Concepts and Terminology for International Comparisons of Prices and Real Incomes

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Abstract

The 2011 Phase of the International Comparison Program (ICP) is nearing completion and PPPs and real incomes for over 180 countries covering all the regions of the world will be released by the end of of 2013. Despite the increasing coverage of the ICP and the popularity of PPPs among international community not much attention has been focused on formalisation of the concepts and terms used in the publications of the ICP and making them consistent with the terminology used by the national statistical offices in the publication of the CPI and national accounts aggregates at current and constant prices. The ICP introduces a new raft of terms including purchasing power parities of currencies; price level indices; real expenditure or volume comparisons across countries. Extrapolation of PPPs over time introduces further complexities in terms of comparing "real aggregates" at current and constant prices which can be confused with nationally published aggregates at current and constant prices. The main objective of the paper is to formalise the concepts and terms used in international comparisons in manner that is consistent with the terminology used by price and national accounts statisticians in different countries. The paper also examines the role of normalisation in the computation of PPPs and consequent changes in the interpretation of price level indices. The paper will also attempt to formalise the concepts needed to discuss PPPs, price movements over time; real and nominal comparisons; and comparisons at current and constant prices.

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1. Introduction

Temporal comparisons of prices are standard for most national statistical offices through the compilation and publication of the consumer price indexes. The concepts, terminology and techniques that underpin the CPI are widely accepted and disseminated through publications such as the CPI Manual, ILO/IMF/OECD/UNECE/Eurostat/The World Bank (2004). In contrast spatial comparisons prices in the form of purchasing power parities of currencies are relatively recent and assumed prominence through the International Comparison Program which started as a research project at the University of Pennsylvania and is now a global statistical initiative under the guidance of the UN Statistical Commission and more recent rounds of the ICP conducted under the auspices of the World bank. The ICP started with a limited coverage of ten countries has now grown into a global project covering 146 and 180 countries in the 2005 and 2011 rounds of ICP respectively. Results from the ICP are in high demand from international organizations such as the International Monetary Fund, World Bank, OECD and Eurostat and from researchers and users from national governments as well as from the private sector. The framework for the ICP is discussed in Rao (2013a) and index number methods used in the 2005 and 2011 rounds of the ICP are discussed in Rao (2013b) and Diewert (2013a,b). A list of important terms used in the ICP can be found in the glossaries attached to the World Bank (2008) and ADB (2007) reports on 2005 ICP.

Since the inception of ICP in 1968, there have been major advances in data related methods as well as the aggregation and linking methods used in the compilation of regional and global comparisons of prices and real incomes. As the ICP is not a mainstream activity in the national statistical offices of the participating countries, the general practice and approach to international comparisons has been somewhat ad hoc. Most of the activities are driven by the coordinating agencies in different regions like the Asian and African Development Banks, OECD, EUROSTAT and the World Bank. It is only in recent years the the applicability of the ICP methodology to intra-country inter-regional comparisons has been recognized especially in larger countries like China, Indonesia and India.

Despite the enormous popularity enjoyed by the ICP largely due to the purchasing power parity data it generates, there is very little appreciation among researchers and users about the conceptual framework that underpins the ICP. In addition most of the ICP related publications tend to be in the form of reports and consulting reports and papers prepared for the meetings of various expert and advisory groups. A major consequence is the limited dissemination of the vast range of concepts, terminology and methods to the researchers, statisticians working in national statistical offices and users from the wider community. A few exceptions are the recent publications of Balk (2008) and Rao (2008).

An added dimension to the ICP is the popular and widely used Penn World Tables (PWT) which provides interpolated and extrapolated PPPs and real income data for over 150 countries covering a period of 50 years. In a way it is PWT that is instrumental in raising the profile of ICP and the PPP data compiled as a part of the ICP. As PWT has a time dimension, several additional concepts and measures are necessary to present the results over time and space.

For an uninitiated user of PPPs and results from the ICP and PWT there are only a handful of sources where a clear statement of the concepts and terms used in the ICP can be found. Kravis *et al* (1982) is still a valuable source but more recent sources include ADB (2007), World Bank (2008), World Bank (2013) and the books by Balk (2008) and Rao (2009). Material in these publications is fairly concise and no formal definitions and rigorous discussion of the implications can be found in these publications. Often even the researchers closely associated with ICP and national price statisticians involved in the ICP exercises tend to be confused between different terms and concepts. The main objective of this paper is to address this problem and to provide a comprehensive review of the concepts and terms used in the ICP and PWT. The paper provides a formal approach to the concepts and delves into the properties and interrelationships between concepts.

