Measuring Dynamic Inflation in Brazil

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April 25, 2013

Abstract

In this article we propose and construct a dynamic measure of consumer cost of living for Brazil. Following the methodology proposed by Reis (2008), our baseline model starts from a representative agent who lives for many periods, which is forward looking and decides each period between consumption 14 types of nondurable goods and services, 5 durable goods and two financial assets. This Dynamic Price Index (DPI) takes into account the rate of return of savings deposit and equity. Consumption of Durable and non-durable goods are considered as separate decisions, and are treated accordingly. We construct the monthly DPI for Brazil from 2002 to 2011 using data from the Brazilian CPI calculated by the Brazilian Institute of Economics of the Getulio Vargas Foundation.

The calculated monthly variation of the DPI is more volatile, less persistent and less serially correlated than the CPI. The accumulated difference between the CPI and the DPI over this 11-year period is 21%.

JEL Classification: E31, C43, D91.

Keywords: Dynamic price index, intertemporal consumer decision, cost of living price index.

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1 Introduction

Consumer inflation is one of the key economic indicators. It serves as crucial information for investors, workers, managers and public policy makers. Research in this area aims at providing more accurate measures of cost of living thus leading to better indexing of government benefit programs, taxes and assessment of economic growth.

The basic framework for Consumer Price Indexes (CPI's) based on the economic approach is a representative consumer who lives for one period, faces no uncertainty about prices, preferences or income, and decides on which goods and services to allocate his budget. Konus (1924) was the first to define a Cost of Living Price Index, henceforth COLI, as the compensating variation in response to price changes to keep the initial welfare level (utility) fixed.

Several countries have their CPI constructed with the COLI framework in mind. Accordingly, Bureau of Labor Statistics (2007) states that "the concept of COLI provides the CPI's measurement objective".

Although it is widely employed, the static framework suffer from known deficiencies. For example, it fails to account for the fact that when the price of a good rises, consumers substitute away from this good both to other goods and into future consumption. Hence even using static measures of inflation that correct for the static substitution problem, we may not be able to assess the full impact of changes in the price of housing or other durables.

To mitigate these dynamic problems in the static COLI framework, Reis (2005) proposed a Dynamic Cost of Living Price Index, DPI for short, based on the modern theory of consumption by Deaton (1992). It assumes a consumer who lives for many periods and is subjected to shocks in the price of goods and services he consumes. The DPI is then the compensating variation that keeps the lifetime utility unchanged not the one period stage utility. Besides solving the intertemporal substitution bias for goods, by incorporating intertemporal prices, a dynamic measure of inflation provides a unifying framework to incorporate the price of financial instruments in the cost of living.

The difference between the CPI and the DPI tends to be bigger[1] the more the consumer cares about consumption in the future, for example a consumer who needs to save for retirement, or as a related application, an university that wants to provide the same level of educational services to future generations etc. The application provided in this paper is closely related to the case of a consumer who saves for retirement.

In this paper, we calculate a version of the DPI for households in Brazil using the series of the CPI calculated by the Brazilian Institute of Economics at Getulio Vargas Foundation.

[1]When financial returns and prices of durable goods follow random walk processes the CPI and the DPI are equal. See Reis (2005) for this result.

We consider 19 types of goods and services and 2 financial assets. The first financial asset is equity with returns equal to the value weighted index IBOVESPA. The second financial asset is the savings deposit called "Poupanca" in portuguese. It is the most common financial asset held by households in Brazil, because it is simple to invest, its dividends are not subject to income taxes and carry no risk of loss on the nominal investment. Still it could yield negative real return.

We calculate the monthly DPI from january 2002 to december of 2012. The accumulated difference between the CPI and the DPI over this 11-year period is approximately 21%, these two measures are not very correlated, only 0.40 for the monthly variation, and the DPI is almost two times more volatile. Part of the difference between these two measures is explained by the large decrease in the return of savings deposit starting in 2007 and the bigger weight of durable goods in the DPI as they grow slower than other goods due to imports from China. In addition, over the 2008 world crisis the DPI grew less than the CPI because of the large decrease in equity returns.

