Construction of Panels of Real Incomes at current and Constant Prices – An Econometric Approach

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Abstract

The paper presents the general form and some analytical properties of a new method for the construction of a consistent panel of Purchasing Power Parities (PPPs), and real incomes. The econometric approach proposed here improves upon the current practice used in the construction of the Penn World Tables, PWT, and similar tables produced by the World Bank. A state-space formulation is used in combining PPPs for benchmark years constructed by the International Comparison Program (ICP) with PPP predictions from a model of the national price level (or exchange rate deviation index) for all countries and years. Data on price movements available from national sources are also incorporated. The smoothed PPP predictions (and standard errors) obtained through the state-space representation of the model are produced for both ICP- participating and non-participating countries and non-benchmark years. A number of analytical results highlight the properties and flexibility of the method presented. The method is extended to construct panels of PPPs and real incomes at constant prices. The empirical illustration shows the general model can produce variants that: a) result in PPP predictions that accurately track the available ICP's PPPs (benchmarks); or b) preserve the growth rates in price levels implicit in individual countries' national accounts data. A data set for 141 countries for the period 1970 to 2005 is used to illustrate the flexibility of the method and to compare its performance to PWT6.3.

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1. Introduction

The main objective of the paper is to briefly describe the progress in the development of an econometric framework for the extrapolation of purchasing power parities (PPPs) to construct panels of PPPs and real incomes at current and constant prices. The focus has been on the development of a method that can combine: information on PPPs generated by various benchmarks of the International Comparison Program; published data from national sources on movements in prices at the country-level in the form of deflators for the gross domestic product (GDP); and the past efforts in extrapolating PPPs to countries that have not participated in the ICP benchmarks on the basis of PPPs for participating countries and models explaining national price levels. The pioneering work of Summers and Heston (1991) and Heston, Summers and Aten (2006) has led to the widely used Penn World Tables (PWT) which are compiled using some of these elements. In particular, the extrapolations rely to a large degree on the latest benchmark information available and, therefore, the panels could be influenced by specific benchmarks. The work on the *New Generation PWT* to be reported in the paper by Feenstra and Inklaar (2011) is moving in the direction of using PPPs from different benchmarks in the extrapolation process.

Inspired by the enormous contribution made by Heston, Summers and Aten (2012) through the development of PWT, we have started working on the development of a more formal structure for the generation of panels of PPPs and panels of real incomes at current and constant prices. As an initial step, we have been able to formulate an econometric approach leading to a model that can be expressed in a state-space framework. The framework allows for the generation of optimal predictors (in a mean square error sense) of PPPs that make use of information available from a number of different sources (listed above). So, far the main focus has been the development of the model and the study of various analytical properties of the model. The first phase of our work on the generation of a panel of PPPs has been completed. These panels can then be used in generating internationally comparable real incomes at current prices. We are now moving the next phase of constructing panels of real incomes at constant prices and also to extend the extrapolation methodology to the three main components of domestic absorption, viz., consumption; investment and government.

The paper presents results on progress made thus far. Section 2 provides a brief overview of the work completed on the construction of consistent panels of PPPs and real incomes at current prices. The econometric approach used in the construction is discussed along with its analytical properties and numerical results for some selected countries are also presented. The newly established website, UQICD[1], with URL:https://uqicd.economics.uq.edu.au/, is also briefly described. Section 3 is devoted to the problem of construction of panels of real incomes at constant prices.[2] First a heuristic/ad hoc method of constructing such panels is described. This is followed by a description of a state-space approach to the construction of such panels. Some empirical results from these two approaches are presented. The paper is concluded with a few remarks in the last section.

2. An Econometric Approach to the Construction of Panels of PPPs and Real Incomes at Current Prices

This section draws heavily from the descriptions of the method provided in Rao, Rambaldi and Doran (RRD) (2010a, 2010b). In order to avoid duplication of the material contained in these papers, only a brief description of the method is provided here. The Rao, Rambaldi and Doran (RRD) approach is designed to combine PPP data available from all the benchmarks of the International Comparison Program (ICP),

[1] The establishment of the website fulfils one of the main funding requirements from the Australian Research Council.

[2] This problem also translates into the construction of panels of PPPs over time and space with some basic consistency requirements.

since 1970 to the extensive coverage of countries in the 2005 round of the ICP, with the information on deflators at the aggregate GDP level available from the national accounts data published by the countries. In addition to these two main sources, the approach also makes use of the vast literature on the explanation of national price levels[3] in the form of a regression model which is used in extrapolating PPPs for countries that have not participated in each of the benchmarks of the ICP.

2.1 The Model

The basic model consists of the following elements:

1. Let PPP_{it} represent the PPP for the currency of country j in period t. Also let p_{it} = ln (PPP_{it}) be the logarithm of the true PPPs. The observed PPPs from the ICP, in the benchmark years, are related to the true PPPs through the following equation:

$$\tilde{p}_{it} = p_{it} + \xi_{it} \tag{1}$$

where ξ_{it} is a random error accounting for measurement error with the properties:

$$E(\xi_{it}) = 0;\ E(\xi_{it}^2) = \sigma_\xi^2 V_{it} \tag{2}$$

The measurement error variance-covariance is of the form

$$\mathbf{V}_t = \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \sigma_{1t}^2 \mathbf{jj}' + diag(\sigma_{2t}^2, ..., \sigma_{Nt}^2) \end{bmatrix} \tag{3}$$

where $\mathbf{j}$ is a vector of 1's and σ_{it}^2 is the variance of the PPP from the ICP benchmark for country i in period t. Here σ_{1t}^2 is the variance of the reference country. In the empirical implementation of the method, σ_{it}^2 is assumed to be inversely related to the GDP of country i in period t[4].

2. The numerical value of the PPP for the reference/numeraire country, 1, is set at 1. Thus

$$p_{1,t} = 0, \quad t = 1, 2, ..., T \tag{4}$$

3. The key element of the approach is the regression model used in extrapolating PPPs to non-participating countries using PPP data from the ICP benchmarks. The regression model draws on the literature on the explanation of national price levels (Kravis and Lipsey, 1983; Clague, 1988 and Bergstrand, 1991, 1996). A linear model in logarithms of price levels is postulated as below:

$$r_{it} = \ln(PPP_{it} / ER_{it}) = \beta_{0t} + \mathbf{x}_{it}'\beta_s + u_{it} \text{ for all } i = 1, 2, ..., N \text{ and } t = 1, 2, ..., T \tag{5}$$

Deviating from the usual assumptions on the disturbance term, we assume that errors in (5) are spatially autocorrelated[5]. The following specification is used.

[3] National price level is defined as the ratio of PPP to the exchange rate of the currency of a given country.

[4] In order to avoid circularity, nominal GDP converted using market exchange rates is used in the estimation process.

[5] This assumption essentially means that if a country has a national price level above the expected value from the model (5) then all those countries that are in the proximity of the country (where proximity needs to be defined) also exhibit national price levels above the expected levels.

$$\mathbf{u}_t = \phi \mathbf{W}_t \mathbf{u}_t + \mathbf{e}_t$$

where $|\phi| < 1$ and $\mathbf{W}_t (N \times N)$ is a spatial weights matrix. The term spatial in the present contexts refers to socio-economic distance rather than the traditional geographical distance. It follows that $E(\mathbf{u}_t \mathbf{u}_t^{'})$ is proportional to $\Omega_t = \left(\mathbf{I}\text{-}\phi \mathbf{W}_t\right)^{-1} \left(\mathbf{I}\text{-}\phi \mathbf{W}_t\right)^{-1'}$.

