# On Measuring Regional or Global Growth and Inflation 

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#### Abstract

In a paper contributed to the Ottawa Group, Rao and Balk (2013) presented an overview of concepts used in the field of international comparison, such as Nominal GDP, Real GDP, and Price Level Index. The present paper continues by providing a simple, symmetric formula for the calculation of regional or global growth and inflation. What distinguishes our proposal from currently used methods (at international organisations such as Eurostat, OECD, and World Bank) is its top-down approach ensuring consistency between value, quantity, and price developments. The role played by exchange rates and purchasing power parities also becomes clear. An example, based on data recently released by the International Comparison Program, illustrates our proposal.


Keywords: International comparison, world growth, world inflation, exchange rate, purchasing power parity, index number theory.

## JEL classification:

## 1 Introduction

Global inflation is a term that is used in the popular press and also by various international organisations. Eurostat documents, for instance, refer to inflation in the EU region and similar references are made in various OECD documents. However, it is difficult to find formal definitions of global inflation even though it is commonly computed as a weighted average of inflation rates observed in the member countries of these organisations, with weights reflecting the relative sizes of these economies.

The notion of global inflation is also used by researchers. For example, Cicarelli and Mojon (2010) use weighted and unweighted averages of quarterly year-on-year national Consumer Price

Index numbers as measures of global (OECD) inflation, which in turn are used in explaining the comovement of national inflation.

Though written before the economic crisis, Ward's (2001) paper is still relevant as an overview of the conceptual issues concerning global inflation and its relationship with international price levels and purchasing power parities. Specifically Ward emphasized the measurement of global growth and inflation as being complementary targets; his paper is also interesting as a brief inventory of then existing approaches.

## 2 Notation and definitions

International economic comparisons of countries (or regions) are conceptually based on considering each country as an aggregate, consolidated production unit. Using the KLEMS-Y framework, the accounting relation of each country for each time period (conventionally assumed to be a year) is given by

$$
\begin{equation*}
C_{K}+C_{L}+M_{E M S}+\Pi=R \tag{1}
\end{equation*}
$$

where $C_{K}$ denotes capital input cost, $C_{L}$ denotes labour input cost, $M_{E M S}$ denotes the cost of imported intermediate commodities (energy, materials, and services), $R$ denotes the revenue obtained from all the goods and services produced, and $\Pi$ is a remainder term which may or may not be equal to 0, dependent on the way capital input cost has been calculated (see Balk 2010 and Jorgenson and Schreyer 2013 for explanation). It is good to note here that by intermediate commodities are understood all those commodities that need further processing before becoming available for final demand. As Kohli and Natal (2014) observe, also "almost all so-called "finished" products must transit through the domestic production sector and go through a number of changes - such as unloading, transporting, storing, assembling, testing, cleaning, financing, insuring, marketing, wholesaling and retailing - before reaching final demand." Put otherwise, imported intermediate commodities comprise all those commodities to which value is added through the domestic production process.

There are, however, imports that don't need domestic value added to them, such as imported services. Let the import cost of those commodities be denoted by $M_{F}$, and let total import cost be defined as $M \equiv M_{E M S}+M_{F}$.

The fundamental supply-demand equality, firmly entrenched in the National Accounts, is given by

$$
\begin{equation*}
M_{F}+R=E+I+G+X, \tag{2}
\end{equation*}
$$

where, respectively, $E$ is the value of private household consumption, $I$ is the value of investment, $G$ is the value of government consumption, and $X$ is the value of exports. The sum of the first three terms, $E+I+G$ is called domestic absorption.

Using the definition of total import cost $M$, equation (2) can be rewritten as

$$
\begin{equation*}
M+R-M_{E M S}=E+I+G+X \tag{3}
\end{equation*}
$$

For each production unit, revenue minus intermediate input cost is called value added, which at the country level is called gross domestic product (GDP):

$$
\begin{equation*}
G D P \equiv R-M_{E M S} \tag{4}
\end{equation*}
$$

Since value added is additive, $G D P$ is the sum of value added of all the individual production units operating within the borders of the country, which is useful for a variety of analytical questions. Inserting the GDP definition (4) in the supply-demand equation (3) we get the familiar result

$$
\begin{equation*}
M+G D P=E+I+G+X \tag{5}
\end{equation*}
$$

Now suppose for a moment that there is a single world currency and that there are no importexport tax distortions, so that import prices paid are equal to export prices received, then total import cost $\sum M$ would be equal to total export revenue $\sum X$, where the sum is taken over all the countries. Then, consequently, total (or world) GDP would be equal to total (or world) domestic absorption,

$$
\begin{equation*}
\sum G D P=\sum(E+I+G) . \tag{6}
\end{equation*}
$$

Relative GDP, that is the ratio of a country's GDP to world GDP, could then be considered as an important indicator of a country's welfare.

Unfortunately, even if there were a single world currency, the comparison of GDPs between countries is hindered by the fact that for the same commodities different prices are charged in different countries. Thus, before comparing GDPs, any price effects must be removed.

Summarizing, the international comparison of GDPs (or their components) is plagued by currency differences and price differences.

## 3 Concepts for international comparison

Let countries be labeled $1, \ldots, M$. How do we compare the GDP of country $j, G D P_{j}$, expressed in its own currency, to the GDP of country $k, G D P_{k}$, also expressed in its own currency? The first instrument that comes to mind is a set of (market) exchange rates $X R_{j}(j=1, \ldots, M)$, where a certain arbitrary country has been selected as reference. Thus for this country the exchange rate equals 1 by definition. Exchange rates are transitive - that is, no arbitrage assumed -, so that $X R_{k} / X R_{j}$ is the exchange rate of country $k$ 's currency relative to country $j$ 's currency; that is, the number of $k$ currency units that can be obtained for $1 j$ currency unit. Of course, when countries use the same currency, as in case of the euro area, they have the same exchange rate.

In the international comparison literature, the term nominal GDP represents GDP after conversion by means of exchange rates. Thus nominal GDP of country $j$ is defined as

$$
\begin{equation*}
N G D P_{j} \equiv G D P_{j} / X R_{j}(j=1, \ldots, M) \tag{7}
\end{equation*}
$$

Since all these nominal GDP's are expressed in the same currency (namely, that of the reference country) they can be added. Thus, total nominal GDP is

$$
\begin{equation*}
N G D P \equiv \sum_{j=1}^{M} N G D P_{j}=\sum_{j=1}^{M} \frac{G D P_{j}}{X R_{j}} \tag{8}
\end{equation*}
$$

Notice that the magnitude of (total) nominal GDP depends on the reference country selected for the exchange rates. However, as one easily checks, the share of country $j$ in total nominal GDP, $N G D P_{j} / N G D P(j=1, \ldots, M)$ does not depend on a reference country.

The second instrument that can be used to make country-specific GDP magnitudes comparable is a set of purchasing power parities $P P P_{j}(j=1, \ldots, M)$. Methods for computing PPPs, given
prices and quantities of all the countries involved, are surveyed by Balk (2008), (2009). It is important to notice that each $P P P_{j}$ is a function of all the underlying prices and quantities of all the $M$ countries; moreover the PPPs are determined up to a positive scalar, and they are transitive.

In general, the PPP of country $j$ represents the number of currency $j$ units required to purchase a basket of goods and services for which one unit of an actual or artificial reference country currency is required. For instance, if the PPP of Indian rupee is 2.50 relative to the Hong Kong dollar, it means that what can be purchased with one dollar in Hong Kong requires 2.50 rupees in India.

Countries can have the same currency, such as the countries of the euro area, yet the purchasing power of this currency in the different countries does not need to be equal. Thus PPPs are like spatial price indices. Unlike spatial price indices, PPPs carry a dimension: currency $j$ units per reference currency unit.

PPPs can be used to convert country-specific GDPs into comparable constructs, basically in the same way as exchange rates were employed. Thus, real GDP of country $j$ is defined as

$$
\begin{equation*}
R G D P_{j} \equiv G D P_{j} / P P P_{j}(j=1, \ldots, M) \tag{9}
\end{equation*}
$$

Real GDP is comparable over countries and can thus be added. Total real GDP is

$$
\begin{equation*}
R G D P \equiv \sum_{j=1}^{M} R G D P_{j}=\sum_{j=1}^{M} \frac{G D P_{j}}{P P P_{j}} \tag{10}
\end{equation*}
$$

Notice that the magnitude of (total) real GDP is determined up to a positive scalar. However, as one easily checks, the share of country $j$ in total real GDP, $R G D P_{j} / R G D P(j=1, \ldots, M)$ does not depend on a reference country. Real GDP per capita, often used as a measure of welfare, is also determined up to a positive scalar.

We note here that the PPPs are compiled from price data collected by countries participating in an international comparison project along with National Accounts weights for aggregating those data; see Rao (2013) for details on the 2011 round of the International Comparison Program (ICP). The fact that such PPPs refer to a particular year, a so-called benchmark year, implies that (total) real GDP magnitudes also refer to a particular year and therefore are not comparable over time. We return to this issue in the next section.

We now have two sets of instruments, exchange rates and purchasing power parities. Recall that the exchange rates are based on a certain reference country and that the PPPs are determined up to a positive scalar. Let the PPPs be rescaled so that they are based on the same reference country as the exchange rates. Then the price level index of country $j$ is defined as

$$
\begin{equation*}
P L I_{j} \equiv P P P_{j} / X R_{j}(j=1, \ldots, M) \tag{11}
\end{equation*}
$$

The name comes from the fact that a PLI is seen as a measure of the price level of a country relative to the level at which its currency can be converted at the exchange rate. For example, consider the case of Australia versus the USA. At some date the exchange rate was 0.97 AUD per 1 USD. At the same time a BigMac costed in these countries 2.75 AUD and 2.25 USD respectively. Then the BigMac-based PPP for Australia relative to the USA was $2.75 / 2.25=1.22$. The PLI was then $1.22 / 0.97=1.26$.

Using definitions (7) and (9) it appears that

$$
\begin{equation*}
P L I_{j}=N G D P_{j} / R G D P_{j}(j=1, \ldots, M) \tag{12}
\end{equation*}
$$

that is, a price level index is nominal GDP divided by real GDP. This provides another interpretation of the concept. Empirically it appears that over countries the price level index is positively correlated with nominal or real GDP per capita. See Inklaar and Timmer (2014) for a recent study of this phenomenon.

Since PPPs and exchange rates are transitive, the PLIs are also transitive. The PLI of the reference country is by definition equal to 1 . Moving to another reference country leads to different PLIs. Since both PPPs and exchange rates are determined up to a positive scalar, the same holds for the PLIs.

A convenient normalisation is to adjust the set of PPPs by a common positive scalar $\mu$ such that total real GDP, based on the adjusted PPPs, is equal to total nominal GDP, based on the given exchange rates:

$$
\begin{equation*}
\sum_{j=1}^{M} \frac{G D P_{j}}{P P P_{j} / \mu}=\sum_{j=1}^{M} \frac{G D P_{j}}{X R_{j}} . \tag{13}
\end{equation*}
$$

Rewriting expression (13), using the PLI definition of expression (11), the nominal GDP definition of expression (7), and the real GDP definition of expression (9), leads to

$$
\begin{equation*}
1=\frac{\sum_{j=1}^{M} \frac{P L I_{j}}{\mu} R G D P_{j}}{\sum_{j=1}^{M} R G D P_{j}}=\frac{\sum_{j=1}^{M} N G D P_{j}}{\sum_{j=1}^{M} N G D P_{j}\left(\frac{P L I_{j}}{\mu}\right)^{-1}} ; \tag{14}
\end{equation*}
$$

that is, the real-GDP-weighted arithmetic mean and the nominal-GDP-weighted harmonic mean of adjusted PLIs are both equal to 1 . Notice that the weights are adjustment-invariant.

The foregoing, and in particular expressions (8), (10), and (13), represents current Eurostat practice in compiling National Accounts for the EU and Euro regions (without consolidation of flows between member states). PPPs are calculated according to the GEKS method.

## 4 Measurement of global inflation and growth

In this section we explore systematically the concepts of global inflation and growth and their connection with exchange rates and PPPs as discussed in the previous section. We begin with the notion of inflation and growth at the national level.

The introduction of the temporal dimension means that we need a superscript denoting time periods (years). Thus, let $G D P_{j}^{s}$ and $G D P_{j}^{t}$ represent GDP of country $j$ in periods $s$ and $t$ respectively (where without loss of generality it can be assumed that $s$ precedes $t$ ). Even though both aggregates are expressed in the currency unit of country $j$, a direct comparison is considered less useful since the effects of price and quantity change between periods $s$ and $t$ are intertwined. Welfare change is usually defined as the quantity part of nominal GDP change.

To measure this, National Accounts expresses GDP "at constant prices" together with its implicit price deflator ( $=$ nominal GDP divided by constant-price GDP). Notice that for any country there is some reference year for which the implicit price deflator exhibits the value 1. Using these data each country's GDP ratio, for any pair of years, can be decomposed as the product of a price index ( $=$ ratio of deflators) and a quantity index ( $=$ ratio of constant-price GDPs),

$$
\begin{equation*}
\frac{G D P_{j}^{t}}{G D P_{j}^{s}}=P_{G D P}^{j}(t, s) Q_{G D P}^{j}(t, s)(j=1, \ldots, M) \tag{15}
\end{equation*}
$$

The price indices measure inflation and the quantity indices measure growth at the country level. The functional forms may or may not be the same for the various countries. Whether the indices are direct or chained is immaterial to the argument in this paper. All we ask of the two indices is that together they exhaust any temporal GDP ratio.

### 4.1 Using exchange rates

Basically we are mimicking this construction at the global level, thereby using the two comparison concepts discussed in the previous section. Thus we start with total nominal GDP as defined by expression (8), repeated here with a time superscript as

$$
\begin{equation*}
N G D P^{t} \equiv \sum_{j=1}^{M} N G D P_{j}^{t}=\sum_{j=1}^{M} \frac{G D P_{j}^{t}}{X R_{j}^{t}} \tag{16}
\end{equation*}
$$

How can we now decompose a ratio $N G D P^{t} / N G D P^{s}$ in price and quantity components? Here is the first attempt:

$$
\begin{align*}
& \frac{N G D P^{t}}{N G D P^{s}}  \tag{17}\\
& \quad=\frac{\sum_{j=1}^{M} P_{G D P}^{j}(t, s)\left(X R_{j}^{s} / X R_{j}^{t}\right) Q_{G D P}^{j}(t, s) N G D P_{j}^{s}}{\sum_{j=1}^{M} N G D P_{j}^{s}} \\
& \quad=\frac{\sum_{j=1}^{M} N G D P_{j}^{s} Q_{G D P}^{j}(t, s)}{N G D P^{s}} \times \frac{N G D P^{t}}{\sum_{j=1}^{M} N G D P_{j}^{t}\left(P_{G D P}^{j}(t, s) \frac{X R_{j}^{s}}{X R_{j}^{t}}\right)^{-1}} .
\end{align*}
$$

The first equality is obtained by using the definition of $N G D P$ as well as equation (15). This equality makes clear that in the movement from $N G D P^{s}$ to $N G D P^{t}$ three components are involved: price change, exchange rate change, and quantity change. The second equality is obtained by applying the familiar Laspeyres-Paasche decomposition to two components: the combination of price and exchange rate change, and quantity change. Thus, the first factor at the right-hand side is a Laspeyres quantity index; that is, a weighted arithmetic mean of country-specific quantity indices where the weights are period $s$ nominal GDP shares. The second factor is a Paasche index of price-over-exchange-rate; it is a weighted harmonic mean, but now the weights are period $t$ nominal GDP shares.

We can also apply the Paasche-Laspeyres decomposition. Then we obtain

$$
\begin{align*}
& \frac{N G D P^{t}}{N G D P^{s}}  \tag{18}\\
& \quad=\frac{\sum_{j=1}^{M}\left(P_{G D P}^{j}(t, s) \frac{X R_{j}^{s}}{X R_{j}^{t}}\right) N G D P_{j}^{s}}{N G D P^{s}} \times \frac{N G D P^{t}}{\sum_{j=1}^{M} N G D P_{j}^{t}\left(1 / Q_{G D P}^{j}(t, s)\right)} .
\end{align*}
$$

The first factor at the right-hand side is now a Laspeyres index of price-over-exchange-rate. The second factor is a Paasche quantity index; that is, a harmonic mean of country-specific quantity indices where the weights are period $t$ nominal GDP shares.