The intertemporal tradeoff addressed in the DPI index proposed by Reis (2005) was first studied by Alchian and Klein (1973) as the former author points out. Many other authors improve on Alchian and Klein (1973)'s work, but all of them involve somewhat unrealistic assumptions and can be seen as special cases of Reis (2005).

Another line of research is to calculate the welfare implications of price changes given the estimated dynamic behaviour from actual choices made by consumers. The paper by Gowrisankaran and Rysman (2011) proposed a structural estimation of a dynamic demand for camcorders. The authors apply the estimated demand to generate a Cost-Living-Index for this durable good, but as they pointed out, it would probably be infeasible to calculate such indexes within the BLS given its time constraints. Nevo and Griffin (2008) proposes a way to deal with timing and quantity of purchases, which is a dynamic decision by nature and its implications for the CPI. Pollack (1998) also points out that a behavioural model of search and stockpiling would be needed to address problems such as how to incorporate sales in the CPI. The main objective of this literature is estimate a structural model of consumer behaviour and apply these results to cost of living measurement.

The rest of this article is as follows. In the next section we introduce the consumer decision model and formally define the DPI. Then, we present a section on the data used and the calibration procedure and the section that follows presents the results. The last section concludes with a brief discussion of the challenging issues associated with the implementation of this index within a statistics agency.

2 Consumer decision model

The mathematical problem facing the representative consumer at time t consists of choosing the sequence of consumption of nondurable goods $\{C_{t+i}\}$, durable goods $\{S_{t+i}\}$ and assets $\{B_{t+i}\}$ to maximise:

$$\mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i \left(\sum_{j \in ND} \alpha_j \ln(C_{j,t+i}) + \sum_{j \in D} \alpha_j \ln(S_{j,t+i}) \right) \right] \tag{1}$$

subject to:

$$P_{t+i}^T C_{t+i} + R_{t+i}^T S_{t+i} + Q_{t+i}^T B_{t+i} \leq W_{t+i}, \tag{2}$$

$$W_{t+1+i} = D_{t+1+i}^T B_{t+i} + R_{t+1+i}^T \Delta S_{t+i}, \tag{3}$$

$$W_{t+1+i} \geq 0, C_{t+i} \geq 0, S_{t+i} \geq 0, \tag{4}$$

$$\text{for } i = 0, 1, 2, \cdots \text{ and } W_t = A_t \tag{5}$$

The consumer maximises total expected utility which equals the expected discounted sum of period utilities. The consumer faces a constant probability of dying, which combined with impatience, leads to a discount factor $\beta < 1$. The utility at each period takes a Cobb-Douglas form, with a set of taste weights α_j that sum to one across all goods.

The consumer allocates her wealth W_{t+i} each period between acquiring non-durables, collected in the vector C_{t+i} at the price vector P_{t+i}, or durable goods S_{t+i} at the price vector R_{t+i}. She can also buy or sell two one-period financial assets: saving deposits which pay a certain amount next period, e.g. $(1 + r_t)$, or stocks which have some random payoff next period. The stock trade at price $Q_{E,t+i}$ and the portfolio holdings are collected in the vector B_{t+i}.

For simplicity we assume that the consumer starts with wealth equal to some exogenous amount A_0. Then, the sources of wealth are the payoffs from the financial assets plus the market value of the stock of durables after depreciation. The vector of payoffs is $D_{t+1+i}^T = (B_{t+i}(1 + r_{r+i}), Q_{E,t+1+i})$ where $B_{t+i}(1 + r_{r+i})$ is the known payoff from savings deposit and the return from holding equity is $Q_{E,t+1+i}/Q_{E,t+i}$. The diagonal matrix Δ has elements $1 - \delta_j$, where δ_i is the depreciation rate of durable j. The additional constraints in (4) are standard. They require that wealth and consumption of both durables and nondurables to be non-negative.

The only source of uncertainty in this model are prices. Hence, we assume that p^{t+i} is a random vector containing the three set of prices: P_{t+i}, R_{t+i} and Q_{t+i} that follows a finite order Markov process. Therefore there exists a maximum lag k such that prices older than $t - k$ are not relevant to predict future prices.

The set of assumptions on consumer preferences and prices guarantee the existence of a value function $V(W_{t+i}, p^{t+i})$ that is equal to the consumer's maximum expected lifetime utility from $t+i$ onwards.