If estimates of parameters in (5) are available, then predictions of PPPs consistent with price level theory can be generated for any country in any period. These are given by:

$$\hat{p}_{it} = \hat{\beta}_{0t} + \mathbf{x}_{it}' \hat{\beta}_s + ln(ER_{it}) \qquad (6).$$

The point to note here is that unlike the PWT and other extrapolation methods, this approach generates predictions for all the cells (time periods and countries). However, it is trivial to limit the extrapolated PPPs used by the state-space representation to generate the final model predictions to only those from years that correspond to the ICP benchmark years.

4. The last element of the methodology is the information used in extrapolating PPPs over time. Using the US as the reference country, the updating of PPPs from period *t-1* to *t* is through the GDP deflators in the country concerned and in the reference country. Thus,

$$PPP_{i,t} = PPP_{i,t-1} \times \frac{GDPDef_{i,[t-1,t]}}{GDPDef_{US,[t-1,t]}} \qquad (7)$$

Taking logarithms on both sides of (7), and assuming the updating equation (7) holds on average due to measurement error, we have

$$p_{it} = p_{i,t-1} + c_{it} + \eta_{it} \qquad (8)$$

where

$c_{it} = ln\left(\frac{GDPDef_{i,[t-1,t]}}{GDPDef_{US,[t-1,t]}}\right)$; and η_{it} is random error accounting for measurement error in the growth rates. Equation (8) is commonly used constructing panels of PPPs including the PWT and in the construction of the Maddison series[6]. The variance covariance matrix of η_{it} is assumed to be of the same form to the matrix in equation (3).

As the current problem is one of finding predictions for the vectors of PPPs from a variety of sources of noisy information through the ICP benchmarks; regression predictions and, finally, the updating equation in (8). A state-space (SS) representation is suitable for these kinds of problems and the approach proposed formulates all the information in equations (1) to (8) in the form of a set of observation and transition equations on the state vector $\boldsymbol{\alpha}_t$ which is the vector of unknown $\ln(PPP_{it})$. Details of the formulation are provided in RRD (2010b). Under Gaussian assumptions, the Kalman filter predictor, the conditional mean $\hat{\boldsymbol{\alpha}}_t$, conditional on information available at time *t*, is a minimum square

[6] Maddison (2007) presents series that are extrapolated from the 1990 benchmark year.

error predictor of the state vector $\boldsymbol{\alpha}_t$. The Kalman Filter and Smoother are used to obtain the model's predictions of the state vector $\boldsymbol{\alpha}_t$.[7]

2.2 Analytical properties of the Model

In order to provide a better appreciation of the features of the econometric model used here, a number of analytical results pertaining to the model are presented here. In particular, these properties demonstrate the flexibility of the model and show how the model provides intuitively meaningful predictions under specific scenarios. The following properties are stated without proofs but complete proofs are provided in Rao, Rambaldi and Doran (2010b).

2.2.1 Constraining the model to track PPPs for countries participating in the benchmarks

As the ICP is the main source of PPPs for countries participating in different benchmarks and given that respective PPPs are determined using price data collected from extensive price surveys, one may consider it necessary that the econometric method proposed should generate predicted PPPs that are identical to PPPs for the countries participating in different ICP benchmarks. In the model proposed here, this can be achieved by simply setting the variance of the disturbance term in (1) to be equal to zero. In this case a particular property of Kalman filter predictions is that the predicted PPPs will be identical to the benchmark PPPs.8

2.2.2 Constraining the model to preserve movements in the Implicit GDP Deflator

In the currently available PWT and the Maddison series, growth rates in real GDP and movements in the implicit price deflators are preserved. As the GDP deflator data are provided by the countries and given that such deflators are compiled using extensive country-specific data, it is often considered more important that the predicted PPPs preserve the observed growth rates implicit in the GDP deflator. This essential feature can be guaranteed in the econometric approach proposed here and in RRD (2010a, b) by simply stipulating the variance of the error in the updating equation (8) to be zero. It is trivial to show that the national level movements in prices are preserved using the formulae for the fixed interval Kalman Smoother.[9]

We note here that it is not possible to simultaneously constrain the predictors to track the benchmark PPPs as well as the national movements in GDP deflators. One has to choose either one or none of these restrictions when generating panels of extrapolated PPPs. Our recommended approach is to simply use unconstrained equations of our model and thereby not imposing either of the restrictions described above.

2.2.3 Kalman Filter predictions as "weighted averages" of benchmark year only extrapolations

Following Rao, Rambaldi and Doran (2010b), suppose there are M+1 benchmark years. If regression based predictions are used to extrapolate PPPs to non-participating countries in benchmark years and then

[7] Technical details and equations for the Kalman Filter and Smoother are provided in Appendix A.6 and Appendix B of Rao, Rambaldi and Doran (2010b).
[8] This result follows from the work of Doran (1992).
[9] The proof of this property is provided in Appendix B of Rao, Rambaldi and Doran (2010b).

use the implicit price deflators to extrapolate from one year to the next, then it is possible to construct a panel of extrapolated PPPs for each of the benchmark years. In this case, an obviously intuitive approach is to make use of an average of these M+1 panels of PPPs. An important property of the State-Space approach is that the Kalman Filter predictions can be shown to be a weighted average of the M+1 panels of PPPs where the weights are determined by the diagonal elements of the Kalman Gain matrices. The weights can be interpreted as reflecting the reliability of the *j*-th benchmark. Proof of this important property is provided in Appendix B of Rao, Rambaldi and Doran (2010b).

2.2.4 Invariance of the Predicted PPPs to the Choice of the Reference Country

In the description of the model, we used the United States as the reference country. However, the use of US as the reference country is only for illustrative purposes. The relative purchasing powers of currencies of countries should, in principle, be invariant to the choice of the reference country. It can be shown that the model and the state-space approach described above satisfy this important invariance property. The proof of this property is quite involved and it is presented in Appendix A of Rao, Rambaldi and Rao (2010b).

2.3 Empirical Results

The current application of the model covers 141 countries and the years 1970 to 2005. Detailed descriptions of the data used can be found in Rao, Rambaldi and Doran (2010a). The PPP data used in the estimation of the regression model, based on an unbalanced panel of PPPs, covers the benchmarks 1970, 1973, 1975, 1980, 1980, 1985, 1990, 1993, 1996, 1999, 2002 and 2005. Several features of the PPP data are noteworthy. The 1975 benchmark covered 34 countries. The 1980, 1985 and the recent 2005 benchmarks represent a truly global comparisons with PPPs computed using data for all the participating countries. The benchmark 1996 was a global comparison but it may be considered weaker and less reliable than the earlier benchmarks as well as the most recent 2005 benchmark. The intervening benchmarks from 1990 cover only the OECD and EU countries. All the benchmarks prior to 1990 made use of the Geary-Khamis method of aggregation but since then the GiniEKS (GEKS) system has been the main aggregation procedure. In the current empirical analysis no adjustments have been made to the PPPs from different benchmarks.

