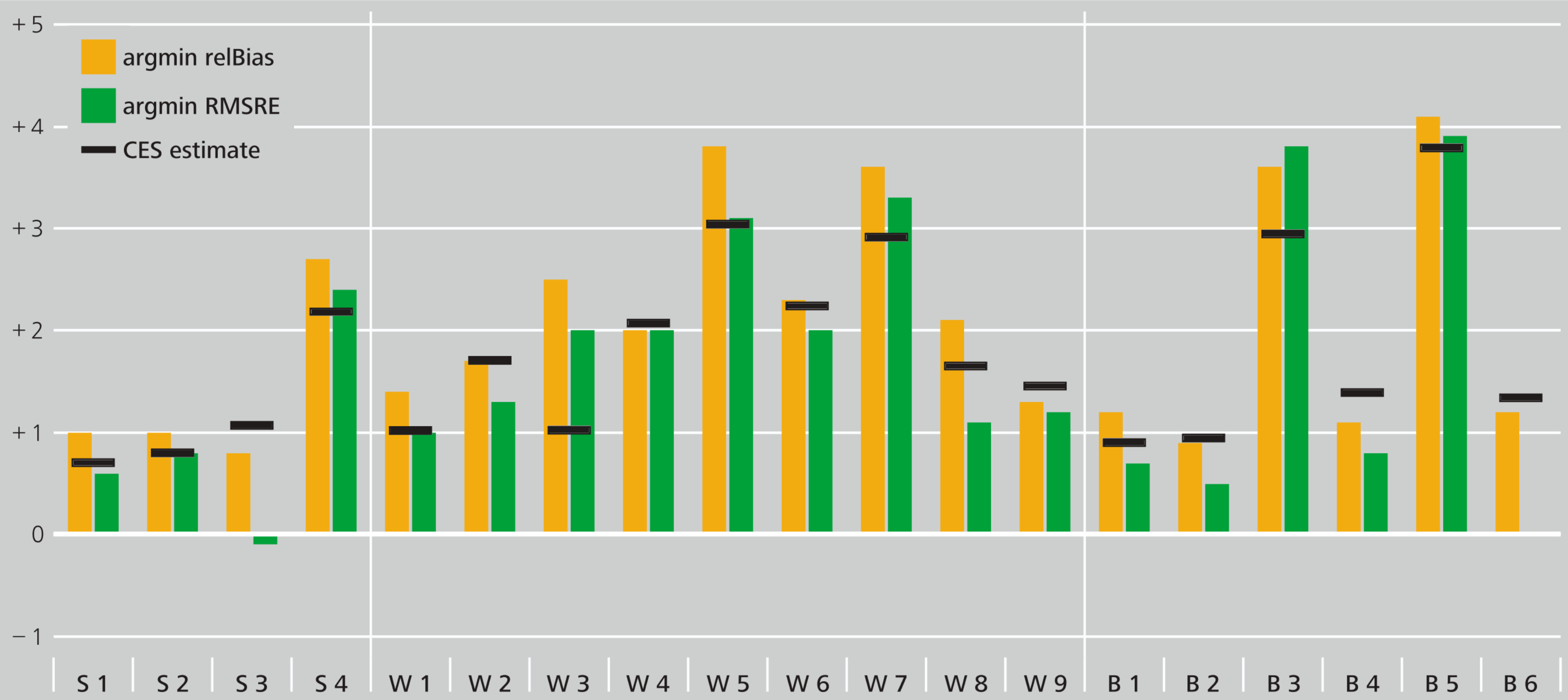
5. Consistency approach
Laspeyres price index: robustness



5. Consistency approach
Paasche price index: robustness

CES estimates robustness check – Paasche price index (inverted scale)



