Chapter 8: Index Calculation

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8 Index Calculation

8.1 Introduction

This chapter details the minimum standards for index calculation methods that are appropriate to fulfil the HICP legal framework. For discussions of alternatives and motivations behind choices made, the reader is referred to the specialist literature, such as ILO *et al.* (2004), v.d. Lippe (2001, 2007) and Balk (2008). A non-specialist introduction is provided by Ralph *et al*. (2015). Generic mathematical formulae are given to specify the principles and to provide an unambiguous base for the implementation in computer programming. In the Appendix a numeric example is provided that illustrates the entire index theory covered in this chapter.

The chapter starts in Section 8.2 with the higher-level indices, defining the HICP as a "Laspeyres-type index". Statistically, household expenditures are organised like an inverse tree, descending from the total to more and more detailed aggregates. There is always some level below which no or insufficient information is available for a further breakdown. This is the level of the elementary aggregates discussed in Section 8.3.

Section 8.4 continues by describing and explaining annual chain-linking, i.e. constructing longer time series. Annual chain-linking has the advantage of allowing the weights, the sample of products, and sample of outlets to be updated at each link, thus ensuring that the HICP is as representative of current consumers' expenditure pattern as possible. The calculation of monthly and annual rates of change follows in Section 8.5.

8.2 Higher-level indices

8.2.1 Legal obligations

The Framework Regulation[1] states in Article 3 (2) that "The harmonised indices shall be annually chain-linked Laspeyres-type indices." The latter term is defined in Article 2 (14) as follows:

> 'Laspeyres-type index' means the price index that measures the average change in prices from the price reference period to a comparison period using expenditure shares from a period prior to the price reference

[1] Regulation (EU) 2016/792 of the European Parliament and of the Council.

period, and where the expenditure shares are adjusted to reflect the prices of the price reference period.

A 'Laspeyres-type index' is defined as:

$$P^{0,t} = \sum \frac{p^t}{p^0} \cdot w^{0,b}.$$

The price of a product is denoted by p, the price reference period is denoted by 0, and the comparison period is denoted by t. Weights (w) are expenditure shares of a period (b) prior to the price reference period, and are adjusted to reflect the prices of the price reference period 0.

Further, the following definitions apply:

- According to Article 2 (16), the 'price reference period' (0) means the period to which the price of the comparison period is compared; for monthly indices, the price reference period is December of the previous year.
- The 'comparison period' (t) means the period for which the index is calculated.
- From Article 3 (1) of Commission Regulation (EU) No 1114/2010 it follows that 'weight reference period' (b) means the previous calendar year.[2]

8.2.2 The HICP concept

Conceptually an HICP, at any ECOICOP level, is designed as a Laspeyres-type index. What does this mean?

Consider a set of N products with prices p_i^τ and quantities q_i^τ $(i = 1, \ldots, N)$ for any time period τ considered. Then, the *short-term*[3] Laspeyres index *with annual weights*[4] for month $m = 1, \ldots, 12$ of current year t, mt being the comparison period, relative to December of the preceding year $t-1$, which is the price reference period[5], is given by[6]

$$P_L^{0t,mt} = \frac{\sum_{i=1}^{N} p_i^{mt} \cdot q_i^{t-1}}{\sum_{i=1}^{N} p_i^{0t} \cdot q_i^{t-1}} = \sum_{i=1}^{N} \frac{p_i^{mt}}{p_i^{0t}} \cdot \frac{p_i^{0t} \cdot q_i^{t-1}}{\sum_{j=1}^{N} p_j^{0t} \cdot q_j^{t-1}}, \qquad (8.1)$$

where the year $t-1$ is the weight reference period.[7]

It is important to realise that in the construction of the HICP, the month of December plays a double role: the month sometimes acts as comparison period, but always acts as price reference period. To distinguish clearly between these two roles, and to avoid complications, the notation is chosen as in Equation (8.1). Thus, being in year t and occurring in the numerator of Equation (8.1), December is labelled as $m = 12$, whereas being in year $t-1$ but occurring in the denominator of Equation (8.1) December is

[2] This is what is meant by "period prior to the price reference period" in Article 2 (14). Though following Article 3 (2) the year before the previous calendar year is in general the weights *data source*, this should not be mistaken for the weight *reference period*.

[3] "Short-term" here denotes the index relative to the price reference period rather than a month-on-month index.

[4] It is because of the annual weights for a monthly index that this exposition deviates from the standard textbook definition of the Laspeyres (price) index.

[5] In this chapter denoted as month 0 of year t.

[6] For reasons of presentation the index numbers are usually multiplied by 100.

[7] The price reference period $0t$ has been labelled "0" in the Framework Regulation, the comparison mt has been labelled "t", and the weight reference period $t-1$ has been labelled "b".

labelled as month $m = 0$ of year t. Put otherwise, each year t is considered as consisting of 13 months, running from December of year $t - 1$ to December of year t.