The dynamic price index is defined by Reis (2005), following Konus (1924):

Definition 2.1 *The dynamic price index $\pi_{t+1} \in \mathbb{R}$ is such that:*

$$V(\pi_{t+1} W_t, p^{t+1}) = V(W_t, p^t)$$

The dynamic price index π_{t+1} measures how much we should compensate the consumer facing the new set of prices p^{t+1} so that he is indifferent between purchasing at each period's prices. Note that, although the stage utility follows a Cobb-Douglas, the value function $V(\cdot)$ may not have a closed form solution. It depends on the consumer's optimal behaviour and how expectations are formed regarding next periods prices.

According to Reis (2005), the DPI has several important theoretical properties. It is well defined, that is, if prices and wealth are positive and finite the DPI exists and is unique. Since the stage utility function is homothetic and time separable, the discounted utility is also homothetic. Hence the DPI is independent of wealth W_t. As consumers engage in intertemporal substitution, the DPI is forward looking, the more persistent are the shocks, the larger their impact on the index. In addition, durable goods affect the DPI through two channels. The first is the change in expenditure and the second is the expected capital gains and losses. If all prices follow random walks and financial asset returns are all i.i.d then the DPI equals the static cost-of-living price index.

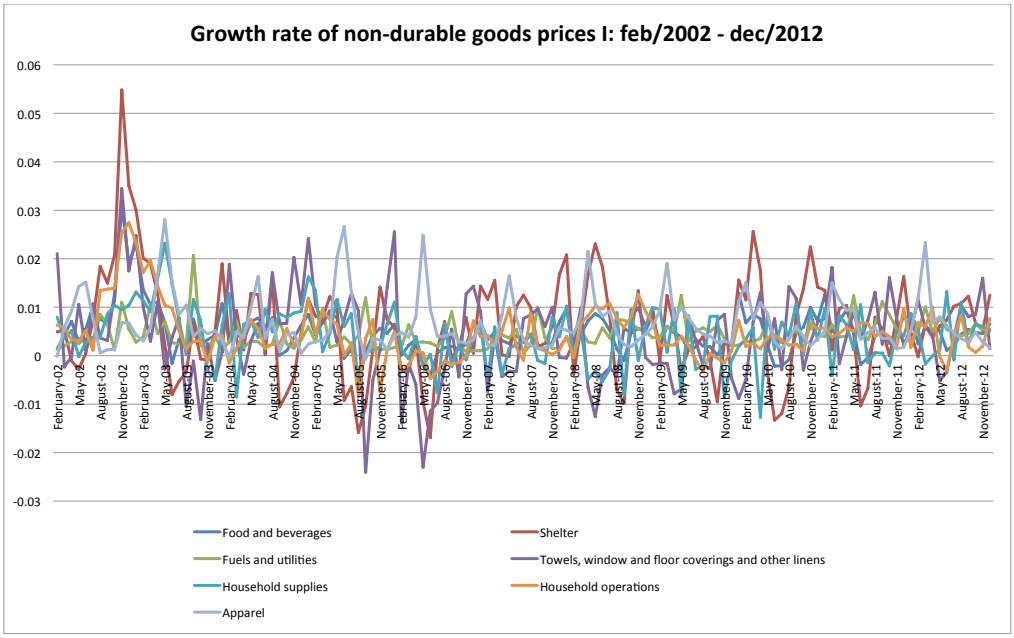
3 Calculating a dynamic measure of inflation for Brazil