These two decompositions are clearly asymmetric. A symmetric decomposition is obtained by taking geometric means of the two price and quantity indices in the previous equations, so that

$$
\begin{align*}
& \frac{N G D P^{t}}{N G D P^{s}} \\
& \quad=\left(\frac{\sum_{j=1}^{M}\left(P_{G D P}^{j}(t, s) \frac{X R_{j}^{s}}{X R_{j}^{t}}\right) N G D P_{j}^{s}}{N G D P^{s}} \frac{N G D P^{t}}{\sum_{j=1}^{M} N G D P_{j}^{t}\left(P_{G D P}^{j}(t, s) \frac{X R_{j}^{s}}{X R_{j}^{t}}\right)^{-1}}\right)^{1 / 2} \\
& \quad \times\left(\frac{\sum_{j=1}^{M} N G D P_{j}^{s} Q_{G D P}^{j}(t, s)}{N G D P^{s}} \frac{N G D P^{t}}{\sum_{j=1}^{M} N G D P_{j}^{t}\left(1 / Q_{G D P}^{j}(t, s)\right)}\right)^{1 / 2} \\
& \quad \equiv(P / X R)_{G D P}^{F}(t, s ; X R) \times Q_{G D P}^{F}(t, s ; X R) . \tag{19}
\end{align*}
$$

Thus we are having here a decomposition in Fisher price-over-exchange-rate and quantity indices. The mnemonic $X R$ as conditioning variable in the two functions expresses the fact that the weights used are nominal GDP shares. Basically expression (19) corresponds to the first approach of Diewert (2014). Notice that for $M=1$ expression (19) reduces to expression (15).

An alternative approach enables us to explicitly break up the total $N G D P$ ratio in three components. Using the logarithmic mean, it appears that

$$
\begin{equation*}
\frac{N G D P^{t}}{N G D P^{s}}=\exp \left\{\sum_{j=1}^{M} \Phi^{j} \ln \left(\frac{N G D P_{j}^{t}}{N G D P_{j}^{s}}\right)\right\} \tag{20}
\end{equation*}
$$

where the weights, adding up to 1 , are defined by

$$
\Phi^{j} \equiv \frac{L\left(\frac{N G D P_{j}^{t}}{N G D P^{t}}, \frac{N G D P_{j}^{s}}{N G D P^{s}}\right)}{\sum_{j=1}^{M} L\left(\frac{N G D P_{j}^{t}}{N G D P^{t}}, \frac{N G D P_{j}^{s}}{N G D P^{s}}\right)}(j=1, \ldots, M)
$$

and the function $L(.,$.$) is the logarithmic mean { }^{1}$ Expression (20) says that the total nominal GDP ratio is a weighted geometric mean of country-specific nominal GDP ratios, the weights being (normalized) logarithmic means of nominal GDP shares in the two periods compared. Of course, when the temporal distance between the periods $s$ and $t$ is large then expression (20) may be replaced by a product of consecutive period ratios (and direct indices by chained indices); but this is immaterial to the argument developed here.

Next equation (15) and the definition of $N G D P_{j}$ is used to obtain the three-factor decomposition

$$
\begin{align*}
& \frac{N G D P^{t}}{N G D P^{s}}=\exp \left\{\sum_{j=1}^{M} \Phi^{j} \ln P_{G D P}^{j}(t, s)\right\} \times \\
& \quad \exp \left\{\sum_{j=1}^{M} \Phi^{j} \ln \left(\frac{X R_{j}^{s}}{X R_{j}^{t}}\right)\right\} \times \exp \left\{\sum_{j=1}^{M} \Phi^{j} \ln Q_{G D P}^{j}(t, s)\right\} . \tag{21}
\end{align*}
$$

[^0]These are three-factor versions of the Sato-Vartia index (see Balk 2002/3). Notice that for $M=1$ expression (21) reduces to expression (15).

Combining the first two right-hand side terms, exchange-rate-based global inflation is defined as

$$
\begin{equation*}
(P / X R)_{G D P}^{S V}(t, s ; X R) \equiv \exp \left\{\sum_{j=1}^{M} \Phi^{j} \ln \left(P_{G D P}^{j}(t, s) \frac{X R_{j}^{s}}{X R_{j}^{t}}\right)\right\} \tag{22}
\end{equation*}
$$

and exchange-rate-based global growth (quantity change) as the remainder,

$$
\begin{equation*}
Q_{G D P}^{S V}(t, s ; X R) \equiv \exp \left\{\sum_{j=1}^{M} \Phi^{j} \ln Q_{G D P}^{j}(t, s)\right\} \tag{23}
\end{equation*}
$$

The pair of indices defined here corrresponds to the pair in expression (19), but the Sato-Vartia indices have a much simpler functional form than the Fisher indices. Moreover, the Sato-Vartia structure in expression (21) enables us to isolate the exchange rate component from the price component in a straightforward way.

One look at the various definitions makes clear that the quantity indices are invariant to the choice of the reference country for the exchange rates, since this choice does not influence the nominal GDP shares. The price-over-exchange-rate indices, however, are not, due to the occurrence of the exchange-rate component. For the case of $(P / X R)_{G D P}^{F}(t, s ; X R)$ the effect of this lack of invariance is convincingly demonstrated in Table 1 of Diewert (2014).

### 4.2 Using PPPs

Instead of total nominal GDP, the second approach considers total real GDP as defined by expression (10), repeated here with a time superscript as

$$
\begin{equation*}
R G D P^{t} \equiv \sum_{j=1}^{M} R G D P_{j}^{t}=\sum_{j=1}^{M} \frac{G D P_{j}^{t}}{P P P_{j}^{t}} \tag{24}
\end{equation*}
$$

Like before, a ratio $R G D P^{t} / R G D P^{s}$ can be symmetrically decomposed as a pair of Fisher indices,

$$
\begin{align*}
& \frac{R G D P^{t}}{R G D P^{s}} \\
& =\left(\frac{\sum_{j=1}^{M}\left(P_{G D P}^{j}(t, s) \frac{P P P_{j}^{s}}{P P P_{j}^{t}}\right) R G D P_{j}^{s}}{R G D P^{s}} \frac{R G D P^{t}}{\sum_{j=1}^{M} R G D P_{j}^{t}\left(P_{G D P}^{j}(t, s) \frac{P P P_{j}^{s}}{P P_{j}^{t}}\right)^{-1}}\right)^{1 / 2} \\
& \times\left(\frac{\sum_{j=1}^{M} R G D P_{j}^{s} Q_{G D P}^{j}(t, s)}{R G D P^{s}} \frac{R G D P^{t}}{\sum_{j=1}^{M} R G D P_{j}^{t}\left(1 / Q_{G D P}^{j}(t, s)\right)}\right)^{1 / 2} \\
& \equiv(P / P P P)_{G D P}^{F}(t, s ; P P P) \times Q_{G D P}^{F}(t, s ; P P P) . \tag{25}
\end{align*}
$$

The first index measures price-over-PPP change from period $s$ to period $t$, and the second index measures quantity change. In both cases the weights are real GDP shares, which is why the mnemonic PPP occurs as conditioning variable. The quantity index $Q_{G D P}^{F}(t, s ; P P P)$ corresponds
to the index advised by Diewert (2014) in his second approach. However, as global inflation index he suggested

$$
\begin{align*}
& P_{G D P}^{F}(t, s ; P P P) \equiv \\
& \qquad\left(\frac{\sum_{j=1}^{M} P_{G D P}^{j}(t, s) R G D P_{j}^{s}}{R G D P^{s}} \frac{R G D P^{t}}{\sum_{j=1}^{M} R G D P_{j}^{t}\left(P_{G D P}^{j}(t, s)\right)^{-1}}\right)^{1 / 2} . \tag{26}
\end{align*}
$$

The advantage of this pair of indices is that both are invariant to the choice of the reference country for the purchasing power parities. The disadvantage is that in general it will be the case that

$$
P_{G D P}^{F}(t, s ; P P P) \times Q_{G D P}^{F}(t, s ; P P P) \neq R G D P^{t} / R G D P^{s}
$$

Put otherwise, a part of the real GDP ratio is left unaccounted for.
Similar to expression (20) we have

$$
\begin{equation*}
\frac{R G D P^{t}}{R G D P^{s}}=\exp \left\{\sum_{j=1}^{M} \Psi^{j} \ln \left(\frac{R G D P_{j}^{t}}{R G D P_{j}^{s}}\right)\right\} \tag{27}
\end{equation*}
$$

where the weights, adding up to 1 , are defined by

$$
\Psi^{j} \equiv \frac{L\left(\frac{R G D P_{j}^{t}}{R G D P^{t}}, \frac{R G D P_{j}^{s}}{R G D P^{s}}\right)}{\sum_{j=1}^{M} L\left(\frac{R G D P_{j}^{t}}{R G D P^{t}}, \frac{R G P_{j}^{s}}{R G D P^{s}}\right)}(j=1, \ldots, M)
$$

Notice the subtle difference with the earlier expression (20). The total real GDP ratio is a weighted geometric mean of country-specific real GDP ratios, the weights being (normalized) logarithmic means of real GDP shares in the two periods compared.

We now use expression (15) and the definition of $R G D P_{j}$. This leads to the three-factor decomposition

$$
\begin{align*}
& \frac{R G D P^{t}}{R G D P^{s}}=\exp \left\{\sum_{j=1}^{M} \Psi^{j} \ln P_{G D P}^{j}(t, s)\right\} \times \\
& \quad \exp \left\{\sum_{j=1}^{M} \Psi^{j} \ln \left(\frac{P P P_{j}^{s}}{P P P_{j}^{t}}\right)\right\} \times \exp \left\{\sum_{j=1}^{M} \Psi^{j} \ln Q_{G D P}^{j}(t, s)\right\} . \tag{28}
\end{align*}
$$

Combining the first two right-hand side terms, PPP-based global inflation is defined by

$$
\begin{equation*}
(P / P P P)_{G D P}^{S V}(t, s ; P P P) \equiv \exp \left\{\sum_{j=1}^{M} \Psi^{j} \ln \left(P_{G D P}^{j}(t, s) \frac{P P P_{j}^{s}}{P P P_{j}^{t}}\right)\right\} \tag{29}
\end{equation*}
$$

and PPP-based global growth (quantity change) is defined as the remainder,

$$
\begin{equation*}
Q_{G D P}^{S V}(t, s ; P P P) \equiv \exp \left\{\sum_{j=1}^{M} \Psi^{j} \ln Q_{G D P}^{j}(t, s)\right\} \tag{30}
\end{equation*}
$$

This quantity index is invariant to the choice of the reference country for the PPPs, since only real GDP shares enter the formula. The price index $(P / P P P)_{G D P}^{S V}(t, s ; P P P)$ is not invariant, since the PPPs play an explicit role. But as yet empirical evidence as to the effect of this lack of invariance is not available.

### 4.3 Relations

It is important to notice that if the normalisation defined by expression (13) is imposed on the data then $N G D P^{t} / N G D P^{s}=R G D P^{t} / R G D P^{s}$. Then expressions (19), (21), (25) and (28) all provide decompositions of the same ratio.

It is interesting to relate expressions (29) to (22), and (30) to (23). Straightforward manipulation, using the price level index definition (11), yields the following expressions,

$$
\begin{gather*}
\frac{(P / P P P)_{G D P}^{S V}(t, s ; P P P)}{(P / X R)_{G D P}^{S V}(t, s ; X R)}=\exp \left\{\sum_{j=1}^{M}\left(\Psi^{j}-\Phi^{j}\right) \ln \left(P_{G D P}^{j}(t, s) \frac{X R_{j}^{s}}{X R_{j}^{t}}\right)\right\}  \tag{31}\\
\times \exp \left\{\sum_{j=1}^{M} \Psi^{j} \ln \left(\frac{P L I_{j}^{s}}{P L I_{j}^{t}}\right)\right\}
\end{gather*}
$$

and

$$
\begin{equation*}
\frac{Q_{G D P}^{S V}(t, s ; P P P)}{Q_{G D P}^{S V}(t, s ; X R)}=\exp \left\{\sum_{j=1}^{M}\left(\Psi^{j}-\Phi^{j}\right) \ln \left(Q_{G D P}^{j}(t, s)\right)\right\} . \tag{32}
\end{equation*}
$$

A full interpretation of these expressions has perhaps to wait for a next version of this paper. In any case, what we see is that the right-hand side of expression (31) consists of two terms. The first is a covariance, between real and nominal share differences and price-over-exchange-rate index numbers; the second term is the inverse of mean price level index change. Expression (32) is also a covariance, but now between real and nominal share differences and country-specific quantity change.

Unlike exchange rates, purchasing power parities are usually not available every year, but are compiled infrequently at so-called benchmark years. PPPs for non-benchmark years are then conveniently estimated by extrapolation. This has a peculiar consequence, as will be demonstrated now.

When the period $t$ PPPs are obtained by extrapolating the period $s$ PPPs, that is, when $P P P_{j}^{t}=P P P_{j}^{s} P_{G D P}^{j}(t, s) / P_{G D P}^{j^{\prime}}(t, s)$ where $j^{\prime}$ is the numeraire for the PPPs, then global inflation according to expression (29) reduces to

$$
\begin{equation*}
(P / P P P)_{G D P}^{S V}(t, s ; P P P) \equiv \exp \left\{\sum_{j=1}^{M} \Psi^{j} \ln P_{G D P}^{j^{\prime}}(t, s)\right\}=P_{G D P}^{j^{\prime}}(t, s), \tag{33}
\end{equation*}
$$

since $\sum_{j=1}^{M} \Psi^{j}=1$. Now there are $M$ choices for the numeraire $j^{\prime}$, so it makes sense to define mean global inflation as the unweighted geometric mean

$$
\begin{equation*}
\bar{P}_{G D P}(t, s) \equiv \prod_{j^{\prime}=1}^{M} P_{G D P}^{j^{\prime}}(t, s)^{1 / M} \tag{34}
\end{equation*}
$$

As one sees, the economic size of the countries does not play any role here. ${ }^{2}$

## 5 Aggregation

### 5.1 The contribution of (groups of) countries

In the previous section we considered the measurement of inflation and growth for an entire set of countries. Though interesting as such, one is usually also interested in the contribution of single countries or groups of countries to global inflation and growth. It is particularly here that we see the advantage of Sato-Vartia indices over Fisher indices. One example is sufficient to demonstrate this.

Consider the exchange-rate-based global quantity index as defined by expression (23). The logarithmic version reads

$$
\begin{equation*}
\ln Q_{G D P}^{S V}(t, s ; X R)=\sum_{j=1}^{M} \Phi^{j} \ln Q_{G D P}^{j}(t, s) \tag{35}
\end{equation*}
$$

Now recall that the logarithm of an index number (in the neighbourhood of 1) can be interpreted as a percentage. Then expression (35) says that the (additive) contribution of country $j$ to global growth is given by the percentage growth experienced by the country itself times its nominal GDP share $\Phi^{j}(j=1, \ldots, M)$.

Next, let the entire set of countries be split into, say, two disjunct subsets $A$ and $B$; that is, $A \cup B=\{1, \ldots, M\}$ and $A \cap B=\emptyset$. Then expression (35) can be decomposed as

$$
\begin{align*}
& \ln Q_{G D P}^{S V}(t, s ; X R) \\
& \quad=\sum_{j \in A} \Phi^{j} \ln Q_{G D P}^{j}(t, s)+\sum_{j \in B} \Phi^{j} \ln Q_{G D P}^{j}(t, s) \\
& \quad=\Phi^{A} \sum_{j \in A} \Phi^{j A} \ln Q_{G D P}^{j}(t, s)+\Phi^{B} \sum_{j \in B} \Phi^{j B} \ln Q_{G D P}^{j}(t, s), \tag{36}
\end{align*}
$$

where $\Phi^{A} \equiv \sum_{j \in A} \Phi^{j}, \Phi^{B} \equiv \sum_{j \in B} \Phi^{j}, \Phi^{j A} \equiv \Phi^{j} / \Phi^{A}(j \in A)$, and $\Phi^{j B} \equiv \Phi^{j} / \Phi^{B}(j \in B)$. Notice that the weights $\Phi^{j A}$ and $\Phi^{j B}$ add up to 1 .

Now expression (36) says that the (additive) contribution of country set $A$ to world growth is given by the mean percentage growth experienced by the set $A$ itself times its nominal GDP share $\Phi^{A}$. Notice, however, that the mean growth of $A, \sum_{j \in A} \Phi^{j A} \ln Q_{G D P}^{j}(t, s)$, is not equal to the logarithm of the Sato-Vartia quantity index of the set $A$, since Sato-Vartia indices are not consistent-in-aggregation (see Balk 2008, 108-113).