The weight reference period, used for the computation of the price indices for all the months of year t, is year $t - 1$ as this is the most recent calendar year. However, these expenditure shares are generally not yet available with sufficient accuracy early in year t when they are required for the first index number computation. Again, Regulation 1114/2010 stipulates that Member States shall

- produce HICPs using sub-index weights which reflect the consumers' expenditure pattern in the weight reference period and aim to be as representative as possible for consumers' expenditure patterns in the previous calendar year (Article 3 (1));
- therefore review and update HICP sub-index weights taking into account preliminary national accounts data on consumption patterns of year $t - 2$ (Article 3 (2));
- review annually whether or not there have been any important and sustained market developments affecting quantities in the sub-divisions of the HICP, between period $t - 2$ and period $t - 1$, in order to estimate weights that are as up-to-date as possible (Article 3 (4)).

In any case,

- HICP weights shall be price-updated to prices of the preceding December (Article 3 (5)).

Therefore, the usual strategy (see Chapter 3) is

1. to make the best estimate of consumers' expenditure patterns in the *weight reference period* year $t - 1$ based on preliminary national accounts data on consumption patterns of year $t - 2$, and
2. to execute price-updating reflecting the prices of the *price reference period* $0t$.

The thus derived weights are denoted by $w_i^{0t,t-1}$. Then, the Laspeyres-type index can be written as a weighted arithmetic mean of price relatives of products,

$$P^{0t,mt} = \sum_{i=1}^{N} \frac{p_i^{mt}}{p_i^{0t}} \cdot w_i^{0t,t-1}, \tag{8.2}$$

with weights adding up to unity. The difference between the Laspeyres index of Equation (8.1) and the Laspeyres-type index of Equation (8.2), consequently, lies in the weights.

Equation (8.2) corresponds to the expression in Article 2 (14) of the Framework Regulation. The target is to compare the prices of the current month to December of the previous year, based on annual expenditure weights from the previous calendar year. The question addressed in what follows is which estimate of the weights in the weight reference period should be used for a Laspeyres-type index.

8.2.3 Deriving the weights

The weights $w_i^{0t,t-1}$ do not correspond to observable expenditure shares, as they depend on prices from the price reference period and quantities from a period prior to the price reference period. They are called mixed-period weights. They can be derived from the observed[8] annual expenditures shares of year $t-2$

$$v_i^{t-2} = \frac{\left(p_i^{t-2} \cdot q_i^{t-2}\right)}{\sum_{j=1}^{N}\left(p_j^{t-2} \cdot q_j^{t-2}\right)} \tag{8.3}$$

in two ways.

Option 1 just uses v_i^{t-2} as the best approximation for the true but unknown weight w_i^{t-1}:

$$w_i^{t-1} := v_i^{t-2} = \frac{\left(p_i^{t-2} \cdot q_i^{t-2}\right)}{\sum_{j=1}^{N}\left(p_j^{t-2} \cdot q_j^{t-2}\right)}. \tag{8.3a}$$

In option 2 the expenditure shares are inflated by the price change between year $t-2$ and year $t-1$:

$$w_i^{t-1} := \frac{v_i^{t-2} \cdot \frac{p_i^{t-1}}{p_i^{t-2}}}{\sum_{j=1}^{N} v_j^{t-2} \cdot \frac{p_j^{t-1}}{p_j^{t-2}}} = \frac{\left(p_i^{t-1} \cdot q_i^{t-2}\right)}{\sum_{j=1}^{N}\left(p_j^{t-1} \cdot q_j^{t-2}\right)}. \tag{8.3b}$$

If goods and services are substitutes at such a rate that the expenditure on one product relative to another one is independent of the relative prices (Cobb-Douglas preferences), option 1 is the preferred approach. If goods and services are perfect complements, i.e. there is no substitutability between them and they are consumed in fixed proportions (Leontief preferences), the best approximation is the price-updated weight of option 2. Of course, the degree of substitutability may vary across products and the choice of either of the two options is not a trivial task.[9]

On this basis the price-updating of the weights to December $t-1$, the price reference period, should be carried out as follows:[10]

$$w_i^{0t,t-1} = \frac{w_i^{t-1} \cdot \frac{p_i^{0t}}{p_i^{t-1}}}{\sum_{j=1}^{N} w_j^{t-1} \cdot \frac{p_j^{0t}}{p_j^{t-1}}}, \tag{8.4}$$

where the (theoretical) expression using prices and quantities depends on whether Equation (8.3a) or Equation (8.3b) is used to estimate w_i^{t-1}. If the expression based on Equation (8.3a) is substituted in Equation (8.2), the resulting formula is known as the Young index.[11] Equation (8.3b) instead yields the Lowe index.