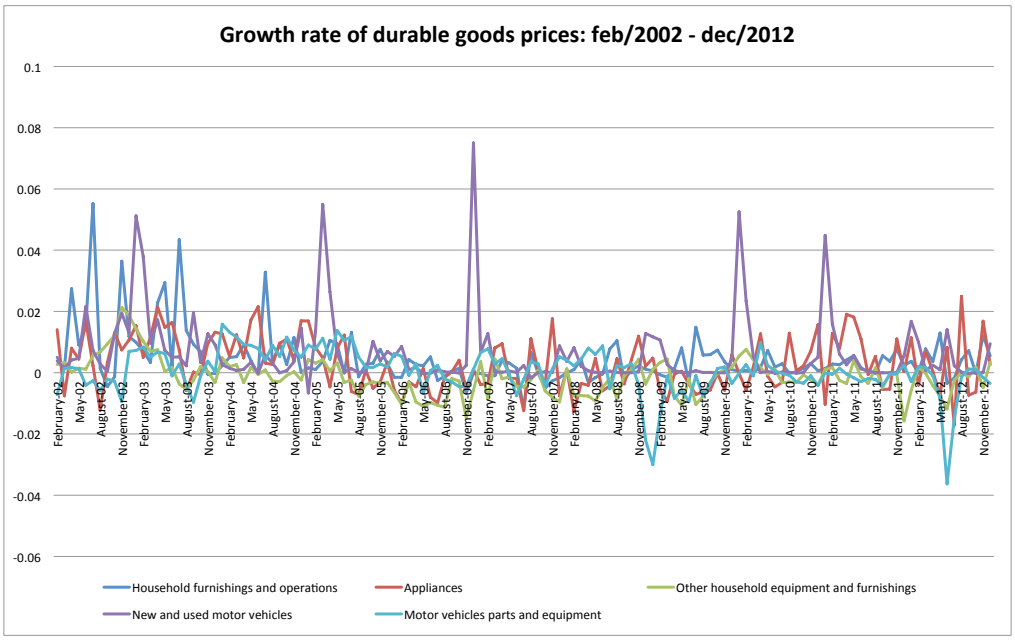
In order to provide a first pass to the problem we consider only broad categories of goods and assets for which there are more reliable time-series. There are 19 categories from the CPI and 2 types of financial assets, savings deposit and equity. This section explains the source of these data.

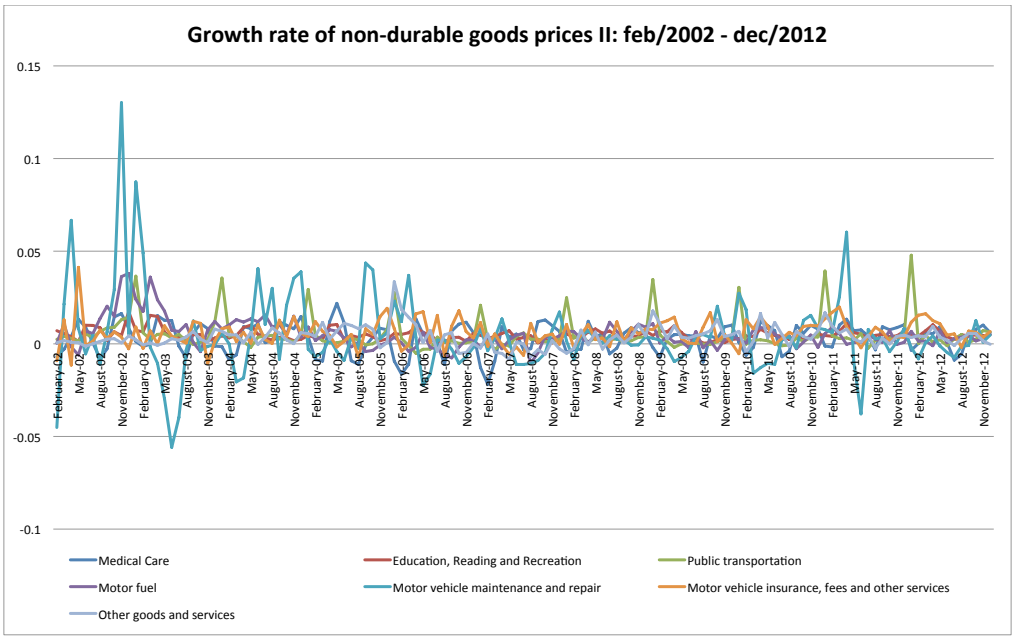
3.1 Data and calibration

The CPI calculated by IBRE contains 4 aggregation levels: group, sub-group, item and sub-item. At the sub-group level it contains 25 series where 18 of which are non-durable goods or services.

We consider all goods and services is included in the CPI calculated by IBRE/FGV at sub-group level with the exception of Food and Beverages included at group level and Transportation included at item level. They cover the period from january of 2002 to december of 2012 at monthly frequency. The growth rate of prices of non-durables goods and services is plotted in figure 1 and 2 and of durables goods in figure 3. Note that during this period we see a large variation on the price of food items, motor vehicle maintenance and repair, Shelter, for non-durables and new and used motor vehicles for durables.