The variables used in the regression model (5) for national price levels can be classified under two categories. As the regression used here is a panel data regression, a number of dummy variables designed to capture country-specific episodes that may influence the exchange rates or PPPs or both and to capture fixed effects were introduced. The second set of variables used are structural and drawn from the works of Kravis and Lipsey (1983), Clague (1988), Ahmad (1996), Bergstrand (1996) and Heston, Summers and Aten (2006). A complete list is available in Rao, Rambaldi and Doran (2010a).

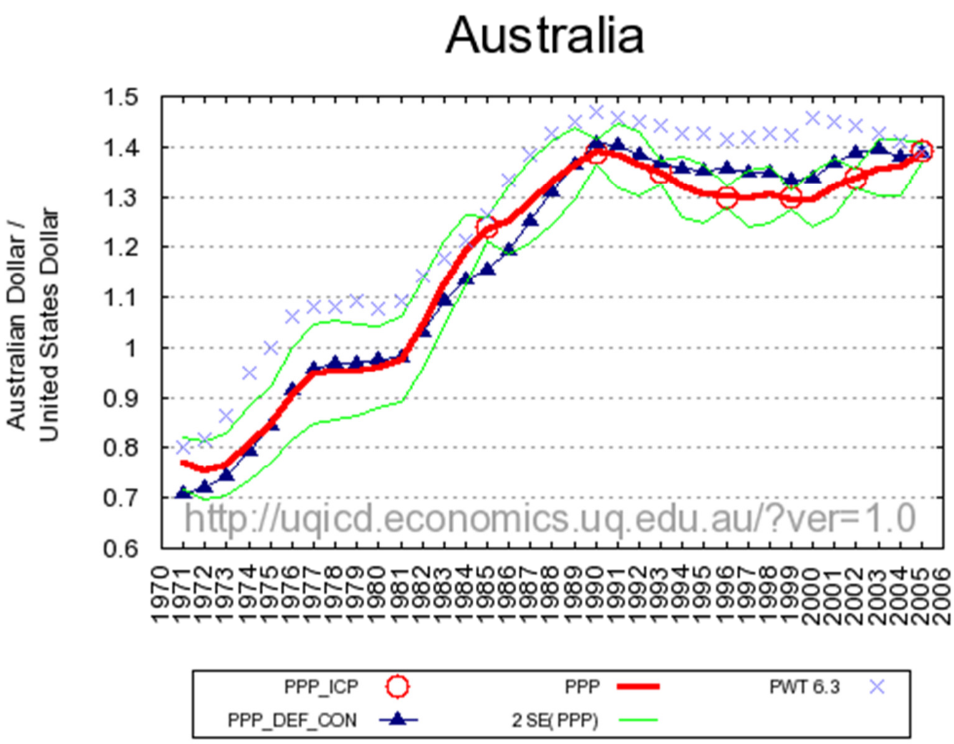
The spatial autocorrelation, which is a special feature of our approach, is introduced through the spatial-weights matrix. The spatial weights are computed using a measure of socio-economic distance constructed by extracting a common factor (using principal components analysis). Measures of *trade closeness*, *geographical proximity* and *cultural and colonial closeness* dummies were used in constructing the elements of the spatial autocorrelation matrix.[10] We find the estimate of the spatial correlation parameter to be 0.59 and statistically significant indicating the presence of strong positive spatial autocorrelation in the price level regression model.

2.3.1 Dissemination of PPP Predictions through UQICD website

As the predicted panel of PPPs covers 141 countries and a 35-year period from 1971 to 2005 along with the associated standard errors, it was decided that these PPPs will be made available through a dedicated website. As a part of the requirements of funding from the Australian Research Council, the website UQICD, the *University of Queensland International Comparison Data*, was established late in 2010 and was made publicly available only in the month of April, 2011. The URL for the website is: https://uqicd.economics.uq.edu.au/ . The website provides interactive tools to choose the countries, years and variables the user wishes to download data.

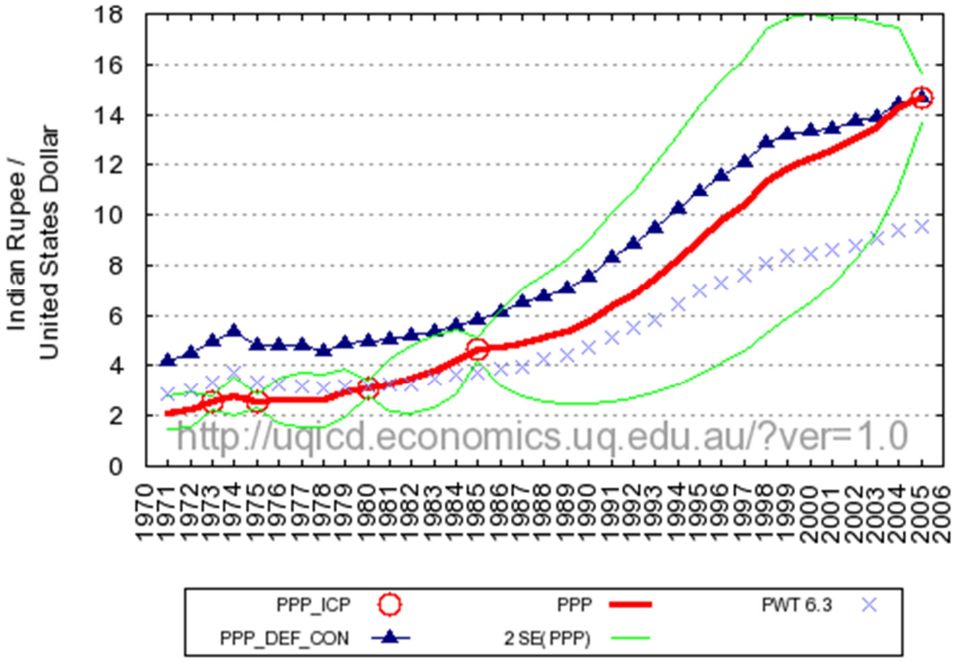
Consistent with the general econometric approach described here, extrapolated PPPs are available under two alternative scenarios. The first, which is our preferred option, provides extrapolations without imposing any prior restrictions with respect to tracking either the benchmarks or the implicit GDP deflators. The second series, however, is a PPP series which is constrained to track only the movements in the implicit GDP deflator.

A special feature of the website is the availability of comparative data in the form of easily interpretable charts. We present charts for three selected countries, viz., Australia, China and India.

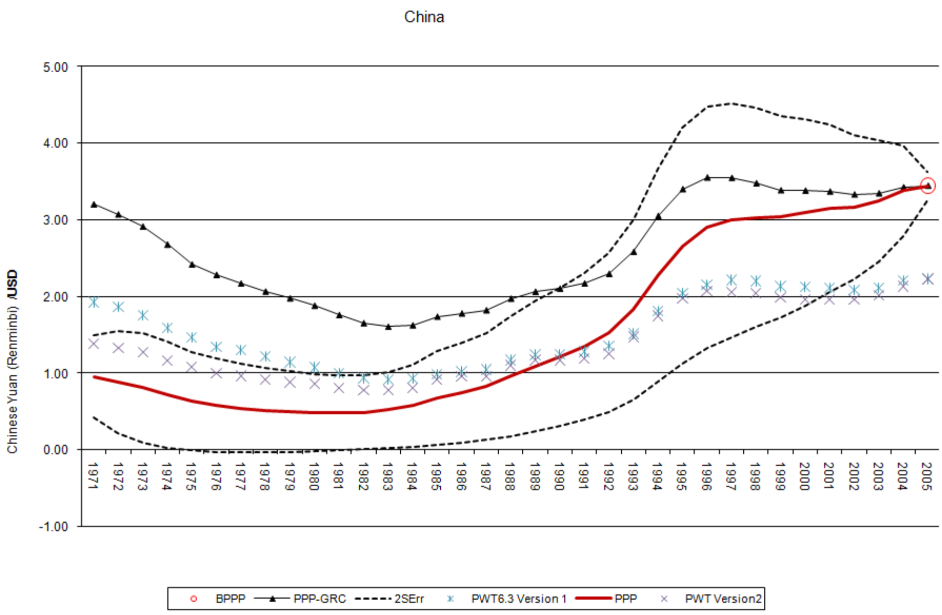


[10] Details of this approach can be found in Rambaldi, Rao and Ganegodage (2010).

