the review of income and wealth

Review of Income and Wealth 2012
DOI: 10.1111/roiw.12005

# POST-LASPEYRES: THE CASE FOR A NEW FORMULA FOR COMPILING CONSUMER PRICE INDEXES

BY PAUL ARMKNECHT AND MICK SILVER*

*International Monetary Fund*

Consumer price indexes (CPIs) are commonly compiled at the higher (weighted) level using Laspeyres-type arithmetic averages. This paper questions the suitability of such formulas and considers two counterpart alternatives that use geometric averaging, the Geometric Young and the (price-updated) Geometric Lowe. The paper provides a formal decomposition and understanding of the differences between the two. Empirical results are provided using United States CPI data. The findings lead to an advocacy of quite simple variants of a hybrid formula suggested by Lent and Dorfman that use the same data as Laspeyres-type indexes but substantially reduce their bias.

**JEL Codes**: C43, C81

**Keywords**: Consumer Price Index, Geometric Young, index numbers, Laspeyres, Lowe index formula, superlative, Young

## 1. INTRODUCTION

Most national statistical offices (NSOs) use in practice what they often describe as "Laspeyres-type" index formulas for aggregating their consumer price index (CPI) at the higher (weighted) level. These Laspeyres-type indexes include the Young and the Lowe indexes, both of which have serious shortcomings. It is argued here that Laspeyres-type indexes can be replaced at little cost by more suitable formulas that use the same data, and can be compiled in real time.

A Laspeyres price index can be defined as a period 0-weighted arithmetic average of price changes between periods 0 and $t$. However, it takes time to compile the results of a household expenditure survey, so in practice statistical agencies use prior period $b$ survey weights to rebase a CPI that runs from the price reference period 0 ($b < 0 < t$). The Young index has as its weights the preceding survey period $b$ expenditure shares and the Lowe index uses period $b$ weights

*Note*: The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management. Paul Armknecht was formerly the Price Statistics Advisor for the IMF Caribbean Regional Technical Assistance Centre and Deputy Division Chief of the Real Sector Division of the IMF Statistics Department and Mick Silver is a Senior Economist in the IMF Statistics Department. The authors thank Walter Lane (U.S. Bureau of Labor Statistics) for his comments on an earlier draft and his assistance with the provision of data for the empirical section. Acknowledgements for helpful comments are also due to Bert Balk (Erasmus University Rotterdam), Erwin Diewert (University of British Columbia), John Greenlees (U.S. Bureau of Labor Statistics), Jens Mehrhoff (Deutsche Bundesbank), and two anonymous referees. The usual disclaimers apply.
*Correspondence to: Mick Silver, International Monetary Fund, 700 19th Street NW, Washington DC 20431, USA (msilver@imf.org).


Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main St, Malden, MA, 02148, USA.

price-updated (and normalized) to the price reference period 0. Laspeyres is rarely used in practice for compiling CPIs.[1]

This paper outlines in Section 2.1 the features of the widely used arithmetically-based Lowe and Young formulas. Both are considered to have major shortcomings. The Lowe index is principally used for CPI compilation in spite of theory and evidence of severe upward bias. However, the Lowe index has the virtue, as a fixed quantity basket index, of being simple to explain. Analytical shortcomings with the Young index include an uncertainty *a priori* about the extent and nature of its deviations from Laspeyres and relatively poor axiomatic properties.

Section 2.2 continues by considering the nature of and case for the geometric equivalents of Young and Lowe indexes, that is, the Geometric Young (sometimes referred to as the Cobb–Douglas) and Geometric Lowe indexes. These geometrically-based indexes share the advantage of their arithmetic counterparts of being computable in real time and are thus practical alternatives to arithmetic versions. Existing empirical work on the differences between these formulas is outlined in Section 2.3.

In Section 3 we focus on these geometric formulations. To better understand their properties, a formal exact decomposition is derived for the difference between the Geometric Young and Geometric Lowe indexes. However, the empirical arbiter of which is the most suitable is their proximity to a superlative index, such as the Törnqvist index, something also considered in this section.[2]

Section 4 provides empirical results using CPI data from the United States. The relationships between the Laspeyres–Paasche interval and the arithmetically-weighted Young and Lowe indexes are considered, followed by an examination of the relationship between the Törnqvist index and the Geometric Young and Geometric Lowe indexes. We find the Geometric Young index, which is consistent with unitary elasticity of substitution, has a downward bias. The U.S. data over the period studied demonstrate inelastic substitution (Greenlees, 2011). However, this bias can be substantially offset by averaging. The averaging of such indexes has a formal justification from Lent and Dorfman (2009) and we consider variants of this approach. Of note is that the Lowe price index, as used in the U.S. and many

[1]Hansen (2007) notes that in the joint UNECE/ILO survey on the *CPI Manual* of the 47 respondents as at September 2007, 32 national statistical offices used the (price-updated) Lowe index and 15 the original (presumably survey period) Young weights. A few larger countries including Germany, Korea, and Japan use Laspeyres by retrospective revisions.

[2]The *Consumer Price Index* (*CPI*) *Manual* (ILO *et al.*, 2004) recommends superlative price indexes—the Fisher, Törnqvist, and Walsh indexes—as the target formulas for the higher-level indexes. These formulas generally produce similar results, use geometric averaging, and symmetric weights based on quantity or expenditure information from both the reference and current periods. They derive their support as superlative indexes from economic theory. A utility function underlies the definition of (constant utility) cost of living index (COLI) in economic theory. Different index number formulas can be shown to correspond with different functional forms of the utility function. Laspeyres, for example, corresponds to a highly restrictive Leontief form. The underlying functional forms for superlative indexes, including Fisher and Törnqvist, are flexible: they are second-order approximations to other (twice-differentiable) homothetic forms around the same point. It is the generality of functional forms that superlative indexes represent that allows them to accommodate substitution behavior and be desirable indexes. The Fisher price index is also recommended on axiomatic grounds and from a fixed quantity basket perspective (ILO *et al.*, 2004).

other countries, is found to have a bias (against superlative indexes) *about 80 times* that of some of these variants, all of which can be computed in real time using the same database as the Lowe index.

Laspeyres itself has the advantage of being an upper bound to a theoretical cost-of living index (COLI). The widely used Lowe index is likely to fall above Laspeyres. Its main advantage is that as a fixed quantity basket index it is easy to explain; biased but easily explained. We propose alternative formulas that can be readily computed in real time.

## 2. Higher-Level Price Index Number Formulas Used in Practice

### 2.1. *Arithmetic Formulas*

The Laspeyres price index is given by:

$$(1) \qquad I_L^t = \sum_{i=1}^{n} \frac{p_i^t q_i^0}{p_i^0 q_i^0} = \sum_{i=1}^{n} \frac{p_i^0 q_i^0 \left(\dfrac{p_i^t}{p_i^0}\right)}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n} s_i^0 \left(\frac{p_i^t}{p_i^0}\right), \quad \text{where} \quad s_i^0 = \frac{p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}.$$

The first term of equation (1) is a standard representation of the Laspeyres formula as a fixed quantity basket index with $p_i^0$ and $q_i^0$ denoting, respectively, prices and quantities in period 0 for $i = 1, \ldots, n$ products/elementary aggregates. In practice CPIs are compiled as a weighted average of price relatives, given by the second and third terms in equation (1), where the weights are the expenditure shares in period 0, $s_i^0$.