The relative taste weight α_j equals the relative shares in the Brazilian household's expenditure for the seven state capitals where the data is collected. In the time frame considered in this article, IBRE has revised these weights two times based on a consumer expenditure survey conducted by IBRE/FGV in 2002/2003 and by IBGE in 2008/2009. IBRE/FGV started using the set of weights calculated from their own consumer expenditure survey in january 2004 and from IBGE's in january 2012. Table 1 shows these revisions.

	CPI-BR/FGV weights		
CPI Component	jan/02	jan/04	jan/12
Non-durables			
Food and beverages	25,4	27,5	29,3
Shelter	10,1	9,7	10,4
Fuels and utilities	8,8	11,2	9,8
Towels, window and floor coverings and other linens	0,3	0,3	0,2
Household supplies	4,7	4,2	3,9
Household operations	3,1	2,6	3,2
Apparel	4,8	5,4	4,7
Medical Care	11,7	10,4	10,5
Education, Reading and Recreation	9,4	8,7	8,6
Public transportation	4,1	5,0	6,0
Motor fuel	4,7	4,0	3,7
Motor vehicle maintenance and repair	0,6	0,5	0,6
Motor vehicle insurance, fees and other services	1,5	1,1	1,2
Other goods and services	3,5	4,4	4,6
Durables			
Household furnishings and operations	1,4	0,9	0,8
Appliances	2,0	2,3	1,2
Other household equipment and furnishings	0,7	0,6	0,6
New and used motor vehicles	2,3	0,6	0,5
Motor vehicles parts and equipment	0,7	0,5	0,4

We consider 5 types of durable goods: Household furnishings and operations, Appliances, Other household equipment and furnishings, New and used motor vehicles and Motor vehicles parts and equipment. Note that it does not contain a price index for shelter as it is not included in any CPI calculated in Brazil to this date. In fact the only index for shelter for Brazil starts in 2010 and we have too few observations to include it in this implementation of the DPI.

Over these three revisions we see an increase in the weight of Food and beverages (which include meals outside home), Transportation and decrease in Education spending to cite a few changes. For durables, there is an overall decrease in the weight of all categories. The largest drop in spending of durable goods is on Appliances, from 1.4% to .8% in 2012.

The depreciation rate for each category is taken from the Fixed Assets Table of the Bureau of Economic Analysis. The annual rate of depreciation is converted to match the monthly frequency of the price data.

We measure equity returns using the value weighted index of stocks at BOVESPA called Ibovespa. The other asset used in this implementation is the savings deposit. This type of asset is very important in Brazil as the government regulates its return (from the sample period 6%

plus TR, a referential rate of inflation calculated by the federal government), it's free of income tax and riskless.

Figure 1 plots both return rates. We had to plot these series in different axis because the savings deposit return varies from .5% to only 1% and the IBOVESPA return from approximately -25% to 17%. From this figure we can see that the overall variation from the savings deposit is small and only due to inflation.