[8] The brackets indicate that only the product of price times quantity can be observed but neither prices nor quantities separately.
[9] Which of the two options performs better in practice can retrospectively be evaluated by comparing their outcomes to the one of using actual $t-1$ expenditures shares available later.
[10] In practice, price-updating may be executed at a higher aggregation level involving indices rather individual prices.
[11] Under *homogeneous* Cobb-Douglas preferences the Young index coincides with the Laspeyres index because expenditure shares remain constant. Yet, it should be borne in mind that the HICP is designed to assess price stability and it is not intended to be a cost of living index (see Recital (12) of the Framework Regulation).

When the correlation between expenditures and relative prices is negative, as it is usually the case, the Lowe index will tend to exceed the Laspeyres index. However, it is more difficult to generalise about the relationship between the Young index and the Laspeyres index. The Young could be greater or less than the Laspeyres depending on how sensitive the expenditures are to changes in relative prices.

8.2.4 Consistency in aggregation

A feature of the Laspeyres-type index is its *consistency in aggregation*. Suppose that the set of all products N is divided into mutually disjoint subsets N_h $(h = 1, \dots, H)$. Then, the following is true:

$$P^{0t,mt} = \sum_{h=1}^{H} \left(\sum_{i_h=1}^{N_h} \frac{p_{i_h}^{mt}}{p_{i_h}^{0t}} \cdot \frac{w_{i_h}^{0t,t-1}}{\sum_{j_h=1}^{N_h} w_{j_h}^{0t,t-1}} \right) \cdot \left(\sum_{i_h=1}^{N_h} w_{i_h}^{0t,t-1} \right) = \sum_{h=1}^{H} P_h^{0t,mt} \cdot w_h^{0t,t-1}. \tag{8.5}$$

Thus, the overall Laspeyres-type index is a weighted arithmetic mean of the Laspeyres-type indices for the subsets of products, defined as

$$P_h^{0t,mt} = \sum_{i_h=1}^{N_h} \frac{p_{i_h}^{mt}}{p_{i_h}^{0t}} \cdot \frac{w_{i_h}^{0t,t-1}}{\sum_{j_h=1}^{N_h} w_{j_h}^{0t,t-1}}. \tag{8.6}$$

The weights

$$w_h^{0t,t-1} = \sum_{i_h=1}^{N_h} w_{i_h}^{0t,t-1}. \tag{8.7}$$

are the mixed-period expenditure shares of the *subsets*. Put otherwise, the overall Laspeyres-type index can be calculated in one stage from the product price relatives, as in Equation (8.2), or in two stages, as in Equation (8.5): from product price relatives to subset Laspeyres-type indices and then from these subset indices to the overall index. For statistical practice this is a very useful feature.

The geographical aggregation to the euro area and European Union works in the same way, where h then denotes Member States rather than subsets of products (see Chapter 11).

8.2.5 Higher-level compilation

In practice, the higher-level index compilation is based on product categories at ECOICOP and lower levels (elementary product groups and elementary aggregates). Correspondingly, price relatives are replaced by price indices for these categories ("sub-indices"). Equations (8.2) and (8.4) make clear that quantity data are not required. What is needed are estimates of the expenditure shares for the weight reference period, sub-indices for price-updating those shares, and sub-indices for the comparison month relative to the price reference period. The indices allowed for elementary aggregation are the topic of Section 8.3.

8.3 Indices for elementary aggregates

8.3.1 Legal obligations

Commission Regulation (EC) No 1749/96 states in Article 7 that "HICPs shall be compiled using either of the two formulae given in paragraph 1 of Annex II to this Regulation or an alternative comparable formula which does not result in an index which differs systematically from an index compiled by either of the given formulae by more than one tenth of one percentage point on average over one year against the previous year." Said Annex defines the formulae to be used in compiling elementary aggregates:[12]

> 1. When compiling price indices for elementary aggregates either the ratio of arithmetic mean prices
>
> $$\frac{\frac{1}{N}\sum p^t}{\frac{1}{N}\sum p^0}$$
>
> or the ratio of geometric mean prices
>
> $$\frac{(\prod p^t)^{\frac{1}{N}}}{(\prod p^0)^{\frac{1}{N}}},$$
>
> where p^t is the current price, p^0 the reference price and N the number of such prices in the elementary aggregate, shall be used. An alternative formula may be used provided that it fulfils the comparability requirement laid down in Article 7.
>
> 2. The arithmetic mean of price relatives
>
> $$\frac{1}{N}\sum \frac{p^t}{p^0}$$
>
> should not normally be used, as it will in many circumstances result in failure to meet the comparability requirement. It may be used exceptionally where it can be shown not to fail the comparability requirement.