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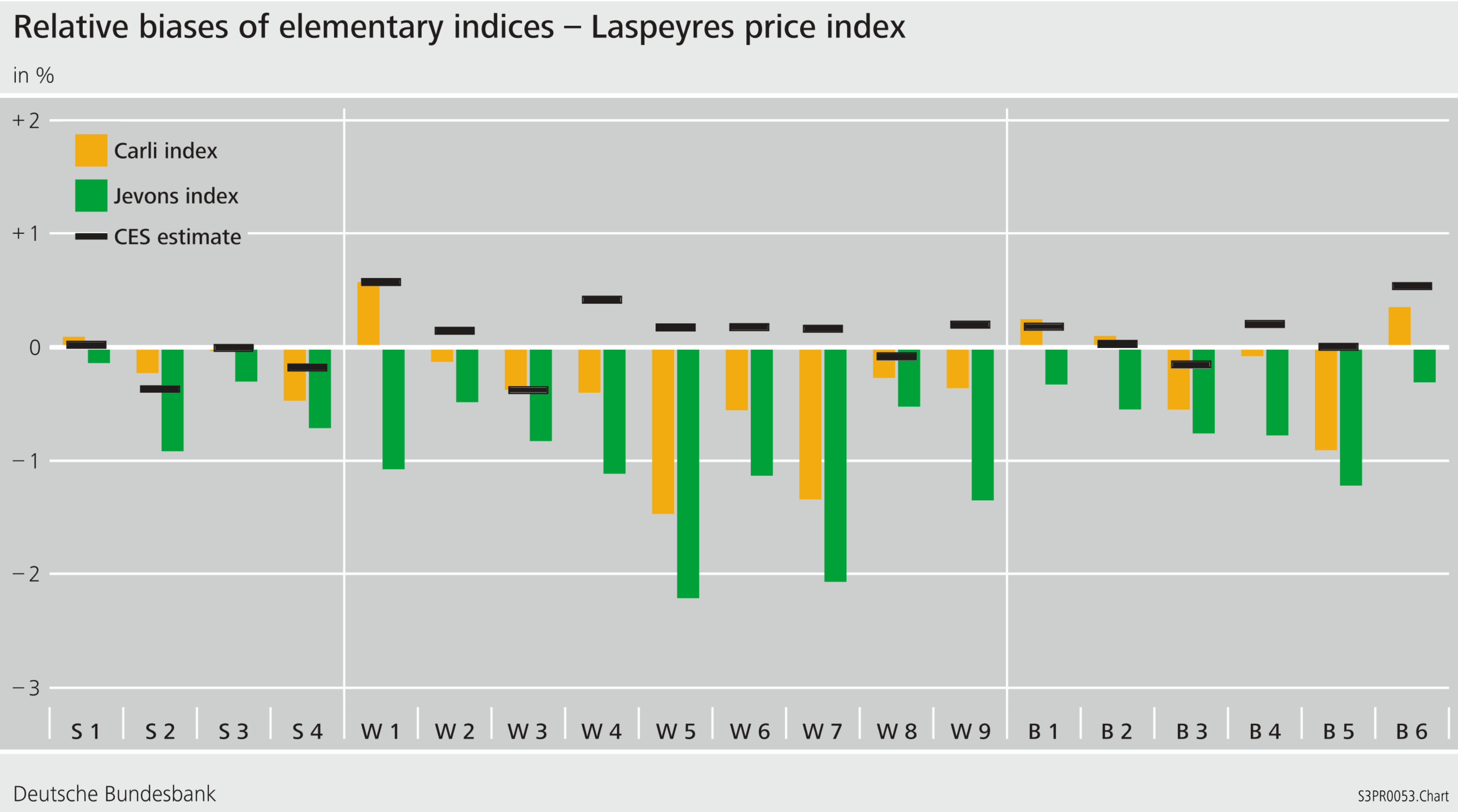
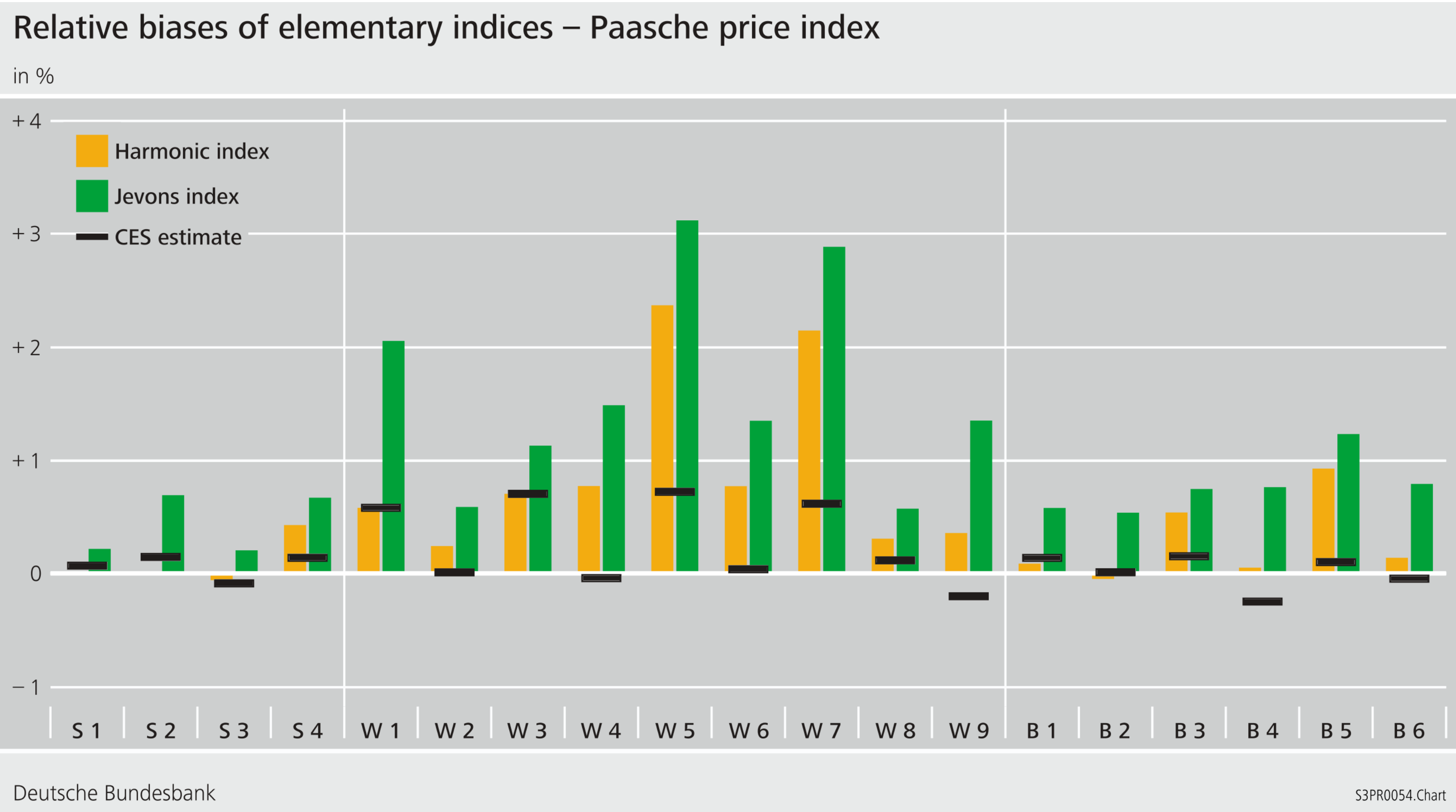
5. Consistency approach
Laspeyres price index: bias