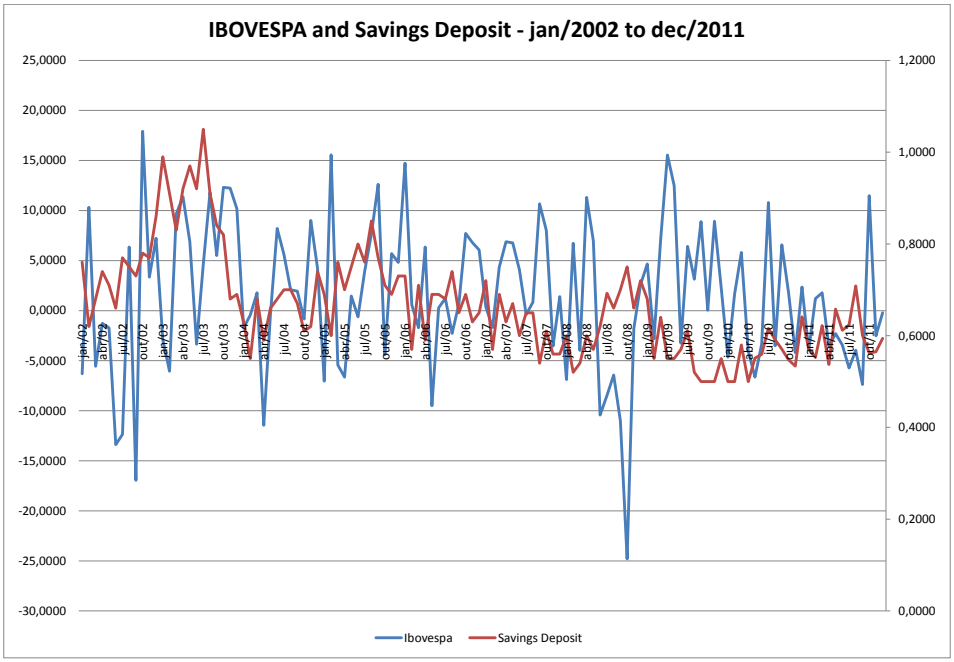


Figure 1: Assets - IBOVESPA and Savings Deposit.

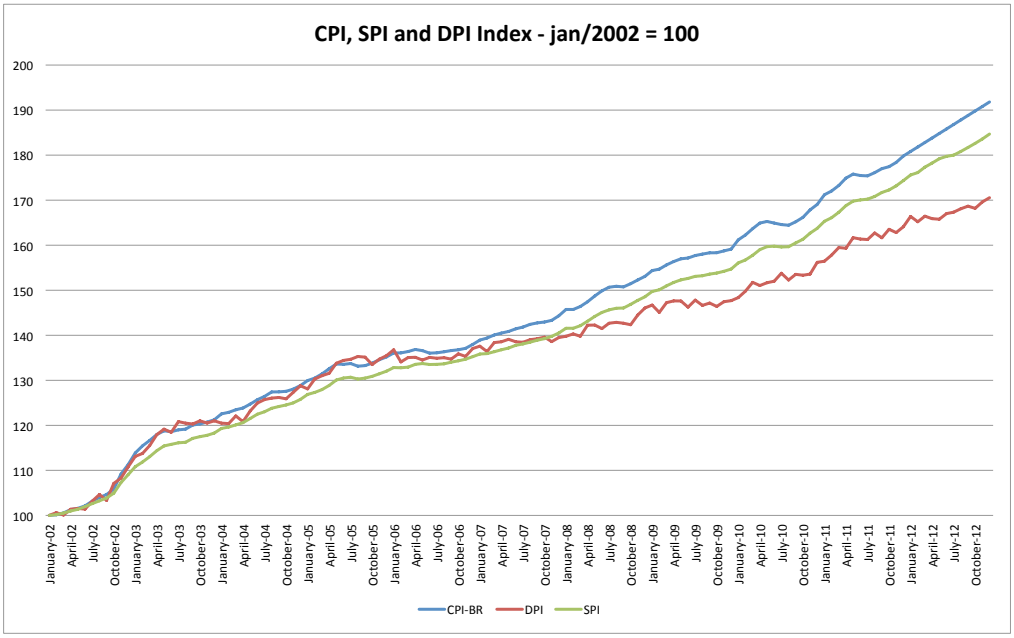
To forecast the model we follow the procedure adopted by Reis (2005) which assume that the first difference of log-prices and returns follow a first order Markov process to estimate several VAR models. In the implementation presented in the next section we estimate an unrestricted VAR(1) model of all series.

