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**Geometric Indexes and Substitution Bias in the CPI**

**Paper Prepared by Gabrielle Bishop (Macroeconomics Research Section, ABS)**

**Presented by Marcel van Kints ABS**

**Introduction**

The Consumer Price Index (CPI) compiled by the Australian Bureau of Statistics (ABS) provides a general measure of changes in the prices of consumer products acquired by Australian households. The CPI is used for a variety of purposes; contributing to the development of economic policy, the adjustment of wages and pensions and in individual contract escalation. The principal purpose of the Australian CPI is to measure inflation faced by consumers to support macroeconomic policy making.

The CPI is calculated as a weighted arithmetic average of the percentage price changes for a specified "basket" of products, with the weights reflecting an item's relative importance in household consumption at a fixed point in time. Currently, weights for the 85 Expenditure Classes (EC) in the CPI are updated every six years using Household Expenditure Survey (HES) data[1]. The CPI is generally described as a 'Laspeyres-type' index or more accurately as an arithmetic Lowe index[2].

The weights for a CPI based on a fixed basket should be updated frequently to take account of changing consumer behaviour and to mitigate for substitution bias at the upper levels of aggregation. The CPI manual[3], (ILO 2004), states that the bias in a Lowe index is likely to increase with the age of the weights. In the context of the Australian CPI, this paper discusses a change in methods in an attempt to minimise substitution bias in the index.

(Details of the ABS CPI aggregation methods and some of the weaknesses in these methods that have been highlighted in recent times are given in APPENDIX 1)

**Outline**

A recent International Monetary Fund (IMF) Working Paper[4] discusses the use of alternatives to the arithmetic Lowe formula and includes some empirical analysis of CPI data from the United States. Section 1 of this paper presents the results of ABS analysis which attempts to establish an estimate of any upward bias inherent in the ABS CPI that could be attributed to the infrequent nature of expenditure weight updates. Section 2 presents further empirical analysis of how any upward bias might be mitigated by use of the alternative methods of aggregation suggested in the IMF paper. In an attempt to assess the appropriateness of these methods, Section 3 of the paper presents some comparisons of expenditure patterns from the three most recent ABS Household Expenditure Surveys (HES) and, based on supermarket scanner data, some basic analysis of changes in expenditure relative to price change. Section 4 gives a summary and offers some concluding remarks.

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[1] Expenditure weights for the lower levels of aggregation are derived from a variety of sources and in many cases are updated more frequently

[2] A Laspeyres price index can be defined as a period 0-weighted arithmetic average of price changes between periods 0 and $t$. However, it takes time to compile the results of a household expenditure survey, so in practice statistical agencies use a prior period $b$ survey weights to rebase a CPI that runs from the price reference period 0 ($b < 0 < t$). The Lowe index uses period $b$ weights price-updated (and normalized) to the price reference period 0.

[3] The 2003 ICLS resolution concerning consumer price indices recommends that weights should be updated once every five years to ensure their relevance.

[4] Post Laspeyres: The Case for a New Formula for Compiling Consumer Price Indexes Prepared by Paul Armknecht and Mick Silver; April 2012

**SECTION 1: ABS Analysis of Upper Level Substitution Bias**

The ABS constructed a retrospective superlative-type index to provide an estimate of potential upper level substitution bias[5] in the fixed-weight ABS CPI. The analysis attempts to provide an estimate of the upward bias that can be attributed to consumer substitution between expenditure classes; this is the lowest level for which weighting information (from the HES) is available and this is the level at which the expenditure weights remain fixed between CPI reviews. Therefore, the analysis captures substitution bias from one expenditure class to another, but not within a given expenditure class.

The analysis began with an attempt to construct a Paasche index for a continuous time period from the June quarter 2000 to the June quarter 2011 based on the historic data used to compile the ABS CPI. A superlative index based on this Paasche index and the published ABS CPI was compiled for the same periods. The results are shown in Figure 1 below.

Using these indexes, an estimate of upper level substitution bias in the CPI was obtained by comparing the superlative (Fisher-type) index with the all groups CPI (Laspeyres-type) index.

The all-groups CPI index increased by 41.3% from the June quarter 2000 to the June quarter 2011 while the superlative index increased by 37.7% over the same period; an estimate of the upper level substitution (upward) bias of 3.6 percentage points.

To estimate the average annual upper level substitution bias, the indexes can be expressed as Compound Annual Growth Rates (CAGR).

$$\text{Laspeyres}_{CAGR} = ((I_{L,JQ2011}/I_{L,JQ2000})^{(1/11)} - 1) * 100$$

$$= ((141.3/100.0)^{(1/11)} - 1) * 100$$

$$= 3.19\%$$

$$\text{Fisher}_{CAGR} = ((I_{F,JQ2011}/I_{F,JQ2000})^{(1/11)} - 1) * 100$$

$$= ((137.7/100.0)^{(1/11)} - 1) * 100$$

$$= 2.95\%$$

The average annual upper level substitution bias was calculated as $\text{Laspeyres}_{CAGR}$ - $\text{Fisher}_{CAGR}$ = 0.24%. The CPI for the period June quarter 2000 to the June quarter 2011 was potentially upwardly biased by 0.24 percentage points per year on average due to the inability to take account of upper level substitution. These results were consistent with studies by other national statistical offices.

The results showed that the longer the period between re-weights, the larger the potential upper level substitution bias in the index.

(More detail on this analysis and the drivers for it is given in APPENDIX 2)

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[5] Of the five main sources of bias in the CPI (detailed in the ABS CPI Concepts Sources and Methods reference manual) this analysis focused on upper level substitution bias only.

Figure 1: Quarterly indexes of ABS CPI, Paasche and Fisher type indexes



Base: June 2000 (=100)
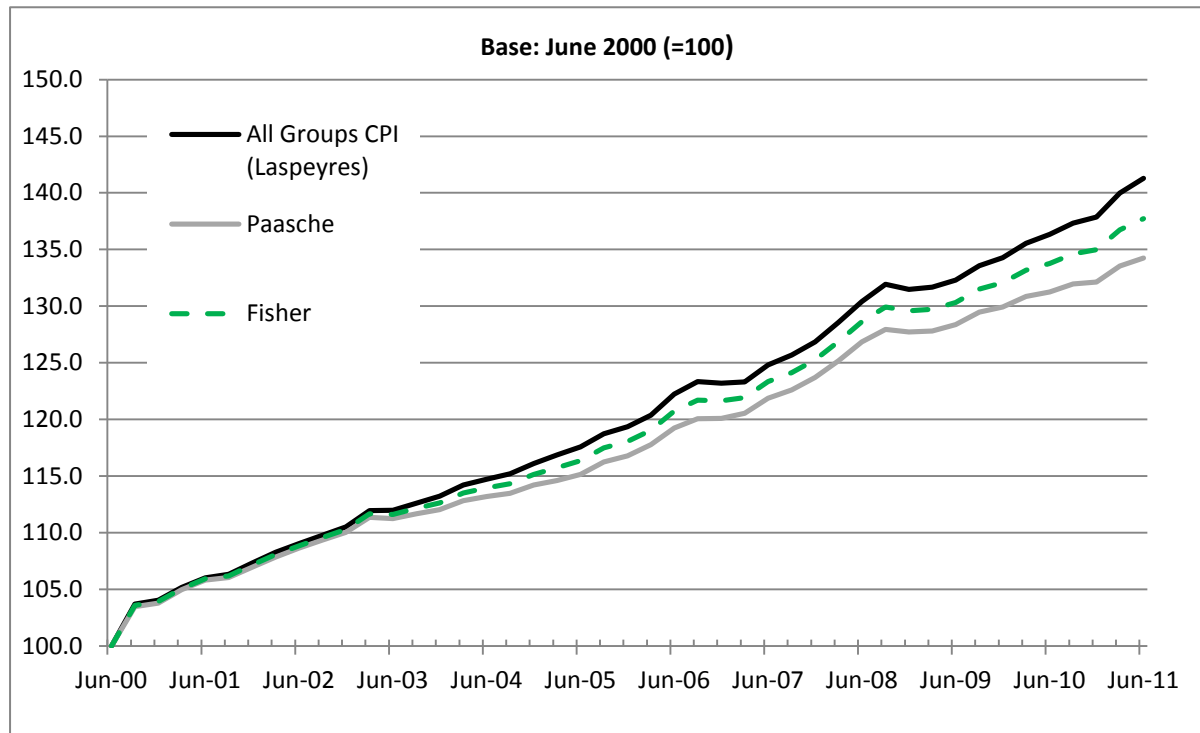
**SECTION 2: Geometric Aggregation of the ABS CPI Data**

To reduce the upper level substitution bias in the CPI, the 16th series CPI review recommended that "subject to availability of the additional funding required, the frequency of CPI weight updates will be increased from six-yearly to four-yearly, via a more frequent Household Expenditure Survey (HES)". While increasing the frequency of re-weights from six to four years will reduce the bias, it will not remove all the substitution bias present. This section discusses proposed additional methods for reducing upper level substitution bias in the CPI.