It takes time to compile and process household expenditure survey data, so there is a lag between the expenditures share survey period, $b$, and their first use in the index, commencing at the price reference period 0. Thus, in practice, the Laspeyres is generally not used for real time CPI compilation and expenditure shares from the earlier period $b$ may be used to weight period 0 to period $t$ price changes. The resulting Young price index is given by:

$$(2) \qquad I_Y^t = \sum_{i=1}^{n} s_i^b \left(\frac{p_i^t}{p_i^0}\right), \quad \text{where} \quad s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}.$$

More typically, weights are price-updated between period $b$ and the price reference period 0 to effect fixed period $b$ quantities. The resulting Lowe index is given by:

$$(3) \qquad I_{Lo}^t = \frac{\sum_{i=1}^{n} \left[ p_i^b q_i^b \dfrac{p_i^0}{p_i^b} \right] \dfrac{p_i^t}{p_i^0}}{\sum_{i=1}^{n} \left[ p_i^b q_i^b \dfrac{p_i^0}{p_i^b} \right]} = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} p_i^0 q_i^b \left(\dfrac{p_i^t}{p_i^0}\right)}{\sum_{i=1}^{n} p_i^0 q_i^b}.$$

The expression in square brackets in the first term is the period $b$ expenditures, $p_i^b q_i^b$, price-updated to period 0. The second term shows the Lowe index to be a period $b$ fixed-quantity basket price index, and the third term to be a weighted average of price changes where the weights are hybrid period 0 prices and period $b$ quantities, with little economic meaning. Price-updating the expenditure shares for price changes is not to make the weights more up-to-date, but to transform the index from a fixed period $b$ expenditure share-weighted index of price changes to a fixed period $b$ quantity basket price index.

Balk and Diewert (2003), from the perspective of the economic theory of index numbers, establish the substitution bias of a Lowe CPI (see also ILO *et al.*, 2004, chapters 15 and 17; Balk, 2010). Not only is the Lowe index shown to have a likely upward substitution bias against a Laspeyres index, but the Laspeyres index has an upward substitution bias against a superlative index. ILO *et al.* (2004, chapter 16) demonstrate that the Lowe index, however, has good axiomatic properties.[3]

The Young index fails the circularity and time reversal tests (ILO *et al.*, 2004, appendix 15.3 and chapter 16). The Young index between periods 0 and $t$ will exceed its time antithesis, that is, its inverse between period $t$ and 0, and in this sense is positively biased.[4] ILO *et al.* (2004, chapter 15) demonstrate how the discrepancy between Laspeyres and Young is difficult to gauge. It is based on the covariance of the difference between expenditure shares between period $b$ and 0 and the deviations of period 0 to $t$ relative prices from their mean.[5] A positive covariance would put Young above Laspeyres and negative covariance below Laspeyres, possibly closer to a superlative index. Analytical shortcomings with the Young index are thus the uncertainty *a priori* about the extent and nature of its deviations from Laspeyres and its relatively poor axiomatic properties.

## 2.2. *Geometric Counterparts*

For elementary-level indexes, the *CPI Manual* recommends the use of the (geometric) Jevons index if weights are not available for individual varieties in the sample (ILO *et al.*, 2004, chapter 20). Using a geometric formula at the higher level would be compatible with the currently widely used Jevons index at the lower level and would have the benefit of maintaining consistency in aggregation.

Equations (4) and (5) are the geometric counterparts to (2) and (3) and can be readily adopted by statistical offices since they use the same weights and price relatives as the Young and Lowe indexes. The Geometric Young price index is given by:

[3]It passes the time reversal test and is transitive. However, as pointed out by ILO *et al.* (2004, paragraph 1.64), "Achieving transitivity by arbitrary holding the quantities constant, especially over a very long period of time, does not compensate for the potential biases introduced by using out-of-date quantities."

[4]It will exceed its time antithesis by a term equal to the Young index times the weighted variance of deviations of price relatives (between periods 0 and $t$) and their mean. Since the variance must be positive, the Young must exceed the inverse of its time antithesis except when there is no price change dispersion, a case that negates the purpose of an index number.

[5]The concern is whether the *share* of expenditure increases over periods 0 and $b$ with relative price increases over periods 0 and $t$. This would require long-run trends in prices and, for Young to be above (below) Laspeyres, very elastic (inelastic) demand (ILO *et al.*, 2004, pp. 275–76).

$$(4) \qquad I_{GY}^{t} = \prod_{i=1}^{n} \left( \frac{p_i^t}{p_i^0} \right)^{s_i^b}, \quad \text{where} \quad s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}.$$

The geometric version of the Lowe price index with its price-updated weights is given by:

$$(5) \qquad I_{GLo}^{t} = \prod_{i=1}^{n} \left( \frac{p_i^t}{p_i^0} \right)^{s_i^{b0}}, \quad \text{where} \quad s_i^{b0} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}.$$

The (superlative) Törnqvist index is given by:

$$(6) \qquad I_{T}^{t} = \prod_{i=1}^{n} \left( \frac{p_i^t}{p_i^0} \right)^{\left(s_i^0 + s_i^t\right)/2}$$

for which current period expenditure shares, $s_i^t$, are not available in real time. It is apparent that for constant expenditure shares over periods 0 and $t$, consistent with unitary elasticity of substitution, the Geometric Young index given by equation (4) equals the Törnqvist index given by (6).

Balk (2010) shows that the Lowe price index is likely to be greater than or equal to the Geometric Young index. On the one hand he demonstrates that the substitution bias of the Lowe relative to the Geometric Young cannot be determined conclusively. On the other hand, if prices are rising from period $b$ to 0 and from 0 to $t$, it is likely that the Lowe index substitution bias is greater than that for the Geometric Young. The *CPI Practical Guide* supports the use of the Geometric Young formulas (UNECE *et al.*, 2009, p. 160).[6] The *CPI Manual* considers the Geometric Young index to be a serious practical possibility for CPI compilation, since the requisite weights are available in real time, and it is less susceptible to bias. With unitary elasticity of substitution, the Geometric Young can be shown to lie within the Laspeyres–Paasche interval. The Geometric Young index, as its name suggests, corresponds to cost-of-living indexes for utility-maximizing households with Geometric Young preferences. The *CPI Manual* cites as its main concern the unlikelihood of it gaining general acceptance in the foreseeable (then 2004) future since it cannot be interpreted as a fixed quantity basket index (ILO *et al.*, 2004, chapter 1, paragraphs 1.40 and 9.137).

The price updating of the weights has no rationale for the Geometric Lowe. Its standing is so low that neither the *CPI Practical Guide* nor the *CPI Manual* mention it. However, the (arithmetic) Lowe is widely used in practice. It is invariably described in terms of a weighted average of price changes, albeit with little

[6]It does so in a footnote: the *CPI Practical Guide* focused on helping implement good practice rather than as a platform for change, but the authors/editors of the *Guide* nonetheless considered the matter sufficiently important to footnote this point. As shown later, the Geometric Young is a good proxy for superlative indexes if the elasticity of substitution is unity.

reference to such weights given in the last term of equation (3), which have little economic meaning. There is a *prima facie* case for some formal and empirical analysis of the geometric counterpart to the arithmetic Lowe.

### 2.3. *Available Empirical Work*

Given the concern about arithmetic formulations and some positive aspects of geometric ones, we consider some of the available, albeit limited, empirical work on how close different formulations lie to a superlative index.

Hansen (2007), using Danish CPI data for 1996 to 2003, found increases for the Young and Lowe indexes of 17.49 and 18.01 percent, respectively, compared with an increase in the Törnqvist index of 17.08 percent.[7] The Geometric Young index was *below* the Törnqvist at 16.51 percent. The differences between Young and Lowe are not always trivial.[8] The annual inflation rate for 2004/05 and 2005/06 *increased* from 1.80 to 1.88 percent using Young but *decreased* from 1.99 to 1.90 percent using Lowe.

Greenlees and Williams (2010), in a major study of the U.S. CPI over December 1990 through December 2008, found Lowe and Young increases to be quite similar, at 18.88 and 18.24 percent respectively, but the (chained) Törnqvist was much lower at 16.78 percent.[9] The Geometric Young index was closer to, and again below, the chained Törnqvist at 15.84 percent.

Pike *et al*. (2009, table 10)—using New Zealand CPI data for June 2006 to June 2008 with weights of 2003/04 and 2006/07, respectively, price-updated to June 2006 and June 2008 quarters (the New Zealand CPI is quarterly)—found Lowe and Young to differ, showing over this period increases of 6.26 and 5.60 percent, respectively. These arithmetic formulations were significantly higher than the 4.83 percent increase for the Geometric Young index which appeared to understate the 5.73 percent increase measured by a retrospective Fisher index.