The difference is subtle and the effect therefore might not be great. By substituting the definition of $\Phi^{j}$ into the definition of $\Phi^{j A}$ we see that

$$
\Phi^{j A}=\frac{L\left(\frac{N G D P_{j}^{t}}{N G D P^{t}}, \frac{N G D P_{j}^{s}}{N G D P^{s}}\right)}{\sum_{j \in A} L\left(\frac{N G D P_{j}^{t}}{N G D P^{t}}, \frac{N G D P_{j}^{s}}{N G D P^{s}}\right)}(j \in A) .
$$

[^1]Recall that $N G D P^{\tau}=\sum_{j=1}^{M} N G D P_{j}^{\tau}(\tau=s, t)$. The Sato-Vartia weights for the elements of subset $A$, however, would be given by

$$
\tilde{\Phi}^{j A}=\frac{L\left(\frac{N G D P_{j}^{t}}{N G D P_{A}^{t}}, \frac{N G D P_{j}^{s}}{N G D P_{A}^{s}}\right)}{\sum_{j \in A} L\left(\frac{N G D P_{j}^{t}}{N G D P_{A}^{t}}, \frac{N G D P_{j}^{s}}{N G D P_{A}^{s}}\right)}(j \in A),
$$

where $N G D P_{A}^{\tau}=\sum_{j \in A} N G D P_{j}^{\tau}(\tau=s, t)$. The logarithm of the Sato-Vartia quantity index of the set $A$ is then given by $\sum_{j \in A} \tilde{\Phi}^{j A} \ln Q_{G D P}^{j}(t, s)$.

Similar definitions of course hold for country set $B$. The effect of the inconsistency-in-aggregation is then given by observing that

$$
\begin{equation*}
\ln Q_{G D P}^{S V}(t, s ; X R) \neq \Phi^{A} \sum_{j \in A} \tilde{\Phi}^{j A} \ln Q_{G D P}^{j}(t, s)+\Phi^{B} \sum_{j \in B} \tilde{\Phi}^{j B} \ln Q_{G D P}^{j}(t, s) \tag{37}
\end{equation*}
$$

### 5.2 Nominal GDP components

As we have seen in Section 2, GDP consists of five components, one of which is negative. Rewriting expression (5), adding time and country labels, we obtain

$$
\begin{equation*}
G D P_{j}^{t}=F_{j}^{t}-M_{j}^{t}(j=1, \ldots, M) \tag{38}
\end{equation*}
$$

where $F_{j}^{t} \equiv E_{j}^{t}+I_{j}^{t}+G_{j}^{t}+X_{j}^{t}$ is final demand. Then

$$
\begin{equation*}
N G D P_{j}^{t}=\left(F_{j}^{t}-M_{j}^{t}\right) / X R_{j}^{t}(j=1, \ldots, M) \tag{39}
\end{equation*}
$$

Now $F_{j}^{t}-M_{j}^{t}$ has the same structure as value added. Thus for splitting the ratio $N G D P_{j}^{t} / N G D P_{j}^{s}$ we follow the procedure of Balk (2010, Appendix B). By repeatedly using the logarithmic mean we obtain

$$
\begin{align*}
& \ln \frac{N G D P_{j}^{t}}{N G D P_{j}^{s}}=\ln \frac{X R_{j}^{s}}{X R_{j}^{t}}+\ln \frac{F_{j}^{t}-M_{j}^{t}}{F_{s}^{t}-M_{s}^{t}} \\
& \quad=\ln \frac{X R_{j}^{s}}{X R_{j}^{t}}+\frac{L\left(F_{j}^{t}, F_{j}^{s}\right) \ln \left(F_{j}^{t} / F_{j}^{s}\right)}{L\left(G D P_{j}^{t}, G D P_{j}^{s}\right)}-\frac{L\left(M_{j}^{t}, M_{j}^{s}\right) \ln \left(M_{j}^{t} / M_{j}^{s}\right)}{L\left(G D P_{j}^{t}, G D P_{j}^{s}\right)} \\
& \quad=\ln \frac{X R_{j}^{s}}{X R_{j}^{t}}+\theta_{F}^{j} \ln \left(F_{j}^{t} / F_{j}^{s}\right)-\theta_{M}^{j} \ln \left(M_{j}^{t} / M_{j}^{s}\right) \tag{40}
\end{align*}
$$

where $\theta_{F}^{j} \equiv L\left(F_{j}^{t}, F_{j}^{s}\right) / L\left(G D P_{j}^{t}, G D P_{j}^{s}\right)$ and $\theta_{M}^{j} \equiv L\left(M_{j}^{t}, M_{j}^{s}\right) / L\left(G D P_{j}^{t}, G D P_{j}^{s}\right)$ are approximately equal to the shares of final demand and imports, respectively, in GDP of country $j$. Notice that $\theta_{F}^{j}-\theta_{M}^{j} \neq 1$ since the logmean $L(., 1)$ is concave.

Next it is assumed that there exist price and quantity indices such that

$$
\begin{align*}
F_{j}^{t} / F_{j}^{s} & =P_{F}^{j}(t, s) Q_{F}^{j}(t, s)(j=1, \ldots, M)  \tag{41}\\
M_{j}^{t} / M_{j}^{s} & =P_{M}^{j}(t, s) Q_{M}^{j}(t, s)(j=1, \ldots, M) . \tag{42}
\end{align*}
$$

Substituting now expressions (41) and (42) into expression (40) and doing some rearrangement delivers

$$
\begin{equation*}
\ln \frac{N G D P_{j}^{t}}{N G D P_{j}^{s}}=\ln \left(\frac{X R_{j}^{s}}{X R_{j}^{t}} \frac{P_{F}^{j}(t, s)^{\theta_{F}^{j}}}{P_{M}^{j}(t, s)^{\theta_{M}^{j}}}\right)+\ln \left(\frac{Q_{F}^{j}(t, s)^{\theta_{F}^{j}}}{Q_{M}^{j}(t, s)^{\theta_{M}^{j}}}\right) . \tag{43}
\end{equation*}
$$

The almost final step is to substitute expression (43) into expression (20). The result is

$$
\begin{equation*}
\ln \frac{N G D P^{t}}{N G D P^{s}}=\sum_{j=1}^{M} \Phi^{j} \ln \left(\frac{X R_{j}^{s}}{X R_{j}^{t}} \frac{P_{F}^{j}(t, s)^{\theta_{F}^{j}}}{P_{M}^{j}(t, s)^{\theta_{M}^{j}}}\right)+\sum_{j=1}^{M} \Phi^{j} \ln \left(\frac{Q_{F}^{j}(t, s)^{\theta_{F}^{j}}}{Q_{M}^{j}(t, s)^{\theta_{M}^{j}}}\right) . \tag{44}
\end{equation*}
$$

The first term at the right-hand side of this equation corresponds to global inflation $(P / X R)_{G D P}^{S V}(t$, $s ; X R)$ as defined by expression (22), and the second term corresponds to global growth $Q_{G D P}^{S V}(t, s$; $X R)$ as defined by expression (23).

The final step consists in the realisation that the final demand price and quantity indices are functions of the price and quantity indices and values of the underlying components; that is,

$$
\begin{align*}
& P_{F}^{j}(t, s)=  \tag{45}\\
& \quad P\left(P_{E}^{j}(t, s), P_{I}^{j}(t, s), P_{G}^{j}(t, s), P_{X}^{j}(t, s), E_{j}^{t}, I_{j}^{t}, G_{j}^{t}, X_{j}^{t}, E_{j}^{s}, I_{j}^{s}, G_{j}^{s}, X_{j}^{s}\right) \\
& \quad(j=1, \ldots, M)
\end{align*}
$$

and

$$
\begin{align*}
& Q_{F}^{j}(t, s)=  \tag{46}\\
& \quad Q\left(Q_{E}^{j}(t, s), Q_{I}^{j}(t, s), Q_{G}^{j}(t, s), Q_{X}^{j}(t, s), E_{j}^{t}, I_{j}^{t}, G_{j}^{t}, X_{j}^{t}, E_{j}^{s}, I_{j}^{s}, G_{j}^{s}, X_{j}^{s}\right) \\
& \quad(j=1, \ldots, M)
\end{align*}
$$

where $P($.$) and Q($.$) are price and quantity indices respectively, and it is assumed that there exist$ price and quantity indices such that

$$
\begin{align*}
E_{j}^{t} / E_{j}^{s} & =P_{E}^{j}(t, s) Q_{E}^{j}(t, s)(j=1, \ldots, M)  \tag{47}\\
I_{j}^{t} / I_{j}^{s} & =P_{I}^{j}(t, s) Q_{I}^{j}(t, s)(j=1, \ldots, M)  \tag{48}\\
G_{j}^{t} / G_{j}^{s} & =P_{G}^{j}(t, s) Q_{G}^{j}(t, s)(j=1, \ldots, M)  \tag{49}\\
X_{j}^{t} / X_{j}^{s} & =P_{X}^{j}(t, s) Q_{X}^{j}(t, s)(j=1, \ldots, M) \tag{50}
\end{align*}
$$

If $P($.$) and Q($.$) have a suitable functional form then it is possible to decompose the right-hand$ side of equation (44) into terms corresponding to the five components of GDP. For example, let $P($.$) and Q($.$) be Sato-Vartia indices; that is,$

$$
\begin{align*}
& \ln P_{F}^{j}(t, s) \equiv \sum_{V=E, I, G, X} \varphi_{V}^{j} \ln P_{V}^{j}(t, s)(j=1, \ldots, M)  \tag{51}\\
& \ln Q_{F}^{j}(t, s) \equiv \sum_{V=E, I, G, X} \varphi_{V}^{j} \ln Q_{V}^{j}(t, s)(j=1, \ldots, M) \tag{52}
\end{align*}
$$

with

$$
\varphi_{V}^{j} \equiv \frac{L\left(\frac{V_{j}^{t}}{F j^{t}}, \frac{V_{j}^{s}}{F_{j}^{s}}\right)}{\sum_{V=E, I, G, X} L\left(\frac{V_{j}^{t}}{F j^{t}}, \frac{V_{j}^{s}}{F_{j}^{s}}\right)}(V=E, I, G, X)(j=1, \ldots, M) .
$$

Then equation (44) turns into

$$
\begin{align*}
& \ln \frac{N G D P^{t}}{N G D P^{s}}= \\
& \quad \sum_{j=1}^{M} \Phi^{j} \ln \left(X R_{j}^{s} / X R_{j}^{t}\right) \\
& \quad+\sum_{j=1}^{M} \sum_{V=E, I, G, X} \Phi^{j} \theta_{F}^{j} \varphi_{V}^{j} \ln P_{V}^{j}(t, s)-\sum_{j=1}^{M} \Phi^{j} \theta_{M}^{j} \ln P_{M}^{j}(t, s) \\
& \quad+\sum_{j=1}^{M} \sum_{V=E, I, G, X} \Phi^{j} \theta_{F}^{j} \varphi_{V}^{j} \ln Q_{V}^{j}(t, s)-\sum_{j=1}^{M} \Phi^{j} \theta_{M}^{j} \ln Q_{M}^{j}(t, s) . \tag{53}
\end{align*}
$$

As we see, for each GDP component there is a price index and a quantity index. In addition to these ten components, there is the contribution of the exchange rates. Recall that this part is numeraire-dependent.

### 5.3 Real GDP components

Unlike exchange rates, purchasing power parities depend on prices and quantities of all the underlying commodities. This implies that in principle each GDP component has its own set of PPPs. We assume that the reference country is the same for all these sets. Then real GDP is equal to

$$
\begin{gather*}
R G D P_{j}^{t}=\frac{G D P_{j}^{t}}{P P P_{j}^{t}}  \tag{54}\\
=\frac{E_{j}^{t}}{P P P_{E j}^{t}}+\frac{I_{j}^{t}}{P P P_{I j}^{t}}+\frac{G_{j}^{t}}{P P P_{G j}^{t}}+\frac{X_{j}^{t}}{P P P_{X j}^{t}}-\frac{M_{j}^{t}}{P P P_{M j}^{t}}(j=1, \ldots, M),
\end{gather*}
$$

where $P P P_{j}^{t}$ is the overall PPP and $P P P_{E j}^{t}, \ldots, P P P_{M j}^{t}$ are the component-specific PPPs. Equation (54) then states that the overall PPP is a (generalized) harmonic mean of the component PPPs.

Equation (54) can also be written as

$$
\begin{equation*}
R G D P_{j}^{t}=\frac{F_{j}^{t}}{P P P_{F j}^{t}}-\frac{M_{j}^{t}}{P P P_{M j}^{t}}(j=1, \ldots, M) \tag{55}
\end{equation*}
$$

where $P P P_{F j}^{t}$ is the final-demand-specific PPP defined by

$$
\begin{equation*}
\frac{F_{j}^{t}}{P P P_{F j}^{t}} \equiv \frac{E_{j}^{t}}{P P P_{E j}^{t}}+\frac{I_{j}^{t}}{P P P_{I j}^{t}}+\frac{G_{j}^{t}}{P P P_{G j}^{t}}+\frac{X_{j}^{t}}{P P P_{X j}^{t}}(j=1, \ldots, M) \tag{56}
\end{equation*}
$$

For splitting the ratio $R G D P_{j}^{t} / R G D P_{j}^{s}$ we basically follow the same procedure as in the previous subsection. The result is

$$
\begin{align*}
& \ln \frac{R G D P_{j}^{t}}{R G D P_{j}^{s}} \\
&=\frac{L\left(F_{j}^{t} / P P P_{F j}^{t}, F_{j}^{s} / P P P_{F j}^{s}\right) \ln \left(F_{j}^{t} P P P_{F j}^{s} / F_{j}^{s} P P P_{F j}^{t}\right)}{L\left(R G D P_{j}^{t}, R G D P_{j}^{s}\right)} \\
&-\frac{L\left(M_{j}^{t} / P P P_{M j}^{t}, M_{j}^{s} / P P P_{M j}^{s}\right) \ln \left(M_{j}^{t} P P P_{M j}^{s} / M_{j}^{s} P P P_{M j}^{t}\right)}{L\left(R G D P_{j}^{t}, R G D P_{j}^{s}\right)} \\
&=\vartheta_{F}^{j} \ln \left(F_{j}^{t} P P P_{F j}^{s} / F_{j}^{s} P P P_{F j}^{t}\right)-\vartheta_{M}^{j} \ln \left(M_{j}^{t} P P P_{M j}^{s} / M_{j}^{s} P P P_{M j}^{t}\right), \tag{57}
\end{align*}
$$

where $\vartheta_{F}^{j} \equiv L\left(F_{j}^{t} / P P P_{F j}^{t}, F_{j}^{s} / P P P_{F j}^{s}\right) / L\left(R G D P_{j}^{t}, R G D P_{j}^{s}\right)$, and $\vartheta_{M}^{j} \equiv L\left(M_{j}^{t} / P P P_{M j}^{t}, M_{j}^{s} / P P\right.$ $\left.P_{M j}^{s}\right) / L\left(R G D P_{j}^{t}, R G D P_{j}^{s}\right)$ are approximately equal to the shares of real final demand and real imports, respectively, in real GDP of country $j$. Notice that $\vartheta_{F}^{j}-\vartheta_{M}^{j} \neq 1$ since the logmean $L(., 1)$ is concave.

Substituting now expressions (41) and (42) into expression (57) and doing some rearrangement delivers

$$
\begin{equation*}
\ln \frac{R G D P_{j}^{t}}{R G D P_{j}^{s}}=\ln \left(\frac{\left(P_{F}^{j}(t, s) P P P_{F j}^{s} / P P P_{F j}^{t}\right)^{\vartheta_{F}^{j}}}{\left(P_{M}^{j}(t, s) P P P_{M j}^{s} / P P P_{M j}^{t}\right)^{\vartheta_{M}^{j}}}\right)+\ln \left(\frac{Q_{F}^{j}(t, s)^{\vartheta_{F}^{j}}}{Q_{M}^{j}(t, s)^{\vartheta_{M}^{j}}}\right) . \tag{58}
\end{equation*}
$$

Then we substitute this into expression (27), which results in

$$
\begin{align*}
\ln \frac{R G D P^{t}}{R G D P^{s}}= & \sum_{j=1}^{M} \Psi^{j} \ln \left(\frac{\left(P_{F}^{j}(t, s) P P P_{F j}^{s} / P P P_{F j}^{t}\right)^{\vartheta_{F}^{j}}}{\left(P_{M}^{j}(t, s) P P P_{M j}^{s} / P P P_{M j}^{t}\right)^{\vartheta_{M}^{j}}}\right)  \tag{59}\\
& +\sum_{j=1}^{M} \Psi^{j} \ln \left(\frac{Q_{F}^{j}(t, s)^{\vartheta_{F}^{j}}}{Q_{M}^{j}(t, s)^{\vartheta_{M}^{j}}}\right)
\end{align*}
$$

The first term at the right-hand side of this equation corresponds to global inflation $(P / P P P)_{G D P}^{S V}$ $(t, s ; X R)$ as defined by expression (29), and the second term corresponds to global growth $Q_{G D P}^{S V}(t, s$; $P P P)$ as defined by expression (30).