A recent IMF Working Paper (Armknecht and Silver, 2012), proposes that "Laspeyres-type indexes can be replaced at little cost by more suitable formulas that use the same data and can be compiled in real time".

In the geometric version of the Laspeyres index, a weighted geometric average is taken of the price relatives using the expenditure shares of period 0 as weights. It is defined below:

$$I^{0:t}_{GeoLaspeyres} = \prod_{i=1}^{n} \left[\frac{p_i^t}{p_i^0}\right]^{s_i^0} \text{, where } s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$$

In practice, the expenditure shares of time 0 are not available and expenditure shares from an earlier period b are used to weight period 0 to period t price changes. The above equation can be written as a geometric Young, the geometric counterpart to the arithmetic Young.

$$I^{0:t}_{GeoYoung} = \prod_{i=1}^{n} \left[\frac{p_i^t}{p_i^0}\right]^{s_i^b} \text{, where } s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}$$

The geometric counterpart of the Lowe index with its price-updated weights is given by:

$$I^{0:t}_{GeoLowe} = \prod_{i=1}^{n} \left[\frac{p_i^t}{p_i^0}\right]^{s_i^{b0}} \text{, where } s_i^{b0} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$
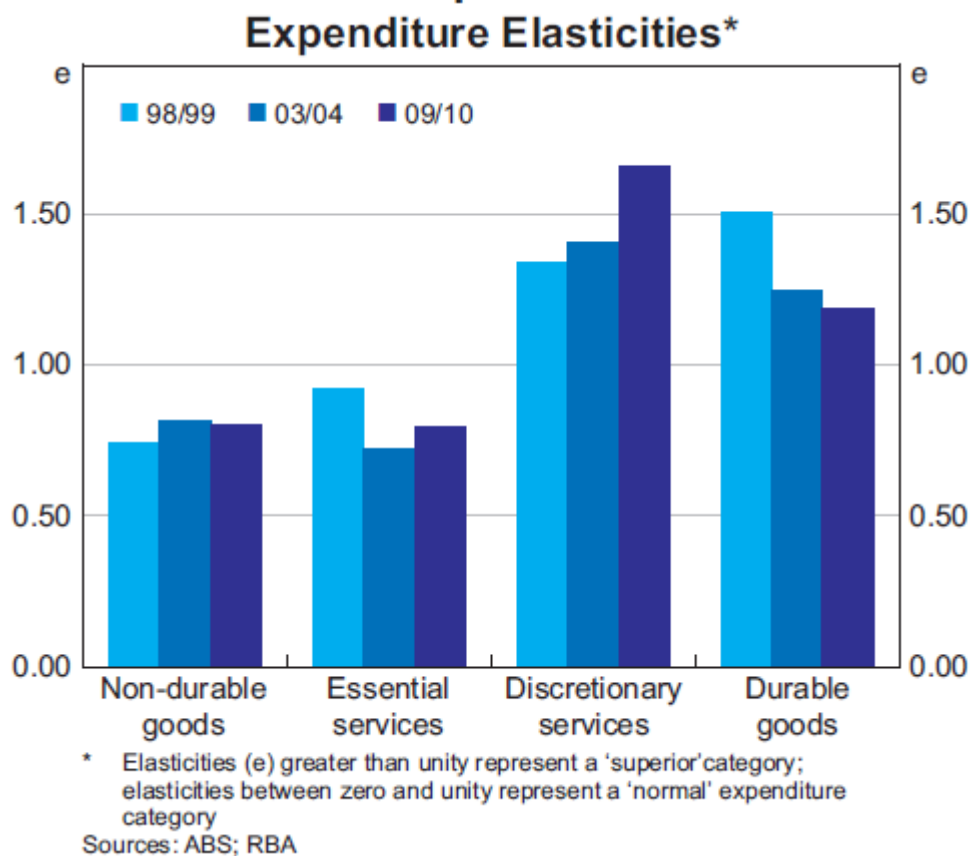
The advantages of the geometric mean are it is:

  i.    Not as sensitive as arithmetic means to the extreme values,
  ii.   It is circular, i.e., fulfils a multi-period transitivity property that the product of index change going from a period 1 to a period 2 times the price index change going from period 2 to a period 3 equals the price index going directly from period 1 to 3, and
  iii.  Is more likely to lie between the Laspeyres and Paasche bounds; a desirable property.

There are some disadvantages to the geometric mean approach. From an ABS perspective, the main disadvantage of geometric aggregation will be in its complexity. The ABS currently publishes a significant amount of information on the index points' contribution of individual expenditure classes to the all-groups index. This calculation will be less straightforward to explain to users with aggregation using a geometric mean. While users will still be provided with the price indexes and weights, allowing them to calculate a geometric mean for themselves, this would require a greater technical understanding and could lead to an increase in the demand for the ABS to produce customised indexes.

However, as the upper level expenditure weights are not updated until new household expenditure information is available (currently every six years),  holding the expenditure shares constant may be a more valid assumption than assuming that people buy the same quantity of goods and services in every index reference period between re-weights. It is for this reason that the possibility of moving to a geometrically aggregated CPI should be examined.

The geometric mean still holds the concept of a "fixed basket", but holds the expenditure shares fixed from period to period, not the quantities. It implicitly has the assumption of unit elasticity – it assumes people substitute away from products with high levels of inflation to equivalent products with relatively lower inflation, keeping the proportion of expenditure on each item constant. The elasticity of expenditure was recently analysed by Jääskelä and Callan (2011) who concluded that almost all classes of products in the ABS Household Expenditure Survey were elastic to some degree. Figure 2 (Jaaskela and Callan, 2011), shows the expenditure elasticities; non-durable goods and essential services are generally less elastic and discretionary services and durable goods more elastic. In the case of non-durable goods and essential services, the elasticity coefficient (denoted 'e') is above 0.5 and in for discretionary services and durable goods the elasticity coefficient is above 1. This suggests that an assumption of unit elasticity using the geometric mean would be a closer approximation to reality than the current arithmetic mean approach which assumes goods and services are completely inelastic.

Figure 2: Expenditure elasticities



Expenditure Elasticities*

* Elasticities (e) greater than unity represent a 'superior' category; elasticities between zero and unity represent a 'normal' expenditure category
Sources: ABS; RBA

<u>The Impact of Compiling the CPI using Geometric Aggregation of Expenditure Classes</u>

Figure 3 shows the arithmetic price indexes: the Laspeyres-type (published CPI) and the Paasche-type. The target index is a Fisher index, which lies between the Laspeyres and Paasche indexes. Also shown are the geometric Young and geometric Lowe indexes.