So while Lowe and Young may generate similar results, their difference from a superlative Törnqvist index is marked and of concern. The Geometric Young index generally falls below (and there is some evidence that it is closer to) the superlative Törnqvist index.

Given that the Geometric Young and Geometric Lowe are practical contenders for the CPI aggregation formula, we now present a formal analysis as to why they might differ.

[7]Cited Törnqvist indexes are approximations as the current period weights are expenditure shares over a period longer than the current month or quarter $t$, due to lack of expenditure data (the expenditure survey not being continuous) and inadequate sample sizes for the single month or quarter.

[8]Rebasing took place in 1994 (for January 1996–December 1999), 1996 (December 1999–December 2002), 1999 (December 2002–December 2005), and 2003 (December 2005–December 2006). The lag for the price-updating varies from 2 to 3.5 years over the links of the index.

[9]The Lowe and Young indexes are based on the U.S. Consumer Price Index for All Urban Consumers, or CPI-U. Its weights, as from 2002, cover a two-year period and are revised every two years. For example, the weights in January 2010 are expenditures from 2007–08 that were price-updated to December 2009. There is approximately a two-year lag from the midpoint of the survey period to the price reference period.

## 3. Why Geometric Higher-Level Price Index Numbers Differ

### 3.1. *Geometric Young vs. Geometric Lowe*

Following on from equation (4), we first define a Geometric Young price index as:

$$(7)\qquad I_{GY}^{0\to t}=\prod_{i=1}^{n}\left(\frac{p_i^t}{p_i^0}\right)^{s_i^b}\quad\text{and}\quad \ln\left(I_{GY}^{0\to t}\right)=\sum_{i=1}^{n}s_i^b\ln\frac{p_i^t}{p_i^0}=\sum_{i=1}^{n}s_i^b y_i$$

where $s_i^b=\dfrac{p_i^b q_i^b}{\sum_{i=1}^{n}p_i^b q_i^b}$ are period $b$ expenditure shares and $y_i\equiv\ln\left(\dfrac{p_i^t}{p_i^0}\right)$ is the natural logarithm of the $i$-th price relative.

The difference between the logarithms of a Geometric Lowe and a Geometric Young price index is given by:

$$(8)\qquad \ln(I_{GLo}^{0\to t})-\ln(I_{GY}^{0\to t})=\frac{\sum_{i=1}^{n}p_i^b q_i^b\left(\frac{p_i^0}{p_i^b}\right)\ln\left(\frac{p_i^t}{p_i^0}\right)}{\sum_{i=1}^{n}p_i^b q_i^b\left(\frac{p_i^0}{p_i^b}\right)}-\frac{\sum_{i=1}^{n}p_i^b q_i^b\ln\left(\frac{p_i^t}{p_i^0}\right)}{\sum_{i=1}^{n}p_i^b q_i^b}$$

$$=\frac{\sum_{i=1}^{n}s_i^b x_i y_i}{\sum_{i=1}^{n}s_i^b x_i}-\frac{\sum_{i=1}^{n}s_i^b y_i}{\sum_{i=1}^{n}s_i^b}$$

where $x_i\equiv\dfrac{p_i^0}{p_i^b}$. Adopting a Bortkiewicz (1923) decomposition:[10]

$$(9)\qquad \ln\left(I_{GLo}^{0\to t}\right)-\ln\left(I_{GY}^{0\to t}\right)=\frac{\sum_{i=1}^{n}s_i^b x_i y_i}{\sum_{i=1}^{n}s_i^b x_i}-\overline{y}_i^{s^b}=\rho_{x,y}^{s^b}cv_x^{s^b}\sigma_y^{s^b}$$

where $\overline{y}^{s^b}$ is the $s^b$-weighted mean of $y_i\equiv\ln\left(\dfrac{p_i^t}{p_i^0}\right)$, that is, a geometric Young price index.

$$(10)\qquad \frac{I_{GLo}^{0\to t}}{I_{GY}^{0\to t}}=\exp\left(\rho_{x,y}^{s^b}cv_x^{s^b}\sigma_y^{s^b}\right)$$

[10]See Bortkiewicz (1923, pp. 374–75) for the first application of this decomposition technique: we define $\sum uv/\sum u=\sigma_u\sigma_v\rho_{u,v}/\overline{u}+\overline{v}=\text{cov}(u,v)/\overline{u}+\overline{v}$ and $\sum suv/\sum su$ as $s$-weighted terms for the decomposition. Equation (10) can be formulated as a covariance, $\dfrac{I_{GLo}^{0\to t}}{I_{GY}^{0\to t}}=\exp\left(\dfrac{\text{cov}_{x_i,y_i}^{s^b}}{\overline{x}_i^{s^b}}\right)$, a preferred stance pointed out by Jens Mehrhoff to an earlier draft since the dispersion in $cv_x^{s_i^b}$ is in part counter-balanced by the dispersion of $x$ in the denominator of $\rho_{x_i,y_i}^{s^b}$. Our position is that the correlation coefficient is meaningful in its own right, but we draw attention to the point.

where $\rho_{x,y}^{s^b}$ is the period $b$ weighted ($s^b$) correlation coefficient between price relatives $x_i$ and $y_i$ (that extend, respectively, from $b \rightarrow 0$ and $0 \rightarrow t$); $cv_x^{s^b} = \sigma_x^{s^b} / \bar{x}^{s^b}$ is the period $b$ weighted $(s_i^b)$ coefficient of variation for $x_i$, for which $\sigma_x^{s^b}$ is the standard deviation and $\bar{x}^{s^b}$ is the $s^b$-weighted mean of $x_i$, that is, a Laspeyres price index between periods $b$ and 0.[11]

First, it is apparent from (10) that $\rho_{x,y}^{s^b}$ dictates whether $I_{GLo}^{0\rightarrow t}$ is larger (positive) or smaller (negative) than $I_{GY}^{0\rightarrow t}$. For (weighted) price changes between periods $b$ and 0 to be positively correlated with (weighted logarithms of) price changes between periods 0 and $t$, there must be some persistent uni-directional long-run price change over period $b$ to $t$. *A priori*, a sign cannot be unambiguously attached to this correlation coefficient.

Second, the magnitude of $I_{GLo}^{0\rightarrow t} / I_{GY}^{0\rightarrow t}$ is determined by:

(1) The magnitude of $\rho_{x,y}^{s^b}$. Ratios of $I_{GLo}^{0\rightarrow t}$ to $I_{GY}^{0\rightarrow t}$ nearer unity would be expected from countries with longer time lags in utilizing and updating the weights, that is, will decrease the correlation between price changes between periods $b$ and 0 and periods 0 and $t$.

(2) The dispersion of price changes, $cv_x^{s^b}$ and $\sigma_y^{s^b}$. It is well established in economic theory and empirical work that dispersion in relative prices increases with increases in inflation.[12] The Geometric Lowe will drift from the Geometric Young with higher rates of inflation. Note that $\sigma_y^{s^b}$ is likely to be the most potent driver of the drift since it is not corrected, as is the coefficient of variation, $cv_x^{s^b}$, for changes in the mean. $\sigma_y^{s^b}$ is concerned with the often larger index changes between period 0 to $t$, than the constant $cv_x^{s^b}$ over period $b$ to 0.[13]

(3) The multiplicative nature of terms on the right-hand side of equation (10). For example, the two formulas will yield very similar indexes in any month $t$ at which $\rho$ is near zero, regardless of the values of $cv_x^{s^b}$ and $\sigma_y^{s^b}$.

Third, we do not depict the difference between Geometric Young and Geometric Lowe indexes as substitution bias. It is clear from equation (10) that the differences stem from a correlation between price changes in one period and the (logarithm of) price changes in a subsequent period: not a correlation between price and quantity changes. It is the latter that defines substitution bias.

## 3.2. *Comparisons with a Superlative Price Index*

Having examined how a Geometric Young differs from a Geometric Lowe price index, we turn to consider how both indexes, given by equations (4) and (5),

[11]Also see Balk (2010, pp. 731–32). He derives similar conclusions to those in this section using a slightly different approach.