India



China



These figures show the results for Australia, India and China. Australia is a typically developed country with fairly reliable benchmarks. The series marked in "red" is our preferred series where no constraints are imposed. These series are generated making use of all the data available including the 2005 benchmark. The PWT 6.3 series do not make use of the 2005 benchmark[11]. The series in "blue" is generated after imposing the movements in the GDP deflator.

Once the desired PPP series is chosen from the alternatives available, it is possible to compile real GDP aggregates at current prices by converting aggregates in national currency units into a common currency unit. For example if PPP_{it} is the PPP for country i in period t using, say, the US as the reference country, the real GDP at current prices, denoted by $RGDP_{it}$ is given by

$$RGDP_{it} = \frac{GDP_{it}}{PPP_{it}} \tag{9}$$

The RGDP series are also available from UQICD for 141 countries covered by the database and for the periods 1971 to 2005.

3. Panels of Real Incomes at Constant Prices[12]

3.1 Basic notation and definitions

In this sub-section we briefly discuss various income measures that are central to the theme of the paper. As the work focuses on the aggregate, gross domestic product (GDP), we let GDP_{jt} represent GDP in country j in period t expressed in local or national currency units. These GDP aggregate measures are not comparable across countries or over time as they are influenced by prices in the respective countries and time periods.

Let XR_{jt} and PPP_{jt} respectively denote the exchange rate and the purchasing power parity of the currency of country j which is equivalent to one unit of currency of a reference or numeraire country.[13] The *nominal and real GDP* of country j in period t, respectively, denoted as *NGDP* and *RGDP* are defined as:

$$NGDP_{jt} = \frac{GDP_{jt}}{XR_{jt}} \text{ and } RGDP_{jt} = \frac{GDP_{jt}}{PPP_{jt}}$$

The *NGDP* adjusts for differences in currency units. In contrast, *RGDP* adjusts for differences in currency units as well as purchasing powers of currencies based on differences in price levels observed in different countries. We note a few features of the real GDP series.

1. $RGDP_{jt}$ is comparable and additive across countries at a given period t. It is possible to compute regional totals for the period t.

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[11] For a more appropriate comparison, it is necessary to compare the series generated using our approach but without using the 2005 benchmark with the series in the PWT. Some graphs highlighting the performance of the new approach and its performance are included in the introduction page of UQICD website. Similar graphs for other countries are available from the authors. It is generally found that the econometric approach suggested here seems to generate reasonable approximations to the 2005 benchmark compared to PWT 6.3.

[12] The material presented in this section can best be described as "work in progress" but included in this paper in the spirit of the Workshop.

[13] We drop the subscript for the reference country to keep the notation simple.

2. $RGDP_{jt}$ is not comparable to $RGDP_{ks}$ for $s \neq t$ and any j and k. Thus $RGDP_{jt}$ may be termed *real GDP* series at *current (period t) prices.* However, it may be difficult to identify a set of prices which are used as reference prices in deriving the real GDP series.[14]
3. $RGDP_{jt}$ and PPP_{jt} are typical outputs of the ICP for a given benchmark year.
4. $RGDP_{jt}$ is obtained by deflating the GDP by a *suitable price deflator*, here it is PPP_{jt}.

By (1) – (4), we refer to PPP_{jt} and $RGDP_{jt}$ series for periods t = 1,2,3,…, T and j=1,…M, as a panels of PPPs and real incomes at *current or period t prices* to emphasize the fact that these PPPs and real GDP aggregates are not comparable over time. The problem of construction of these series at *current prices* has been satisfactorily addressed by the PWT or by the econometric approach proposed in Rao, Rambaldi and Rao (2010a, 2010b), and for the purpose of this paper we will denote by $P\hat{P}P_{it}$ represent the predictions of PPP_{it} constructed from either of these approaches.[15]

Now let $PPP_{jt}^{k\tau}$ represent the PPP for the currency of country j in period t with reference country k and reference period τ. Then, <u>the real GDP expressed at constant τ year prices with reference country k</u> is given by:

$$CRGDP_{jt}^{k\tau} = \frac{GDP_{jt}^{k\tau}}{PPP_{jt}^{k\tau}}$$

Here GDP in period t is adjusted for price movements over time (from the reference or base year, τ*)* and across space to adjust for price level differences between country j and the reference country, k. *CRGDP* by construction can be summed over countries as well as time periods.[16]

Given these definitions and the underlying notation, the main problem is one of constructing panels of PPPs and real incomes at *constant* prices. In this paper we examine a few alternative ways of generating PPPs and real incomes *at constant prices.*

3.2 A Heuristic Approach

Let $CRGDP_{it}^{\tau}$ represents real GDP of country i in period t expressed in constant period τ dollars (or constant period τ dollars). Then we have by definition

$$CRGDP_{it}^{\tau} = R\hat{G}DP_{i\tau} \times \frac{CGDP_{it}}{CGDP_{i\tau}} \tag{10}$$

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[14] See Feenstra, Ma and Rao (2009) for a definition of real income comparisons at a set of reference prices and for examples where deflated series could be interpreted as real income comparisons at some reference prices. For example, the GK based real GDP figures could be considered as real income comparisons obtained at GK *international prices* along with a *Leontief* utility function and real series obtained by using the Tornqvist index as the deflator corresponding to real income comparisons based on translog cost function.

[15] Any panel of PPPs at current prices can be used as a starting point.

[16] These series are similar to the GDP series at constant prices produced by national statistical offices except that the focus in such cases is on a single country.

where $RG\hat{D}P_{i\tau} = \dfrac{GDP_{i\tau}}{P\hat{P}P_{i\tau}}$.

It is possible to compute $CRGDP_{it}^{\tau}$ starting from a different reference years, say, s*. Then we have

$$CRGDP_{(i,t)}^{\tau,s^*} = \frac{GDP_{i,s^*}}{P\hat{P}P_{i,s^*}} \times \frac{CGDP_{i,t}}{CGDP_{i,s^*}} \times \frac{USDef_{\tau}}{USDef_{s^*}} \forall s^* \tag{11}$$

Equation (11) starts with real GDP for country *i* in period *s** expressed in current period *s** prices with US as the numeraire currency. The real GDP is then extrapolated to period *t* using the growth in *CGDP*, GDP at constant prices thus leading to a real GDP for country *i* in period *t* but expressed in period *s** prices and in US dollars. The last part of the expression in (11) makes an adjustment for price movements from period *s** to period *t* in the US.