Both the geometric Young and the geometric Lowe indexes are lower than the published CPI. The geometric Lowe index very closely follows the Fisher index, only starting to deviate from December 2008. From that point until June 2011, the geometric Lowe remained slightly above the Fisher index but below the published CPI index. The geometric Young index was lower than the geometric Lowe and at all times was below the Fisher but above the Paasche index. From December 2007 to December 2008 the geometric Young was as low as the Paasche index; significantly lower than the superlative Fisher index. The geometric Young index started to deviate from the Fisher index much earlier than the geometric Lowe index (in December 2002 as opposed to June 2006).
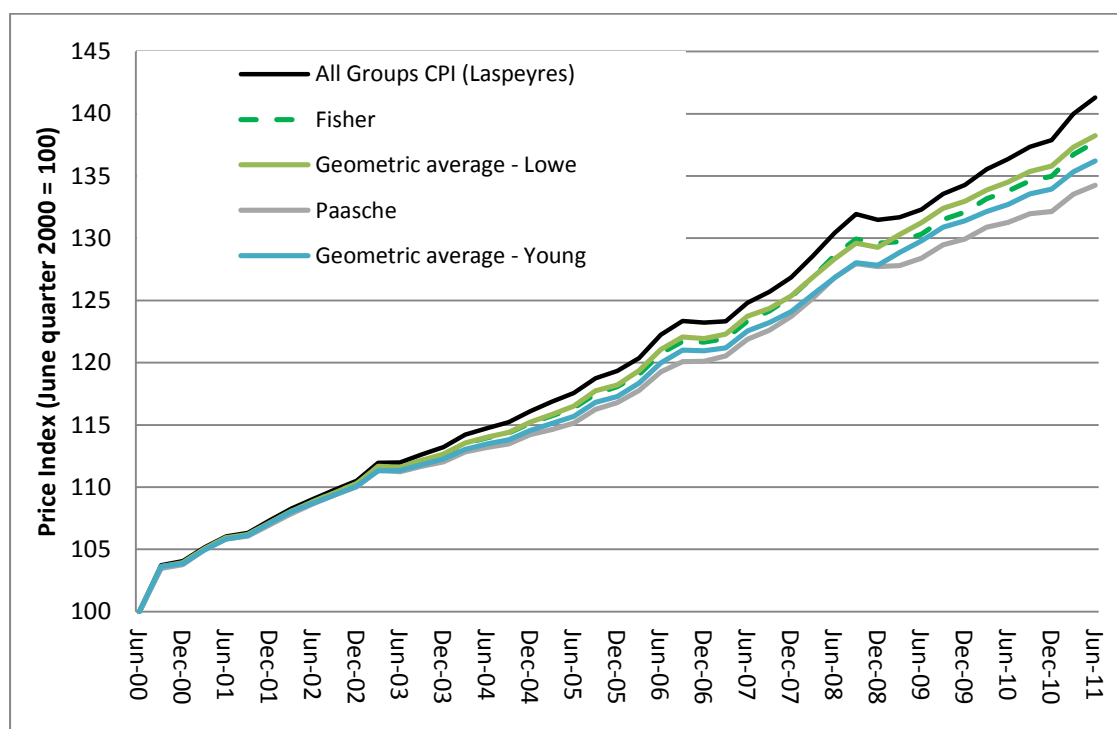
It is interesting to note that while the geometric Lowe index has little conceptual support – neither the CPI Practical Guide nor the CPI Manual mention it - it is a much closer fit to the  superlative Fisher than the geometric Young. This result replicates the results found in the IMF paper (Armknecht and Silver, 2012) where the geometric Lowe produced the second smallest average

monthly differences (out of twelve indexes) from the target indexes[6] while the geometric Young had much larger monthly differences. The geometric Young still performed better than the arithmetic Lowe which was the furthest from the superlative.

Another interesting point is that the geometric Young is at all times lower than the Fisher index, sometimes as low as the Paasche index. Using this method to calculate the headline CPI would possibly introduce a downward bias (the Paasche is considered to be the lower bound of the true 'cost of living index').

(More detail on the methods applied for this analysis and some more detailed results are presented in APPENDIX 3)

Figure 3: Comparison of the All groups CPI, Paashe, Fisher and Weighted Geometric Means



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**SECTION 3: Comparing Changes in Expenditures with Changes in Prices**

The previous section discusses the application of the geometric mean at the higher levels of index aggregation within the ABS CPI. The impact of geometric mean aggregation is to shift the level of the CPI downwards. This is as expected since the ABS CPI is constructed as an arithmetic Lowe index and the geometric mean form of bilateral index aggregation gives a result that is always lower than an arithmetic equivalent (apart from the trivial case where no change is observed).

Given that the geometric aggregation methods are in general consistent with unitary elasticity of substitution (Armknecht and Silver, 2012) the ABS questions whether this is appropriate for the ABS CPI. The analysis in Section 2 does not conclusively determine this although some evidence of the elasticity of certain products is presented. Using ABS HES data and supermarket scanner data, this section examines further the assumption that consumer purchasing behaviour is more consistent with the properties of geometric mean aggregation.

*ABS Household Expenditure Survey (HES)*

The table in Figure 6 below contains a summary of analysis into the movement in the levels of household expenditure compared with the movement in the level of prices. The expenditure values are taken from the three most recent ABS HES (1998-99, 2003-04 and 2009-10) and reduced to 'per capita' and price deflated. The price movements are taken from the published CPI. The comparison was made across 84 expenditure classes. (All of the individual comparisons are shown in Table 4.1 in Appendix 4).

From the 1998-99 survey to the 2003-04 survey, expenditure rose in 58 classes and prices rose in 73 classes. In the 58 classes where expenditure rose, prices rose by an average of 11 per cent. In the 26 classes where expenditure fell, prices rose by an average of 38.9 per cent.

From the 2003-04 survey to the 2009-10 survey, expenditure rose in 56 classes and prices rose in 68 classes. In the 56 classes where expenditure rose, prices rose by an average of 14.5 per cent. In the 28 classes where expenditure fell, prices rose by an average of 29.5 per cent.

With expenditures falling in the classes that have seen the greatest rise in prices and vice versa, this would suggest that the assumption of consumers switching preferences to products with lower levels of inflation might hold true.

|  | Total Expenditure Classes | Deflated Per Capita Expenditure Increase | Average Percentage Change in Prices | Deflated Per Capita Expenditure Decrease | Average Percentage Change in Prices |
|---|---|---|---|---|---|
| 1998-99 to 2003-04 | 84 | 58 | 11 | 26 | 38.9 |
| 2003-04 to 2009-10 | 84 | 56 | 14.5 | 28 | 29.5 |

*Scanner Data*

The ABS has been able to acquire a time series of four-weekly blocks of supermarket scanner data covering a period of around 14 months from late 2011 to early 2013 (fifteen 4-weekly blocks of data). The supermarket classifications allow these data to be partitioned broadly in line with ABS expenditure classes and below that into groups of more closely defined products that could be considered to be "elementary aggregates' (EAs). These data contain both price and expenditure information and some fairly crude analyses of elasticity were carried out using these data.

The analysis considered the relationship between price and expenditure in these 'elementary aggregate' groups. Relative price movements were calculated as an RGEKS[7] index and these were compared with movements in the unadjusted expenditures. All fourteen movements in prices and expenditures between successive four-weekly blocks of data were computed and comparisons made between the movement in prices and the movement in expenditures for each of the theoretical EAs at each point. The change in prices and expenditures across the complete time span was also calculated for each EA. For each comparison, if price and expenditure were moving in opposite directions, the relationship was described as elastic; otherwise it was described as inelastic.

The results of the analysis are presented at a very general level but can still provide some assessment of the relationship between prices and expenditures at the more detailed levels.

A summary of the results given below in figure 6 shows that in between 50 and 70 per cent of cases, the method used suggests that the demand of these products is elastic. A further comparison, across the full time series of data (i.e. time 't' to time 't-14' [ approx. fourteen months]), for the 338 'elementary aggregates' considered, shows this negative correlation holding 70% of the time.