[12]Early empirical research in this area includes Glejser (1965), Vining and Elwertowski (1976), and Parks ((1978). Most of the evidence on this relationship relies on regressions of relative price dispersion on inflation with a common finding of a positive relationship, although this finding is not universal. The main two theoretical models to explain the relationship are signal extraction models in which inflation which is not correctly anticipated by economic agents leading to erroneous output levels (Friedman, 1977; Hercowitz, 1982; Lastrapes, 2006), and models with price-setting behavior and price-rigidities that vary across markets (see Ball and Mankiw, 1995). Other models include search cost theory (see Van Hoomisson, 1988).

[13]A finding of an association between the dispersion in relative prices and their mean also applies to the coefficient of variation as a measure of dispersion (Balk, 1983; Reinsdorf, 1994; Silver and Ioannidis, 2001).

differ from a superlative Törnqvist price index given by equation (6). The ratio of a Geometric Lowe to a Törnqvist price index is by extension of equation (10):

$$(11) \qquad \frac{I_{Glo}^{0\to t}}{I_T^{0\to t}} = \exp\left[\left(\rho_{x,y}^{s^b} cv_x^{s^b} \sigma_y^{s^b}\right) + \left(\bar{y}_i^{s^b} - \bar{y}_i^{s^{(s^0+s^t)/2}}\right)\right]$$

and the ratio of a Geometric Young to Törnqvist price index, by definition, equations (4) and (6) by:

$$(12) \qquad \frac{I_{GY}^{0\to t}}{I_T^{0\to t}} = \frac{\prod_{i=1}^{n}\left(\frac{p_i^t}{p_i^0}\right)^{s^b}}{\prod_{i=1}^{n}\left(\frac{p_i^t}{p_i^0}\right)^{(s^0+s^t)/2}}.$$

The Geometric Young and Törnqvist are equal if the shares in period $b$ are equal to the average of the shares in periods 0 and $t$, that is, $s^b = (s^0 + s^t)/2$ . As the index is progressively compiled across periods 0 to $t$, the implicit assumption is of a price elasticity of substitution of unity for comparisons between $b$ to 0 and continuing through to $t$. To evaluate the suitability of $I_{GY}^t$ as an estimate of $I_T^t$ we need to evaluate the elasticity of substitution in terms of its proximity to unity and its changes over time. One approach is to use a formula that simply assumes it is constant over time. The Lloyd–Moulton (constant elasticity of substitution, CES) index is given by:[14]

$$(13) \qquad I_{LM}^{0\to t} = \left[\sum_{i=1}^{n} s^b \left(\frac{p_i^t}{p_i^0}\right)^{1-\eta}\right]^{1/(1-\eta)}$$

for which $\eta$ is the elasticity of substitution. The formulation is quite flexible: the Young index is consistent with $\eta$ tending to zero and the Geometric Young index is consistent with $\eta$ tending to unity. Greenlees (2011) used an approach proposed by Feenstra and Reinsdorf (2007) to estimate $\eta$ for U.S. data. He found values of $\eta$ lie between 0 and 1, that is, inelastic substitution, though he also found occasional anomalous years. For 1999/2000 to 2005/06 the estimated $\eta$ varied between 0.521 and 0.655, but was close to (not significantly different from) unity for 2006/07 at 0.981, and close to (not significantly different from) zero for 2007/08 at 0.192.These findings are at odds with an assumption of constant elasticity and, all the more, constant unitary elasticity. A finding of inelastic substitution argues against the implicit fixed baskets of (arithmetic) Lowe and Young indexes and implies that items with relatively higher price trends receive less importance in $I_{GY}^{0\to t}$ than in $I_T^{0\to t}$, that is, $I_{GY}^{0\to t} < I_T^{0\to t}$.

[14]Period $b$ weights are used here for a price comparison in real time from period 0, since only these weights are then available. A true Lloyd–Moulton index uses period 0 weights. This requires that $s^b = s^0$; Greenlees (2011) used a formula akin to (13) and price-updated the weights, from period $b$ to 0.

Lent and Dorfman (2009) derive their formulation from a Taylor approximation to CES and superlative indexes. They find that a weighted average of the arithmetic Laspeyres, $I_{Las}^{0\to t}$, and Geometric Laspeyres, $I_{GLas}^{0\to t}$, indexes (called an AG Mean index) can approximate a superlative target index:

$$I_{AG}^{0\to t} = \eta^{\tau}\prod_{i=1}^{n}\left(\frac{p_i^t}{p_i^0}\right)^{s_i^0} + \left(1-\eta^{\tau}\right)\sum_{i=1}^{n} s_i^0\left(\frac{p_i^t}{p_i^0}\right). \tag{14}$$

The weights are not restricted to be constant. The authors demonstrate that the AG Mean can provide a close approximation to a superlative (Fisher and Törnqvist) price index when $0 \leq \eta \leq 1$. Estimators of $\eta$ vary to be compatible with the target index number. For a Fisher price index as the target, $I_{AG}^{0\to t} = I_F^{0\to t}$, equation (14) is given by:

$$I_F^{0\to t} = \eta^{\tau} I_{GLas}^{0\to t} + \left(1-\eta^{\tau}\right) I_{Las}^{0\to t}. \tag{15}$$

Solving (15) for $\eta^{\tau}$:

$$\eta^{\tau} = \frac{\left(I_F^{0\to t} - I_{Las}^{0\to t}\right)}{\left(I_{GLas}^{0\to t} - I_{Las}^{0\to t}\right)}. \tag{16}$$

The estimated weights in equation (16) and used in equation (15) can vary. Lent and Dorfman (2009) suggest that a moving average of $\eta^{\tau}$ be used over $\tau = t - T$, $t$ to smooth any volatility. We consider in the next (empirical) section how well the currently used arithmetic Lowe and Young indexes compare to (the bounds of) a superlative index.[15] We then look at whether their geometric counterparts do any better and, if so, why they differ, leading to a consideration of simple geometric averages of these arithmetic and geometric indexes, the Lloyd–Moulton and real-time variants of the Lent–Dorfman approach.

[15]There is an intriguing possibility that a positively biased Lowe index at the higher level may counterbalance a negatively-biased Jevons index at the elementary level and that the widely used Jevons and Lowe as respective elementary and higher-level aggregation formulas may together provide a good approximation to a superlative CPI. The argument is contingent on first establishing the conditions under which Jevons would be negatively-biased against a superlative index. These are not immediately obvious since Jevons has, unlike a superlative index, no weights. However, Balk (2008, p. 182) demonstrates the conditions under which Jevons is less than Törnqvist. First, if the sampling of (matched) prices is with probability proportionate to references period value shares, the Jevons price index is then an (approximate) unbiased estimator of the Geometric Young population price index. Second, the Geometric Young will be less than the Törnqvist population price index, unless reference and current period value shares are equal, in which case it will equal Törnqvist. The *Consumer Price Index Manual* (ILO *et al.*, 2004, chapter 20) gives only weak support to Jevons on the basis of such suppositions. Further, the magnitude of the bias at the higher level is likely to be very different to that of the lower level, and thus not counterbalance it. Not only are different formulas involved at these levels, but also substitution behavior at the higher level should differ from that at the elementary level.

TABLE 1
EXPENDITURE SURVEY DATES AND THEIR USE AS WEIGHTS IN THE U.S. URBAN CPI

| Mean Annual Expenditures | Basis of Weights for: |
|---|---|
| 1993–1995 | Jan98–Dec01 |
| 1999–2000 | Jan02–Dec03 |
| 2001–2002 | Jan04–Dec05 |
| 2003–2004 | Jan06–Dec07 |
| 2005–2006 | Jan08–Dec09 |
| 2007–2008 | Jan10–Dec11 |
| 2008–2009 | Jan11–Dec12 |

## 4. Empirical Results

### 4.1. *The Data*

The data used are the elementary aggregate indexes for the U.S. Urban CPI and their weights over the period January 2002 to December 2010, provided by the U.S. Bureau of Labor Statistics (BLS). The elementary aggregate indexes are for about 211 item strata (product groups). We stress that the compilation of the U.S. Urban CPI is based on 211 item strata (product groups) *for 38 area strata*, that is, 8,018 cells. The indexes for the individual item/area strata are for the large part derived using weighted geometric means, while the aggregation across areas uses the Lowe formula.[16] Our analysis is, for simplicity, of the effect of using a different formula to measure the U.S. Urban CPI if only the 211 weights for product groups were available, as is the case with many countries.