It is easy to see that for a given τ expression in (11) can be calculated for *s** = 1,2,…,T and these expressions will all be numerically different. Hence a possible measure of the CRGDP in period τ dollars is given by

$$CRGDP_{(i,t)}^{\tau} = \prod_{s^*=1}^{T} [CRGDP_{(i,t)}^{\tau,s^*}]^{1/T} \tag{12}$$

The measure in (12) provides a measure of real GDP at constant period τ dollars for country *i* in period *t*. However, the main problem with this approach is that it is not invariant to the choice of the numeraire or reference country. It may be possible to modify (12) and construct series based on different reference countries and then possibly average them.

3.3 A State-Space approach to the Problem

In this section we try to extend the state-space approach developed in Section 2 by observing that the price level (*PPP/ER*) can also be expressed as a ratio of real GDP to nominal GDP. We have

$$R_{it} = \frac{PPP_{it}}{ER_{it}} = \frac{PPP_{it}}{ER_{it}} \times \frac{GDP_{it}}{GDP_{it}} = \frac{NGDP_{it}}{RGDP_{it}} \tag{13}$$

where *NGDP* represents nominal GDP, GDP in national currency units converted using exchange rates. In contrast *RGDP* is the real GDP (at current prices) is GDP converted using PPP's in period *t*. Equation (13) can be expressed in logarithms as:

$$r_{it} = \ln(R_{it}) = \ln(NGDP_{it}) - \ln(RGDP_{it}) \tag{14}$$

which can be written as:

$$rgdp_{it} = ngdp_{it} - r_{it} \tag{15}$$

where lower-case letters represent, respectively, the logarithms. Further, for a fixed τ, we let $crgdp_{it}^{\tau}$ denote the real GDP at constant period τ prices. This can be obtained by adjusting *rgdp* for movements in prices of the reference country. Thus

$$crgdp_{it}^{\tau} = rgdp_{it} + c_t \tag{16}$$

where

$c_t = ln(\frac{GDPDef_{1,\tau}}{GDPDef_{1,t}})$ is an adjustment factor for movements in the prices of the reference country denoted here by 1. By definition, $crgdp_{1\tau}^{\tau} = rgdp_{1\tau}$.

Further, the growth in constant price real GDP can be expressed as:

$$crgdp_{it}^{\tau} = crgdp_{it-1}^{\tau} + g_{it} \tag{17}$$

where $g_{it} = ln(\frac{CGDP_{i,t}^{\tau}}{CGDP_{i,t-1}^{\tau}})$ is the observed growth rate in GDP at constant τ year prices in country *i*.

Using the extrapolated PPPs, real GDP at current prices and their standard errors obtained from the application of the state-space approach described in Section 2, we can express equations (16) and (17) in a state-space form with observation equations written using estimated PPPs from Section 2 and the transition equation in (17) based on observed growth rates in different countries computed using constant price GDP series. The covariance structures can be derived using the corresponding structures in Section 2 and the Kalman Filter and Smoother can be used in deriving predicted $crgdp_{i\tau}^{\tau}$**.**

3.2.1 The State Space Model

Observation Equation

The observation equation maps the state vectors of unobservables $\mathbf{crgdp}_t^{\tau}$ to the vector of observations denoted by $\mathbf{y}_t$.

$$\mathbf{y}_t = \mathbf{Z}_t \mathbf{crgdp}_t^{\tau} + \xi_t \tag{18}$$

where $\mathbf{y}_t$ is an ($N \times 1$) vector of observations of $\mathbf{crgdp}_t^{\tau}$

The observed values are generated using:

$y_{it} = rgdp_{it} + c_t$

$$rgdp_{it} = ln(\frac{NGDP_{it}}{P\hat{P}P_{it}}) = ln(NGDP_{it}) - ln(P\hat{P}P_{it}) \tag{19}$$

where,

$P\hat{P}P_{it}$ is the estimated PPP_{it} in current prices from RRD.

$crgdp_t^{\tau}$ is the unobserved state vector.

$\mathbf{Z}_t$ is an identity matrix of size N. Then the vector of observations takes the form

$$y_t = \begin{bmatrix} crgdp_{1t}^{\tau} \\ crgdp_{2t}^{\tau} \\ . \\ . \\ crgdp_{Nt}^{\tau} \end{bmatrix}$$

where,

$crgdp_{it}^{\tau} = rgdp_{it} + c_t$ that is,

$$crgdp_t^{\tau} = ln(\frac{NGDP_{it}}{P\hat{P}P_{it}}) + ln(\frac{GDP\ Def_{1,\tau}}{GDP\ Def_{1,t}})$$

The error ξ_t in equation (18) has the following properties:

$$\xi_t \sim (0, \sigma_{\xi}^2 H_t) \tag{20}$$

where

σ_{ξ}^2 is a constant of proportionality

Let us look at how $\mathbf{H}_t$ is determined.

$$Var(crgdp^{\tau}{}_t) = Var(rgdp_t) + Var(c_t) = \sigma_H^2 H_t$$

Therefore, there are two more sources of error, the first is the error in the estimate of PPP_{it} and the second is the error in national accounts. For the purpose of the current study, $\mathbf{H}_t$ will be specified as the sum of a diagonal matrix containing the estimated variances of $P\hat{P}P_{it}$ and a non-diagonal matrix, $\mathbf{V}_{t'}$, which is inversely proportional to the level of development of each country and anchored in the reference country:

$$V_{t'} = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_{1t}^2 \iota\iota' + diag(\sigma_{2t}^2, \sigma_{3t}^2, \ldots, \sigma_{Nt}^2) \end{bmatrix}$$

where,

ι is a column of N-1 ones

$$\sigma_{it}^2 = \frac{1}{ERGDP_{it}}$$

$ERGDP_{it}$ is a per capita income in the currency of the reference country but adjusted using exchange rates. Exchange rates are well known to accentuate the division between developed and developing countries' income.

This form of $\mathbf{V}_{t'}$ was derived using the RRD approach and it is a sufficient form to preserve the variance of the $P\hat{P}P_{it}$ to the choice of the reference country

$$H_t = \sigma_H^2 \begin{bmatrix} Var(\hat{p}_{1t}^{RRD}) & 0 & 0 \\ 0 & . & 0 \\ 0 & 0 & Var(\hat{p}_{Nt}^{RRD}) \end{bmatrix} + V_{t'} \quad (21)$$

Transition Equation

The transition equation is derived from (number?) and an additional random term to account for measurement error in the computation of growth rates in national accounts. The transition equation is given by

$$crgdp_{it}^{\tau} = crgdp_{i,t-1}^{\tau} + g_{it} + \eta_t \quad (22)$$

where $g_{it} = ln(\frac{CGDP_{i,t}}{CGDP_{i,t-1}})$

and η_t is a vector of random disturbances centred at zero with covariance $\mathbf{Q}_t$. $\eta_t \sim (0, \sigma_\eta^2 Q_t)$

with $Q_t = \sigma_\eta^2 V_{t'}$

σ_η^2 is a constant of proportionality

Finally, the model contains the following constraint for the numeraire country and reference period"

$$crgdp_{1\tau}^{\tau} \equiv ngdp_{1\tau} \quad (23)$$

That is, in the base year τ, the observation of $crgdp_{1\tau}^{\tau}$ has variance with a value of zero. Thus, the first row and column of H_τ are zero vectors.