Figure 6: Summary of findings of the analysis into the elasticity within EAs by expenditure class

| Expenditure Class | Number of EAs | Total t to t-1 Observations | Of which, Elastic | Percentage |
|---|---|---|---|---|
| I | 13 | 182 | 118 | 65% |
| II | 13 | 182 | 91 | 50% |
| III | 6 | 84 | 37 | 44% |
| IV | 7 | 98 | 65 | 66% |
| V | 13 | 182 | 123 | 68% |
| VI | 11 | 154 | 74 | 48% |
| VII | 35 | 490 | 300 | 61% |
| VIII | 34 | 476 | 273 | 57% |
| IX | 22 | 308 | 179 | 58% |
| X | 21 | 294 | 159 | 54% |
| XI | 5 | 70 | 40 | 57% |
| XII | 4 | 56 | 33 | 59% |
| XIII | 30 | 420 | 239 | 57% |
| XIV | 26 | 364 | 219 | 60% |
| XV | 15 | 210 | 145 | 69% |
| XVI | 19 | 266 | 158 | 59% |
| XVII | 34 | 476 | 307 | 64% |
| XVIII | 5 | 70 | 45 | 64% |

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[7] The price index constructed uses a 13 period window over the 15 periods of data and therefore the 'rolling' element applies only to the later periods.

## SECTION 4: Summary and Concluding Remarks

Summary

A considerable amount of research has been carried out to try to quantify any negative effects that the ABS CPI might suffer from due to the infrequent updating of expenditure information for the derivation of index weights. The work has been the subject of much discussion and this paper attempts to summarise and put into context the work and the discussion to date.

The analysis undertaken to compare the ABS CPI with a superlative index quite obviously illustrates that if the superlative is to be accepted as the 'target index' then the ABS CPI is upwardly biased with respect to the target.

Constructing the ABS CPI using geometric aggregation at the higher levels (expenditure class and above) shifts the level downwards compared with the published CPI. Relative to the superlative, the geometric Lowe index is closer than any of the other variants, although, as stated, it has little conceptual support. The geometric Young index, which it has been stated is "consistent with unitary elasticity of substitution", performs less well on these grounds and is actually closer to the Paasche index (the lower bound of the superlative). These results should come as no surprise as, generally, geometric averaging produces a result that is lower than an arithmetic equivalent based on equivalent data. Whether this should be interpreted as replicating price elasticity/substitution/cost-minimising behaviour in a 'real world' sense is however, is not so clear cut.

The examination of the expenditure information from the ABS HES for the three most recent periods shows rises in the levels of household expenditure that generally outstrip the rise in prices. However, there are falls in the expenditure levels in around a third of the classes and these are generally within the classes where prices have risen the most.

The analysis of the supermarket scanner data provides a rough but useful summary of the price and quantity (expenditure) relationship within a number of expenditure classes. Based on these findings, the period to period (four-weekly periods) movement in price and expenditure are in the opposite direction between 50 and 70% of the time.

The best way to minimise upper-level substitution bias in the ABS CPI is to ensure that the expenditure weights are up to date. The ABS derives expenditure weights for the CPI from its HES which is conducted every six years. The best way therefore to minimise the upper-level substitution bias is to implement a more frequent HES and the ABS has a proposal to move to a four-yearly HES. As this paper has discussed, additional methods have been proposed to mitigate these biases between household expenditure surveys. However, their suitability is yet to be confirmed. Evidence of rapid growth in household spending against a less rapid rise in prices at the broad level of expenditure class presents challenges when trying to determine the flexibility of household demand as prices are changing. The analysis of the price and quantity relationship at the more detailed levels using the scanner data provides a clearer indication that the demand for some products is elastic while for others it is not.

## Concluding Remarks

Increasing the frequency of the HES to provide more timely expenditure weights for the CPI is viewed by the ABS as the best approach to improve the quality of the CPI.

It has been proposed that geometric averaging at the higher levels of aggregation can improve the accuracy of a CPI between household expenditure surveys. The ABS is not yet convinced that this method does, in practice, improve the accuracy of the CPI. This is because of the unitary elasticity of substitution assumption. For the CPI the question remains whether substitution occurs at the upper levels of the index, say between automotive fuel and vegetables when relative prices change.

Whether the results of the brief pieces of analysis presented here are sufficient to justify the application of a geometric mean formula at the higher levels of aggregation is debatable and the ABS is interested in any similar analysis carried out elsewhere.

The ABS will continue to experiment with this approach as well as undertake work on other initiatives to develop and improve the CPI. The aim is to make greater use of available data to further analyse the suitability of geometric aggregation.

**APPENDIX 1: ABS CPI Aggregation Methods and Recent Issues of Concern**

The ABS has a policy of continually assessing samples of products that are used in the compilation of the CPI. Essentially there are three levels of maintaining the representativeness of the index:

(i) Sample maintenance - ongoing updating and replacement of specifications, respondents, and weights for the prices collected in the CPI, which ensures that the structure of respondent samples and specifications remains relevant.

(ii) Sample review - a complete assessment of the sample used to represent all products in the commodity classification; covering respondents, products, pricing procedures and relative weights based on consumer expenditure. The end product of the sample review may be a new or revised sample (respondents, specifications and collection methods), the confirmation of the existing sample or a change to the index structure below the Expenditure Class (EC) level.

(iii) Index reviews - periodic (six-yearly) reviews of the overall index structure and the price collection methodology and updates to the weighting pattern based on Household Expenditure Survey (HES) data.

The ABS CPI is calculated using arithmetic aggregation at the published levels (expenditure class and above). The weights used in the CPI are (in most cases) derived from the HES and as such can only be updated when new HES information becomes available. Currently, there are new HES data once every six years which results in reweights of the CPI at six yearly intervals. This is outside the five years between reweights recommended by the ILO (2003).

Whilst all levels of the ABS CPI used to be calculated using arithmetic aggregation, as part of the 13th series review in 1997, the CPI changed the elementary aggregation from arithmetic mean of price relatives (Carli index) to a geometric mean of price relatives (Jevons index). This is internationally accepted as a more robust method (ILO, 2003). Most national statistical offices, like the ABS, use arithmetic aggregation at the upper levels.

The current aggregation approach is a Lowe index (Laspeyres type) which fixes the quantities in period b using the following definition.

$$I_{Young}^t = \sum_{i=1}^{n} s_i^{bi} \frac{p_i^t}{p_i^0}, where\ s_i^{bi} = \frac{p_i^0 q_i^{bi}}{\sum_{i=1}^{n} p_i^0 q_i^{bi}}$$

Where p = price of good i at time t
q = quantity of good i at time t
bi = weight reference period of good i

The equation above shows the quantities remaining fixed in the period they are observed (bi)[8]. The relative expenditure share over time (published as points contributions) are price-updated each period to reflect the price change on the selected basket.

To measure price change, a statistical agency needs to decide what "fixed basket of goods" to price. This "fixed basket" specifies both the type and number of items to be held constant and priced each month. The current approach used in price index aggregation when weighting information is available is to hold the base period *quantities* constant. This assumes there is no consumer substitution from period to period. This is generally seen as a reasonable assumption in the short term where people are less likely to be able to shift preferences but does not hold in the long term which leads to an upward bias. Since the ABS does not update the CPI weights in the short-term (due to the unavailability of up to date expenditure data) the upward bias can start to have a significant effect on the headline CPI.

Holding quantities fixed in a certain period can lead to the basket not representing actual expenditure when expenditure patterns change.  A recent Australian example was the volatility introduced into the CPI due to the spikes in banana prices in 2006 and 2011 following tropical cyclones where the price of bananas increased by over 300% temporarily. The current arithmetic aggregation approach assumed consumers still purchased the same quantity of bananas they purchased when prices were at their pre-cyclone levels. In reality, consumers reduced the quantity of bananas they purchased and substituted to other fruit. In these circumstances, the arithmetic aggregation method led to increased volatility in the CPI, although this did not lead to any long term bias.

A more serious issue was raised by the Reserve Bank of Australia (RBA) in its submission to the 16th series CPI review. The RBA noted that re-weighting the CPI every six years is not sufficiently frequent to capture economically significant changes at the upper levels in the consumption basket and this has a marked effect on the overall CPI result. The largest effect of this is seen with the price index of audio, visual and computing equipment which has fallen persistently relative to other goods and services over recent years. As a result, the effective influence of this index in the CPI diminishes as the index moves further away from the most recent re-weighting period (shown in Figure 2 below). This is unlikely to be consistent with current household spending patterns, which is evident with the update of the expenditure weights as shown by the dark lines which represent the introduction of a new CPI series.