The final step again consists in substituting expressions (45) and (46) into expression (59). If $P($.$) and Q($.$) have a suitable functional form then it is possible to decompose the right-hand side$ of equation (59) into terms corresponding to the five components of GDP.

## 6 Estimates of regional and global growth and inflation

In this section we report estimates of regional and global price change and economic growth over the period 2005 to 2011. The period chosen is largely determined by the data available from the International Comparison Program (ICP) at the World Bank. ICP conducts international comparisons periodically, and the last two rounds of the ICP were in the benchmark years 2005 and 2011. The ICP is a worldwide statistical program to collect comparative price and national accounts and compile estimates of purchasing power parities (PPPs) of currencies and real expenditures for the whole range of final goods and services that comprise GDP including consumer goods and services, government services and capital goods (see http://icp.worldbank.org for extensive details). Results from the 2005 and 2011 ICP are available respectively from World Bank (2008)
and World Bank (2014). The methodology and the conceptual framework that underpins the ICP are described in Rao (2013).

The 2005 ICP covered 146 economies whereas the 2011 ICP had an increased coverage of 199 economies. In implementing the measures of regional and global inflation and economic growth proposed in this paper, we focus on 141 countries that are common to both rounds of ICP. ${ }^{3}$ As a result our world estimates refer to these 141 economies and the regional groupings used here coincide with those used in the ICP. The regions used are: Asia and the Pacific; Africa; Caribbean; CIS; Eurostat-OECD; Latin America; West Asia and the singleton countries Iran and Georgia. Egypt and Sudan participated in both Africa and the West Asian region but for the purpose of our computations we have included them in Africa. Similarly, the Russian Federation is included in the Eurostat-OECD region and not in the CIS region. Readers must exercise caution in interpreting results for the Asia-Pacific region as countries like Australia, Japan, South Korea and New Zealand are included in the Eurostat-OECD region (the list of the countries in the ICP regions is available from http://go.worldbank.org/PCQ6V9LWZ0). The basic data used to obtain the results are presented in the Appendix. The Caribbean and Pacific Islands did not participate in 2005 and thus we are unable to provide estimates for these regions.

Table 1 provides the regional and global inflation estimates using equations (19) and (22), which are exchange rate based. The PPP based counterparts are obtained using equations (25) and (29), respectively. The PPPs are normalised according to expression (13), so that the weighted mean price levels equal 1 for each region and the world. Recall that then $N G D P^{t} / N G D P^{s}=$ $R G D P^{t} / R G D P^{s}$. The regional and global growth estimates are obtained by dividing this ratio by the corresponding inflation estimate.

The computations show that the use of our proposed Sato-Vartia indices produce almost identical estimates to those obtained with the Fisher type indices. However, their advantage is that they are simpler to compute and enable the straightforward decompositions such as separating the exchange rate component from the price component of the movement in total nominal GDP between two time periods. For all regions but one (West Asia), inflation is estimated to be higher using exchange rates than purchasing power parities. This then leads to lower growth estimates using exchange rates than those obtained using purchasing power parities. The exchange rate based growth estimates for the aggregate ( 141 countries) is found to be close to $17 \%$ from 2005 to 2011, while the PPP based growth is estimated as $25 \%$. Using PPP based estimates, the fastest growing region was Asia and the Pacific ( $62 \%$ ), while the slowest growing region was EU-OECD (7\%). Using exchange rates the Asia-Pacific growth was $61 \%$ and the EU-OECD was $6.5 \%$.

Table 2 shows a further decomposition of global inflation into the portion due to the movement in domestic prices and that due to changes in exchange rates or purchasing power parities. Here we use the decompositions in equations (21) and (28) where three components are identified; namely, the change due to the movement in domestic prices, the change due to the movement in exchange rates or purchasing power parities, and the change due to global growth. The movement in domestic prices are weighted functions of the change in the domestic GDP deflator. The weights can be exchange rate based - as in equation (21) - or purchasing power parities based - as in equation (28). The results show that the domestic price changes are measured as higher when using PPP weights. The proportion of the change due to non-domestic factors are measured as higher when exchange rates are used.

We believe that the set of PPP based measures corresponds to what Ward (2001) envisaged,

[^2]Table 1: Regional and Global Growth and Inflation, 2005 to 2011

|  |  | Exchange Rate Based |  |  |  | PPP Based |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ICP Region | $\frac{N G D P_{2011}}{N G D P_{2005}}$ | Price <br> CHANGE <br> $\left(\right.$ Fisher $^{1}{ }^{1}$ | Growth <br> (Fisher) | Price <br> Change <br> $(\mathrm{SV})^{2}$ | Growth (SV) | Price <br> Change <br> (Fisher) $^{3}$ | Growth <br> (FiSher) | Price <br> Change $(\mathrm{SV})^{4}$ | Growth (SV) |
| Asia and the Pacific | 2.5297 | 1.5722 | 1.6090 | 1.5717 | 1.6095 | 1.5648 | 1.6166 | 1.5644 | 1.6170 |
| Africa | 2.1443 | 1.4286 | 1.5010 | 1.4285 | 1.5011 | 1.4018 | 1.5297 | 1.4016 | 1.5300 |
| CIS | 2.4622 | 1.9576 | 1.2578 | 1.9577 | 1.2577 | 1.9549 | 1.2595 | 1.9548 | 1.2596 |
| EuroStat-OECD | 1.2888 | 1.2098 | 1.0653 | 1.2098 | 1.0653 | 1.2024 | 1.0719 | 1.2024 | 1.0719 |
| Latin America | 2.5991 | 1.9725 | 1.3177 | 1.9727 | 1.3175 | 1.9609 | 1.3255 | 1.9611 | 1.3253 |
| Iran | 2.7520 | 2.1138 | 1.3019 | 2.1138 | 1.3019 | 2.1138 | 1.3019 | 2.1138 | 1.3019 |
| West Asia | 2.3175 | 1.5821 | 1.4648 | 1.5821 | 1.4648 | 1.6051 | 1.4438 | 1.6053 | 1.4436 |
| Georgia | 2.1408 | 1.6377 | 1.3072 | 1.6377 | 1.3072 | 1.6377 | 1.3072 | 1.6377 | 1.3072 |
| World | 1.5388 | 1.3196 | 1.1661 | 1.3194 | 1.1663 | 1.2358 | 1.2451 | 1.2358 | 1.2452 |

${ }^{1}$ Equation (19). ${ }^{2}$ Equation (22). ${ }^{3}$ Equation (25). ${ }^{4}$ Equation (29) .

Table 2: Components of Global Inflation

| ICP Region | Domestic Price Change $^{1}$ | Exchange Rate Change $^{1}$ | Domestic Price Change $^{2}$ | PPP Change $^{2}$ |
| :--- | :---: | :---: | :---: | :---: |
| Asia and the Pacific | 1.3945 | 1.1271 | 1.4354 | 1.0899 |
| Africa | 1.6089 | 0.8878 | 1.6257 | 0.8621 |
| CIS | 2.1326 | 0.9180 | 2.1478 | 0.9102 |
| EuroStat-OECD | 1.1104 | 1.0896 | 1.1244 | 1.0693 |
| Latin America | 1.6809 | 1.1736 | 1.6968 | 1.1557 |
| Iran | 2.5035 | 0.8444 | 2.5035 | 0.8444 |
| West Asia | 1.5661 | 1.0103 | 1.6155 | 0.9937 |
| Georgia | 1.5237 | 1.0748 | 1.5237 | 1.0748 |
|  |  |  |  | 1.3058 |
| World | 1.2148 | 1.0861 |  | 0.9464 |

${ }^{1}$ Equation (21). ${ }^{2}$ Equation (28).
whereby the Sato-Vartia indices possess the virtue of simple decomposability. The pair formed by the last column of Table 1 (Growth) and the next to last column of Table 2 (Domestic Price Change) are symmmetric, but don't exhaust the world nominal/real GDP development. The gap is closed by the last column of Table 2 (PPP Change).

Table 3 illustrates the decomposition discussed in Section 5.1. The logarithmic Sato-Vartia quantity index numbers, which can be interpreted as percentage changes, for the whole world as well as the various regions are given in columns C1. Exponentiating the numbers of the C1 columns produces the two columns labelled "Growth (SV)" in Table 1. The C2 columns then provide the decomposition of the world index numbers, in the bottom row, according to the right-hand side of equation (36). The right-hand side of expression (37) is given in columns C3. The bottom row is the sum of the group contributions. The difference with the bottom row of columns C 2 is the effect of the inconsistency-in-aggregation of the Sato-Vartia indices. For all practical purposes this effect appears to be negligible.

Table 3: An Illustration of the Inconsistency-in-Aggregation Effect

|  | C1 ${ }^{1}$ | $\mathrm{C} 1^{2}$ | $\mathrm{C} 2{ }^{1}$ | $\mathrm{C} 2{ }^{2}$ | C3 ${ }^{1}$ | C3 ${ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Asia and the Pacific | 0.4759 | 0.4806 | 0.0720 | 0.1260 | 0.0717 | 0.1258 |
| Africa | 0.4062 | 0.4252 | 0.0078 | 0.0129 | 0.0078 | 0.0128 |
| CIS | 0.2293 | 0.2308 | 0.0066 | 0.0106 | 0.0065 | 0.0106 |
| EuroStat-OECD | 0.0632 | 0.0694 | 0.0462 | 0.0389 | 0.0463 | 0.0390 |
| Latin America | 0.2758 | 0.2817 | 0.0122 | 0.0157 | 0.0122 | 0.0157 |
| Iran | 0.2638 | 0.2638 | 0.0015 | 0.0033 | 0.0015 | 0.0033 |
| West Asia | 0.3817 | 0.3672 | 0.0074 | 0.0118 | 0.0074 | 0.0117 |
| Georgia | 0.2679 | 0.2679 | 0.0000 | 0.0001 | 0.0000 | 0.0001 |
| World | 0.1538 | 0.2193 | 0.1538 | 0.2193 | 0.1535 | 0.2191 |
| $\begin{aligned} & C 1=\sum_{j \in A} \tilde{\Phi}^{j A} \ln Q_{G D P}^{j}(2011,2005) \text { with } \tilde{\Phi}^{j A} \text { defined below equation (36). } \\ & C 2=\Phi^{A} \sum_{j \in A} \Phi^{j A} \ln Q_{G D P}^{j}(2011,2005) \text { with } \Phi^{j A} \text { defined below equation (36). } \\ & C 3=\Phi^{A} \sum_{j \in A} \tilde{\Phi}^{j A} \ln Q_{G D P}^{j}(2011,2005)=\text { RHS of equation (37). } \end{aligned}$ |  |  |  |  |  |  |

## 7 Conclusion

Using the concepts of Nominal GDP, Real GDP, and Price Level Index from the field of international comparisons, the present paper provides a simple, symmetric formula for the calculation of regional or global growth and inflation. What distinguishes our proposal from currently used methods (at international organisations such as Eurostat, OECD, and World Bank) is its top-down approach ensuring consistency between value, quantity, and price developments. The role played by exchange rates and purchasing power parities also becomes clear.

The application uses data for 141 countries for the last two rounds of the International Comparison Program, regarding 2005 and 2011. The exchange rate based growth estimates for the aggregate ( 141 countries) is found to be $17 \%$ from 2005 to 2011, while the PPP based growth is estimated as $25 \%$. Using PPP based estimates, the fastest growing region was Asia and the Pacific (48\%), while the slowest growing region was EU-OECD (6.3\%). Global inflation movements are due to changes in domestic prices as well as changes in the relative worth of currencies. When using exchange-rate based weights to compute movements, the measured domestic price change components are smaller for all regions than they are when using purchasing power parities. We also show the importance of using appropriately derived weights when measuring regional growth, and the effect of the inconsistency-in-aggregation of the Sato-Vartia indices, which we found to be negligible.

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## Appendix: Data Used for Calculations