3.2.2 Estimation

Now that the model is specified in state space form, the constants of proportionality σ_ξ^2 and σ_η^2 can be estimated using maximum likelihood methods. Given starting values of σ_ξ^2 and σ_η^2, a Newton-Raphson iterative procedure is employed to maximise the likelihood function. The Kalman filter equations are built into the computation of the likelihood function. Given estimates of σ_ξ^2 and σ_η^2, the Kalman filter and smoother will compute smoothed values, $\mathbf{crgdp}_t^{\tau}$ and its covariance matrix, $\mathbf{P}_t^{crgdp}$. Assuming $crgdp_{it}^{\tau}$ terms are log-normally distributed, the standard errors of $crgdp_{it}^{\tau}$ are given by:

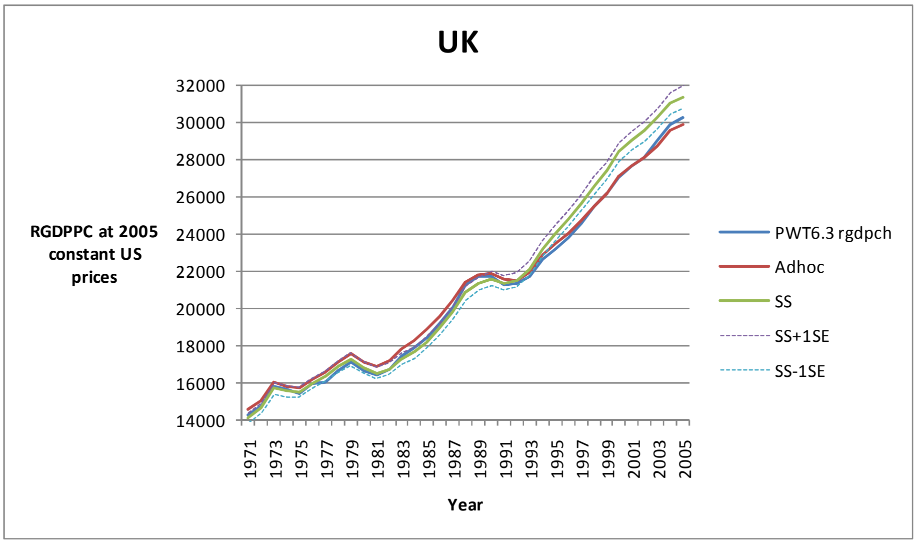
$$SE_{it} = \sqrt{exp(2(crgdp_{it}^{\tau})) \times exp(P_{it}^{crgdp}) \times exp(P_{it}^{crgdp} - 1)}$$

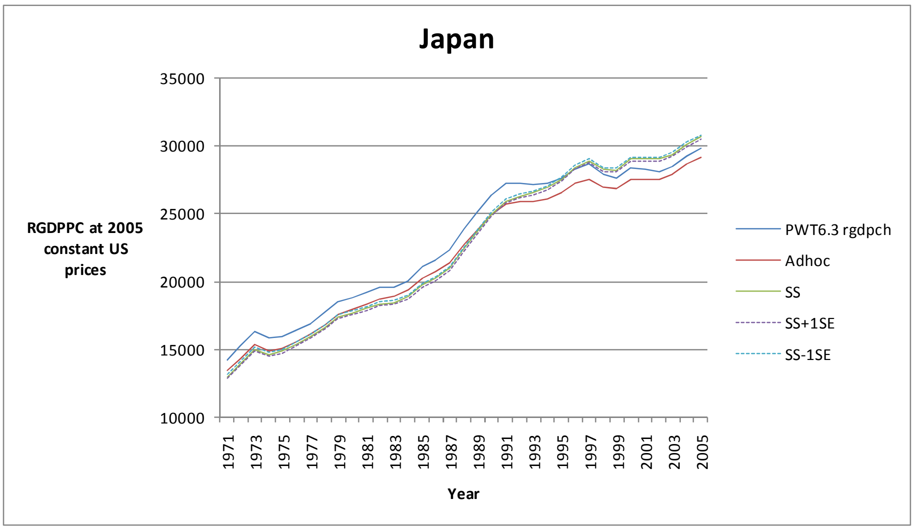
where,

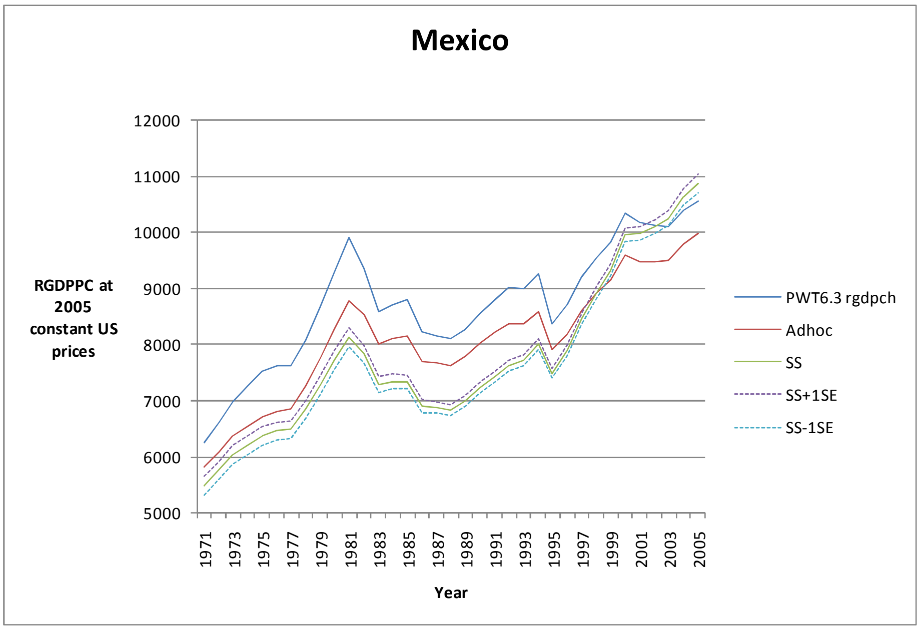
P_{it}^{crgdp} is the ith diagonal element of $\mathbf{P}_t^{crgdp}$.