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[8] Most, but not all weighting information comes from the HES. Some of the exceptions represent data from periods other than the period represented by the HES, hence the use of period 'bi' to indicate the weight reference period of good i.

Figure 2: Effective weight of AVC equipment*
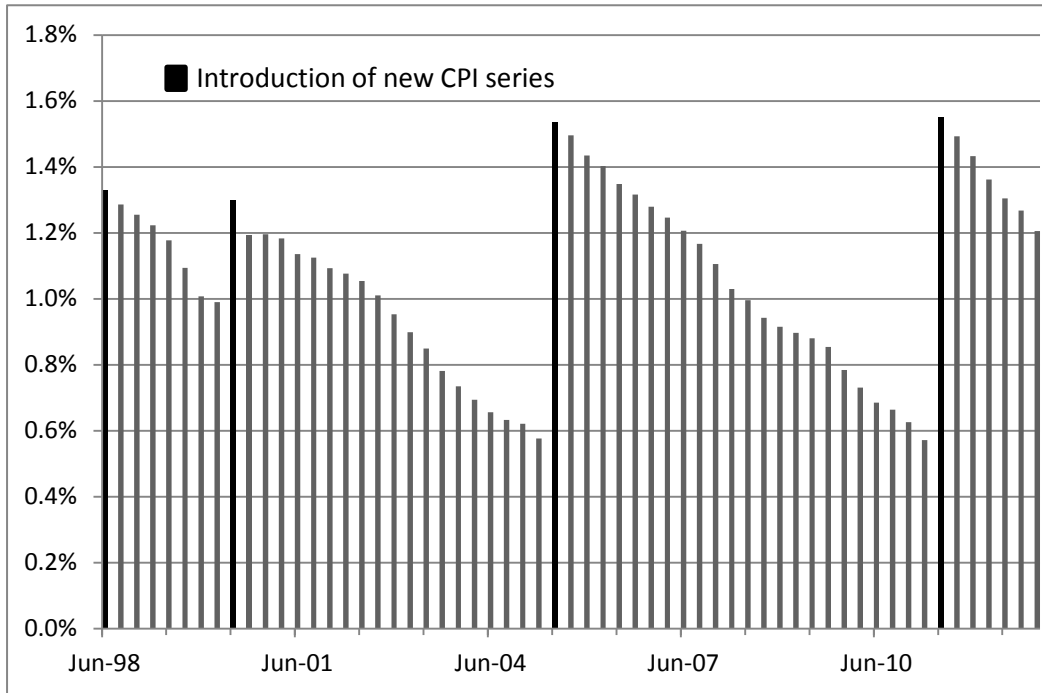


*Audio, visual and computing equipment expenditure share, price updated each period
Source: ABS, RBA

**APPENDIX 2: Extract from the ABS CPI Concepts, Sources and Methods Reference Guide**

# MAINTAINING THE RELEVANCE OF THE CONSUMER PRICE INDEX

**INTRODUCTION**

11.1 In order to measure the price change in the Consumer Price Index (CPI) excluding any quality or quantity changes, the ABS uses a fixed basket of goods and services. However, as consumer expenditure patterns change over time in a dynamic economy, the fixed basket used in the CPI runs the risk of becoming unrepresentative and can lead to bias. There are a number of different types of bias that may affect price indexes. The ABS applies significant effort to address these biases. This chapter includes the strategies the ABS uses to minimise the effect of substitution bias on the CPI and an estimation of one type of bias, the upper-level substitution basis.

**LIMITATIONS OF FIXED BASKET PRICE INDEXES**

11.2 The production of a price index by reference to a fixed basket of goods and services has several advantages. Firstly, the concept is easy to understand; price the same basket of goods and services at two different periods, and compare the total price of the basket. Secondly, by fixing both the items within the basket and their quantities, the resulting values provide a measure of pure price change that is free from compositional change. In application, this process is more complex than the basket analogy would suggest. In practice, the transactions occurring in the market place are frequently changing. This observation reveals a dilemma, namely how can a price index use a fixed basket to measure pure price change and at the same time remain both contemporary and representative of the market?

**ABS STRATEGY FOR REVIEWING AND MAINTAINING PRICE INDEXES**

11.3 The ABS has a policy of continual assessment of the samples of consumer goods and services that it uses in the CPI. Essentially there are three levels of maintaining representation of an index:

(i) Sample maintenance - ongoing updating and replacement of specifications, respondents, and weights for the prices collected in the CPI, which ensures that the structure of respondent samples and specifications remains relevant.

(ii) Sample review - a complete reassessment of the sample used to represent all products in the commodity classification; covering companies, products, pricing procedures and relative weights based on consumer expenditure. The end product of the sample review may be a new or revised sample (respondents, specifications and collection methods), the confirmation of the existing sample or a change to the index structure below the Expenditure Class (EC) level.

(iii) Index reviews - periodic (six-yearly) reviews of the overall index structure and the price collection methodology and updates to the weighting pattern based on Household Expenditure Survey (HES) data.

**ITEM SUBSTITUTION, INDEX FORMULAS AND THE FREQUENCY OF CPI WEIGHT UPDATES**

11.4 Item substitution occurs when households react to changes in relative prices by choosing to reduce purchases of goods and services showing higher relative price change and instead buy more of those showing lower relative price change.

11.5 Under such circumstances, a fixed-basket Laspeyres index will overstate the price change of the whole basket as it does not take account of changes in the substitutions that consumers make in response to relative price changes. For example, if the price of beef were to increase more than the price of chicken, one would expect consumers to purchase more chicken and less beef. As a fixed-base index would continue to price the original quantities of beef and chicken, the price change faced by consumers would be overstated.

11.6 Item substitution bias is due to changes in the pattern of household consumption which takes place over time as a result of both demand and supply changes. The longer the period between weight revision periods, the more time there is for consumers to substitute towards or away from goods and services in reaction to relative price changes and as a result of changes in income. Similarly, supply conditions (and therefore the availability, or otherwise, of certain goods and services) can change substantially over the period in which the weights are fixed.

11.7 Like most CPIs, the Australian CPI is calculated using a base-weighted modified Laspeyres index formula (known as Lowe index) which keeps quantities fixed between major revisions but allows prices to vary. A Laspeyres (or in most cases a Laspeyres-type) index measures the change in the cost of purchasing the same basket of goods and services in the current period as was purchased in a specified base period. The weights reflect expenditures from a historical period, the base period.

11.8 There is a family of indexes called superlative indexes. Superlative indexes make use of both beginning-of-period and end-of-period information on both prices and quantities (expenditures), thereby accounting for substitution across items. However, in order to construct a superlative index both price and quantity (expenditure) data are required for both periods under consideration.

11.9 Superlative indexes can only be produced retrospectively once the required weighting data is available. Given that current period expenditure data for households is not available on a sufficiently timely basis (generally not available until 12 months after the reference period), a superlative formula cannot be used in the routine production of the CPI, which is why statistical agencies rely on fixed baskets. Most, if not all, statistical agencies use a Laspeyres-type index. The requirement for end-of-period information in real time is the reason a superlative index is an impractical option for statistical offices for the compilation of the CPI.

**ESTIMATION OF THE UPPER LEVEL SUBSTITUTION BIAS**

11.10 The ABS has constructed a retrospective superlative-type index to provide an estimation of potential item (upper level) substitution bias in the fixed-weight Australian CPI. While there are five main sources of bias in CPIs, this analysis focuses on one type only - upper level item substitution bias - and therefore the results in the analysis should not be taken to equate to the total bias in the CPI, which will be the cumulative impact of all sources of bias. This analysis can only be conducted retrospectively when new HES data is available - currently every six years.

11.11 Superlative indexes allow for substitution as they make use of weights for both the earlier and later periods under consideration (basically averaging across historical and current expenditures to derive a 'representative' set of weights for the period) whereas the Laspeyres index uses only base period weights.