Since January 2002, the weights (mean annual expenditure shares) for the U.S. Urban CPI are updated every two years. The weights cover two years and are two years old when they are price updated and introduced into the CPI. The weight dates and their associated introduction date appear in Table 1. Mean annual 1993–95 expenditures for the 211 CPI item strata were the basis of the CPI weights for the four years for the U.S. Urban CPI from January 1998 through December 2001. From January 2002 the U.S. Urban CPI has been a chained Lowe index with biannual links, and we date our analysis from January 2002 for consistency.

The same data on weights used for calculating the Urban CPI are used in this study for alternative formulas, linked together at two-year intervals. The Young index simulation uses the two-year old weights without any price updating. The simulated Laspeyres index is calculated using the two-year mean annual weights and the average prices for the same period, for example weights for 1999–2000 and a price reference period of 1999–2000. The Paasche price index simulation uses the 1999–2000 price reference period and weights from 2001–02. The simulated Törnqvist uses the 1999–2000 price reference period and the average share weights from 1999–2000 and 2001–02.

[16]For the 18 product groups using arithmetic means, see BLS, January 2008 *CPI Detailed Report*, table 3 (http://www.bls.gov/cpi/#tables).

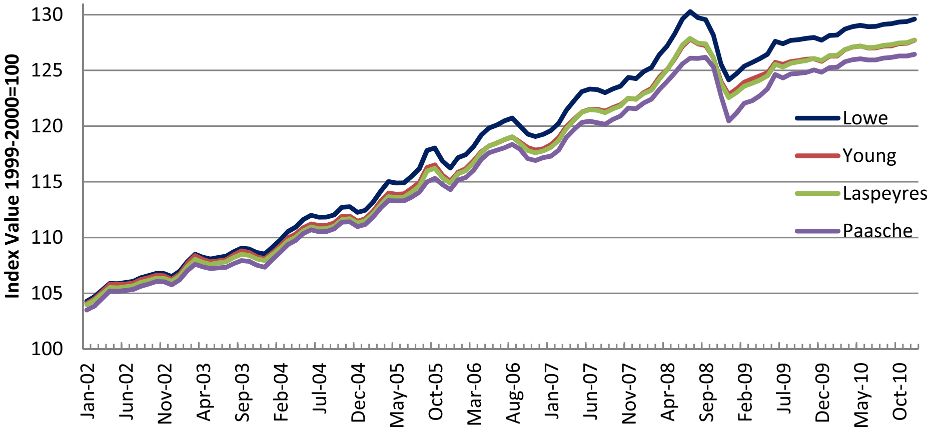


Figure 1. Arithmetic Indexes

The results of our estimates of the chained Lowe index show the same price change as the U.S. Urban CPI, and the chained Törnqvist index is very close to the BLS,[17] findings in themselves of interest.[18]

## 4.2. *Results*

Figure 1 shows the standard arithmetic price indexes: Lowe, Laspeyres, and Young and the harmonic Paasche. The target index is a superlative Fisher price index (not shown in Figure 1), a symmetric (geometric) average of Laspeyres and Paasche price indexes that lies between them.[19] The (arithmetic) Lowe price index,

[17]The U.S. chained Törnqvist C-CPI-U is calculated in real time as a preliminary Geometric Young index and, when subsequent data on expenditure share data become available, the Geometric Young element is revised to a Törnqvist index. Greenlees (2011) develops an operationally feasible formula that can out-perform the Geometric Young component. He employs a constant-elasticity of substitution (CES) index number formula. First, support for the use of CES assumptions are validated by the closeness of the Sato–Vartia price index to superlative indexes. Second, he derives estimates of the elasticity of substitution ($\eta$) using the Feenstra and Reinsdorf (2007) approach, for use in a Lloyd–Moulton CES formula, equation (13). A clear improvement over using the Geometric Young index is demonstrated. While the Lent–Dorfman estimates are not provided by Greenlees, he refers to deriving such estimates using his data and, encouragingly, yielding similar estimates of $\eta$ as those from the Feenstra and Reinsdorf approach (Greenlees, 2011, pp. 1–18).

[18]We calculated indexes over the period December 1998 to December 2010 except for the Lent–Dorfman and Lloyd–Moulton, which could only be calculated for January 2002 to December 2010. Our calculated monthly Lowe index compared with the U.S. Urban CPI shows growth of 24.2 vs. 23.8 percent, respectively, between January 2002 and December 2010. For our biannually chained Törnqvist index compared with the BLS monthly chained Törnqvist index (C-CPI-U), the price growth was 22.5 vs. 21.1 percent. We found correlations of 0.99931 and 0.97770 between the two series for the levels and monthly annual changes, respectively. See http://www.bls.gov/cpi/super_paris.pdf for details of the BLS methodology.

[19]Lowe and Young are calculated following BLS weighting procedures; for example, for January 2006–December 2007we use 2003/04 expenditure weights, price-updated for Lowe, but not for Young. Laspeyres uses available weights most closely aligned with the reference period, in this example, for 2005/06. Paasche (and thus Fisher), and Törnqvist use available counterpart symmetric weights most closely aligned to the current period, in this case, 2007/08. As noted in the comparisons of our simulated results with the BLS indexes provided in footnote 18, this does not detract from the analysis.

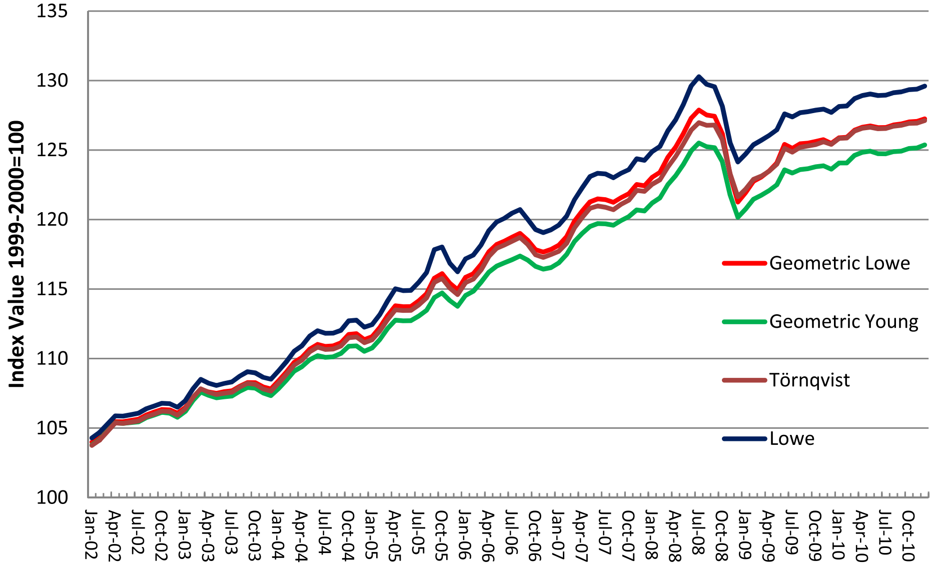


Figure 2. Geometric Indexes

widely used by many countries for their CPI, is above Laspeyres. It performs poorly against the Young. Young is much closer to Laspeyres and, thus, to the desirable Laspeyres–Paasche interval.

The differences between the results of the formulas are not large. Some of this is due to the more frequent (biannual) updating of weights undertaken by the U.S. Bureau of Labor Statistics than in many other countries. Yet, the differences are not insubstantial, especially given that the CPI is used extensively to escalate payments for pensioners, rents, wages, alimony, child support, and other such obligations. The Fisher price index increased in 2010 compared with 2002 at a (compound) annual average rate of 2.31 percent, compared with a Lowe price index increase of 2.49 percent and a Young price index increase of 2.35.