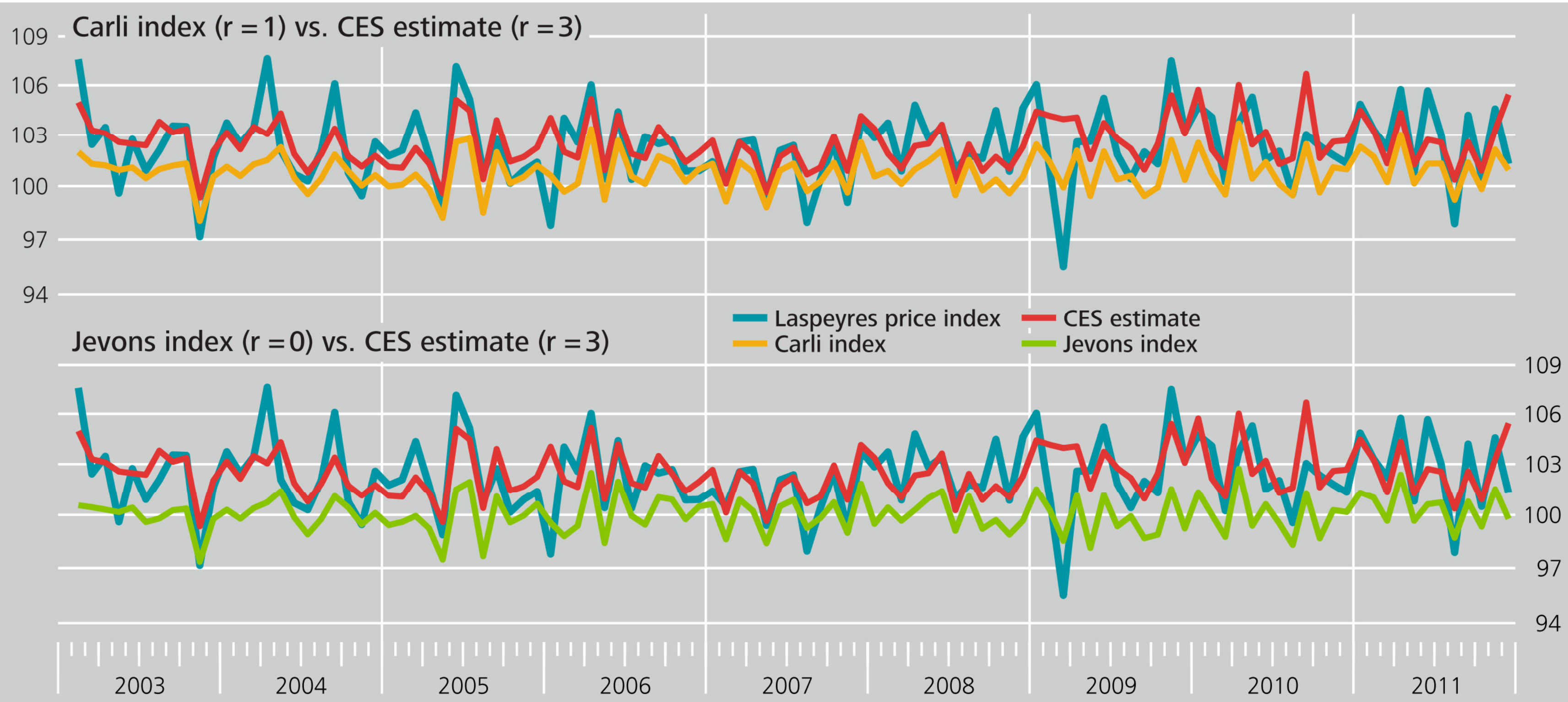
5. Consistency approach Paasche price index: bias