| WB | GDP05 | XR05 | PPP05 | NGDP05 | RGDP05 | GDP11 | XR11 | PPP11 | NGDP11 | RGDP11 | PGDP ${ }^{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code |  |  |  |  |  |  |  |  |  |  |  |
| USA | $1.32 \mathrm{E}+13$ | $1.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ | $1.32 \mathrm{E}+13$ | $1.32 \mathrm{E}+13$ | $1.57 \mathrm{E}+13$ | $1.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ | $1.57 \mathrm{E}+13$ | $1.57 \mathrm{E}+13$ | $1.12 \mathrm{E}+00$ |
| GBR | $1.27 \mathrm{E}+12$ | $5.50 \mathrm{E}-01$ | $6.50 \mathrm{E}-01$ | $2.32 \mathrm{E}+12$ | $1.96 \mathrm{E}+12$ | $1.52 \mathrm{E}+12$ | $6.24 \mathrm{E}-01$ | $7.00 \mathrm{E}-01$ | $2.43 \mathrm{E}+12$ | $2.17 \mathrm{E}+12$ | $1.17 \mathrm{E}+00$ |
| AUT | $2.46 \mathrm{E}+11$ | $8.04 \mathrm{E}-01$ | $8.70 \mathrm{E}-01$ | $3.05 \mathrm{E}+11$ | $2.82 \mathrm{E}+11$ | $3.00 \mathrm{E}+11$ | 7.19E-01 | $8.30 \mathrm{E}-01$ | $4.18 \mathrm{E}+11$ | $3.62 \mathrm{E}+11$ | $1.11 \mathrm{E}+00$ |
| BEL | $3.04 \mathrm{E}+11$ | $8.04 \mathrm{E}-01$ | $9.00 \mathrm{E}-01$ | $3.78 \mathrm{E}+11$ | $3.38 \mathrm{E}+11$ | $3.68 \mathrm{E}+11$ | 7.19E-01 | $8.40 \mathrm{E}-01$ | $5.12 \mathrm{E}+11$ | $4.38 \mathrm{E}+11$ | $1.13 \mathrm{E}+00$ |
| DNK | $1.54 \mathrm{E}+12$ | $6.00 \mathrm{E}+00$ | $8.52 \mathrm{E}+00$ | $2.58 \mathrm{E}+11$ | $1.81 \mathrm{E}+11$ | $1.79 \mathrm{E}+12$ | $5.37 \mathrm{E}+00$ | $7.69 \mathrm{E}+00$ | $3.34 \mathrm{E}+11$ | $2.33 \mathrm{E}+11$ | $1.15 \mathrm{E}+00$ |
| FRA | $1.72 \mathrm{E}+12$ | $8.04 \mathrm{E}-01$ | $9.20 \mathrm{E}-01$ | $2.14 \mathrm{E}+12$ | $1.87 \mathrm{E}+12$ | $2.01 \mathrm{E}+12$ | 7.19E-01 | $8.40 \mathrm{E}-01$ | $2.79 \mathrm{E}+12$ | $2.39 \mathrm{E}+12$ | $1.11 \mathrm{E}+00$ |
| DEU | $2.26 \mathrm{E}+12$ | $8.04 \mathrm{E}-01$ | 8.90E-01 | $2.81 \mathrm{E}+12$ | $2.54 \mathrm{E}+12$ | $2.64 \mathrm{E}+12$ | 7.19E-01 | $7.80 \mathrm{E}-01$ | $3.68 \mathrm{E}+12$ | $3.39 \mathrm{E}+12$ | $1.06 \mathrm{E}+00$ |
| ITA | $1.44 \mathrm{E}+12$ | $8.04 \mathrm{E}-01$ | $8.70 \mathrm{E}-01$ | $1.79 \mathrm{E}+12$ | $1.65 \mathrm{E}+12$ | $1.58 \mathrm{E}+12$ | 7.19E-01 | $7.70 \mathrm{E}-01$ | $2.20 \mathrm{E}+12$ | $2.05 \mathrm{E}+12$ | $1.11 \mathrm{E}+00$ |
| LUX | $2.98 \mathrm{E}+10$ | $8.04 \mathrm{E}-01$ | $9.20 \mathrm{E}-01$ | $3.71 \mathrm{E}+10$ | $3.24 \mathrm{E}+10$ | $4.16 \mathrm{E}+10$ | $7.19 \mathrm{E}-01$ | $9.10 \mathrm{E}-01$ | $5.78 \mathrm{E}+10$ | $4.57 \mathrm{E}+10$ | $1.25 \mathrm{E}+00$ |
| NLD | $5.13 \mathrm{E}+11$ | $8.04 \mathrm{E}-01$ | $9.00 \mathrm{E}-01$ | $6.38 \mathrm{E}+11$ | $5.70 \mathrm{E}+11$ | $5.98 \mathrm{E}+11$ | 7.19E-01 | $8.30 \mathrm{E}-01$ | $8.32 \mathrm{E}+11$ | $7.21 \mathrm{E}+11$ | $1.08 \mathrm{E}+00$ |
| NOR | $1.96 \mathrm{E}+12$ | $6.44 \mathrm{E}+00$ | $8.84 \mathrm{E}+00$ | $3.04 \mathrm{E}+11$ | $2.22 \mathrm{E}+11$ | $2.75 \mathrm{E}+12$ | $5.60 \mathrm{E}+00$ | $8.97 \mathrm{E}+00$ | $4.90 \mathrm{E}+11$ | $3.06 \mathrm{E}+11$ | $1.33 \mathrm{E}+00$ |
| SWE | $2.77 \mathrm{E}+12$ | $7.47 \mathrm{E}+00$ | $9.24 \mathrm{E}+00$ | $3.71 \mathrm{E}+11$ | $3.00 \mathrm{E}+11$ | $3.48 \mathrm{E}+12$ | $6.49 \mathrm{E}+00$ | $8.82 \mathrm{E}+00$ | $5.36 \mathrm{E}+11$ | $3.95 \mathrm{E}+11$ | $1.13 \mathrm{E}+00$ |
| CHE | $4.77 \mathrm{E}+11$ | $1.25 \mathrm{E}+00$ | $1.74 \mathrm{E}+00$ | $3.83 \mathrm{E}+11$ | $2.74 \mathrm{E}+11$ | $5.85 \mathrm{E}+11$ | $8.88 \mathrm{E}-01$ | $1.44 \mathrm{E}+00$ | $6.59 \mathrm{E}+11$ | $4.06 \mathrm{E}+11$ | $1.08 \mathrm{E}+00$ |
| CAN | $1.37 \mathrm{E}+12$ | $1.21 \mathrm{E}+00$ | $1.21 \mathrm{E}+00$ | $1.13 \mathrm{E}+12$ | $1.13 \mathrm{E}+12$ | $1.73 \mathrm{E}+12$ | $9.90 \mathrm{E}-01$ | $1.24 \mathrm{E}+00$ | $1.75 \mathrm{E}+12$ | $1.39 \mathrm{E}+12$ | $1.15 \mathrm{E}+00$ |
| JPN | $5.01 \mathrm{E}+14$ | $1.10 \mathrm{E}+02$ | $1.30 \mathrm{E}+02$ | $4.54 \mathrm{E}+12$ | $3.87 \mathrm{E}+12$ | $4.69 \mathrm{E}+14$ | $7.98 \mathrm{E}+01$ | $1.07 \mathrm{E}+02$ | $5.87 \mathrm{E}+12$ | $4.36 \mathrm{E}+12$ | $9.24 \mathrm{E}-01$ |
| FIN | $1.57 \mathrm{E}+11$ | $8.04 \mathrm{E}-01$ | $9.80 \mathrm{E}-01$ | $1.96 \mathrm{E}+11$ | $1.61 \mathrm{E}+11$ | $1.89 \mathrm{E}+11$ | $7.19 \mathrm{E}-01$ | $9.10 \mathrm{E}-01$ | $2.63 \mathrm{E}+11$ | $2.07 \mathrm{E}+11$ | $1.12 \mathrm{E}+00$ |
| GRC | $1.92 \mathrm{E}+11$ | $8.04 \mathrm{E}-01$ | 7.00E-01 | $2.39 \mathrm{E}+11$ | $2.74 \mathrm{E}+11$ | $2.08 \mathrm{E}+11$ | 7.19E-01 | $6.90 \mathrm{E}-01$ | $2.90 \mathrm{E}+11$ | $3.02 \mathrm{E}+11$ | $1.16 \mathrm{E}+00$ |
| ISL | $1.03 \mathrm{E}+12$ | $6.30 \mathrm{E}+01$ | $9.71 \mathrm{E}+01$ | $1.63 \mathrm{E}+10$ | $1.06 \mathrm{E}+10$ | $1.64 \mathrm{E}+12$ | $1.16 \mathrm{E}+02$ | $1.34 \mathrm{E}+02$ | $1.42 \mathrm{E}+10$ | $1.23 \mathrm{E}+10$ | $1.54 \mathrm{E}+00$ |
| IRL | $1.63 \mathrm{E}+11$ | $8.04 \mathrm{E}-01$ | $1.02 \mathrm{E}+00$ | $2.02 \mathrm{E}+11$ | $1.60 \mathrm{E}+11$ | $1.61 \mathrm{E}+11$ | 7.19E-01 | $8.30 \mathrm{E}-01$ | $2.24 \mathrm{E}+11$ | $1.94 \mathrm{E}+11$ | $9.73 \mathrm{E}-01$ |
| MLT | $4.95 \mathrm{E}+09$ | $8.05 \mathrm{E}-01$ | $5.80 \mathrm{E}-01$ | $6.14 \mathrm{E}+09$ | $8.53 \mathrm{E}+09$ | $6.74 \mathrm{E}+09$ | 7.19E-01 | $5.60 \mathrm{E}-01$ | $9.38 \mathrm{E}+09$ | $1.20 \mathrm{E}+10$ | $1.21 \mathrm{E}+00$ |
| PRT | $1.54 \mathrm{E}+11$ | $8.04 \mathrm{E}-01$ | 7.10E-01 | $1.92 \mathrm{E}+11$ | $2.17 \mathrm{E}+11$ | $1.72 \mathrm{E}+11$ | 7.19E-01 | $6.30 \mathrm{E}-01$ | $2.39 \mathrm{E}+11$ | $2.73 \mathrm{E}+11$ | $1.09 \mathrm{E}+00$ |
| ESP | $9.04 \mathrm{E}+11$ | $8.04 \mathrm{E}-01$ | 7.70E-01 | $1.12 \mathrm{E}+12$ | $1.17 \mathrm{E}+12$ | $1.04 \mathrm{E}+12$ | 7.19E-01 | 7.10E-01 | $1.45 \mathrm{E}+12$ | $1.47 \mathrm{E}+12$ | $1.10 \mathrm{E}+00$ |
| TUR | $6.49 \mathrm{E}+11$ | $1.34 \mathrm{E}+00$ | $8.70 \mathrm{E}-01$ | $4.83 \mathrm{E}+11$ | $7.46 \mathrm{E}+11$ | $1.30 \mathrm{E}+12$ | $1.67 \mathrm{E}+00$ | $9.90 \mathrm{E}-01$ | $7.75 \mathrm{E}+11$ | $1.31 \mathrm{E}+12$ | $1.57 \mathrm{E}+00$ |
| AUS | $9.27 \mathrm{E}+11$ | $1.31 \mathrm{E}+00$ | $1.39 \mathrm{E}+00$ | $7.08 \mathrm{E}+11$ | $6.67 \mathrm{E}+11$ | $1.43 \mathrm{E}+12$ | $9.69 \mathrm{E}-01$ | $1.51 \mathrm{E}+00$ | $1.47 \mathrm{E}+12$ | $9.46 \mathrm{E}+11$ | $1.29 \mathrm{E}+00$ |
| NZL | $1.62 \mathrm{E}+11$ | $1.42 \mathrm{E}+00$ | $1.54 \mathrm{E}+00$ | $1.14 \mathrm{E}+11$ | $1.05 \mathrm{E}+11$ | $2.07 \mathrm{E}+11$ | $1.27 \mathrm{E}+00$ | $1.49 \mathrm{E}+00$ | $1.63 \mathrm{E}+11$ | $1.39 \mathrm{E}+11$ | $1.19 \mathrm{E}+00$ |
| ZAF | $1.59 \mathrm{E}+12$ | $6.36 \mathrm{E}+00$ | $3.87 \mathrm{E}+00$ | $2.50 \mathrm{E}+11$ | $4.11 \mathrm{E}+11$ | $2.94 \mathrm{E}+12$ | $7.26 \mathrm{E}+00$ | $4.77 \mathrm{E}+00$ | $4.05 \mathrm{E}+11$ | $6.16 \mathrm{E}+11$ | $1.53 \mathrm{E}+00$ |
| ARG | $5.32 \mathrm{E}+11$ | $2.90 \mathrm{E}+00$ | $1.27 \mathrm{E}+00$ | $1.83 \mathrm{E}+11$ | $4.19 \mathrm{E}+11$ | $1.84 \mathrm{E}+12$ | $4.11 \mathrm{E}+00$ | $2.70 \mathrm{E}+00$ | $4.48 \mathrm{E}+11$ | $6.82 \mathrm{E}+11$ | $2.30 \mathrm{E}+00$ |
| BOL | $7.70 \mathrm{E}+10$ | $8.07 \mathrm{E}+00$ | $2.23 \mathrm{E}+00$ | $9.55 \mathrm{E}+09$ | $3.45 \mathrm{E}+10$ | $1.66 \mathrm{E}+11$ | $6.94 \mathrm{E}+00$ | $2.95 \mathrm{E}+00$ | $2.39 \mathrm{E}+10$ | $5.63 \mathrm{E}+10$ | $1.64 \mathrm{E}+00$ |
| BRA | $2.15 \mathrm{E}+12$ | $2.43 \mathrm{E}+00$ | $1.36 \mathrm{E}+00$ | $8.82 \mathrm{E}+11$ | $1.58 \mathrm{E}+12$ | $4.14 \mathrm{E}+12$ | $1.67 \mathrm{E}+00$ | $1.47 \mathrm{E}+00$ | $2.48 \mathrm{E}+12$ | $2.82 \mathrm{E}+12$ | $1.51 \mathrm{E}+00$ |
| CHL | $6.96 \mathrm{E}+13$ | $5.60 \mathrm{E}+02$ | $3.34 \mathrm{E}+02$ | $1.24 \mathrm{E}+11$ | $2.09 \mathrm{E}+11$ | $1.21 \mathrm{E}+14$ | $4.84 \mathrm{E}+02$ | $3.48 \mathrm{E}+02$ | $2.51 \mathrm{E}+11$ | $3.49 \mathrm{E}+11$ | $1.39 \mathrm{E}+00$ |
| COL | $3.40 \mathrm{E}+14$ | $2.32 \mathrm{E}+03$ | $1.08 \mathrm{E}+03$ | $1.47 \mathrm{E}+11$ | $3.14 \mathrm{E}+11$ | $6.22 \mathrm{E}+14$ | $1.85 \mathrm{E}+03$ | $1.16 \mathrm{E}+03$ | $3.36 \mathrm{E}+11$ | $5.35 \mathrm{E}+11$ | $1.37 \mathrm{E}+00$ |
| ECU | $4.15 \mathrm{E}+10$ | $1.00 \mathrm{E}+00$ | $4.20 \mathrm{E}-01$ | $4.15 \mathrm{E}+10$ | $9.88 \mathrm{E}+10$ | $7.68 \mathrm{E}+10$ | $1.00 \mathrm{E}+00$ | $5.30 \mathrm{E}-01$ | $7.68 \mathrm{E}+10$ | $1.45 \mathrm{E}+11$ | $1.46 \mathrm{E}+00$ |
| MEX | $9.48 \mathrm{E}+12$ | $1.09 \mathrm{E}+01$ | $7.13 \mathrm{E}+00$ | $8.70 \mathrm{E}+11$ | $1.33 \mathrm{E}+12$ | $1.44 \mathrm{E}+13$ | $1.24 \mathrm{E}+01$ | $7.67 \mathrm{E}+00$ | $1.16 \mathrm{E}+12$ | $1.88 \mathrm{E}+12$ | $1.33 \mathrm{E}+00$ |
| PRY | $5.40 \mathrm{E}+13$ | $6.18 \mathrm{E}+03$ | $2.01 \mathrm{E}+03$ | $8.73 \mathrm{E}+09$ | $2.69 \mathrm{E}+10$ | $1.09 \mathrm{E}+14$ | $4.19 \mathrm{E}+03$ | $2.23 \mathrm{E}+03$ | $2.60 \mathrm{E}+10$ | $4.88 \mathrm{E}+10$ | $1.51 \mathrm{E}+00$ |
| PER | $2.62 \mathrm{E}+11$ | $3.30 \mathrm{E}+00$ | $1.49 \mathrm{E}+00$ | $7.94 \mathrm{E}+10$ | $1.76 \mathrm{E}+11$ | $4.98 \mathrm{E}+11$ | $2.75 \mathrm{E}+00$ | $1.52 \mathrm{E}+00$ | $1.81 \mathrm{E}+11$ | $3.27 \mathrm{E}+11$ | $1.26 \mathrm{E}+00$ |
| URY | $4.25 \mathrm{E}+11$ | $2.45 \mathrm{E}+01$ | $1.33 \mathrm{E}+01$ | $1.74 \mathrm{E}+10$ | $3.20 \mathrm{E}+10$ | $8.97 \mathrm{E}+11$ | $1.93 \mathrm{E}+01$ | $1.53 \mathrm{E}+01$ | $4.64 \mathrm{E}+10$ | $5.87 \mathrm{E}+10$ | $1.50 \mathrm{E}+00$ |
| VEN | $3.04 \mathrm{E}+11$ | $2.11 \mathrm{E}+00$ | $1.15 \mathrm{E}+00$ | $1.44 \mathrm{E}+11$ | $2.64 \mathrm{E}+11$ | $1.36 \mathrm{E}+12$ | $4.29 \mathrm{E}+00$ | $2.71 \mathrm{E}+00$ | $3.16 \mathrm{E}+11$ | $5.01 \mathrm{E}+11$ | $3.57 \mathrm{E}+00$ |
| BHR | $5.06 \mathrm{E}+09$ | $3.76 \mathrm{E}-01$ | $2.50 \mathrm{E}-01$ | $1.35 \mathrm{E}+10$ | $2.02 \mathrm{E}+10$ | $1.09 \mathrm{E}+10$ | $3.76 \mathrm{E}-01$ | $2.10 \mathrm{E}-01$ | $2.90 \mathrm{E}+10$ | $5.20 \mathrm{E}+10$ | $1.36 \mathrm{E}+00$ |
| CYP | $1.35 \mathrm{E}+10$ | $7.90 \mathrm{E}-01$ | 7.20E-01 | $1.70 \mathrm{E}+10$ | $1.87 \mathrm{E}+10$ | $1.79 \mathrm{E}+10$ | $7.20 \mathrm{E}-01$ | $6.70 \mathrm{E}-01$ | $2.48 \mathrm{E}+10$ | $2.67 \mathrm{E}+10$ | $1.18 \mathrm{E}+00$ |
| IRN | $1.72 \mathrm{E}+15$ | $8.96 \mathrm{E}+03$ | $2.67 \mathrm{E}+03$ | $1.92 \mathrm{E}+11$ | $6.43 \mathrm{E}+11$ | $5.61 \mathrm{E}+15$ | $1.06 \mathrm{E}+04$ | $4.66 \mathrm{E}+03$ | $5.28 \mathrm{E}+11$ | $1.20 \mathrm{E}+12$ | $2.50 \mathrm{E}+00$ |
| IRQ | $7.35 \mathrm{E}+13$ | $1.47 \mathrm{E}+03$ | $5.59 \mathrm{E}+02$ | $5.00 \mathrm{E}+10$ | $1.32 \mathrm{E}+11$ | $2.24 \mathrm{E}+14$ | $1.41 \mathrm{E}+03$ | $5.17 \mathrm{E}+02$ | $1.59 \mathrm{E}+11$ | $4.34 \mathrm{E}+11$ | $2.06 \mathrm{E}+00$ |
| ISR | $5.73 \mathrm{E}+11$ | $4.49 \mathrm{E}+00$ | $3.72 \mathrm{E}+00$ | $1.28 \mathrm{E}+11$ | $1.54 \mathrm{E}+11$ | $8.97 \mathrm{E}+11$ | $3.58 \mathrm{E}+00$ | $3.94 \mathrm{E}+00$ | $2.51 \mathrm{E}+11$ | $2.28 \mathrm{E}+11$ | $1.16 \mathrm{E}+00$ |