2. Notation

We consider the problem of making price comparisons across *I* countries (or regions) based on price and quantity data on *N* commodities. Let $i = 1,2,\ldots, I$ with ($I \geq 3$) denote countries commodities re labeled *n* = 1,2,…,*N*. The price vector for country *i*, expressed in its own currency, is denoted by $p^i \equiv \left(p_1^i, p_2^i, ..., p_N^i \right) \in \Re_{++}^N$. In contrast, the corresponding vector of quantities is denoted by $x^i = \left(x_1^i, x_2^i, ..., x_n^i \right) \in \Re^N \ (i = 1, 2, ...I)$. We assume that the price and quantity vectors refer to a particular period of time: without loss of generality this is assumed to be a particular year.[1] This means that our main purpose to make price and quantity comparisons across countries at a given point of time.

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[1] At this stage we do not include a time subscript. We will introduce it when we discuss comparisons over time and space.

Gross domestic product - Value aggregate

The value aggregate for country i is[2]:

$$V^i = p^i \cdot x^i = \sum_{n=1}^{N} p_n^i \cdot x_n^i \ \ for \ i = 1, 2, ..., I \tag{1}$$

The value aggregate in (1) is expressed in national currency units and, therefore, not comparable across countries. Given that the quantity vector is allowed to have negative values, we assume that

$$p^i \cdot x^j = \sum_{n=1}^{N} p_n^i \, x_n^j > 0 \ \text{for all} \ i, j = 1, 2, ..., N$$

This assumption implies that the value aggregate obtained when the quantity vector of x^j of country j is evaluated at prices p^i of country i is always positive[3]. We note here $p^i \cdot x^j$ is a value aggregate expressed in the currency units of country j.

In general, the value aggregate in (1) may refer to total output or total input or in some cases value added. As the principal aggregate used in the ICP is gross domestic product (GDP) we use GDP as the aggregate in our exposition.[4]

Exchange rates[5]

Exchange rates are generally used in converting V^i into a common currency unit. Let XR^i denote the exchange rate of currency of country i with respect to a reference currency (which is, unless stated otherwise, the US dollar); thus XR^i is the amount of i currency per 1 US\$. We assume that exchange rates are transitive between countries.[6] This means that XR^i / XR^j is the amount of i currency per unit of j currency.

GDP in national currency units

We let GDP^i represent the GDP of country i expressed in its own currency unit.

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[2] From here on, the dot denotes inner product between two vectors.
[3] This assumption ensures that the standard measures like the Laspeyres and Paasche indices are strictly positive.
[4] Though we use GDP as our aggregate, the results and interpretation hold for sub-aggregates like private consumption, government consumption and gross fixed capital formation as well.
[5] Exchange rates refer to the market exchange rates.
[6] Thus we assume that there is no arbitrage in currency conversion.

Nominal GDP

In the international comparison literature, the term *nominal GDP* represents GDP after conversion into a common currency unit using exchange rates. Thus nominal GDP is defined as

$$NGDP^i = \frac{V^i}{XR^i} = \frac{p^i \cdot x^i}{XR^i} = \frac{GDP^i}{XR^i}.$$ (2)

Nominal GDP is expressed in the units of the reference currency and therefore additive across countries.

Here the term *nominal* indicates that these value aggregates are not adjusted for price level differences across countries even though they are converted into a common currency.

3. Concepts and terms used in spatial comparisons at a given point of time

In this section we discuss the basic terminology used in cross-country comparisons at a given point of time. In the ICP the time point refers to the benchmark year when the comparisons are conducted.

3.1 Binary/bilateral versus multilateral comparisons

The most important concept associated with international comparisons is *multilateral comparisons*. Binary or bilateral comparisons refer to a comparison of two countries under consideration. In this case most standard index number methods like the Laspeyres, Paashe and Fisher can be used (see Balk 2009, p. 62).

In the case of multilateral comparisons, the interest is focused on the complete set of binary comparisons and therefore there is a need to ensure internal consistency. If I^{jk} refers to a comparison of country k relative to country j, then multilateral comparisons require that all the elements of the following matrix

$$I_{IxI} = \begin{bmatrix} I^{11} & I^{12} & I^{13} & ..... & I^{1I} \\ I^{21} & I^{22} & I^{23} & ..... & I^{2I} \\ I^{31} & I^{32} & I^{33} & ..... & I^{3I} \\ . & . & . & ..... & . \\ I^{I1} & I^{I2} & I^{I3} & ..... & I^{II} \end{bmatrix}$$ (3)

are filled. In the case of multilateral comparisons *transitivity* which is a condition for internal consistency is imposed on the formula used in computing I^{jk} ($j,k = 1,2,\ldots,I$). The condition states that for any choice of three countries j, k and l the index must satisfy that:

$$I^{jk} = I^{jl} \times I^{lk} \qquad (4)$$

The transitivity condition in conjunction with identity test property implies that the standard *country reversal test* in the context of binary comparisons is automatically satisfied. We note here that most standard binary index number formulae do not satisfy transitivity.[7]

3.2 Purchasing Power Parities

Though transitivity is a technical internal consistency requirement it leads to one of the most commonly used terms in international comparisons, *purchasing power parities* (PPPs) *of currencies*. We state the following result that leads to PPPs.