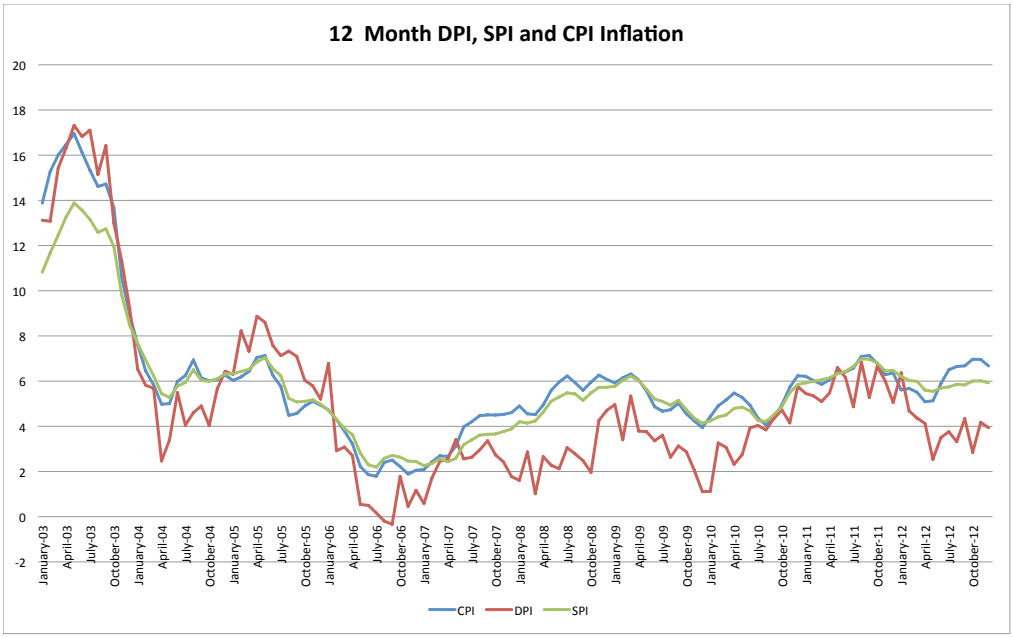
The last step is to solve the consumer decision problem presented in the section 2. As our implementation involves too many variables, we decided to take a first order approximation around the non-stochastic steady state. If we had decided to calculate the value function and prices followed a Markov process with k states, the function would depend on $1 + 21k$ variables which is just too large.

Lastly, the discount factor is set at .9936 to match a 8% annual real rate of return.

4 Dynamic Brazilian inflation

In figure 5 we present the monthly CPI, SPI and the DPI index, where the DPI is calculated using the procedure described in the last section and the SPI is the standard price index when consumer utility function is Cobb-Douglas. In addition, in figure 6 we present the 12-month percentage change for those price indexes. The difference between the DPI and the SPI comes from the fact that returns and prices are not random walks (Reis (2005) proposition 4), hence the value function is not Cobb-Douglas.





A brief inspection of both graphs show that these DPI and the CPI are different but follow the same trend. Still, the correlation between their monthly variation is only 0.40. One reason why these two measures are different comes from the high volatility of the DPI. While the CPI's monthly variation has standard deviation of only 0.46, the standard deviation of the DPI is 0.86. Another part of the difference comes from the fact that CPI variation is more persistent than DPI's. The serial correlation for the CPI is 0.59 and for the DPI is -0.21[2]. Over the 10 years studied in this paper the accumulated difference between the CPI and the DPI is of approximately 21%. On the other hand, the accumulated difference between the SPI and the DPI is approximately 14%.

From the beginning of the calculation period, jan/2002, until january/2007 both indexes were close. The biggest difference between them in this period was 3 p.p. in april/2004. After this month, the DPI grew at slower rates than the CPI until october/2010 and in the year 2012. Still, the bigger growth of the DPI during late 2010 and 2011 is not enough to make the difference between then smaller.

During the years 2007 and 2008 we see a large devaluation of IBOVESPA of about 16% caused by the world crisis. On the other hand, during 2007 the savings deposit return decreased from .72 in january to .56, but in the end of 2008 it had already grown to the same .72 as december of 2007. Overall the impact of equity and savings deposit lowered the DPI comparing to the goods and services portion of the CPI.

[2]The serial correlation of the SPI monthly variation is 0.61, the standard deviation .34. The SPI is more persistent than both the CPI and the DPI and also less volatile.

Another part of the difference between the DPI and the CPI can be explained by the price behaviour of durable goods during this period. Durable goods have bigger weight in the DPI than in the CPI because consumers derive utility from consuming its services flow and they serve as a technology to transfer wealth from one period to another. During this time period durable goods grew at slower rates than nondurable goods. In some cases, the average price decreased over this time period, i.e. Electrical appliances and Equipment and own transportation.