11.12 The estimate of upper level substitution bias has been made at relatively high levels of aggregation. The analysis is calculated based on the amount of consumer substitution between expenditure classes as this is the lowest level for which reliable weighting information (from the HES) is available and this is the level at which the underlying quantity weights remain fixed between CPI reviews. Thus, the analysis captures substitution from one expenditure class to another, e.g. from beef and veal to poultry, but not within a given expenditure class, e.g. from beef to veal. The substitution within an expenditure class is called lower level substation bias which is minimised through regular sample maintenance, sample reviews and choice of index formulas.

11.13 Two superlative indexes have been constructed and linked together to form one continuous series. The first index was constructed on the 14th series CPI basis between the June quarter 2000 and the June quarter 2005 and the second index was constructed on the 15th series CPI basis between the June quarter 2005 and the June quarter 2011.

11.14 Using the expenditure class at the weighted average of eight capital cities level, i) Laspeyres-type, ii) Paasche-type, and iii) superlative Fisher-type indexes have been calculated at the All groups CPI level. The indexes have all been calculated with the base period June quarter 2000 = 100.0. The Fisher index is regarded as the best practical approximation of a 'true' (or 'ideal') price index, being the geometric average of the Laspeyres and Paasche indexes.

11.15 The Laspeyres-type index is equivalent to the published All groups CPI re-referenced to the June quarter 2000. There may be some differences in the movements compared to the All groups CPI due to rounding.

11.16 The Paasche and Fisher-type indexes were a retroactively modelled analytical series and are not replacing the published Australian Consumer Price Index which is designed to measure price inflation for the household sector as a whole.

11.17 The Paasche-type and superlative Fisher-type indexes were constructed using the same structure as the All groups CPI as published at the time to allow for direct comparison. The indexes from the June quarter 2000 to the June quarter 2005 were derived using the 14th series classification consisting of 88 expenditure classes. The index numbers from the June quarter 2005 to the June quarter 2011 were derived using the 15th series classification consisting of 90 expenditure classes.

11.18 Using these indexes, an estimate of upper level substitution bias in the CPI was obtained by subtracting the superlative (Fisher-type) index from the All groups CPI (Laspeyres-type) index. The

Fisher index is regarded as the best practical approximation of a 'true' (or 'ideal') price index, being the geometric average of the Laspeyres and Paasche indexes.

11.19 For the Paasche-type index, to estimate current period weights each quarter, the ABS applied a linear model between the re-weighting periods (June quarter 2000 - June quarter 2005 and June quarter 2005 - June quarter 2011). In calculating the Paasche-type index the June quarter 2011 weight for the Fruit expenditure class was modified to adjust for the effect of cyclone Yasi.

## ANALYSIS OF THE UPPER LEVEL SUBSTITUTION BIAS

11.20 The analysis found the total upper level substitution bias of the All groups CPI (as measured by the difference between the Laspeyres-type index and the Fisher-type index) was 3.6 percentage points after 11 years due to the inability of the fixed-base index to take account of the item substitution effect. The All groups CPI, calculated using a fixed-weight direct Laspeyres-type index increased by a total of 41.3% from June quarter 2000 to June quarter 2011. The retrospective superlative index, calculated using the Fisher-type index, increased by 37.7% over the same period.

11.21 To estimate the average annual upper level substitution bias, the indexes can be expressed as Compound Annual Growth Rates (CAGR).

LaspeyresCAGR

$$= ((I_{L,JQ11} / I_{L,JQ00}) (1/11) - 1) * 100$$
$$= ((141.3/100.0) (1/11) - 1) * 100$$
$$= 3.19\%$$

FisherCAGR

$$= ((I_{F,JQ11} / I_{F,JQ00}) (1/11) - 1) * 100$$
$$= ((137.7/100.0) (1/11) - 1) * 100$$
$$= 2.95\%$$

11.22 The average annual upper level substitution bias was calculated as LaspeyresCAGR - FisherCAGR = 3.19% - 2.95% = 0.24%. The CPI for the period June quarter 2000 to the June quarter 2011 was potentially upwardly biased by 0.24 of a percentage point per year on average due to the inability to take account of the upper level item substitution effect. These results are consistent with studies by other national statistical agencies.

11.23 The results show that the longer the period between re-weights, the larger the potential upper level item substitution bias effect on the index. Table 11.1 illustrates that the average annual substitution bias increases at a faster rate the longer the period between re-weights. The re-weighting periods in this analysis were June quarter 2000 and June quarter 2005.

**11.1 Average Annual item substitution bias**(a)

| Time since re-weight | Laspeyres$_{CAGR}$ - Fisher$_{CAGR}$ |
|---|---|
| (b)1 year | 0.16 |
| 2 years | 0.08 |
| 3 years | 0.12 |
| 4 years | 0.15 |
| 5 years | 0.22 |
| (c)6 years | 0.25 |
| Annual average between June quarter 2000 and June quarter 2011 | 0.24 |

(a) This takes the average of the average annual item substitution bias for the period June quarter 2000 - June quarter 2005 and the period June quarter 2005 - June quarter 2011.
(b) This figure includes the banana price increase in March 2006 which was a result of cyclone Larry.
(c) The six year average annual item substitution bias is only based on the index numbers for June quarter 2005 to June quarter 201.

11.24 The result for 1 year since re-weight was caused by the introduction of the GST and cyclone Larry and can be considered atypical. Excluding this, it can be seen that the average annual item substitution bias increases over time and also increases at a faster rate, especially after the fourth year. This finding is consistent with the Statistics New Zealand (SNZ) analysis which showed that item substitution bias is considerably greater when NZ CPI weights are updated at six-yearly rather than three-yearly intervals.

11.25 While there are five main sources of bias in CPIs, this analysis focuses on one type only - upper level item substitution bias - and therefore the results in the analysis should not be taken to equate to the total bias in the CPI, which will be the cumulative impact of all sources of bias.

**APPENDIX 3: Empirical Analysis of the Impact of Compiling the ABS CPI Using Geometric Mean Formulae.**

*Data sources*

By virtue of the geometric Young and geometric Lowe having the same data requirements as their arithmetic counterparts, both the geometric Young and geometric Lowe indexes were calculated using the data that are used to calculate the existing CPI. That is, 85 quarterly value aggregates from September quarter 2000 to September quarter 2012 to calculate the price movements and the relevant HES data to calculate the base period weights. This 12 year period involved three different CPI structures (14th series, 15th series and 16th series). As a result, some changes had to be made to move these three series onto a comparative basis so they could be linked.

The difference between the two indexes is the data used to calculate the base period weights. The geometric Young index used (mostly) HES period weights (period *b*), while the geometric Lowe index used (mostly) HES period weights, price-updated to period *0*.

14th series: the 98/99 HES was used as the basis for weights for June quarter 2000 – March quarter 2005.
15th series: the 03/04 HES was used as the basis for weights for June quarter 2005 – March quarter 2011.
16th series: the 09/10 HES was used as the basis for weights for June quarter 2011 to present.

The geometric series are to be compared to the ideal Fisher index which is a geometric average of the Laspeyres-type and Paasche-type indexes. Due to Paasche indexes needing final period weighting information, the Paasche-type (and accordingly the Fisher-type) index cannot extend beyond June quarter 2011 until new weighting information is available.

The fruit price index was also calculated to see how the geometric mean handled the volatile index compared with the arithmetic mean. A monthly (fruit is one of the expenditure classes in the CPI which are priced more frequently than quarterly, due to the volatility of prices) index is calculated from the elementary aggregate components for the months September 2005 to February 2013.

The time series spanned more than a decade and as the CPI underwent multiple reviews and changes over this time, was based on multiple CPI structures; the 14th, 15th and 16th series. So that the different series structures could be comparable, some changes had to be made to the 14th and 15th series to make it resemble as closely as possible the 16th series structure. The 14th series structure did not differ from the 15th series considerably with the main difference being that the 'miscellaneous' group was removed and these expenditures were redistributed to appropriate existing CPI groups. However, as an outcome of the 16th series review, there were many changes between the 15th and 16th series.