| WB | GDP05 | XR05 | PPP05 | NGDP05 | RGDP05 | GDP11 | XR11 | PPP11 | NGDP11 | RGDP11 | PGDP ${ }^{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code |  |  |  |  |  |  |  |  |  |  |  |
| JOR | $8.64 \mathrm{E}+09$ | $7.09 \mathrm{E}-01$ | $3.80 \mathrm{E}-01$ | $1.22 \mathrm{E}+10$ | $2.27 \mathrm{E}+10$ | $2.23 \mathrm{E}+10$ | 7.10E-01 | $2.90 \mathrm{E}-01$ | $3.14 \mathrm{E}+10$ | $7.69 \mathrm{E}+10$ | $1.65 \mathrm{E}+00$ |
| KWT | $2.36 \mathrm{E}+10$ | $2.92 \mathrm{E}-01$ | $2.10 \mathrm{E}-01$ | $8.08 \mathrm{E}+10$ | $1.12 \mathrm{E}+11$ | $4.43 \mathrm{E}+10$ | $2.76 \mathrm{E}-01$ | $1.70 \mathrm{E}-01$ | $1.61 \mathrm{E}+11$ | $2.61 \mathrm{E}+11$ | $1.67 \mathrm{E}+00$ |
| LBN | $3.30 \mathrm{E}+13$ | $1.51 \mathrm{E}+03$ | $8.48 \mathrm{E}+02$ | $2.19 \mathrm{E}+10$ | $3.89 \mathrm{E}+10$ | $6.18 \mathrm{E}+13$ | $1.51 \mathrm{E}+03$ | $8.39 \mathrm{E}+02$ | $4.10 \mathrm{E}+10$ | $7.36 \mathrm{E}+10$ | $1.30 \mathrm{E}+00$ |
| OMN | $1.19 \mathrm{E}+10$ | $3.85 \mathrm{E}-01$ | $2.30 \mathrm{E}-01$ | $3.09 \mathrm{E}+10$ | $5.17 \mathrm{E}+10$ | $2.69 \mathrm{E}+10$ | $3.85 \mathrm{E}-01$ | $1.90 \mathrm{E}-01$ | $7.00 \mathrm{E}+10$ | $1.42 \mathrm{E}+11$ | $1.60 \mathrm{E}+00$ |
| QAT | $1.57 \mathrm{E}+11$ | $3.64 \mathrm{E}+00$ | $2.75 \mathrm{E}+00$ | $4.30 \mathrm{E}+10$ | $5.70 \mathrm{E}+10$ | $6.24 \mathrm{E}+11$ | $3.64 \mathrm{E}+00$ | $2.42 \mathrm{E}+00$ | $1.71 \mathrm{E}+11$ | $2.58 \mathrm{E}+11$ | $1.49 \mathrm{E}+00$ |
| SAU | $1.23 \mathrm{E}+12$ | $3.75 \mathrm{E}+00$ | $2.41 \mathrm{E}+00$ | $3.28 \mathrm{E}+11$ | $5.11 \mathrm{E}+11$ | $2.51 \mathrm{E}+12$ | $3.75 \mathrm{E}+00$ | $1.84 \mathrm{E}+00$ | $6.70 \mathrm{E}+11$ | $1.36 \mathrm{E}+12$ | $1.42 \mathrm{E}+00$ |
| SYR | $1.51 \mathrm{E}+12$ | $5.31 \mathrm{E}+01$ | $1.97 \mathrm{E}+01$ | $2.84 \mathrm{E}+10$ | $7.64 \mathrm{E}+10$ | $3.05 \mathrm{E}+12$ | $4.74 \mathrm{E}+01$ | $2.13 \mathrm{E}+01$ | $6.43 \mathrm{E}+10$ | $1.43 \mathrm{E}+11$ | $1.62 \mathrm{E}+00$ |
| EGY | $5.38 \mathrm{E}+11$ | $5.78 \mathrm{E}+00$ | $1.62 \mathrm{E}+00$ | $9.32 \mathrm{E}+10$ | $3.32 \mathrm{E}+11$ | $1.37 \mathrm{E}+12$ | $5.93 \mathrm{E}+00$ | $1.62 \mathrm{E}+00$ | $2.31 \mathrm{E}+11$ | $8.46 \mathrm{E}+11$ | $1.85 \mathrm{E}+00$ |
| BGD | $3.71 \mathrm{E}+12$ | $6.43 \mathrm{E}+01$ | $2.26 \mathrm{E}+01$ | $5.76 \mathrm{E}+10$ | $1.64 \mathrm{E}+11$ | $7.97 \mathrm{E}+12$ | $7.42 \mathrm{E}+01$ | $2.32 \mathrm{E}+01$ | $1.07 \mathrm{E}+11$ | $3.44 \mathrm{E}+11$ | $1.49 \mathrm{E}+00$ |
| BTN | $3.61 \mathrm{E}+10$ | $4.41 \mathrm{E}+01$ | $1.57 \mathrm{E}+01$ | $8.19 \mathrm{E}+08$ | $2.29 \mathrm{E}+09$ | $8.56 \mathrm{E}+10$ | $4.67 \mathrm{E}+01$ | $1.69 \mathrm{E}+01$ | $1.83 \mathrm{E}+09$ | $5.08 \mathrm{E}+09$ | $1.39 \mathrm{E}+00$ |
| BRN | $1.59 \mathrm{E}+10$ | $1.66 \mathrm{E}+00$ | $9.00 \mathrm{E}-01$ | $9.53 \mathrm{E}+09$ | $1.76 \mathrm{E}+10$ | $2.06 \mathrm{E}+10$ | $1.26 \mathrm{E}+00$ | $7.20 \mathrm{E}-01$ | $1.64 \mathrm{E}+10$ | $2.86 \mathrm{E}+10$ | $1.23 \mathrm{E}+00$ |
| KHM | $2.58 \mathrm{E}+13$ | $4.09 \mathrm{E}+03$ | $1.28 \mathrm{E}+03$ | $6.29 \mathrm{E}+09$ | $2.01 \mathrm{E}+10$ | $5.21 \mathrm{E}+13$ | $4.06 \mathrm{E}+03$ | $1.35 \mathrm{E}+03$ | $1.28 \mathrm{E}+10$ | $3.87 \mathrm{E}+10$ | $1.37 \mathrm{E}+00$ |
| LKA | $2.49 \mathrm{E}+12$ | $1.00 \mathrm{E}+02$ | $3.52 \mathrm{E}+01$ | $2.48 \mathrm{E}+10$ | $7.08 \mathrm{E}+10$ | $6.56 \mathrm{E}+12$ | $1.11 \mathrm{E}+02$ | $3.87 \mathrm{E}+01$ | $5.93 \mathrm{E}+10$ | $1.70 \mathrm{E}+11$ | $1.81 \mathrm{E}+00$ |
| HKG | $1.43 \mathrm{E}+12$ | $7.78 \mathrm{E}+00$ | $5.69 \mathrm{E}+00$ | $1.84 \mathrm{E}+11$ | $2.51 \mathrm{E}+11$ | $1.94 \mathrm{E}+12$ | $7.78 \mathrm{E}+00$ | $5.46 \mathrm{E}+00$ | $2.50 \mathrm{E}+11$ | $3.56 \mathrm{E}+11$ | $1.08 \mathrm{E}+00$ |
| IND | $3.69 \mathrm{E}+13$ | $4.41 \mathrm{E}+01$ | $1.47 \mathrm{E}+01$ | $8.37 \mathrm{E}+11$ | $2.52 \mathrm{E}+12$ | $9.01 \mathrm{E}+13$ | $4.67 \mathrm{E}+01$ | $1.51 \mathrm{E}+01$ | $1.93 \mathrm{E}+12$ | $5.96 \mathrm{E}+12$ | $1.53 \mathrm{E}+00$ |
| IDN | $2.77 \mathrm{E}+15$ | $9.70 \mathrm{E}+03$ | $3.93 \mathrm{E}+03$ | $2.86 \mathrm{E}+11$ | $7.05 \mathrm{E}+11$ | $7.42 \mathrm{E}+15$ | $8.77 \mathrm{E}+03$ | $3.61 \mathrm{E}+03$ | $8.46 \mathrm{E}+11$ | $2.06 \mathrm{E}+12$ | $1.90 \mathrm{E}+00$ |
| KOR | $8.45 \mathrm{E}+14$ | $1.02 \mathrm{E}+03$ | $7.89 \mathrm{E}+02$ | $8.25 \mathrm{E}+11$ | $1.07 \mathrm{E}+12$ | $1.21 \mathrm{E}+15$ | $1.11 \mathrm{E}+03$ | $8.55 \mathrm{E}+02$ | $1.09 \mathrm{E}+12$ | $1.41 \mathrm{E}+12$ | $1.14 \mathrm{E}+00$ |
| LAO | $2.91 \mathrm{E}+13$ | $1.07 \mathrm{E}+04$ | $2.99 \mathrm{E}+03$ | $2.74 \mathrm{E}+09$ | $9.75 \mathrm{E}+09$ | $6.65 \mathrm{E}+13$ | $8.03 \mathrm{E}+03$ | $2.47 \mathrm{E}+03$ | $8.28 \mathrm{E}+09$ | $2.70 \mathrm{E}+10$ | $1.44 \mathrm{E}+00$ |
| MAC | $9.45 \mathrm{E}+10$ | $8.01 \mathrm{E}+00$ | $5.27 \mathrm{E}+00$ | $1.18 \mathrm{E}+10$ | $1.79 \mathrm{E}+10$ | $2.95 \mathrm{E}+11$ | $8.02 \mathrm{E}+00$ | $4.59 \mathrm{E}+00$ | $3.68 \mathrm{E}+10$ | $6.43 \mathrm{E}+10$ | $1.46 \mathrm{E}+00$ |
| MYS | $5.44 \mathrm{E}+11$ | $3.79 \mathrm{E}+00$ | $1.73 \mathrm{E}+00$ | $1.44 \mathrm{E}+11$ | $3.14 \mathrm{E}+11$ | $8.84 \mathrm{E}+11$ | $3.06 \mathrm{E}+00$ | $1.46 \mathrm{E}+00$ | $2.89 \mathrm{E}+11$ | $6.06 \mathrm{E}+11$ | $1.24 \mathrm{E}+00$ |
| MDV | $1.40 \mathrm{E}+10$ | $1.28 \mathrm{E}+01$ | $8.13 \mathrm{E}+00$ | $1.09 \mathrm{E}+09$ | $1.72 \mathrm{E}+09$ | $3.47 \mathrm{E}+10$ | $1.46 \mathrm{E}+01$ | $8.53 \mathrm{E}+00$ | $2.38 \mathrm{E}+09$ | $4.07 \mathrm{E}+09$ | $1.51 \mathrm{E}+00$ |
| NPL | $5.89 \mathrm{E}+11$ | $7.14 \mathrm{E}+01$ | $2.27 \mathrm{E}+01$ | $8.26 \mathrm{E}+09$ | $2.60 \mathrm{E}+10$ | $1.37 \mathrm{E}+12$ | $7.40 \mathrm{E}+01$ | $2.46 \mathrm{E}+01$ | $1.86 \mathrm{E}+10$ | $5.58 \mathrm{E}+10$ | $1.82 \mathrm{E}+00$ |
| PAK | $6.50 \mathrm{E}+12$ | $5.95 \mathrm{E}+01$ | $1.91 \mathrm{E}+01$ | $1.09 \mathrm{E}+11$ | $3.40 \mathrm{E}+11$ | $1.83 \mathrm{E}+13$ | $8.63 \mathrm{E}+01$ | $2.44 \mathrm{E}+01$ | $2.12 \mathrm{E}+11$ | $7.51 \mathrm{E}+11$ | $2.31 \mathrm{E}+00$ |
| PHL | $5.68 \mathrm{E}+12$ | $5.51 \mathrm{E}+01$ | $2.18 \mathrm{E}+01$ | $1.03 \mathrm{E}+11$ | $2.61 \mathrm{E}+11$ | $9.71 \mathrm{E}+12$ | $4.33 \mathrm{E}+01$ | $1.79 \mathrm{E}+01$ | $2.24 \mathrm{E}+11$ | $5.44 \mathrm{E}+11$ | $1.30 \mathrm{E}+00$ |
| SGP | $2.20 \mathrm{E}+11$ | $1.66 \mathrm{E}+00$ | $1.08 \mathrm{E}+00$ | $1.32 \mathrm{E}+11$ | $2.04 \mathrm{E}+11$ | $3.35 \mathrm{E}+11$ | $1.26 \mathrm{E}+00$ | $8.90 \mathrm{E}-01$ | $2.66 \mathrm{E}+11$ | $3.76 \mathrm{E}+11$ | $1.11 \mathrm{E}+00$ |
| THA | $7.09 \mathrm{E}+12$ | $4.02 \mathrm{E}+01$ | $1.59 \mathrm{E}+01$ | $1.76 \mathrm{E}+11$ | $4.45 \mathrm{E}+11$ | $1.05 \mathrm{E}+13$ | $3.05 \mathrm{E}+01$ | $1.24 \mathrm{E}+01$ | $3.46 \mathrm{E}+11$ | $8.52 \mathrm{E}+11$ | $1.25 \mathrm{E}+00$ |
| VNM | $9.42 \mathrm{E}+14$ | $1.59 \mathrm{E}+04$ | $4.71 \mathrm{E}+03$ | $5.94 \mathrm{E}+10$ | $2.00 \mathrm{E}+11$ | $2.85 \mathrm{E}+15$ | $2.05 \mathrm{E}+04$ | $6.71 \mathrm{E}+03$ | $1.39 \mathrm{E}+11$ | $4.24 \mathrm{E}+11$ | $2.11 \mathrm{E}+00$ |
| DJI | $1.26 \mathrm{E}+11$ | $1.78 \mathrm{E}+02$ | $8.47 \mathrm{E}+01$ | $7.09 \mathrm{E}+08$ | $1.49 \mathrm{E}+09$ | $2.20 \mathrm{E}+11$ | $1.78 \mathrm{E}+02$ | $9.40 \mathrm{E}+01$ | $1.24 \mathrm{E}+09$ | $2.34 \mathrm{E}+09$ | $1.22 \mathrm{E}+00$ |
| AGO | $2.46 \mathrm{E}+12$ | $8.72 \mathrm{E}+01$ | $4.45 \mathrm{E}+01$ | $2.82 \mathrm{E}+10$ | $5.53 \mathrm{E}+10$ | $9.78 \mathrm{E}+12$ | $9.39 \mathrm{E}+01$ | $6.83 \mathrm{E}+01$ | $1.04 \mathrm{E}+11$ | $1.43 \mathrm{E}+11$ | $2.14 \mathrm{E}+00$ |
| BWA | $5.08 \mathrm{E}+10$ | $5.11 \mathrm{E}+00$ | $2.42 \mathrm{E}+00$ | $9.93 \mathrm{E}+09$ | $2.10 \mathrm{E}+10$ | $1.05 \mathrm{E}+11$ | $6.84 \mathrm{E}+00$ | $3.76 \mathrm{E}+00$ | $1.53 \mathrm{E}+10$ | $2.78 \mathrm{E}+10$ | $1.59 \mathrm{E}+00$ |
| BDI | $1.21 \mathrm{E}+12$ | $1.08 \mathrm{E}+03$ | $3.43 \mathrm{E}+02$ | $1.12 \mathrm{E}+09$ | $3.52 \mathrm{E}+09$ | $2.97 \mathrm{E}+12$ | $1.26 \mathrm{E}+03$ | $4.26 \mathrm{E}+02$ | $2.36 \mathrm{E}+09$ | $6.98 \mathrm{E}+09$ | $1.35 \mathrm{E}+00$ |
| CMR | $8.75 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.51 \mathrm{E}+02$ | $1.66 \mathrm{E}+10$ | $3.49 \mathrm{E}+10$ | $1.20 \mathrm{E}+13$ | $4.72 \mathrm{E}+02$ | $2.27 \mathrm{E}+02$ | $2.55 \mathrm{E}+10$ | $5.29 \mathrm{E}+10$ | $1.14 \mathrm{E}+00$ |
| CPV | $9.67 \mathrm{E}+10$ | $8.87 \mathrm{E}+01$ | $6.94 \mathrm{E}+01$ | $1.09 \mathrm{E}+09$ | $1.39 \mathrm{E}+09$ | $1.48 \mathrm{E}+11$ | $7.93 \mathrm{E}+01$ | $4.86 \mathrm{E}+01$ | $1.86 \mathrm{E}+09$ | $3.04 \mathrm{E}+09$ | $1.04 \mathrm{E}+00$ |
| CAF | $7.12 \mathrm{E}+11$ | $5.27 \mathrm{E}+02$ | $2.64 \mathrm{E}+02$ | $1.35 \mathrm{E}+09$ | $2.70 \mathrm{E}+09$ | $1.04 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.56 \mathrm{E}+02$ | $2.21 \mathrm{E}+09$ | $4.08 \mathrm{E}+09$ | $9.92 \mathrm{E}-01$ |
| TCD | $3.51 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.08 \mathrm{E}+02$ | $6.65 \mathrm{E}+09$ | $1.69 \mathrm{E}+10$ | $5.74 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.50 \mathrm{E}+02$ | $1.22 \mathrm{E}+10$ | $2.29 \mathrm{E}+10$ | $1.29 \mathrm{E}+00$ |
| COG | $3.21 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.69 \mathrm{E}+02$ | $6.09 \mathrm{E}+09$ | $1.19 \mathrm{E}+10$ | $6.81 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.89 \mathrm{E}+02$ | $1.44 \mathrm{E}+10$ | $2.35 \mathrm{E}+10$ | $1.59 \mathrm{E}+00$ |
| ZAR | $3.41 \mathrm{E}+12$ | $4.74 \mathrm{E}+02$ | $2.14 \mathrm{E}+02$ | $7.19 \mathrm{E}+09$ | $1.59 \mathrm{E}+10$ | $1.44 \mathrm{E}+13$ | $9.19 \mathrm{E}+02$ | $5.22 \mathrm{E}+02$ | $1.57 \mathrm{E}+10$ | $2.77 \mathrm{E}+10$ | $3.02 \mathrm{E}+00$ |
| BEN | $2.30 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.20 \mathrm{E}+02$ | $4.36 \mathrm{E}+09$ | $1.05 \mathrm{E}+10$ | $3.44 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.14 \mathrm{E}+02$ | $7.29 \mathrm{E}+09$ | $1.61 \mathrm{E}+10$ | $1.21 \mathrm{E}+00$ |
| GNQ | $4.33 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.87 \mathrm{E}+02$ | $8.22 \mathrm{E}+09$ | $1.51 \mathrm{E}+10$ | $7.93 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.95 \mathrm{E}+02$ | $1.68 \mathrm{E}+10$ | $2.69 \mathrm{E}+10$ | $1.47 \mathrm{E}+00$ |
| ETH | $1.05 \mathrm{E}+11$ | $8.67 \mathrm{E}+00$ | $2.25 \mathrm{E}+00$ | $1.22 \mathrm{E}+10$ | $4.68 \mathrm{E}+10$ | $5.06 \mathrm{E}+11$ | $1.69 \mathrm{E}+01$ | $4.92 \mathrm{E}+00$ | $2.99 \mathrm{E}+10$ | $1.03 \mathrm{E}+11$ | $2.58 \mathrm{E}+00$ |
| GAB | $4.57 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.56 \mathrm{E}+02$ | $8.67 \mathrm{E}+09$ | $1.79 \mathrm{E}+10$ | $8.85 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $3.18 \mathrm{E}+02$ | $1.88 \mathrm{E}+10$ | $2.78 \mathrm{E}+10$ | $1.62 \mathrm{E}+00$ |
| GMB | $1.78 \mathrm{E}+10$ | $2.86 \mathrm{E}+01$ | $7.56 \mathrm{E}+00$ | $6.24 \mathrm{E}+08$ | $2.36 \mathrm{E}+09$ | $2.65 \mathrm{E}+10$ | $2.95 \mathrm{E}+01$ | $9.94 \mathrm{E}+00$ | $8.98 \mathrm{E}+08$ | $2.66 \mathrm{E}+09$ | $1.23 \mathrm{E}+00$ |
| GHA | $9.73 \mathrm{E}+09$ | $9.06 \mathrm{E}-01$ | $3.70 \mathrm{E}-01$ | $1.07 \mathrm{E}+10$ | $2.63 \mathrm{E}+10$ | $5.98 \mathrm{E}+10$ | $1.51 \mathrm{E}+00$ | $7.00 \mathrm{E}-01$ | $3.96 \mathrm{E}+10$ | $8.55 \mathrm{E}+10$ | $3.88 \mathrm{E}+00$ |
| GNB | $3.02 \mathrm{E}+11$ | $5.27 \mathrm{E}+02$ | $2.17 \mathrm{E}+02$ | $5.73 \mathrm{E}+08$ | $1.39 \mathrm{E}+09$ | $4.57 \mathrm{E}+11$ | $4.72 \mathrm{E}+02$ | $2.20 \mathrm{E}+02$ | $9.68 \mathrm{E}+08$ | $2.07 \mathrm{E}+09$ | $1.22 \mathrm{E}+00$ |
| GIN | $1.07 \mathrm{E}+13$ | $3.64 \mathrm{E}+03$ | $1.22 \mathrm{E}+03$ | $2.94 \mathrm{E}+09$ | $8.78 \mathrm{E}+09$ | $3.37 \mathrm{E}+13$ | $6.66 \mathrm{E}+03$ | $2.52 \mathrm{E}+03$ | $5.07 \mathrm{E}+09$ | $1.34 \mathrm{E}+10$ | $2.73 \mathrm{E}+00$ |