Result: A matrix of multilateral comparisons, derived using some index number formula, satisfies the transitivity condition in (3) *if and only if* there exist positive real numbers say $\pi^1, \pi^2, \ldots, \pi^I$ such that

$$I^{jk} = \frac{\pi^k}{\pi^j} \ \text{for all} \ j,k = 1,2,\ldots,I \qquad (5)$$

Equation (5) implies that $\pi^1, \pi^2, \ldots, \pi^I$ are like general price levels and I^{jk} may be interpreted as purchasing power parity between currencies of countries k and j. This leads to the term PPP between currencies of two countries with $PPP^{jk} = I^{jk}$.

It can be seen from (5) that solution for $\pi^1, \pi^2, \ldots, \pi^I$ is not unique as any constant (positive) multiple of a given solution also satisfies (5). Therefore it is a standard practice to normalize by setting one of the π's equal to one. If π^1 is set equal to 1, then the currency of the first country is the reference currency. In this case, purchasing power parities are given by:

$$PPP^1 = 1; \ PPP^2 = \pi^2; \ \ldots, PPP^I = \pi^I$$

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[7] This failure of many standard index number formulae to satisfy transitivity has led researchers to come up with alternative index number formulae. Some popular index number formulae are the GEKS, Geary-Khamis, weighted CPD and Iklé methods. These are not discussed in this paper. Interested reader may refer to a range of papers in Rao (2009) including Balk (2009) and papers in the ICP Book (World Bank, 2013).

In this case we can interpret the PPP for a given country as the number of currency units required to buy a basket of goods and services which can be purchased with one unit of currency of the reference (in this case the first) country.

PPP versus spatial price index numbers

The basic structure and formulae that underpin the PPPs are similar to the price index numbers in the time series context. However, PPPs cannot be directly interpreted as spatial price index number except in the case where all the countries, or the countries for comparison, have the same currency unit. Each PPP measures the number of currency units of a certain country that have the same purchasing power of as one currency unit of the reference country. A few points can be made here:

- PPP is made up of two components viz. the price level component and the currency unit component.

- If the currency units of two countries j and k are the same, then PPP^{jk} can be interpreted as a spatial price index between the two countries. For example, PPP^{jk} between two countries from the Euro-zone can be interpreted as a price index.

- If PPP^{jk}'s are computed using price data that is converted into a common currency unit using exchange rates thus adjusting for differences in currency units, the resulting PPP^{jk}'s can be interpreted as spatial price index numbers.

This conceptual difference between PPPs and spatial price index numbers has led the practitioners to the concept of price level indexes.

3.3 Real value aggregates or Real GDP

Since PPPs provide amounts of currencies that have purchasing power equivalent to one unit of currency of reference currency, we can use PPPs to convert the GDP of each country to an aggregate that can be compared across countries. This aggregate is referred to as the *real GDP* in ICP terms. Its definition is[8]

$$\text{Real GDP of country } i = RGDP^i = \frac{GDP^i}{PPP^{1i}} \tag{6}$$

As PPPs reflect the purchasing power of currencies reflecting the prices prevailing in different countries, real GDP is GDP adjusted for price level differences across countries. Therefore it is possible to talk

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[8] Strictly speaking we need to include a superscript to indicate that country 1 is the reference country.

about relative GDP levels between countries and also to compare real per capita GDP across countries and use it as a measure of standard of living.

We note here that the PPPs used in (6) are derived using price data collected from countries participating in the ICP along with National Accounts weights for aggregating those data.[9] The fact that these PPPs refer to a particular year[10] in which price data are collected implies that real GDP according to (6) also refers to a particular year and therefore is not comparable over time. We return to this in section 4.

3.4 Price Level Indexes (PLIs)

The price level index for country j relative to a certain reference country, say country 1, is defined as the ratio of PPP to the exchange rate. It is given by[11]

$$PLI^{1j} = \frac{PPP^{1j}}{XR^{1j}} \tag{7}$$

PLI is seen as a measure of price level relative to the level at which the currency can be converted at the exchange rate. Consider the case of Australian dollar versus the US dollar. The current exchange rate between is 0.97 AUD per 1 USD. Suppose a BigMac costs 2.25 USD and 2.75 AUD in these countries respetively. Then the PPP for AUD with USD as the reference currency is 2.75/2.25=1.22. The PLI based on the price BigMac alone is then 1.22/0.97=1.26.

We note the following properties of the PLIs:

- By definition, the PLI of the reference country is always equal to 1.
- When the PLI of a country is high, it is difficult to know whether prices in the reference country are low or prices in the country under consideration are high.
- The PLIs for a certain country based on two different reference countries are not simply comparable.
- PLIs are transitive. This follows from the transitivity of PPPs and XRs.