To calculate Geometric Lowe index using same expenditure data and weights (price updated) as the published CPI:

$$I_{GLowe}^t = \prod_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right)^{s_i^{b0}}, where \; s_i^{b0} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$$

To calculate Geometric Young index using the same expenditure data as the published CPI but different weighting data (not price updated):

$$I^t_{GeoYoung} = \prod_{i=1}^{n} \left( \frac{p^t_i}{p^0_i} \right)^{s^b_i}, where \; s^b_i = \frac{p^b_i q^b_i}{\sum_{i=1}^{n} p^b_i q^b_i}$$

To calculate Laspeyres, Fisher and Paasche indexes:

The Laspeyres index used is the published CPI series (calculated using arithmetic mean). The Paasche and Fisher indexes used were calculated as part of the 16th series CPI review project on the frequency of reweighting. It should be noted that the Paasche and Fisher indexes used are not true Paasche and Fisher indexes. It was not possible to calculate a true Paasche index as it was not possible to derive quarterly expenditure weights therefore nor was it possible to derive a true Fisher index. The key point to note is that the index levels at the quarters that lie in between the weights updates (at 2000, 2005 and 2011) are estimated via quadratic interpolation (Diewert et al,2008). This was the most appropriate way to approximate a superlative index which is our 'target' index in terms of reducing substitution bias.

The time series used link period value aggregates and link period price indexes to join the 14th series to the 15th series and then the 15th series to the 16th series. Link periods were also used where necessary for various expenditure classes when there had been breaks in the series.

*Results*

Figure 3 shows the arithmetic price indexes: the Laspeyres-type (published CPI) and the Paasche-type. The target index is a Fisher index, which lies between the Laspeyres and Paasche indexes. Also shown are the geometric Young and geometric Lowe indexes.

Both the geometric Young and the geometric Lowe indexes are lower than the published CPI. The geometric Lowe index very closely follows the Fisher index, only starting to deviate from December 2008. From that point until June 2011, the geometric Lowe remained slightly above the Fisher index but below the published CPI index. The geometric Young index was lower than the geometric Lowe and at all times was below the Fisher but above the Paasche index. From December 2007 to December 2008 the geometric Young was as low as the Paasche index; significantly lower than the superlative Fisher index. The geometric Young index started to deviate from the Fisher index much earlier than the geometric Lowe index (in December 2002 as opposed to June 2006).

It is interesting to note that while the geometric Lowe index has little conceptual support – neither the CPI Practical Guide nor the CPI Manual mention it - it is a much closer fit to the superlative Fisher than the geometric Young. This result replicates the results found in the IMF paper (Armknecht and Silver, 2012) where the geometric Lowe produced the second smallest average monthly differences (out of twelve indexes) from the target indexes[9] while the geometric Young had much larger monthly differences. The geometric Young still performed better than the arithmetic Lowe which was the furthest from the superlative.

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[9] Fisher and Törnqvist indexes

Another interesting point is that the geometric Young is at all times, lower than the Fisher index, sometimes as low as the Paasche index. Using this method to calculate the headline CPI would possibly introduce a downward bias (the Paasche is considered to be the lower bound of the true 'cost of living index').

Figure 4: Comparison of the All groups CPI, Paashe, Fisher and Weighted Geometric Means
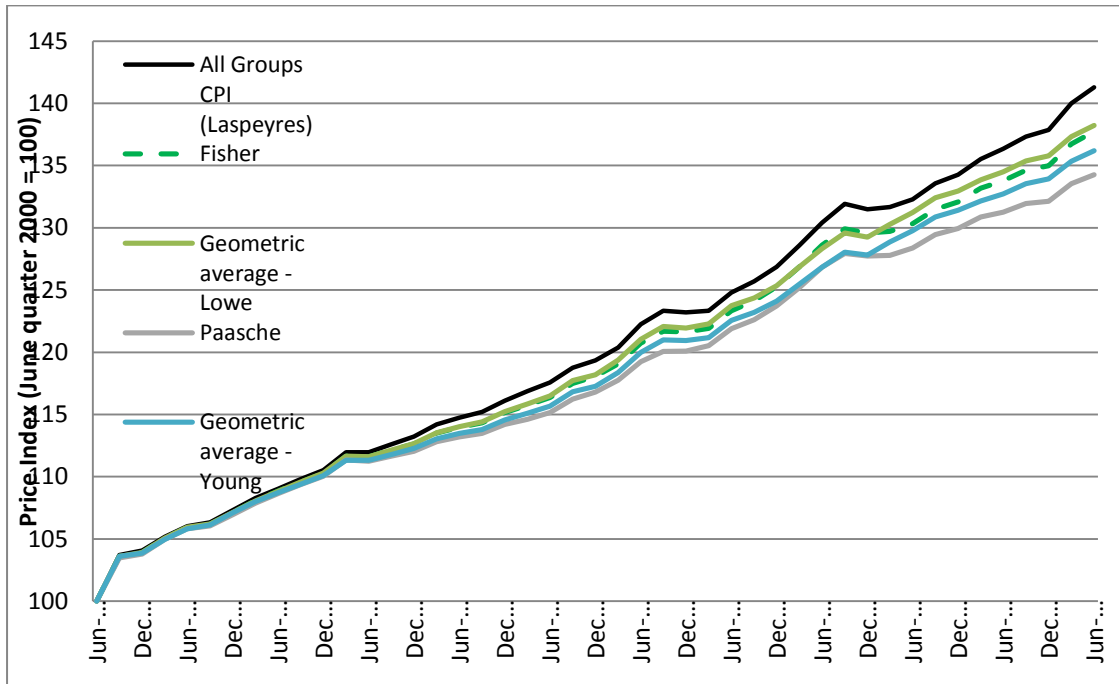


Another key advantage of the geometric mean is its ability to cope with temporary price spikes. The figure below shows how the geometric average reduces the volatility introduced into the index due to the price spikes in bananas in 2006 and 2011. Calculating the fruit price index from the elementary aggregate components, the impact of the banana price spike is greatly reduced using the geometric mean which would lead to less spurious volatility in the CPI and improved volume estimates in the national accounts. Of note, when banana prices return to normal levels the differences between the price indexes are minimal.

Figure 5: Monthly CPI fruit index
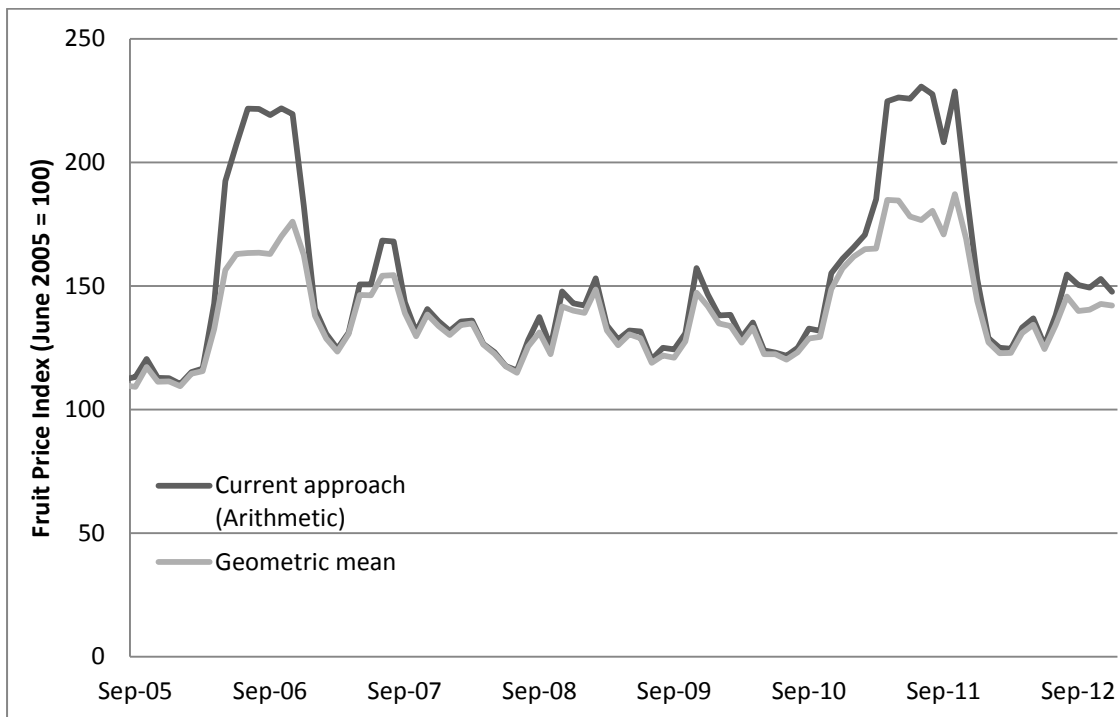
**APPENDIX 4:**