| WB | GDP05 | XR05 | PPP05 | NGDP05 | RGDP05 | GDP11 | XR11 | PPP11 | NGDP11 | RGDP11 | PGDP ${ }^{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code |  |  |  |  |  |  |  |  |  |  |  |
| CIV | $8.63 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.87 \mathrm{E}+02$ | $1.64 \mathrm{E}+10$ | $3.01 \mathrm{E}+10$ | $1.14 \mathrm{E}+13$ | $4.72 \mathrm{E}+02$ | $2.28 \mathrm{E}+02$ | $2.41 \mathrm{E}+10$ | $4.98 \mathrm{E}+10$ | $1.24 \mathrm{E}+00$ |
| KEN | $1.42 \mathrm{E}+12$ | $7.56 \mathrm{E}+01$ | $2.95 \mathrm{E}+01$ | $1.87 \mathrm{E}+10$ | $4.80 \mathrm{E}+10$ | $2.99 \mathrm{E}+12$ | $8.88 \mathrm{E}+01$ | $3.43 \mathrm{E}+01$ | $3.36 \mathrm{E}+10$ | $8.71 \mathrm{E}+10$ | $1.61 \mathrm{E}+00$ |
| LSO | $8.70 \mathrm{E}+09$ | $6.36 \mathrm{E}+00$ | $3.49 \mathrm{E}+00$ | $1.37 \mathrm{E}+09$ | $2.49 \mathrm{E}+09$ | $1.83 \mathrm{E}+10$ | $7.26 \mathrm{E}+00$ | $3.92 \mathrm{E}+00$ | $2.52 \mathrm{E}+09$ | $4.68 \mathrm{E}+09$ | $1.57 \mathrm{E}+00$ |
| LBR | $6.08 \mathrm{E}+08$ | $1.00 \mathrm{E}+00$ | $4.90 \mathrm{E}-01$ | $6.08 \mathrm{E}+08$ | $1.24 \mathrm{E}+09$ | $1.15 \mathrm{E}+09$ | $1.00 \mathrm{E}+00$ | $5.20 \mathrm{E}-01$ | $1.15 \mathrm{E}+09$ | $2.21 \mathrm{E}+09$ | $1.13 \mathrm{E}+00$ |
| MDG | $1.01 \mathrm{E}+13$ | $2.00 \mathrm{E}+03$ | $6.50 \mathrm{E}+02$ | $5.04 \mathrm{E}+09$ | $1.55 \mathrm{E}+10$ | $2.01 \mathrm{E}+13$ | $2.03 \mathrm{E}+03$ | $6.74 \mathrm{E}+02$ | $9.91 \mathrm{E}+09$ | $2.98 \mathrm{E}+10$ | $1.69 \mathrm{E}+00$ |
| MWI | $3.26 \mathrm{E}+11$ | $1.18 \mathrm{E}+02$ | $3.95 \mathrm{E}+01$ | $2.75 \mathrm{E}+09$ | $8.27 \mathrm{E}+09$ | $8.80 \mathrm{E}+11$ | $1.57 \mathrm{E}+02$ | $7.63 \mathrm{E}+01$ | $5.62 \mathrm{E}+09$ | $1.15 \mathrm{E}+10$ | $2.16 \mathrm{E}+00$ |
| MLI | $2.80 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.40 \mathrm{E}+02$ | $5.31 \mathrm{E}+09$ | $1.17 \mathrm{E}+10$ | $5.04 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.10 \mathrm{E}+02$ | $1.07 \mathrm{E}+10$ | $2.40 \mathrm{E}+10$ | $1.33 \mathrm{E}+00$ |
| MRT | $5.80 \mathrm{E}+11$ | $2.66 \mathrm{E}+02$ | $9.90 \mathrm{E}+01$ | $2.18 \mathrm{E}+09$ | $5.86 \mathrm{E}+09$ | $1.20 \mathrm{E}+12$ | $2.81 \mathrm{E}+02$ | $1.16 \mathrm{E}+02$ | $4.27 \mathrm{E}+09$ | $1.04 \mathrm{E}+10$ | $1.54 \mathrm{E}+00$ |
| MUS | $1.81 \mathrm{E}+11$ | $2.95 \mathrm{E}+01$ | $1.47 \mathrm{E}+01$ | $6.13 \mathrm{E}+09$ | $1.23 \mathrm{E}+10$ | $3.10 \mathrm{E}+11$ | $2.87 \mathrm{E}+01$ | $1.59 \mathrm{E}+01$ | $1.08 \mathrm{E}+10$ | $1.95 \mathrm{E}+10$ | $1.35 \mathrm{E}+00$ |
| MAR | $5.20 \mathrm{E}+11$ | $8.87 \mathrm{E}+00$ | $4.88 \mathrm{E}+00$ | $5.87 \mathrm{E}+10$ | $1.07 \mathrm{E}+11$ | $7.90 \mathrm{E}+11$ | $8.09 \mathrm{E}+00$ | $3.68 \mathrm{E}+00$ | $9.76 \mathrm{E}+10$ | $2.15 \mathrm{E}+11$ | $1.13 \mathrm{E}+00$ |
| MOZ | $1.52 \mathrm{E}+11$ | $2.31 \mathrm{E}+01$ | $1.09 \mathrm{E}+01$ | $6.58 \mathrm{E}+09$ | $1.39 \mathrm{E}+10$ | $3.65 \mathrm{E}+11$ | $2.91 \mathrm{E}+01$ | $1.60 \mathrm{E}+01$ | $1.26 \mathrm{E}+10$ | $2.28 \mathrm{E}+10$ | $1.62 \mathrm{E}+00$ |
| NER | $1.80 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.27 \mathrm{E}+02$ | $3.41 \mathrm{E}+09$ | $7.91 \mathrm{E}+09$ | $3.03 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.21 \mathrm{E}+02$ | $6.41 \mathrm{E}+09$ | $1.37 \mathrm{E}+10$ | $1.28 \mathrm{E}+00$ |
| NGA | $1.47 \mathrm{E}+13$ | $1.31 \mathrm{E}+02$ | $6.00 \mathrm{E}+01$ | $1.12 \mathrm{E}+11$ | $2.46 \mathrm{E}+11$ | $6.40 \mathrm{E}+13$ | $1.55 \mathrm{E}+02$ | $7.44 \mathrm{E}+01$ | $4.14 \mathrm{E}+11$ | $8.60 \mathrm{E}+11$ | $1.75 \mathrm{E}+00$ |
| RWA | $1.44 \mathrm{E}+12$ | $5.58 \mathrm{E}+02$ | $1.86 \mathrm{E}+02$ | $2.58 \mathrm{E}+09$ | $7.74 \mathrm{E}+09$ | $3.81 \mathrm{E}+12$ | $6.00 \mathrm{E}+02$ | $2.61 \mathrm{E}+02$ | $6.35 \mathrm{E}+09$ | $1.46 \mathrm{E}+10$ | $1.65 \mathrm{E}+00$ |
| STP | $1.32 \mathrm{E}+12$ | $1.06 \mathrm{E}+04$ | $5.56 \mathrm{E}+03$ | $1.25 \mathrm{E}+08$ | $2.37 \mathrm{E}+08$ | $4.59 \mathrm{E}+12$ | $1.76 \mathrm{E}+04$ | $8.53 \mathrm{E}+03$ | $2.61 \mathrm{E}+08$ | $5.39 \mathrm{E}+08$ | $2.73 \mathrm{E}+00$ |
| SEN | $4.59 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.52 \mathrm{E}+02$ | $8.71 \mathrm{E}+09$ | $1.82 \mathrm{E}+10$ | $6.81 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.36 \mathrm{E}+02$ | $1.44 \mathrm{E}+10$ | $2.88 \mathrm{E}+10$ | $1.22 \mathrm{E}+00$ |
| SLE | $4.70 \mathrm{E}+12$ | $2.89 \mathrm{E}+03$ | $1.07 \mathrm{E}+03$ | $1.63 \mathrm{E}+09$ | $4.38 \mathrm{E}+09$ | $1.28 \mathrm{E}+13$ | $4.35 \mathrm{E}+03$ | $1.55 \mathrm{E}+03$ | $2.94 \mathrm{E}+09$ | $8.23 \mathrm{E}+09$ | $1.96 \mathrm{E}+00$ |
| NAM | $4.62 \mathrm{E}+10$ | $6.36 \mathrm{E}+00$ | $4.26 \mathrm{E}+00$ | $7.26 \mathrm{E}+09$ | $1.08 \mathrm{E}+10$ | $9.17 \mathrm{E}+10$ | $7.26 \mathrm{E}+00$ | $4.66 \mathrm{E}+00$ | $1.26 \mathrm{E}+10$ | $1.97 \mathrm{E}+10$ | $1.53 \mathrm{E}+00$ |
| SDN | $5.15 \mathrm{E}+10$ | $2.44 \mathrm{E}+00$ | $1.08 \mathrm{E}+00$ | $2.11 \mathrm{E}+10$ | $4.77 \mathrm{E}+10$ | $1.49 \mathrm{E}+11$ | $2.67 \mathrm{E}+00$ | $1.22 \mathrm{E}+00$ | $5.60 \mathrm{E}+10$ | $1.22 \mathrm{E}+11$ | $2.02 \mathrm{E}+00$ |
| SWZ | $1.64 \mathrm{E}+10$ | $6.36 \mathrm{E}+00$ | $3.29 \mathrm{E}+00$ | $2.58 \mathrm{E}+09$ | $4.99 \mathrm{E}+09$ | $2.88 \mathrm{E}+10$ | $7.26 \mathrm{E}+00$ | $3.90 \mathrm{E}+00$ | $3.97 \mathrm{E}+09$ | $7.39 \mathrm{E}+09$ | $1.55 \mathrm{E}+00$ |
| TZA | $1.60 \mathrm{E}+13$ | $1.13 \mathrm{E}+03$ | $3.96 \mathrm{E}+02$ | $1.42 \mathrm{E}+10$ | $4.04 \mathrm{E}+10$ | $3.76 \mathrm{E}+13$ | $1.57 \mathrm{E}+03$ | $5.22 \mathrm{E}+02$ | $2.39 \mathrm{E}+10$ | $7.19 \mathrm{E}+10$ | $1.58 \mathrm{E}+00$ |
| TGO | $1.12 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.40 \mathrm{E}+02$ | $2.12 \mathrm{E}+09$ | $4.65 \mathrm{E}+09$ | $1.74 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.15 \mathrm{E}+02$ | $3.69 \mathrm{E}+09$ | $8.09 \mathrm{E}+09$ | $1.27 \mathrm{E}+00$ |
| TUN | $4.20 \mathrm{E}+10$ | $1.30 \mathrm{E}+00$ | $5.80 \mathrm{E}-01$ | $3.23 \mathrm{E}+10$ | $7.24 \mathrm{E}+10$ | $6.59 \mathrm{E}+10$ | $1.41 \mathrm{E}+00$ | $5.90 \mathrm{E}-01$ | $4.68 \mathrm{E}+10$ | $1.12 \mathrm{E}+11$ | $1.28 \mathrm{E}+00$ |
| UGA | $1.61 \mathrm{E}+13$ | $1.78 \mathrm{E}+03$ | $6.20 \mathrm{E}+02$ | $9.01 \mathrm{E}+09$ | $2.59 \mathrm{E}+10$ | $3.91 \mathrm{E}+13$ | $2.52 \mathrm{E}+03$ | $8.34 \mathrm{E}+02$ | $1.55 \mathrm{E}+10$ | $4.69 \mathrm{E}+10$ | $1.54 \mathrm{E}+00$ |
| BFA | $2.88 \mathrm{E}+12$ | $5.27 \mathrm{E}+02$ | $2.00 \mathrm{E}+02$ | $5.46 \mathrm{E}+09$ | $1.44 \mathrm{E}+10$ | $4.91 \mathrm{E}+12$ | $4.72 \mathrm{E}+02$ | $2.14 \mathrm{E}+02$ | $1.04 \mathrm{E}+10$ | $2.30 \mathrm{E}+10$ | $1.26 \mathrm{E}+00$ |
| ZMB | $3.20 \mathrm{E}+10$ | $4.46 \mathrm{E}+00$ | $2.42 \mathrm{E}+00$ | $7.18 \mathrm{E}+09$ | $1.32 \mathrm{E}+10$ | $9.33 \mathrm{E}+10$ | $4.86 \mathrm{E}+00$ | $2.38 \mathrm{E}+00$ | $1.92 \mathrm{E}+10$ | $3.92 \mathrm{E}+10$ | $2.00 \mathrm{E}+00$ |
| FJI | $5.08 \mathrm{E}+09$ | $1.69 \mathrm{E}+00$ | $1.43 \mathrm{E}+00$ | $3.01 \mathrm{E}+09$ | $3.56 \mathrm{E}+09$ | $6.73 \mathrm{E}+09$ | $1.79 \mathrm{E}+00$ | $1.04 \mathrm{E}+00$ | $3.75 \mathrm{E}+09$ | $6.47 \mathrm{E}+09$ | $1.29 \mathrm{E}+00$ |
| ARM | $2.24 \mathrm{E}+12$ | $4.58 \mathrm{E}+02$ | $1.79 \mathrm{E}+02$ | $4.90 \mathrm{E}+09$ | $1.26 \mathrm{E}+10$ | $3.78 \mathrm{E}+12$ | $3.73 \mathrm{E}+02$ | $1.87 \mathrm{E}+02$ | $1.01 \mathrm{E}+10$ | $2.02 \mathrm{E}+10$ | $1.33 \mathrm{E}+00$ |
| AZE | $1.28 \mathrm{E}+10$ | $9.45 \mathrm{E}-01$ | $3.30 \mathrm{E}-01$ | $1.35 \mathrm{E}+10$ | $3.87 \mathrm{E}+10$ | $5.22 \mathrm{E}+10$ | 7.90E-01 | 3.60E-01 | $6.62 \mathrm{E}+10$ | $1.45 \mathrm{E}+11$ | $1.94 \mathrm{E}+00$ |
| BLR | $6.51 \mathrm{E}+13$ | $2.15 \mathrm{E}+03$ | $7.79 \mathrm{E}+02$ | $3.02 \mathrm{E}+10$ | $8.35 \mathrm{E}+10$ | $2.96 \mathrm{E}+14$ | $4.97 \mathrm{E}+03$ | $1.89 \mathrm{E}+03$ | $5.96 \mathrm{E}+10$ | $1.57 \mathrm{E}+11$ | $3.05 \mathrm{E}+00$ |
| ALB | $8.37 \mathrm{E}+11$ | $9.99 \mathrm{E}+01$ | $4.86 \mathrm{E}+01$ | $8.38 \mathrm{E}+09$ | $1.72 \mathrm{E}+10$ | $1.31 \mathrm{E}+12$ | $1.01 \mathrm{E}+02$ | $4.55 \mathrm{E}+01$ | $1.30 \mathrm{E}+10$ | $2.88 \mathrm{E}+10$ | $1.19 \mathrm{E}+00$ |
| GEO | $1.19 \mathrm{E}+10$ | $1.81 \mathrm{E}+00$ | 7.40E-01 | $6.58 \mathrm{E}+09$ | $1.61 \mathrm{E}+10$ | $2.38 \mathrm{E}+10$ | $1.69 \mathrm{E}+00$ | 8.60E-01 | $1.41 \mathrm{E}+10$ | $2.76 \mathrm{E}+10$ | $1.52 \mathrm{E}+00$ |
| KAZ | $7.55 \mathrm{E}+12$ | $1.33 \mathrm{E}+02$ | $5.76 \mathrm{E}+01$ | $5.68 \mathrm{E}+10$ | $1.31 \mathrm{E}+11$ | $2.68 \mathrm{E}+13$ | $1.47 \mathrm{E}+02$ | $8.02 \mathrm{E}+01$ | $1.83 \mathrm{E}+11$ | $3.34 \mathrm{E}+11$ | $2.50 \mathrm{E}+00$ |
| KGZ | $9.85 \mathrm{E}+10$ | $4.10 \mathrm{E}+01$ | $1.14 \mathrm{E}+01$ | $2.40 \mathrm{E}+09$ | $8.68 \mathrm{E}+09$ | $2.80 \mathrm{E}+11$ | $4.61 \mathrm{E}+01$ | $1.78 \mathrm{E}+01$ | $6.07 \mathrm{E}+09$ | $1.58 \mathrm{E}+10$ | $2.15 \mathrm{E}+00$ |
| BGR | $4.52 \mathrm{E}+10$ | $1.57 \mathrm{E}+00$ | 5.90E-01 | $2.87 \mathrm{E}+10$ | $7.65 \mathrm{E}+10$ | $7.52 \mathrm{E}+10$ | $1.41 \mathrm{E}+00$ | $6.60 \mathrm{E}-01$ | $5.34 \mathrm{E}+10$ | $1.14 \mathrm{E}+11$ | $1.42 \mathrm{E}+00$ |
| MDA | $3.95 \mathrm{E}+10$ | $1.26 \mathrm{E}+01$ | $4.43 \mathrm{E}+00$ | $3.13 \mathrm{E}+09$ | $8.91 \mathrm{E}+09$ | $8.20 \mathrm{E}+10$ | $1.17 \mathrm{E}+01$ | $5.53 \mathrm{E}+00$ | $6.98 \mathrm{E}+09$ | $1.48 \mathrm{E}+10$ | $1.75 \mathrm{E}+00$ |
| RUS | $2.17 \mathrm{E}+13$ | $2.83 \mathrm{E}+01$ | $1.27 \mathrm{E}+01$ | $7.68 \mathrm{E}+11$ | $1.71 \mathrm{E}+12$ | $5.60 \mathrm{E}+13$ | $2.94 \mathrm{E}+01$ | $1.74 \mathrm{E}+01$ | $1.91 \mathrm{E}+12$ | $3.23 \mathrm{E}+12$ | $2.08 \mathrm{E}+00$ |
| TJK | $7.21 \mathrm{E}+09$ | $3.12 \mathrm{E}+00$ | 7.40E-01 | $2.31 \mathrm{E}+09$ | $9.74 \mathrm{E}+09$ | $3.01 \mathrm{E}+10$ | $4.61 \mathrm{E}+00$ | $1.74 \mathrm{E}+00$ | $6.52 \mathrm{E}+09$ | $1.73 \mathrm{E}+10$ | $2.82 \mathrm{E}+00$ |
| CHN | $1.87 \mathrm{E}+13$ | $8.19 \mathrm{E}+00$ | $3.45 \mathrm{E}+00$ | $2.28 \mathrm{E}+12$ | $5.42 \mathrm{E}+12$ | $4.82 \mathrm{E}+13$ | $6.46 \mathrm{E}+00$ | $3.51 \mathrm{E}+00$ | $7.45 \mathrm{E}+12$ | $1.37 \mathrm{E}+13$ | $1.38 \mathrm{E}+00$ |
| UKR | $4.42 \mathrm{E}+11$ | $5.12 \mathrm{E}+00$ | $1.68 \mathrm{E}+00$ | $8.62 \mathrm{E}+10$ | $2.63 \mathrm{E}+11$ | $1.30 \mathrm{E}+12$ | $7.97 \mathrm{E}+00$ | $3.43 \mathrm{E}+00$ | $1.64 \mathrm{E}+11$ | $3.80 \mathrm{E}+11$ | $2.67 \mathrm{E}+00$ |
| CZE | $3.12 \mathrm{E}+12$ | $2.40 \mathrm{E}+01$ | $1.44 \mathrm{E}+01$ | $1.30 \mathrm{E}+11$ | $2.17 \mathrm{E}+11$ | $3.87 \mathrm{E}+12$ | $1.77 \mathrm{E}+01$ | $1.35 \mathrm{E}+01$ | $2.18 \mathrm{E}+11$ | $2.87 \mathrm{E}+11$ | $1.06 \mathrm{E}+00$ |
| SVK | $4.95 \mathrm{E}+10$ | $1.03 \mathrm{E}+00$ | 5.50E-01 | $4.81 \mathrm{E}+10$ | $9.00 \mathrm{E}+10$ | $6.95 \mathrm{E}+10$ | 7.19E-01 | 5.10E-01 | $9.67 \mathrm{E}+10$ | $1.36 \mathrm{E}+11$ | $1.08 \mathrm{E}+00$ |
| EST | $1.10 \mathrm{E}+10$ | $8.04 \mathrm{E}-01$ | $5.00 \mathrm{E}-01$ | $1.36 \mathrm{E}+10$ | $2.19 \mathrm{E}+10$ | $1.57 \mathrm{E}+10$ | 7.19E-01 | $5.20 \mathrm{E}-01$ | $2.19 \mathrm{E}+10$ | $3.02 \mathrm{E}+10$ | $1.32 \mathrm{E}+00$ |
| LVA | $9.01 \mathrm{E}+09$ | $5.65 \mathrm{E}-01$ | $3.00 \mathrm{E}-01$ | $1.60 \mathrm{E}+10$ | $3.00 \mathrm{E}+10$ | $1.44 \mathrm{E}+10$ | $5.01 \mathrm{E}-01$ | $3.50 \mathrm{E}-01$ | $2.87 \mathrm{E}+10$ | $4.11 \mathrm{E}+10$ | $1.55 \mathrm{E}+00$ |
| HUN | $2.20 \mathrm{E}+13$ | $2.00 \mathrm{E}+02$ | $1.29 \mathrm{E}+02$ | $1.10 \mathrm{E}+11$ | $1.71 \mathrm{E}+11$ | $2.77 \mathrm{E}+13$ | $2.01 \mathrm{E}+02$ | $1.24 \mathrm{E}+02$ | $1.38 \mathrm{E}+11$ | $2.24 \mathrm{E}+11$ | $1.25 \mathrm{E}+00$ |