The following table shows PLI's for a selected set of countries from the global comparisons reported in World Bank (2008, p.23).

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[9] For details of the process of price collection and subsequent aggregation see Rao (2013a)'s paper on the ICP framework.

[10] This is usually referred to as the benchmark year in ICP.

[11] We deviate from the standard convention where the reference country is not shown in the price level index formula. For example ADB (2007) defines it in Appendix C simply as $PLI^j = PPP^j / XR^j$ suppressing the reference country.

Table 1: Price Level Indexes for Selected Countries, 2005
(Price level in USA = 1.0)

Country	PPP	XR	PLI with USA = 1	PLI with world average. =1
Australia	1.39	1.31	1.06	1.32
Denmark	8.52	5.95	1.42	1.76
India	14.67	44.10	0.33	0.41
Brazil	1.36	2.43	0.56	0.69
South Africa	3.87	6.36	0.61	0.76
USA	1	1	1	1.24

3.5 Price Level Indexes relative to World Average =1

A standard practice to address the interpretation problem associated with PLIs[12] in the table above is to express them relative to the world average. The idea is simply to compute the world average PLI and then measure PLI's relative to this world average. Essentially the process will result in PPPs for each country that are defined against a basket of world currencies such that the nominal world GDP at exchange rates equals the real world GDP which is the sum total of real GDP for each country in the comparison. The method can be described as below[13]:

Start with PLI of country j with reference country 1:

$$PLI^{1j} = \frac{PPP^j}{XR^j} \text{ for } j=1,2,\ldots,I$$

Define weighted average of PLI's from all the I countries in the international comparison. Given that the PLI's are based on country 1 as the reference country, we denote the weighted average as μ^1. We have

$$\text{World average of } PLIs = \mu^1 = \sum_{j=1}^{I} PLI^{1j} \frac{RGDP^{1j}}{\sum_{j=1}^{I} RGDP^{1j}} \tag{8}$$

where the weights reflect the relative sizes of real GDP in different countries. It is useful to find the following equivalent expression. Substituting the definition of PLI in (7) and the expression of real GDP, we have

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[12] For example, give a high PLI for a certain country relative to the reference country it is difficult to know whether the prices in comparison country are high or the prices are low in the reference country.

[13] Material in this subsection draws from a short note prepared by the first author for the Asian Development Bank which appeared as Appendix C in the 2005 ICP Asia-Pacific Final Report published by ADB (2007).

$$\text{World average of } PLIs = \mu^1 = \sum_{j=1}^{I} \frac{PPP^{1j}}{XR^{1j}} \frac{GDP^j / PPP^{1j}}{\sum_{j=1}^{I} RGDP^{1j}} = \frac{\sum_{j=1}^{I} GDP^j / XR^{1j}}{\sum_{j=1}^{I} GDP^j / PPP^{1j}} \tag{9}$$

Here μ^1 can be seen as an adjustment factor for PPP's such that we transform them from PPPs with country 1 as the reference to country to PPPs with world average basket, $PPP^{Wj} = PPP^{1j} / \mu^1$. Then we have the world average PLI equal to 1 by construction and the world real GDP using the new PPPs equal to world nominal GDP expressed in the currency of country 1. The following remarks are useful:

1. PPPs at the world basket: $PPP^{Wj} = PPP^{1j} / \mu^1$;

2. PLI for country j against world average $= PLI^{Wj} = PPP^{Wj} / XR^{1j}$; $\tag{10}$

3. World average PLIs at new PPPs =

$$\sum_{j=1}^{I} \frac{PPP^{Wj}}{XR^{1j}} \frac{GDP^j / PPP^{Wj}}{\sum_{j=1}^{I} RGDP^{Wj}} = \sum_{j=1}^{I} \frac{\left(PPP^{1j}/\mu^1\right)}{XR^{1j}} \frac{GDP^j / \left(PPP^{1j}/\mu^1\right)}{\sum_{j=1}^{I} GDP^j / \left(PPP^{1j}/\mu^1\right)} = \sum_{j=1}^{I} \frac{\left(PPP^{1j}/\mu^1\right)}{XR^{1j}} \frac{GDP^j / PPP^{1j}}{\sum_{j=1}^{I} GDP^j / PPP^{1j}} = 1$$

4. The world real GDP at the new PPPs is equal to world nominal GDP at exchange rates with country 1 as the reference,

$$\sum_{j=1}^{I} GDP^j / PPP^{Wj} = \sum_{j=1}^{I} GDP^j / XR^{1j}$$

5. The PLIs defined using world average are invariant to the choice of the reference currency express nominal GDP. This means that if we were to repeat the whole process using country 2 as the reference and define PLIs using (10) we will get the same price levels as using country 1 as the reference country (due to transitivity of PPPs and XRs).

6. We note that PPP^{Wj} is difficult to explain to practitioners as this PPP does not correspond to the currency of any particular country. So our recommendation is that PPPs are published using one of the comparison countries as reference but use PLIs according to the world average.