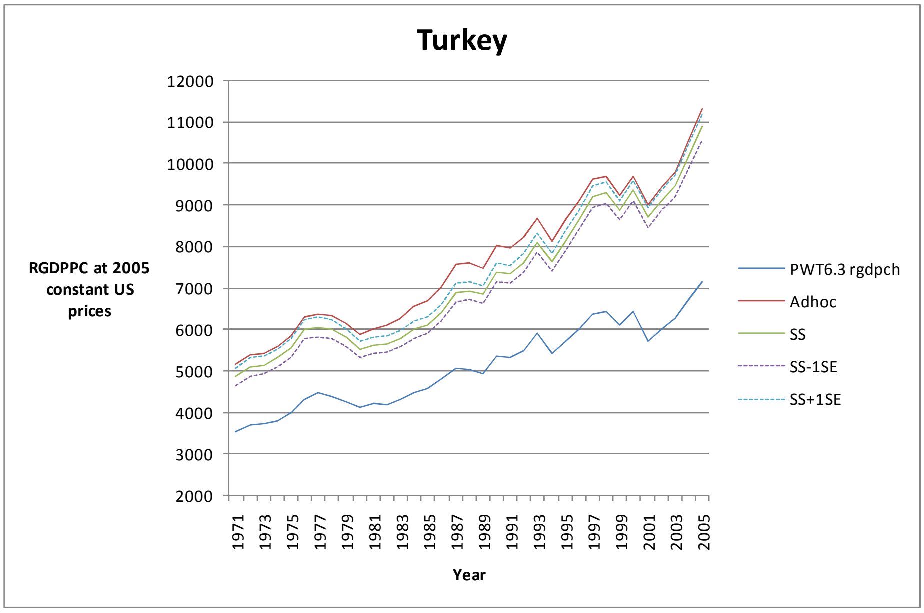
3.4 Empirical Results using OECD Data

For purposes of illustrating the heuristic and the state-space formulations used in deriving constant price real GDP expressed in the prices of a specific year, say τ, we make use of OECD data covering 27 countries and the period 1970-2005. The empirical application makes use of PPPs derived using methods discussed in Section 2 as an input into the calculations. The real GDP per capita derived using the heuristic/ad hoc method as well as the state-space model are presented for a selected set of countries. These are the UK, Japan, Mexico and Turkey.









The above figures clearly show that the heuristic and state-space models are feasible and can be used as a starting point for the generation of panels of real gdp series at constant prices. As the state-space approach makes use of an econometric model, standard errors of the predicted constant price real GDP can be obtained. In the case of the United Kingdom, all the series appear to be close until the year 1990 and then these series appear to diverge. The State-Space approach seems to give higher estimates of real GDP per capita at constant 2005 prices. The ad hoc approach of ours and PWT series are reasonably close. In the case of Japan, PWT series is above the other two series until 1995 and the ad hoc approach resulting in lower real GDP per capita at constant prices towards the end of the study period. In the case of Mexico, PWT is well above the other series with the ad hoc above the S-S series. The case of Turkey is interesting.

This is a country that has experienced high levels of inflation. In this case, the PWT series is well below the ad hoc and S-S series.

Though the levels of real GDP at constant prices appear to be different, the underlying growth patterns appear to be very similar. A close examination of the underlying data series and movements in prices is needed if one has to provide an explanation of the results.

These methods in their current formulation are not invariant to the choice of the numeraire currency. The state-space model is likely to be invariant to the choice of reference country but the formulation does not guarantee invariance to the choice of the base year for the constant price comparisons. Both the ad hoc and state-space approaches need further examination and refinement.

3.4 A Constrained GEKS Approach to Consistent Panels of PPPs and Real Incomes

In this section we pursue a totally different strategy and present a new approach and method of compiling PPPs with time-space dimensions. In this section we assume that PPP matrices (for comparisons at current prices) are available for each of the periods. In this case we assume that a procedure similar to RRD is already implemented and thus a panel of $P\hat{P}P_{it}$ i=1,…,M and t=1,…,T is available to start the proposed procedure.

In view of the space-time nature of the approach, we introduce further notation to what has been introduced in Section 2. Let the time periods be indexed by t = 1,2,…,T and countries be indexed by j = 1,2,…,M. Let PPP_{jk}^{ts} denote the PPP for country k in period s expressed relative to the reference country j and reference period t. Let Π represent a (TM x TM) matrix of PPPs over space and time. Then we can write Π as

$$\Pi = \begin{bmatrix} \Pi^{11} & \Pi^{12} & \ldots & \Pi^{1T} \\ \Pi^{21} & \Pi^{22} & \ldots & \Pi^{2T} \\ \ldots & \ldots & \ldots & \ldots \\ \Pi^{T1} & \Pi^{T2} & \ldots & \Pi^{TT} \end{bmatrix} \tag{24}$$

where Π^{ts} represents a (M x M) matrix showing PPPs for countries period s with countries in period t used as reference countries.

Elements of block-diagonal matrices

The matrix in equation (24) involves two types of information. The first refers to the block-diagonal matrices, Π^{tt} for t = 1,2,…,T. For example if t is the year 2005, then this matrix provides PPPs for all pairs of countries in the benchmark year. Thus PPPs in this block diagonal matrices represent PPPs at current prices (see Section 2 for a description of these concepts). We assume that the block diagonal matrices satisfy transitivity property. Transitivity of Π^{tt} implies the existence of a vector of constants, say $\pi^t = \left[\pi_1^t, \pi_2^t, ..., \pi_M^t\right]$ such that

$$PPP_{jk}^{tt} = \frac{\pi_k^t}{\pi_j^t} \tag{25}$$

The source of information for these block diagonal matrices is the ICP for different benchmark years and studies like RRD (2010) or the PWT which provide extrapolations of PPPs from the benchmark years to non-benchmark years. Without loss of generality we can assume that all the Π^{tt} matrices satisfy transitivity and, therefore, can be expressed in form similar to (25).

Elements of off-diagonal matrices

Elements of the off-diagonal matrices are not directly observed nor are available from any of the standard extrapolation studies. These matrices refer to PPP for a country *k* in given period s relative to a reference country *j* in the reference period *t*. We propose the following procedure to fill these elements. Let us for example consider

PPP_{jk}^{12}. This is PPP for country *k* in period 2 relative to country *j* in period 1. We can derive this comparison either using a comparison between *j* and *k* in period 1 or in period 2. We can update the period 1 comparison, PPP_{jk}^{11} using the implicit deflator d_k^{12} which represents movements in prices of country *k* from period 1 to 2. In this case, we have

$$PPP_{jk}^{12} = PPP_{jk}^{11} \cdot d_k^{12} \tag{26}$$

Alternatively, we could start with comparisons in period 2 and adjust PPP_{jk}^{22} backwards using d_j^{21} representing the implicit price deflator in country *j* measuring change from period 2 to period 1. This in turn gives and alternative to (26) in the form:

$$PPP_{jk}^{12} = PPP_{jk}^{22} \cdot d_k^{21} \tag{27}$$

As both of these are equally satisfactory, we make use of the geometric mean of (26) and (27) to measure PPP_{jk}^{12}.

$$PPP_{jk}^{12} = \left[\left(PPP_{jk}^{11} \cdot d_k^{12}\right)\left(PPP_{jk}^{22} \cdot d_j^{21}\right)\right]^{1/2} \tag{28}$$

Substituting (25) into (28), we can express the general element in the off-diagonal matrices in logarithmic form as:

$$\ln PPP_{jk}^{ts} = \frac{1}{2}\left[\left(\ln \pi_k^s - \ln \pi_j^s\right) + (\ln \pi_k^t - \ln \pi_j^t) + (\ln d_k^{ts} + \ln d_j^{st})\right] \tag{29}$$

Using the form in (28) we can fill all the off-diagonal blocks thus completing the matrix Π. However, Π is not transitive. To solve this we propose to use the standard GEKS approach with a slight modification.