5. Consistency approach
Laspeyres price index: time series

Laspeyres price index for W 5 – Red wine (New World)

Previous month = 100, log scale



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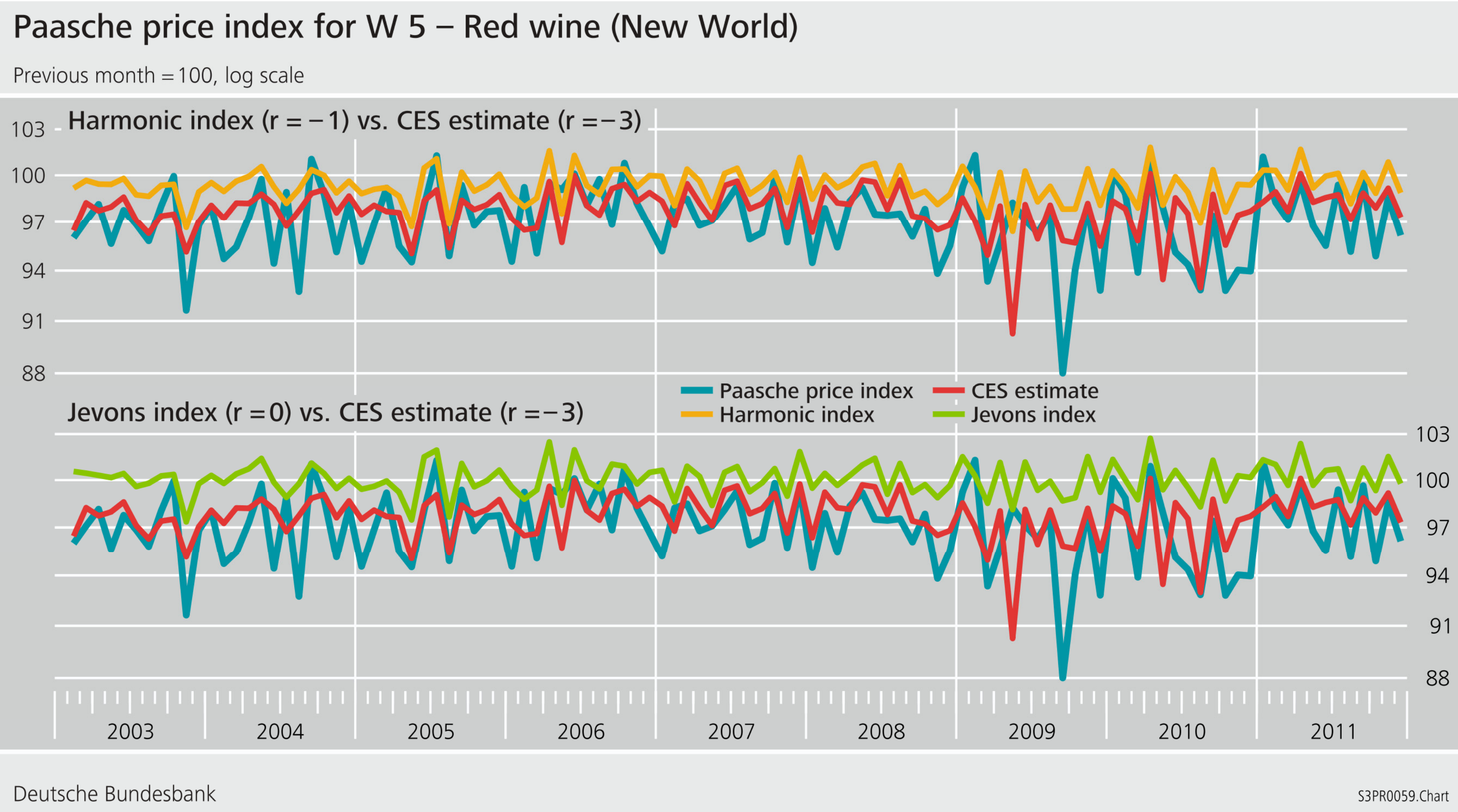
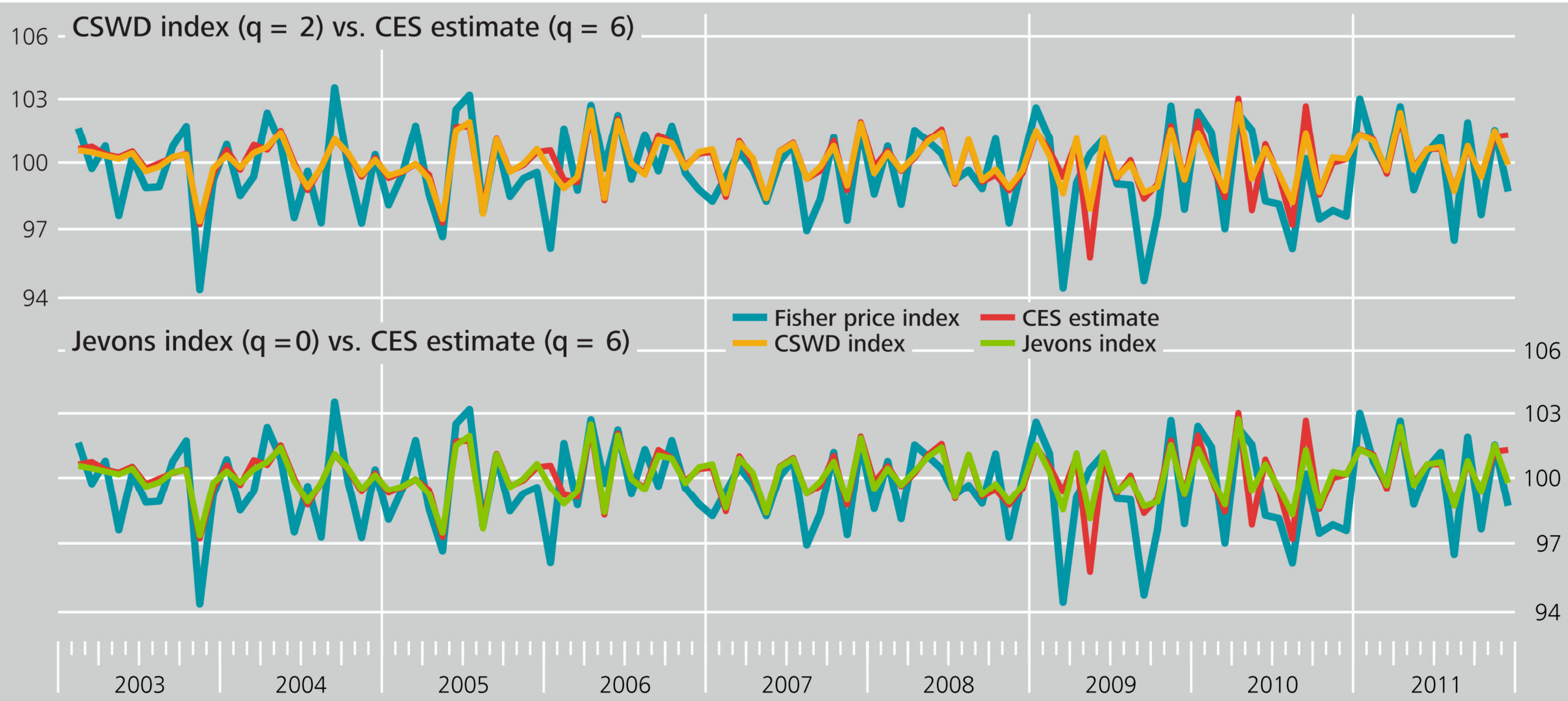
5. Consistency approach
Paasche price index: time series