7. See Table 1 for PLIs with USA and World averages equal to 1 respectively. In the last column it can be seen that prices in USA are 24% above world average. All the PLIs are adjusted accordingly.

3.4 Real GDP and constant price comparisons

The real GDP of country j defined in (6) is the standard measure derived by deflating the GDP by a suitable price deflator. Thus RGDP is comparable across countries as the aggregate is adjusted for price level differences. In National Accounts, the term constant price comparisons is used to represent deflated real aggregates measured over time. However in the ICP and related publications, the term *constant prices GDP* comparison is used when a single set of reference prices are used to evaluate quantity data from different countries. For example, let π_n $(n = 1, 2, ..., N)$ represent a set of commodity prices[14], which we refer to as *international prices* used in measuring GDP at constant prices. We let CGDP represent GDP at constant prices and defined for each country j as:

$$CGDP^j = \sum_{n=1}^{N} \pi_n x_n^j = \pi \cdot x^j \tag{11}$$

As quantity vectors q^j are all evaluated at a fixed price vector π, CGDP is comparable across countries and can be added to form regional and global totals. Several remarks are in order.

1. International prices are usually expressed in the currency units of a selected reference country. If country 1 is used as the reference country then CGDPs are all in the currency units of country 1. In this case it is possible to compute an *implicit* PPP for country j as:

$$PPP^{1j} = \frac{GDP^j}{\sum_{n=1}^{N} \pi_n x_n^j} = \frac{p^j \cdot x^j}{\pi \cdot x^j} \tag{12}$$

    Superscript "1' in the expression simply implies that country 1's currency is used as the reference currency.

2. International prices for different commodities can also be represented using a numeraire commodity as is common in exchange equilibrium models. For example if "rice", listed as commodity 1 in the list, is used as the numeraire then π_n expresses the number of units of rice that are equivalent in price to one unit of commodity n. By construction, $\pi_1 = 1$. In this case the implicit PPP defined in (12) simply represents the number of units of commodity 1 (rice) that can be purchased with one unit of currency of country j at international prices.

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[14] We follow the notation used in Balk (2009).

3. In computing CGDP in (10) we need to specify the international prices. These are usually computed as international average prices. The Geary-Khamis and Iklé methods are the simplest examples of this approach to international comparisons. We briefly describe below the Geary-Khamis (GK) method and some recent extensions to that approach.

Geary (1958)-Khamis (1972) (GK) Method

The GK method defines international average prices and PPPs through the following system of inter-related equations.

$$\pi_n = \frac{\sum_{i=1}^{I} p_n^i x_n^i / PPP^i}{\sum_{i=1}^{I} x_n^i} \quad (n = 1, 2, ..., N)$$

$$PPP^i = \frac{p^i \cdot x^i}{\pi \cdot x^i} \quad (i = 1, 2, ..., I)$$

(13)

This system of (*N+I*) equations in as many unknowns has a unique solution up to a factor of proportionality. It can be solved directly or indirectly after setting one of the unknowns at a given level. If the currency of country 1 is selected as reference, then $PPP^1 = 1$. Alternatively if we select a commodity, say the first, to be the numeraire then $\pi_n = 1$. Some remarks here.

- We note here that the Iklé system is similar to GK system in (13) except that the international average prices π_n are defined as harmonic means.

- The GK system satisfies *strong additivity*[15] in that:

$$\pi \cdot x^i = \frac{p^i \cdot x^i}{PPP^i} = \frac{GDP^i}{PPP^i}$$

This equation implies that when the GK method is used there exist a set of international average prices such that constant price valuations of each country's quantity vector equals RGDP which is GDP deflated by the PPP derived from the GK system.

- There are two general criticisms against the GK method. The first concerns the use of "plutocratic" weights in the definition of international average prices in (13). As these are defined as quantity-share weighted averages of national prices of commodity *n*, larger countries tend to have a higher weight and consequently the international average prices may resemble larger quantities.

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[15] For a definition of *additivity and strong additivity* see Balk (2009) and Sakuma, Rao and Kurabayashi (2009).

- The second criticism against GK refers to the fact that PPP in equation (12) is defined as the ratio of two value aggregates derived by valuing x^i at the international average prices and the national prices, p^i. The GK definition of PPP does not allow for any substitution to reflect the differences in relative prices implicit in the international average price vector π. This aspect has received considerable attention recently in the works of Neary (2004), Feenstra, Ma and Rao (2009), Balk (2009) and Feenstra, Ma, Neary and Rao (2012). These papers allow for substitution possibilities by modifying the GK system.

- We also note here that not all RGDP can be expressed in the form of *constant price comparisons*. For example when PPPs are computed using the GEKS[16] then it is not possible to find a set of *international prices* at which the constant price GDP comparisons equal RGDP comparisons derived using GEKS.