| WB | GDP05 | XRO5 | PPP05 | NGDP05 | RGDP05 | GDP11 | XR11 | PPP11 | NGDP11 | RGDP11 | PGDP ${ }^{1}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Code |  |  |  |  |  |  |  |  |  |  |  |
| LTU | $7.13 \mathrm{E}+10$ | $2.77 \mathrm{E}+00$ | $1.48 \mathrm{E}+00$ | $2.57 \mathrm{E}+10$ | $4.82 \mathrm{E}+10$ | $1.07 \mathrm{E}+11$ | $2.48 \mathrm{E}+00$ | $1.57 \mathrm{E}+00$ | $4.31 \mathrm{E}+10$ | $6.81 \mathrm{E}+10$ | $1.32 \mathrm{E}+00$ |
| MNG | $3.04 \mathrm{E}+12$ | $1.21 \mathrm{E}+03$ | $4.17 \mathrm{E}+02$ | $2.52 \mathrm{E}+09$ | $7.29 \mathrm{E}+09$ | $1.11 \mathrm{E}+13$ | $1.27 \mathrm{E}+03$ | $5.37 \mathrm{E}+02$ | $8.76 \mathrm{E}+09$ | $2.06 \mathrm{E}+10$ | $2.27 \mathrm{E}+00$ |
| HRV | $2.63 \mathrm{E}+11$ | $5.95 \mathrm{E}+00$ | $3.94 \mathrm{E}+00$ | $4.43 \mathrm{E}+10$ | $6.69 \mathrm{E}+10$ | $3.34 \mathrm{E}+11$ | $5.34 \mathrm{E}+00$ | $3.80 \mathrm{E}+00$ | $6.24 \mathrm{E}+10$ | $8.78 \mathrm{E}+10$ | $1.21 \mathrm{E}+00$ |
| SVN | $2.87 \mathrm{E}+10$ | $8.04 \mathrm{E}-01$ | $6.10 \mathrm{E}-01$ | $3.57 \mathrm{E}+10$ | $4.71 \mathrm{E}+10$ | $3.63 \mathrm{E}+10$ | $7.19 \mathrm{E}-01$ | $6.30 \mathrm{E}-01$ | $5.05 \mathrm{E}+10$ | $5.76 \mathrm{E}+10$ | $1.14 \mathrm{E}+00$ |
| MKD | $2.95 \mathrm{E}+11$ | $4.93 \mathrm{E}+01$ | $1.91 \mathrm{E}+01$ | $5.99 \mathrm{E}+09$ | $1.55 \mathrm{E}+10$ | $4.62 \mathrm{E}+11$ | $4.42 \mathrm{E}+01$ | $1.87 \mathrm{E}+01$ | $1.04 \mathrm{E}+10$ | $2.47 \mathrm{E}+10$ | $1.28 \mathrm{E}+00$ |
| BIH | $1.72 \mathrm{E}+10$ | $1.57 \mathrm{E}+00$ | $7.30 \mathrm{E}-01$ | $1.09 \mathrm{E}+10$ | $2.36 \mathrm{E}+10$ | $2.57 \mathrm{E}+10$ | $1.41 \mathrm{E}+00$ | $7.20 \mathrm{E}-01$ | $1.83 \mathrm{E}+10$ | $3.57 \mathrm{E}+10$ | $1.26 \mathrm{E}+00$ |
| POL | $9.84 \mathrm{E}+11$ | $3.24 \mathrm{E}+00$ | $1.90 \mathrm{E}+00$ | $3.04 \mathrm{E}+11$ | $5.18 \mathrm{E}+11$ | $1.52 \mathrm{E}+12$ | $2.96 \mathrm{E}+00$ | $1.82 \mathrm{E}+00$ | $5.11 \mathrm{E}+11$ | $8.32 \mathrm{E}+11$ | $1.18 \mathrm{E}+00$ |
| SRB | $1.68 \mathrm{E}+12$ | $6.67 \mathrm{E}+01$ | $2.72 \mathrm{E}+01$ | $2.52 \mathrm{E}+10$ | $6.19 \mathrm{E}+10$ | $3.18 \mathrm{E}+12$ | $7.33 \mathrm{E}+01$ | $3.73 \mathrm{E}+01$ | $4.34 \mathrm{E}+10$ | $8.53 \mathrm{E}+10$ | $1.68 \mathrm{E}+00$ |
| ROM | $3.00 \mathrm{E}+11$ | $2.91 \mathrm{E}+00$ | $1.42 \mathrm{E}+00$ | $1.03 \mathrm{E}+11$ | $2.11 \mathrm{E}+11$ | $6.03 \mathrm{E}+11$ | $3.05 \mathrm{E}+00$ | $1.61 \mathrm{E}+00$ | $1.98 \mathrm{E}+11$ | $3.74 \mathrm{E}+11$ | $1.64 \mathrm{E}+00$ |
| 1 GDP deflator, 2005 1 |  |  |  |  |  |  |  |  |  |  |  |


[^0]:    ${ }^{1}$ For any two strictly positive real numbers $a$ and $b$ their logarithmic mean is defined by $L(a, b)=(a-b) / \ln (a / b)$ if $a \neq b$ and $L(a, a)=a$. The properties of this mean are discussed in Balk (2008, 134-136). See Balk $(2008,85)$ for the derivation of expression (20).

[^1]:    ${ }^{2}$ This result was also derived by Rao and Rambaldi (2013) as the closed form GEKS solution from which a full panel of space-time consistent PPPs can be computed satisfying fixity of PPPs in a given year.

[^2]:    ${ }^{3}$ Annual PPP estimates are available from projects such as the Penn World Tables (Feenstra et al. 2013) and UQICD (Rao et al. 2014), which use the ICP data since its inception in 1970 and produce annual extrapolated series for countries, even if they have not consistently participated in the ICP