5. Consistency approach
Fisher price index: time series

Fisher price index for W 5 – Red wine (New World)

Previous month = 100, log scale



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6. Discussion
Summary

− The existing approaches to index numbers including but not restricted to **the axiomatic approach** are **of little guidance in choosing the elementary index** corresponding to the characteristics of the index at the second stage.

− In **the CIA approach**, it is shown that the solution to the problem of elementary indices that **correspond to a desired aggregate index** depends on the empirical correlation between prices and quantities, in particular on the elasticity of substitution.

− **The importance of the elementary level** and the elementary index cannot be emphasised enough; biases of these indices at this level are **more severe than the pros and cons of the formula at the aggregate level**.

− This is because if prices and quantities are trending relatively smoothly, **chaining will reduce the spread between the Paasche and Laspeyres indices**.

6. Discussion
Conclusion

- In addition, the problem of aggregational consistency demonstrates the need for **a weighting at the lowest possible level**.
- This would mean that, in the trade-off between **estimated weights/weights from secondary sources** on the one hand and the elementary bias of unweighted indices on the other, the balance would often tip **in favour of weighting**.
- **The biases at the elementary level** can, in some cases, reach such large dimensions that they become **relevant for the aggregate index**.
- There is **a "price" to be paid at the upper level** for suboptimal index formula selection at the lower level; thus, the need for **two-staged price indices to be accurately constructed** becomes obvious.
- **Disaggregation is a panacea!**

- Insofar as **no information on weights** is available, **studies on substitution** can help in **guiding the choice of the optimal elementary index** for a given measurement target.
- Often, even an expert judgement on substitutability **outperforms the test approach**.