4. Real GDP Comparisons at current and constant prices – Comparisons over space and time

In this section we turn our attention to a relatively new area in international comparisons which deals with comparisons of prices and real incomes across countries and over time. The ICP work in the past has largely focused on real income comparisons across countries in the benchmark year. Since its inception, ICP has completed seven phases at 1970, 1973, 1975, 1980, 1985, 1993 and 2005. The most recent ICP comparison for the benchmark year 2011 is currently underway and is expected to be completed by the end of 2012 or early in 2013. Results from these comparisons have largely been considered in isolation of each other and no major effort has gone into the compilation of PPPs over space and time. The PWT constructed by Summers and Summers (1991) and more recently by Heston, Summers and Aten (2012) where PPP results from the ICP benchmarks have been extrapolated over space and time to provide a panel of PPPs and real income for 180 countries and the sixty year period of 1950 to 2010.

Given that benchmark comparisons will be produced on a more regular basis, the need to establish appropriate terminology is becoming apparent. Before we move in this direction, we present results on PPPs and real incomes compiled by the Asian Development Bank for the year 2009 along with results from the 2005 ICP Asia-Pacific for a selected set of countries. Results for 2005 are based on price data collected through extensive price surveys for the benchmark year and the national accounts data used as weights. Detailed results for the 2005 benchmark are available from ADB (2007). Results for 2009 are

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[16] See Balk (2009), Diewert (2013a) for details of the GEKS (Gini-Eltetö-Köves-Szulc) method.

based on special price surveys conducted in 2009 for a subset of selected products from the 2005 ICP. Results for the 2009 are available in ADB (2012). Table 2 contains a selction.

Table 2: PPPs and Real Incomes for Selected Asia-Pacific Countries, 2005 and 2009
Reference Currency – Hong Kong dollar

Country	2005					2009			
	PPP	XR	PLI*	Real per capita GDP		PPP	XR	PLI*	Real per capita GDP
Bangladesh	3.98	8.27	86	7,215		4.16	8.91	76	10,888
China, PR	0.61	1.05	103	23,938		0.64	0.88	117	40,706
Hong Kong	1	1	180	202,941		1	1	162	231,611
India	2.58	5.67	82	12,090		2.70	6.25	70	19,500
Malaysia	0.31	0.49	112	65,217		0.29	0.45	104	82,524
Thailand	2.80	5.17	97	39,070		2.47	4.42	91	52,753
Vietnam	829	2039.12	73	12,185		950.68	2201.95	70	20,389

Note: (*) PLIs are defined relative to regional average with Asia = 100.
Source: ADB (2007) and ADB (2012)

In this table we have results from two benchmark years. The left panel refers to 2005 and figures in this panel are all comparable across countries. The PPPs and exchange rates refer to the year 2005 and the real GDP per capita in column 4 are derived by adjusting for price level differences in 2005. As all the figures, with the exception of PLIs, are normalized relative to Hong Kong these figures may be interpreted as reading in HK dollars. The right panel gives PPPs and exchange rates based on prices prevailing in 2009 and the real per capita figures in 2009 are based on HK dollars and prices in 2009. As a result, figures in the left and right panels, respectively for 2005 and 2009, are not strictly comparable. For example, we have PPP and exchange rate for HK dollar as 1 for both 2005 and 2009 but these are not the same units as the purchasing power of HK dollar as such has changed over time due to inflation experienced in HK.

So, all we can do is to make comparisons within but not across the panels. For example, China has a PLI of 103 in 2005 compared to 117 in 2009. Both of these are relative to the Asian regional

average which has been set equal to 1 in both years. It is necessary to have some sense of the price movement in the Asian region 9regional inflation) between 2005 and 2009 to drawn any conclusion..

The point we wish to stress here is that in the presence of the two panels, it is important to identify the PPP, XR, PLI and real GDP figures with the benchmark years to which they refer. In order to distinguish the results in the two panels for 2005 and 2009, we need a new set of terms and possibly new measures. The material in rest of this section is drawn from Rao and Rambaldi (2012).

Real GDP series at current prices

We consider the case where we have a set of international comparisons covering I countries using data on price of N commodities covering a period of T years. We let PPP_t^j, XR_t^j, and GDP_t^j represent PPP, exchange rate and GDP in national currency units for country j (j=1,2,…,I) in year t (t=1,2,…T). We assume that PPPs and exchange rates are measured with reference country 1.[17] As before, we have the nominal and real GDP defined as:

$$NGDP_t^j = \frac{GDP_t^j}{XR_t^j} \quad \text{and} \quad RGDP_t^j = \frac{GDP_t^j}{PPP_t^j} \qquad j = 1,2,...,I \, and \, t = 1,2,...,T \qquad (14)$$

We note the following features of the real GDP series.

   a. $RGDP_t^j$ are comparable across countries at any given year. These figures can be used in computing regional and global totals.

   b. $RGDP_t^j$ and PPP_t^j are typical outputs of ICP in a given benchmark year.

   c. $RGDP_t^j$ is obtained by deflating GDP of each country by a suitable price deflator, which is PPP_t^j here.

   d. As $RGDP_t^j$ is not comparable to $RGDP_s^j$ for $s \neq t$ and given that they are comparable across countries at given t, this series is referred as **real GDP series at year t or at current prices**.