**Table 4.1 ABS CPI Price Movements Compared with HES Expenditure Movements**

| Expenditure Class | 1998-99 to 2003-04 | | 2003-04 to 2009-10 | |
|---|---|---|---|---|
| | Change in Prices | Change in Deflated Per Capita Expenditure | Change in Prices | Change in Deflated Per Capita Expenditure |
| Milk | 13.1% | -11.9% | 5.2% | -8.0% |
| Cheese | 26% | 10.8% | 37.1% | -4.9% |
| Ice cream and other dairy | 15% | 8.8% | 21.1% | 31.0% |
| Bread | 9% | 3.7% | 33.8% | -13.7% |
| Cakes and biscuits | 19% | 2.3% | 27.7% | 2.9% |
| Breakfast cereals | 18% | 1.0% | 10.1% | 2.2% |
| Other cereal products | 12% | -9.8% | 24.3% | 16.2% |
| Beef and veal | 40% | -6.5% | 3.3% | -6.5% |
| Lamb and mutton | 57% | -1.3% | 39.9% | -11.9% |
| Pork | 27% | 1.0% | 15.6% | 2.8% |
| Poultry | 5% | 34.5% | 9.2% | 20.7% |
| Other fresh processed meat | 24% | 4.6% | 26.7% | 2.2% |
| Fish and other seafood | 10% | 21.9% | 23.0% | 8.5% |
| Fruit | 25% | 15.3% | 107.1% | 0.2% |
| Vegetables | 13% | 12.3% | 39.2% | 4.4% |
| Soft drinks waters juices | 8% | -6.5% | 30.9% | -6.2% |
| Snacks and confectionery | 21% | 5.0% | 25.4% | -5.4% |
| Restaurant meals | 28% | 18.7% | 22.3% | 33.0% |
| Takeaway and fast foods | 27% | -0.1% | 27.7% | -2.5% |
| Eggs | 12% | 9.6% | 16.7% | 10.9% |
| Jams, honey and s'wich spre | 24% | -2.3% | 21.7% | 0.1% |
| Tea, coffee and food drinks | 1% | 7.4% | 15.5% | 20.5% |
| Food additives and cond | 3% | 16.7% | 23.5% | 9.1% |
| Fats and oils | 24% | -2.9% | 28.8% | -7.5% |
| Food n.e.c | 15% | 34.9% | 15.9% | -8.7% |
| Beer | 28% | -5.1% | 28.9% | -2.7% |
| Wine | 13% | 26.3% | 5.2% | 19.8% |
| Spirits | 18% | -0.2% | 36.8% | -3.3% |
| Tobacco EC | 43% | 1.1% | 57.4% | -20.6% |
| Men's outerwear | 1% | 16.6% | -5.0% | 34.8% |
| Women's outerwear | 4% | 14.3% | -7.1% | 42.2% |
| Children's and infants' clo | 8% | 16.7% | -8.0% | 4.0% |
| Men's footwear | 2% | 15.3% | -4.8% | 7.0% |
| Women's footwear | 2% | 26.8% | -9.1% | 39.4% |
| Children's footwear | 5% | 6.3% | 0.2% | 11.5% |
| Clothing access, jewellery | 7% | 29.1% | 21.2% | 40.0% |
| Clothing services & shoe | 24% | -44.7% | 21.1% | -39.0% |
| Rents | 13% | 16.1% | 36.3% | 19.5% |
| Electricity | 28% | 12.9% | 65.7% | -12.7% |

| | | | | |
|---|---|---|---|---|
| Gas and other hhld fuels | 39% | -2.2% | 45.3% | -12.0% |
| Water and sewerage | 18% | 15.2% | 71.9% | -21.9% |
| House purchase | 31% | 30.6% | 21.1% | 7.8% |
| Property rates and charges | 30% | 5.2% | 37.9% | 1.8% |
| House repairs & maintenance | 26% | 44.9% | 18.1% | -6.5% |
| Furniture | 4% | 17.3% | 5.2% | -1.5% |
| Floor and window coverings | 27% | 19.5% | 11.8% | -64.0% |
| Towels and linen | -6% | 14.5% | -6.1% | 84.8% |
| Major household appliances | 4% | 15.0% | -7.7% | 3.1% |
| Small elec hhld appliances | -9% | 55.1% | -1.1% | 2.5% |
| Glassware, tablew, utnsils | -8% | 53.8% | -2.4% | 9.8% |
| Tools | -2% | 41.0% | 6.1% | 14.0% |
| Household cleaning agents | 2% | 12.8% | 7.3% | 3.5% |
| Other Household Supplies | 9% | 22.1% | 11.4% | 19.8% |
| Household services EC | 12% | 44.6% | 30.8% | -5.3% |
| Hospital & medical service | 39% | 2.0% | 40.9% | 12.9% |
| Dental services | 50% | 6.2% | 30.4% | -18.4% |
| Pharmaceuticals | 59% | -10.6% | 4.8% | 23.0% |
| Motor vehicles | 33% | -3.3% | -2.6% | -12.5% |
| Automotive fuel | -34% | 96.3% | 37.9% | -19.3% |
| Motor vehicle repair & ser | 47% | -15.8% | 17.6% | 3.0% |
| Motor vehicle parts & acc | 17% | -8.7% | 18.4% | 66.1% |
| Other motoring charges | -3% | 32.7% | 38.2% | 30.0% |
| Urban transport fares (EC) | 32% | -14.1% | 23.6% | 10.0% |
| Postal | 23% | -13.9% | 25.9% | 7.2% |
| Telecommunication | 47% | 16.3% | -1.1% | 14.6% |
| Audio, visual and comp equi | 201% | -46.9% | -62.5% | 240.1% |
| Audio, visual & comp media | -80% | 612.8% | 8.2% | -20.2% |
| Books | -17% | 44.5% | 7.0% | 5.2% |
| Newspapers and magazines | 12% | -7.8% | 11.6% | 79.3% |
| Sport & recreational equip | 42% | -7.9% | -10.1% | 43.7% |
| Toys, games and hobbies | -11% | 53.2% | -6.6% | 114.2% |
| Sports participation | -29% | 85.8% | 34.5% | 4.2% |
| Pets, pet food and supplies | 16% | 14.9% | 44.3% | -10.0% |
| Pet Services Incl. Veterina | 25% | 54.9% | 27.1% | -3.5% |
| Other recreational activiti | 23% | -12.2% | 31.6% | 7.6% |
| Domestic hol travel & acc | 42% | -3.1% | 16.5% | 12.6% |
| Overseas hol travel & acc | 25% | 2.5% | 6.1% | 60.7% |
| Preschool primary educatn | -4% | 67.9% | 38.3% | 14.4% |
| Secondary education | 25% | 26.8% | 49.6% | 18.2% |
| Tertiary education | 66% | -21.8% | 23.1% | 14.9% |
| Insurance services | 10% | 43.8% | 40.4% | -18.3% |
| Hairdressing & pers'l care | 44% | -2.2% | 17.1% | 29.6% |
| Toiletries & Personal Produ | 56% | -20.7% | -1.9% | 21.4% |
| Child care | 20% | 6.9% | -11.3% | 114.8% |