As outlined in Section 2, arithmetic Lowe and Young indexes have counterpart geometric averages as practical alternatives, the Geometric Lowe and Geometric Young indexes, respectively. Both formulas use the same data and can be compiled in real time.

Figure 2 shows results for the Lowe, Geometric Lowe, Geometric Young, and Törnqvist price indexes. At the beginning of the series, all indexes seem to track each other quite closely. After December 2003, the Geometric Lowe and Geometric Young indexes drift apart. The Geometric Lowe is closer to the Törnqvist from December 2003 to December 2007, providing evidence, at least for this dataset, in its favor against the Geometric Young. Bear in mind that the Geometric Lowe has little conceptual support. The arithmetic Lowe had conceptual support as a fixed quantity basket price index and price updating took place with this in mind. However, price updating the weights for a geometric formulation does not yield a fixed basket index. It is the Geometric Young price index that has a conceptual foundation as a period *b* weighted average of price relatives. But for these data, the

Geometric Young price index has a marked downward bias against the Törnqvist index. The (arithmetic) Lowe is included in Figure 2 for reference. Its comparative strong upwards bias relative to the Törnqvist is apparent.

Two questions arise. First, what factors underlie the difference between the two geometric formulas? And second, is the nature of the bias such that an average of formulas may be more suitable, similar to the averaging of Laspeyres and Paasche indexes? By inspection in Figure 2, the Lowe overstates and the Geometric Young understates the Törnqvist index and an average of the two may be suitable. We consider below, for U.S. inflation rates, this and other combinations of formulas using the Lent and Dorfman (2009) and Lloyd–Moulton framework, outlined in Section 3.2.

### 4.3. *The Geometric Formulas: Differences and Adjustments*

The factors underlying the differences between the Geometric Lowe and Geometric Young are of interest as are averages of formulas that can make them better track superlative indexes. We consider the results for both in turn.

#### 4.3.1. What Factors Underlie the Difference Between the Two Formulas?

Factors underlying the differences between the Geometric Lowe and Geometric Young can be understood based on the decomposition in equation (10). This decomposition is considered in Table 2. For brevity the results are only given from December 2007, re-referenced to December 2007 = 1.0000.

The sign of $\rho_{x,y}^{s^b}$ dictates which of the two formulas' growth exceeds the other. From January 2008 to October 2008 it was positive, leading to the Geometric Lowe growing faster than the Geometric Young, and from November 2008 onward it was negative, leading to the reverse position, as shown in Table 2. These empirical runs in signs reflect long-run trends in price change between periods $b$ to 0 (2004/05 to December 2007) being continued in sub-periods 0 to $t$ (December 2007 to months up to October 2008) and then being reversed in subsequent sub-periods (to months after October 2008). This illustrates the dependency on long-run trends in prices for the relative positioning of the two formulas.

The magnitude of the difference is determined by the magnitude of three factors. The correlation coefficient is not expected to be strong given that price changes in one period are to be related to the logarithms of price changes in a subsequent one; $cv_x^{s_i^b}$ is a one-off factor for period $b$ to 0—the higher it is, other factors equal, the larger the difference. If such dispersion increases over time, then lags between introducing weights from the survey period into the rebased index, between $b$ and 0, will accentuate the difference between the two formulas. Finally, $\sigma_y^{s^b}$ and thus the difference between the two formulas, can be expected to increase over time. Of note is that the three factors are multiplicative: minimize any one factor, such as $cv_x^{s^b}$ by minimizing the time lag in the introduction of weights, and the difference between formulas becomes smaller. The results from Table 2 confirm this.

TABLE 2

Decomposition of Geometric Lowe to Geometric Young Ratio

| | Geometric Young | Geometric Lowe | Geometric Lowe/ Geometric Young | $\rho_{x,y}^{s^b}$ | $\sigma_y^{s^b}$ | $cv_x^{s^b}$ | $\exp\left(\rho_{x,y}^{s^b} cv_x^{s^b} \sigma_y^{s^b}\right)$ |
|---|---|---|---|---|---|---|---|
| 2007 Dec | 1 | 1 | 1 | | | | |
| 2008 Jan | 1.0048 | 1.0052 | 1.0004 | 0.1962 | 0.0133 | 0.1712 | 1.0004 |
| 2008 Feb | 1.0077 | 1.0081 | 1.0003 | 0.1039 | 0.0184 | | 1.0003 |
| 2008 Mar | 1.0147 | 1.0170 | 1.0023 | 0.4543 | 0.0292 | | 1.0023 |
| 2008 Apr | 1.0194 | 1.0232 | 1.0038 | 0.6069 | 0.0366 | | 1.0038 |
| 2008 May | 1.0250 | 1.0316 | 1.0065 | 0.7142 | 0.0530 | | 1.0065 |
| 2008 Jun | 1.0321 | 1.0414 | 1.0091 | 0.7324 | 0.0719 | | 1.0091 |
| 2008 Jul | 1.0367 | 1.0467 | 1.0096 | 0.6892 | 0.0809 | | 1.0096 |
| 2008 Aug | 1.0356 | 1.0434 | 1.0075 | 0.6390 | 0.0685 | | 1.0075 |
| 2008 Sep | 1.0355 | 1.0423 | 1.0066 | 0.6256 | 0.0612 | | 1.0066 |
| 2008 Oct | 1.0294 | 1.0315 | 1.0021 | 0.2621 | 0.0469 | | 1.0021 |
| 2008 Nov | 1.0139 | 1.0064 | 0.9927 | –0.5619 | 0.0767 | | 0.9927 |
| 2008 Dec | 1.0031 | 0.9900 | 0.9869 | –0.6632 | 0.1159 | | 0.9869 |
| 2009 Jan | 1.0077 | 0.9959 | 0.9883 | –0.6329 | 0.1089 | | 0.9883 |
| 2009 Feb | 1.0131 | 1.0026 | 0.9897 | –0.6119 | 0.0987 | | 0.9897 |
| 2009 Mar | 1.0156 | 1.0051 | 0.9897 | –0.6074 | 0.0998 | | 0.9897 |
| 2009 Apr | 1.0178 | 1.0085 | 0.9909 | –0.5676 | 0.0943 | | 0.9909 |
| 2009 May | 1.0203 | 1.0133 | 0.9932 | –0.3853 | 0.0845 | | 0.9944 |
| 2009 Jun | 1.0275 | 1.0248 | 0.9974 | –0.2102 | 0.0725 | | 0.9974 |
| 2009 Jul | 1.0259 | 1.0226 | 0.9968 | –0.2383 | 0.0779 | | 0.9968 |
| 2009 Aug | 1.0276 | 1.0255 | 0.9979 | –0.1581 | 0.0766 | | 0.9979 |
| 2009 Sep | 1.0284 | 1.0256 | 0.9973 | –0.2025 | 0.0794 | | 0.9973 |
| 2009 Oct | 1.0298 | 1.0266 | 0.9969 | –0.2312 | 0.0785 | | 0.9969 |
| 2009 Nov | 1.0300 | 1.0281 | 0.9981 | –0.1457 | 0.0747 | | 0.9981 |
| 2009 Dec | 1.0280 | 1.0259 | 0.9979 | –0.1589 | 0.0766 | | 0.9979 |

### 4.3.2. Formulas that Better Track Superlative Indexes: Lent–Dorfman, Lloyd–Moulton, and Simple Averages

In Table 3 we consider percentage differences in annual growth rates between the target indexes and alternative measures using the simulated U.S. CPI data for January 2002 to December 2010. Lowe has the largest bias of about 0.16 annually from a superlative index and Young performs much better, reducing the bias to about 0.016 annually. Their geometric equivalents show mixed results with the Geometric Lowe being, on average, 0.002 percent below the Fisher while the Geometric Young has an annual growth rate about 0.156 below. An approach based on the Lent and Dorfman (2009) (hereafter L-D) framework uses weighted averages of formulas (equation (14)) to more closely correspond to a superlative index.[20] We use practical real-time variants of this approach.