Construction of such series on an annual basis in the presence of benchmark comparisons conducted once in five years is addressed by the PWT or by the econometric approach proposed in Rao, Rambaldi and Doran (2010a, 2010b).

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[17] We drop the superscript for the reference country to keep the notation simple.

Real GDP series at constant prices[18]

In principle, real GDP series at constant prices can be constructed if we can have PPPs that can be used to adjust for price level differences through time and space. Suppose we are able to compile $PPP_{\tau t}^{jk}$ representing PPP for country k in period t relative to reference country j in reference period τ. The real GDP of country k in period t expressed at constant year τ prices in reference country j is given by:

$$CRGDP_{\tau t}^{jk} = \frac{GDP_t^k}{PPP_{\tau t}^{jk}} \text{ for } k = 1, 2, ..., I \text{ and } t = 1, 2, ...T \tag{15}$$

A few remarks are in order:

1. There are a number of sources from which *real GDP series at current prices* are available. Penn World Tables due to Summers and Heston (1991) and Heston, Summers and Aten (2012) are the primary source of such comparisons. The main problem is one of extrapolation or interpolation between the benchmarks and outside benchmarks. The PWT series have been available for over three decades and they have been instrumental in popularizing the use of PPPs in economic research. Rao, Rambaldi and Doran (2010a, 2010b) used an econometric approach to construct a panel of PPPs and real GDP series at current prices which are made available through the UQICD website (URL: www.uqicd.economics.uq.edu.au) covering 140 countries and the period 1970 to 2005. The World Bank also produces series by extrapolating from the latest benchmark using GDP deflators.

2. The $CRGDP_{\tau t}^{jk}$ makes it possible to compare real per capita income of country k in a certain period t with any other country j in any other period s and these would be internally consistent when the $PPP_{\tau t}^{jk}$ series used in the denominator of (14) are transitive over space and time.

3. There are only limited sources for *real GDP series at constant prices*. Currently only PWT produces such series. The PWT applies Geary-Khamis method to data (in aggregated form for three commodities, viz. consumption, investment and government) for all the countries and all the years to derive a set of *international average prices over time and space* and use these prices to form constant price aggregates.

4. The PWT approach described in (3) fails to satisfy the "fixity principle". The problem may be described using a simple example. Suppose we have 150 countries covering the period 1970 to 2000. Suppose we compute all the $PPP_{\tau t}^{jk}$'s using the PWT approach and the resulting real GDP series at

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[18] Constant prices here refer to those of the reference country j and reference period τ.

year 1990 prices. Now one wants the in PWT to coincide with the real GDP series for 1990 at current 1990 prices. However the PWT approach fails to guarantee such fixity.

5. Rao and Rambaldi (2012) developed a constrained GEKS approach to the construction of consistent panels of PPPs and real incomes over space and time. Their approach is in the development stage and empirical implementation is underway. It is expected that their approach would generate real GDP series at constant prices that maintain fixity with comparisons at current prices.

5. Concluding Remarks

The main objective of this paper was to provide an overview of the concepts and terms used in international comparisons of prices, purchasing power parities and real incomes. Despite the popularity of the ICP results for the 2005 benchmark and the use of PPPs by international organizations and researchers, there is no systematic treatment and description of the basic concepts used in ICP work. One finds somewhat simplistic description of terms like PPPs, price level indexes and real expenditures in publications of the World Bank (2008, 2013) and of organizations like OECD, Eurostat and the Asian and African Development Bank. In this paper, we have provided a more rigorous treatment of these concepts and terms and have attempted to cover comparisons over space and time. In the recent years interest in consistent space-time comparisons has increased and this is an area that deserves further research and resources.

REFERENCES

ADB (2007), *Purchasing Power Parities and Real Expenditures: 2005 International Comparison Program in Asia and the Pacific*, pp. 217. Asian Development Bank, Manila.

ADB (2012), *Purchasing Power Parity Update for Selected Economies in Asia and the Pacific A Research Study*, pp. 126, Asian Development Bank, Manila.

Balk, B.M. (2008), *Price and Quantity Index Numbers : Models for Measuring Aggregate Change and Difference*, pp. 283Cambridge University Press, Cambridge, UK.