GEKS Methodology and transitivity

The GEKS methodology involves the minimisation of sum of squared logarithmic differences between observed PPPs and the PPPs solved out of the system. Let Π^* be the solution of the GEKS method. Then the typical elements of Π^*, PPP^{*ts}_{jk} are obtained by minimising

$$\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{ts} - \ln PPP^{*ts}_{jk}\right]^2 \tag{30}$$

Subjecting to the transitivity of PPP^{*ts}_{jk}. In implementing GEKS we reparametrise the objective function by noting that the matrix Π^* is transitive if and only if there exists a vector π^* of order (TMx1) with a typical element, π^{*s}_{k} associated country k and period s such that

$$\Pi^{*ts}_{jk} = \frac{\pi^{*s}_{k}}{\pi^{*t}_{j}} \text{ for all } j,k \text{ and } t,s \tag{31}$$

Substituting (31) into (30) yields the GEKS objective function in terms of the new parameters:

$$\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{ts} - \ln \pi^{*s}_{k} + \ln \pi^{*t}_{j}\right]^2 \tag{32}$$

Minimisation of (32) yields the standard EKS solution to the problem. In the process we get a transitive matrix of PPPs which are time-space consistent. In this paper we improve this process further by improving additional restrictions on the solutions to ensure consistency of the time-space PPPs from (32) and the observed PPPs for each of the time periods, a form of *fixity*.

GEKS with fixity condition

The main problem with a straightforward minimisation of (32) is that comparisons between countries at a given period of time obtained from GEKS will not be equal to the PPP's matrix in the block diagonal of matrix Π in equation (24). Suppose we have international price comparisons in the form of PPPs for pairs of countries for a given year, say 2005. These comparisons are essentially price comparisons at the prices observed in 2005. When we minimise (32), the resulting comparisons between countries for the year 2005 will not be the same as those observed for 2005 as the new comparisons are affected by comparisons for all pairs of countries for all periods in the exercise. This basically means that price and real income comparisons for 2005 will differ at current 2005 prices and constant 2005 prices. So we implement a refined GEKS by imposing the condition that the price (and hence real income) comparisons for a given year are the same at the current and constant prices. This can be achieved by minimising (32)

$$\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{ts} - \ln \pi^{*s}_{k} + \ln \pi^{*t}_{j}\right]^2$$

subject to additional restrictions:

$$\pi^{*s}_{k} = \delta^{*s}\,\pi^{s}_{k} \text{ for all } k \text{ and } s. \tag{33}$$

If we incorporate restrictions (33) into (32), the GEKS with *fixity* requirement simplifies to one of minimising

$$\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{ts}-\delta^{s}-\ln\pi_{k}^{s}+\delta^{t}+\ln\pi_{j}^{t}\right]^{2} \tag{34}$$

with respect to $\left\{\delta^{1},\delta^{2},...,\delta^{T}\right\}$ *where* $\delta^{s}=\ln\delta*^{s}$.

We note that if $\left\{\delta^{1},\delta^{2},...,\delta^{T}\right\}$ is a solution to the problem, $\left\{c\delta^{1},c\delta^{2},...,c\delta^{T}\right\}$ for any c>0 is also a solution to the problem. Hence we minimise (34) after imposing an identifying restriction. In the discussion below we impose the restriction $\delta^{1}=0$. This means that all the comparisons are anchored on the reference period 1. The final solution can then be considered as price and real income comparisons at *constant year 1 prices*.

The first order conditions for optimisation after imposing $\delta^{1}=0$ yield the following system of (T-1) linear equations:

$$\begin{bmatrix} 1 & -\frac{1}{T-1} & \cdots & -\frac{1}{T-1} \\ -\frac{1}{T-1} & 1 & \cdots & -\frac{1}{T-1} \\ \cdots & \cdots & \cdots & \cdots \\ -\frac{1}{T-1} & -\frac{1}{T-1} & \cdots & 1 \end{bmatrix}\begin{bmatrix}\delta^{2}\\ \delta^{3}\\ \cdots \\ \delta^{T}\end{bmatrix}=\begin{bmatrix}\frac{1}{(T-1)M(M-1)}\sum_{s=1(\neq 2)}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{2s}-\ln\pi_{k}^{s}+\ln\pi_{j}^{2}\right]\\ \cdots \\ \cdots \\ \frac{1}{(T-1)M(M-1)}\sum_{s=1(\neq T)}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{Ts}-\ln\pi_{k}^{s}+\ln\pi_{j}^{T}\right]\end{bmatrix}$$

This leads to the following solution for the unknown constants.

$$\begin{bmatrix}\delta^{2}\\ \delta^{3}\\ \cdots \\ \delta^{T}\end{bmatrix}=\left(\frac{T-1}{T}\right)\begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & \cdots & 2 \end{bmatrix}\begin{bmatrix}\frac{1}{(T-1)M(M-1)}\sum_{s=1(\neq 2)}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{2s}-\ln\pi_{k}^{s}+\ln\pi_{j}^{2}\right]\\ \cdots \\ \cdots \\ \frac{1}{(T-1)M(M-1)}\sum_{s=1(\neq T)}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{Ts}-\ln\pi_{k}^{s}+\ln\pi_{j}^{T}\right]\end{bmatrix} \tag{35}$$

Now we can derive expressions for each of the elements of $\left\{\delta^{2},...,\delta^{T}\right\}$. We note further that typical elements involved in the summation on the RHS of equation (35) involve terms like:

$$\sum_{s=1(\neq 2)}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{2s}-\ln\pi_{k}^{s}+\ln\pi_{j}^{2}\right] \tag{36}$$

which can be further simplified by noting the procedure used in filling the elements of the off-diagonal blocks of matrices as described in equation (29). We have from (29)

$$\ln PPP_{jk}^{ts} = \frac{1}{2}\left[\left(\ln \pi_k^s - \ln \pi_j^s\right) + (\ln \pi_k^t - \ln \pi_j^t) + (\ln d_k^{ts} + \ln d_j^{st})\right]$$

Inserting this expression in (36) allows us to derive simple closed form solutions for the elements of the vector $\left\{\delta^2,\ldots,\delta^T\right\}$. This work is still in progress, and the conjecture is that the solution for the adjustment factors will be essentially functions of only the deflators, d_j^{st} for different values of *j* and for all pairs of time periods *s* and *t*.

Consistent panel of PPPs at constant prices

Based on the solution for the equations in (35), the consistent panel of space-time PPPs constructed using the vector π^* with a typical element π^{*t}_j for *j*=1,2,…,M and *t* =1,2,…,T can be computed as:

$$PPP^{*ts}_{jk} = \frac{\Pi^{*s}_k}{\Pi^{*t}_j} = \frac{\exp(\pi^{*s}_k)}{\exp(\pi^{*t}_j)} = \frac{\exp(\pi_k^s + \delta^s)}{\exp(\pi_j^t + \delta^t)} = \frac{\exp(\pi_k^s)}{\exp(\pi_j^t)} \cdot \frac{\exp(\delta^s)}{\exp(\delta^t)}$$

(37)

$$= \frac{PPP_k^{ss} \cdot \delta^{*s}}{PPP_j^{tt} \cdot \delta^{*t}}$$

The main property of this panel of PPPs is that it satisfies fixity for each of the time periods.

4 Conclusions

The paper provides an overview of the status of research on the development of an econometric approach to the construction of panels of PPPs, real incomes at current and constant prices. While the approach is fully developed in the context of PPPs and real incomes at current prices, there is need for further research into the formulation of the state-space model for the construction of panels of constant price real income series. In particular, it is important to examine the invariance of the results to the choice of the reference country and the reference period. Further examination of the empirical results from the ad hoc/heuristic and the state-space approaches to constant price real income comparisons is necessary.

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