First, the L-D variants use approximations to Laspeyres and Geometric Laspeyres in equation (14) for which real-time data would not be available in

[20]Both Fisher and Törnqvist simulations are used as target superlative indexes; they track each other very closely. Both the Fisher and Törnqvist increase by 22.5 percent over the period January 2002 to December 2010.

TABLE 3

Percentage Differences in Annual Growth Rates between Alternative vs Target Indexes[a]

| | Fisher | Tornqvist |
|---|---|---|
| *Arithmetic formulas* | | |
| Lowe | 0.161 | 0.159 |
| Young | 0.016 | 0.013 |
| *Geometric formulas* | | |
| Geometric Lowe (GLowe) | –0.012 | –0.014 |
| Geometric Young (GY) | –0.156 | –0.159 |
| *Geometric means of formulas* | | |
| GY-Young | –0.070 | –0.073 |
| GY-Lowe | 0.002 | 0.000 |
| GLowe-Young | 0.002 | 0.000 |
| GLowe-Lowe | 0.075 | 0.072 |
| *Lent–Dorfman ($\eta^\tau$ using 2-year lag)*[b] | | |
| GY-Young | –0.046 | –0.047 |
| GY-Lowe | –0.046 | –0.048 |
| GLowe-Young | –0.003 | –0.005 |
| GLowe-Lowe | 0.010 | 0.007 |
| *Lloyd–Moulton ($\eta^\tau$ using 2-year lag)*[b] | | |
| L-M (Greenlees estimate of $\eta^\tau$) | –0.044 | –0.047 |
| L-M (L-D estimate of $\eta^\tau$) | –0.028 | –0.030 |

[a]Covers January 2002 to December 2010.
[b]January 2002 to December 2003 uses a one-year lag.

practice. We use, in turn, Lowe and Young formulas to approximate Laspeyres and Geometric Lowe and Geometric Young formulas to approximate Geometric Laspeyres indexes in (14) and (15).

Second, we calculate estimates of $\eta^\tau$ in (16) based on a selected superlative benchmark, in turn the Fisher and the Törnqvist indexes. The Fisher and Törnqvist indexes cannot be estimated in real time, so the most recently available estimates are used to enable a real-time computation. For example, for January 2008 to December 2009, using our U.S. data, the most recent estimates of Fisher and Törnqvist indexes were those available for January 2004–December 2005, and only this period's data were used for the January 2008 to December 2009 estimates of $\eta^\tau$.[21]

Third, the average $\eta^\tau$ over the months of January 2004 to December 2005 (two-year lag) is used in equation (15) for all months in January 2008 to December 2009, and similarly over other periods. The Lent–Dorfman formulation has the advantage of allowing $\eta^\tau$ to change on a *monthly* basis. However, we constrain such changes to the period of the rebasing of the index and then hold $\eta^\tau$ constant

[21]Estimated $\eta^\tau$ from equation (16) can be seen to be based on the relative distances of arithmetic and geometric (Laspeyres) indexes from Fisher, that is, $\eta^\tau = \frac{\left(I_F^{0\to t} - I_{Las}^{0\to t}\right)}{\left(I_{GLas}^{0\to t} - I_{Las}^{0\to t}\right)} = \frac{\left(I_F^{0\to t} - I_{Las}^{0\to t}\right)}{\left(I_F^{0\to t} - I_{Las}^{0\to t}\right) + \left(I_{GLas}^{0\to t} - I_F^{0\to t}\right)}$ and $\left(1-\eta^\tau\right) = \frac{\left(I_{GLas}^{0\to t} - I_F^{0\to t}\right)}{\left(I_F^{0\to t} - I_{Las}^{0\to t}\right) + \left(I_{GLas}^{0\to t} - I_F^{0\to t}\right)}$. However, in a relatively small number of cases, particularly at the start of the series when all index levels are close to 100.0, the calculated $\eta^\tau$ or $(1 - \eta^\tau)$ is negative. In such cases, absolute values of the distance terms in the equation for $\eta^\tau$ are used.

until the next rebasing. This has practical advantages: the timing of such weight changes concurs with the rebasing of the index and CPI changes are not affected by changes in the weight $\eta^{\tau}$ given in (14) to the different formulas used.

We also calculate Lloyd–Moulton (L-M) indexes, equation (13), using estimates of $\eta^{\tau}$ from Greenlees (2011, table 3) and from using the L-D approach described in this section and in equations (14)–(16).

Table 3 shows percentage differences in annual growth rates over January 2002 to December 2010 for the Lowe, Young, Geometric Lowe, and Geometric Young indexes relative to Fisher and Törnqvist indexes. The results of simple geometric means of these formulas are also given along with those of four different variants of the L-D formulation. Equations (15) and (16) for L-D use Laspeyres and Geometric Laspeyres. The variants in Table 3 use Young and Lowe as real-time approximations to Laspeyres, and Geometric Young and Geometric Lowe as real-time approximations to Geometric Laspeyres; each variant uses our simulated Fisher and Törnqvist indexes as benchmarks.

From Table 3 we can see that in real time, using the same database as used by Lowe, we can cut Lowe's annual bias from 0.161 percent to nearly zero by simply averaging Lowe and the Geometric Young. Table 3 shows the Geometric Young to have a numerically comparable bias to Lowe, but in the opposite direction.[22] Alternatively, if a simple geometric average is taken of the Geometric Lowe and Young indexes, the Lowe bias is also reduced to near zero. To put the matter in context, annual CPI inflation, as measured by the Lowe index, increased by 2.5 percent over this period (average 2002 to average 2010); however, it rose by 2.3 percent according to our estimates of Fisher and Törnqvist. Simultaneously, both the geometric means of Geometric Lowe and Young, and of the Geometric Young and Lowe, also increased by about 2.3 percent.

These simple averages of arithmetic and geometric indexes can be justified in terms of simple forms of L-D indexes, that in turn have a conceptual basis as a Taylor approximation to the Törnqvist (or Fisher) index and that can be calculated in real time. As Lent and Dorfman (2009) note: the systematic updating of the index continuously picks up changes in consumer buying patterns as reflected in the data, while requiring no iterative numerical procedures, and can therefore be easily programmed and automated in a statistical production setting. It is an area for more empirical work. Table 3 provides results for the L-D variants that again improve substantially on the Lowe index. They, for the large part, provide very similar results to the simple geometric means. But the simplicity of calculation and exposition of the latter, on the basis of this data, argues in favor of the geometric means of the simpler formulas.

The last section of Table 3 shows that the L-M index also provides an improvement over the Lowe. Its annual growth rate falls below the target indexes by 0.03–0.04 percent, depending on the estimate of $\eta^{\tau}$ that we use in the simulation. These values are not as low as the Geometric Young and the simple geometric

[22]We calculated the root mean squared error (RMSE) for the annual inflation rates (month-on-same-month in previous year) for various indexes against a Fisher index. To give an indication of scale, our estimated Laspeyres (and Paasche) had RMSEs of 2.86 percent, a rough indication of higher-level substitution effects. The RMSE for Lowe was 12.6 percent. All other indexes had much lower RMSE than Lowe; Young at 6.25 percent was the next highest.

means of arithmetic and geometric indexes. Yet the L-M is still a viable alternative. A major advantage of this approach is that it provides consistency in aggregation—if the same geometric formula is used throughout the index.

For statistical offices that have frequent consumer expenditure surveys, it may be possible to regularly update the estimates of $\eta^{\tau}$. However, statistical offices that have limited resources would be able to calculate the simple geometric averages of available arithmetic and geometric indexes that we have presented here and provide consumer price index measures that, at least on the basis of this work, more closely approximate the target superlative indexes.