Balk, B.M (2009), "Aggregation Methods in International Comparisons: An Evaluation", Chapter 3 in Prasada Rao (ed) *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, pp. 59-85, Edward Elgar Publishing, Cheltenham, UK

Diewert, W.E. (2013a), " Methods of Aggregation above the Basic Heading Level within Regions", Chapter 5 in World Bank (ed) *Measuring the Real Size of the World Economy: The Framework, Methodology, and Results of the International Comparison Program (ICP)*, World Bank, Washington, DC

Diewert, W.E. (2013b), "Methods of Aggregation above the Basic Heading Level: Linking the Regions" Chapter 6 in World Bank (ed) *Measuring the Real Size of the World Economy: The Framework,*

Methodology, and Results of the International Comparison Program (ICP), World Bank, Washington, D.C.

Geary, R.C. (1958), 'A Note on the Comparison of Exchange Rates and Purchasing Power between Countries', *Journal of the Royal Statistical Society:* A, **121**, 97–99.

Feenstra, R., Hong Ma and D.S. Prasada Rao (2009), "Consistent Comparisons of Real Incomes Across time and Space", *Macroeconomic Dynamics*, **13** (Supplement 2), 169-193.

Feenstra, Robert C., Hong Ma, J. Peter Neary and D.S. Prasada Rao, (2012) "Who Shrunk China" Puzzles in Measurement of Real GDP", *Economic Journal,* Accepted manuscript online: 18 DEC 2012 08:48AM EST | DOI: 10.1111/ecoj.12021.

Heston, A, Robert Summers and Bettina Aten (2012), Penn World Table Version 7.1, *Center for International Comparisons of Production, Income and Prices* at the University of Pennsylvania, July 2012.

ILO/IMF/OECD/UNECE/Eurostat/The World Bank (2004), *Consumer price index manual: Theory and Practice*, Geneva, International Labour Office.

Khamis, S.H. (1972), 'A New System of Index Numbers for National and International Purposes', *Journal of the Royal Statistical Society*: A, 135, 96–121.

Kravis, I., A. Heston and R. Summers (1982), *World Product and Income: International Comparisons of Real Gross Product*, The Johns Hopkins University Press, Baltimore.

Neary, J.P. (2004), 'Rationalizing the Penn World Table: True Multilateral Indices for International Comparisons of Real Income', *American Economic Review*, **94**, 1411–1428.

Rao, D.S. Prasada (2009) (ed.), *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, pp. 400, Edward Elgar Publishing, Cheltenham, UK.

Rao, D.S. Prasada (2009), "Generalised Elteto-Koves-Szulc and Country-Product-Dummy Methods for International Comparisons", Chapter 4 in Prasada Rao (ed) *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, pp. 86-120, Edward Elgar Publishing, Cheltenham, UK

Rao, D.S. Prasada (2013a) , "The Framework of the International Comparison Program", Chapter 1 in World Bank (ed) *Measuring the Real Size of the World Economy: The Framework, Methodology, and Results of the International Comparison Program (ICP)*, World Bank, Washington, DC

Rao, D.S. Prasada (2013b) , "Computation of Basic Heading Purchasing Power Parities for Comparisons

within and between Regions", Chapter 4 in World Bank (ed) *Measuring the Real Size of the World Economy: The Framework, Methodology, and Results of the International Comparison Program (ICP)*, World Bank, Washington, DC

Rao, D.S. Prasada, Alicia Rambaldi and Howard Doran (2010a), "Extrapolation of Purchasing Power Parities using Multiple Benchmarks and Auxiliary Information: A New Approach", *Review of Income and Wealth*, Special Issue. Series 56, *Special Issue 1*, June, 2010, pp. S59-S98.

Rao, D.S. Prasada, Alicia Rambaldi and Howard Doran (2010b), "An Econometric Approach to the Construction of Complete Panels of Purchasing Power Parities: Analytical properties and Empirical Results", *2010 World Congress of the Econometric Society*, Shanghai, 17-21 August, 2010.

Rao, D.S. Prasada and Alicia Rambaldi (2012), "Construction of Consistent Panels of PPPs and Real Incomes at Current and Constant prices", International Comparisons Workshop at the University of Pennsylvania, Philadelphia, May 17-18.

Sakuma, I., D.S. Prasada Rao and Y. Kurabayashi (2009), "Additivity, Matrix Consistency and a New Method for International Comparisons of Real Income and Purchasing Power Parities", Chapter 6 in Prasada Rao (ed) *Purchasing Power Parities of Currencies: Recent Advances in Methods and Applications*, pp. 142-159, Edward Elgar Publishing, Cheltenham, UK.

Summers, Robert and Heston, Alan, 1991. "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988," *The Quarterly Journal of Economics*, MIT Press, vol. 106(2), pages 327-68.

World Bank (2008), *Global Purchasing power Parities and Real Expenditures: 2005 International Comparison Program*, pp. 213. World Bank, Washington, D.C.

World Bank (2013), *Measuring the Real Size of the World Economy: The Framework, Methodology, and Results of the International Comparison Program (ICP)* , pp. 691, World Bank, Washington, D.C.