Table 2 presents a more systematic analysis of the factors driving the DPI. The first column shows the static weight of each component. The second column shows the standard deviation of changes in the price index of each item. Equity is by far more volatile than any other good or service. On the third column we have the serial correlation of each component. Then, on the next two columns we have the minimum and the maximum variation of each component. Lastly, we have the dynamic weights of each item calculated for the case where consumer's forecasting model is a VAR(1) and the weights calculated using only the AR(1) estimated parameters.

Unlike equity, savings deposit returns are quite persistent and thus carry a huge impact in the DPI. Although the dynamic weight for the savings deposit in the DPI is 15.29, the maximum change in the monthly return observed in the sample is only 0.2% so the maximum impact in the DPI is of only .31%. Overall, durable goods have a larger weight in the index than non-durable items. The durable item with the largest weight is own transportation with .337.

5 Conclusion

In this article we constructed a measure of consumer inflation for Brasil based on the cost-of-living concept for a consumer who lives for many periods and face uncertainty about prices of goods and services and financial assets.

The constructed measure contains 21 series, 14 of them of non-durable goods and services, 5 of durable goods and 2 financial assets. The time period ranges from 2002 to 2012, a period with important economic dynamics in Brazil. The DPI is very volatile and with correlation of only .40 with the CPI calculated by IBRE/FGV based on the static framework.

There are two main complications on implementing this measure and calculating it on a timely fashion. Both of them are caused by the econometric nature of this index. First, in the Brazilian CPI-FGV in the lowest aggregation level there are 456 items. If we decided to estimate an unrestricted VAR(1) to be the agents forecasting model we would need to estimate over 207,000 parameters which would require 38 years of monthly observations (207,000/456*12). This is just infeasible. The other issue is which financial assets to include. Surveys on finan-

DPI Component	Static Weights - jan/2004	Standard Deviation	Serial Correlation	Minimum Monthly Variation	Maximum Monthly Variation	Dynamic Weights with AR(1)
Non-durables						
Food and beverages	0.275	0.005	0.587	-0.004	0.031	0.417
Shelter	0.097	0.010	0.575	-0.017	0.055	0.195
Fuels and utilities	0.112	0.003	0.037	0.000	0.021	0.078
Towels, window and floor coverings and other linens	0.003	0.009	0.333	-0.024	0.034	0.003
Household supplies	0.042	0.006	0.104	-0.013	0.023	0.042
Household operations	0.026	0.006	0.674	-0.009	0.027	0.049
Apparel	0.054	0.006	0.498	-0.001	0.028	0.072
Medical Care	0.104	0.008	0.395	-0.022	0.022	0.139
Education, Reading and Recreation	0.087	0.003	0.426	0.000	0.017	0.128
Public transportation	0.050	0.009	0.104	-0.005	0.048	0.034
Motor fuel	0.040	0.008	0.694	-0.011	0.038	0.072
Motor vehicle maintenance and repair	0.005	0.022	0.340	-0.056	0.130	0.009
Motor vehicle insurance, fees and other services	0.011	0.007	-0.073	-0.011	0.041	0.014
Other goods and services	0.044	0.006	0.382	-0.008	0.034	0.054
Durables						
Household furnishings and operations	0.009	0.009	0.159	-0.012	0.055	0.234
Appliances	0.023	0.009	0.168	-0.017	0.025	0.309
Other household equipment and furnishings	0.006	0.006	0.632	-0.016	0.021	0.111
New and used motor vehicles	0.006	0.012	0.306	-0.007	0.075	0.337
Motor vehicles parts and equipment	0.005	0.007	0.626	-0.036	0.016	0.075
Financial Assets						
Savings Deposit	-	0.001	-0.467	-0.002	0.002	15.129
Equity	-	0.096	-0.493	-0.233	0.349	1.672

cial holdings are costly and have known problems, hence limits its implementation in several countries.

Our view is that to incorporate more realistic features of consumer behaviour in the consumer price index we must look at how in fact consumers behave. If consumers do not substitute away from goods when the change in price is small, perhaps Laspeyres measures are good enough and we don't need the extra cost associated with calculating a superlative index. On the dynamic side, if consumers do not change their holdings when the return on some asset is lowered or change address when the price of his home increases/decreases perhaps we should not include them in the index. The close investigation of these issues is left to future work.

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