## 5. Concluding Remarks

The widely-used arithmetic Laspeyres-type aggregation formula at the higher (weighted) level for CPIs, the Young and Lowe indexes, have little justification in theory and in practice, something of major concern for this key macroeconomic indicator. The empirical work in Section 4 used U.S. data which benefited from relatively frequent rebasing (biannual chaining) and thus shows only some of the potential bias that may arise from these formulas. Nonetheless, we found the upward drift of the Lowe index against (the already upwardly biased) Laspeyres to be substantial. The Lowe index, like Laspeyres, has the advantage of ease of interpretation as a fixed quantity basket index. It provides a well-defined, but biased, result.

The two geometric formulations most readily available for compilers are the Geometric Young and the Geometric Lowe price indexes. The Geometric Young is easily explained as a weighted geometric average of price changes, using the survey period expenditure shares as weights. However, its bias was of a similar magnitude to that of Lowe, albeit in the opposite direction (Table 3). The Geometric Lowe has no meaningful interpretation. A formal exact decomposition of the difference between the Geometric Lowe and Geometric Young indexes found it to be based on long-run unidirectional price changes, equations (10) and (11), the nature of which made it unreliable as a basis for a predictable relationship between the Geometric Lowe and a Geometric Young or a Törnqvist index.

Table 3 provides the results for simple averages of arithmetic and geometric formulas, L-D and L-M approximations, all of which substantially improve on the Lowe estimates. We believe that L-D is a very real superior approach for countries with frequent weight updates, as would be L-M where consistency in aggregation is very important. However, we find using U.S. data that the weighting of geometric and arithmetic formulas using $\eta^{\tau}$ in the L-D approach, and more directly in the L-M approach, does not improve on simple symmetric geometric means of the two estimates; this is an important finding. It seems the positive bias of Lowe is counterbalanced (for practical index number purposes) by the negative bias of the geometric mean. The advice for statistical offices may be simple. Where regular consumer expenditure surveys are undertaken and reliable estimates of elasticity of substitution possible, statistical offices should evaluate the outcomes of L-D and L-M indexes using estimates of $\eta^{\tau}$ as outlined in this paper, along with the outcomes of quite simple formulations also investigated here. Occam's razor argues for simpler formulations. Where estimates of $\eta^{\tau}$ are not possible, the simple geometric formulations are likely to be a vast improvement on using the Lowe index, an upwardly biased element of some of these simple formulas.

The authors are well aware of the difficulties involved in changing the CPI formula from a long-standing and easily understood one to a more complex one. Similar issues arose when statistical agencies moved from arithmetic formulations to the widely adopted and conceptually sound geometric mean (Jevons index) at the lower level of CPI aggregation (Armknecht, 1996; Silver and Heravi, 2007). However, it is time to debate moving on from Laspeyres-type indexes. One approach is to calculate retrospective indexes to identify the extent of the substitution bias.[23] Yet the CPI is a key economic indicator and users would be better served by a real-time measure that more closely tracks a superlative index. It may well be that the public will accept a more complex formula if it can be demonstrated that it works much better. The Lowe index was found to have as much as *80 times* the bias of some of the geometric means of indexes and L-D variants that could be compiled in real time using the self-same data.

## References


Armknecht, Paul A., "Improving the Efficiency of the U.S. CPI," Working Paper 96/103, International Monetary Fund, Washington DC, 1996.

Balk, Bert M., "Does There Exist a Relation Between Inflation and Relative Price Change Variability? The Effect of the Aggregation Level," *Economic Letters*, 13, 173–80, 1983.

———, *Price and Quantity Index Numbers*, Cambridge University Press, Cambridge, 2008.

———, "Lowe and Cobb–Douglas Consumer Price Indexes and their Substitution Bias," *Jahrbücher für Nationalökonomie und Statistik (Journal of Economics and Statistics)*, 230, 726–40, 2010.

Balk, Bert M. and W. Erwin Diewert, "The Lowe Consumer Price Index and its Substitution Bias," Discussion Paper No. 04–07, Department of Economics, University of British Columbia, Vancouver, 2003.

Ball, Laurence and N. Gregory Mankiw, "Relative-Price Changes as Aggregate Supply Shocks," *Quarterly Journal of Economics*, 110, 161–93, 1995.

Bortkiewicz, L. V., "Zweck und Struktur einer Preisindexzahl," *Nordisk Statistisk Tidsskrift*, 2, 369–408, 1923.

Feenstra, Robert C. and Marshall B. Reinsdorf, "Should Exact Index Numbers Have Standard Errors? Theory and Application to Asian Growth," in Ernst R. Berndt and Charles R. Hulten (eds), *Hard-to-Measure Goods and Services: Essays in Honor of Zvi Griliches*, National Bureau of Economic Research, Chicago, IL, 2007.

Friedman, M., "Nobel Lecture: Inflation and Unemployment," *Journal of Political Economy*, 85, 451–72, 1977.

Glejser, Herbert, "Inflation, Productivity, and Relative Prices: A Statistical Study," *Review of Economics and Statistics*, 47, 76–80, 1965.

Greenlees, John S., "Improving the Preliminary Values of the Chained CPI-U," *Journal of Economic and Social Measurement*, 36(1), 1–18, 2011.

Greenlees, John S. and Elliot Williams, "Reconsideration of Weighting and Updating Procedures in the U.S. CPI," *Jahrbücher für Nationalökonomie und Statistik (Journal of Economics and Statistics)*, 230, 741–58, 2010.

Hansen, Carsten Bolden, "Recalculations of the Danish CPI 1996–2006," Proceedings of the Tenth Meeting of the International Working (Ottawa) Group on Price Indexes, Ottawa, Canada, October 9–12, 2007 (available at: http://www.ottawagroup.org).

Hercowitz, Zvi, "Money and Price Dispersion in the United States," *Journal of Monetary Economics* 10, 25–37, 1982.

ILO (International Labour Organization), IMF, OECD, Eurostat, United Nations, and World Bank, *Consumer Price Index Manual: Theory and Practice*, ILO, Geneva, 2004 (available at: http://www.ilo.org/public/english/bureau/stat/guides/cpi/index.htm).


[23]The U.S. Chained Consumer Price Index for All Urban Consumers or C-CPI-U is a chained Törnqvist. Details and data are available at: http://www.bls.gov/cpi/.

Lastrapes, W. D., "Inflation and the Distribution of Relative Prices: The Role of Productivity and Money Supply Shocks," *Journal of Money, Credit, and Banking*, 38, 2159–98, 2006.
Lent, Janice and Alan H. Dorfman, "Using a Weighted Average of Jevons and Laspeyres Indexes to Approximate a Superlative Index," *Journal of Official Statistics*, 25, 129–49, 2009.
Parks, R., "Inflation and Relative Price Variability," *Journal of Political Economy*, 86, 79–96, 1978.
Pike, Chris, Ben Nimmo, and Ludeth Mariposa, "New Zealand 2006 and 2008 Consumers Price Index Review: Price Updating," Room document, Eleventh Meeting of the International Working (Ottawa) Group on Price Indexes, Neuchatel, Switzerland, May 2009 (available under "Papers" at: http://www.ottawagroup.org/).
Reinsdorf, Marshall B., "New Evidence on the Relation between Inflation and Price Dispersion," *American Economic Review*, 84, 720–31, 1994.
Silver, Mick and Saeed Heravi, "Why Elementary Price Index Number Formulas Differ: Price Dispersion and Product Heterogeneity," *Journal of Econometrics*, 140, 2, 874–83, 2007.
Silver, Mick and Christos Ioannidis, "Inter-Country Differences in the Relationship Between Relative Price Variability and Average Prices," *Journal of Political Economy*, 109, 355–74, 2001.
UNECE (United Nations Economic Commission for Europe), ILO, IMF, OECD, Eurostat, World Bank, and Office for National Statistics United Kingdom, *Practical Guide to Producing Consumer Price Indices*, United Nations, New York and Geneva, 2009 (available at: http://www.imf.org/external/data.htm#guide).
Van Hoomisson, T., "Price Dispersion and Inflation: Evidence from Israel," *Journal of Political Economy*, 96, 6, 1303–14, 1988.
Vining, Daniel R., Jr. and Thomas C. Elwertowski, "The Relationship between Relative Prices and the General Price Level," *American Economic Review*, 66, 699–708, 1976.