According to this Regulation, the arithmetic mean of price relatives must not be used where chaining is more frequent than annual. The *de facto* ban on this index in paragraph 2 (comparability requirement) is for good reasons; the underlying phenomenon is that it does not satisfy the circularity (transitivity) test.[13] This failure implies an indefinite bias, which in practice may show up as chain drift:[14]

[12] Notice that the notation in this quote is adapted to fit the notation used in the Framework Regulation.

[13] The test asks that the chain-linked index defined as the product of the short-term index going from period 0 to 1 times the short-term index going from period 1 to 2 should equal the direct index that compares the prices of period 2 with those of period 0.

[14] This bias would only be definitely upward, i.e. the chain-linked index greater than the direct index and the covariance negative, if $p^2 = p^0$. However, this claim in the literature uses the special case of the so-called time reversal test as an argument, which is known as the country reversal test in interspatial comparisons.

$$\left(\frac{1}{N}\sum\frac{p^2}{p^0}\right)-\left(\frac{1}{N}\sum\frac{p^1}{p^0}\right)\cdot\left(\frac{1}{N}\sum\frac{p^2}{p^1}\right)=\text{Cov}\left[\frac{p^1}{p^0},\frac{p^2}{p^1}\right]. \qquad (8.8)$$

8.3.2 Index theory and practice

Each 5-digit ECOICOP expenditure category is usually built up from subsets, called elementary product groups, and these in turn are built up from elementary aggregates.[15]

Thus, practical HICPs are constructed in two stages:

1. a first stage at the lowest level of aggregation where price information is available but associated expenditure information is not available, and
2. a second stage of aggregation where expenditure information is available at a higher level of aggregation.

The aggregates that pertain to the first stage of aggregation are called *elementary aggregates*. Article 2 of the abovementioned Regulation 1749/96 provides the following definitions:

(i) 'Elementary aggregate index' is a price index for an elementary aggregate comprising only price data.
(j) 'Elementary aggregate' refers to the expenditure or consumption covered by the most detailed level of stratification of the HICP and within which reliable expenditure information is not available for weighting purposes.

Two-stage aggregation implies losing consistency in aggregation at the lower level since a Laspeyres-type index cannot be calculated in the first step of the process but an elementary aggregate index $P_i^{0t,mt}$ is used instead:

$$P^{0t,mt}=\sum_{i=1}^{N}P_i^{0t,mt}\cdot w_i^{0t,t-1}, \qquad (8.9)$$

where $i=1,\dots,N$ from now on denotes elementary aggregates rather than individual products.

Although, due to new and disappearing products, the set of products available for sampling generally varies from month to month, for the ease of exposition the presentation here is restricted to matched-model indices, i.e. it is assumed that there are no missing observations and no changes in the quality of the products sampled so that the sets of prices are perfectly matched.

Thus, consider an elementary aggregate with a set of K common products in any time period considered. For measuring the price change of an elementary aggregate many formulae have been proposed in the literature, of which the Dutot and Jevons index are preferred.

Dutot index:
ratio of arithmetic mean prices

$$P_D^{0t,mt}=\frac{\frac{1}{K}\sum_{k=1}^{K}p_k^{mt}}{\frac{1}{K}\sum_{k=1}^{K}p_k^{0t}} \qquad (8.10)$$

[15] Elementary product groups are sets of products that are sampled in order to represent one or more consumption segments in the HICP (see Chapter 4).

Jevons index:
ratio of geometric mean prices, or
geometric mean of price relatives

$$P_J^{0t,mt} = \frac{\left(\prod_{k=1}^{K} p_k^{mt}\right)^{\frac{1}{K}}}{\left(\prod_{k=1}^{K} p_k^{0t}\right)^{\frac{1}{K}}} = \left(\prod_{k=1}^{K} \frac{p_k^{mt}}{p_k^{0t}}\right)^{\frac{1}{K}} \tag{8.11}$$

Carli index:
arithmetic mean of price relatives

$$P_C^{0t,mt} = \frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{mt}}{p_k^{0t}} \tag{8.12}$$

If the samples remain unchanged throughout the year, as it is assumed here, then the chain-linked Dutot and Jevons index reduce to the respective direct indices. For example using the ratio of arithmetic means:[16]

$$\begin{aligned} CP_D^{0t,mt} &= P_D^{0t,1t} \cdot P_D^{1t,2t} \cdot \ldots \cdot P_D^{(m-1)t,mt} \\ &= \frac{\frac{1}{K}\sum_{k=1}^{K} p_k^{1t}}{\frac{1}{K}\sum_{k=1}^{K} p_k^{0t}} \cdot \frac{\frac{1}{K}\sum_{k=1}^{K} p_k^{2t}}{\frac{1}{K}\sum_{k=1}^{K} p_k^{1t}} \cdot \ldots \cdot \frac{\frac{1}{K}\sum_{k=1}^{K} p_k^{mt}}{\frac{1}{K}\sum_{k=1}^{K} p_k^{(m-1)t}} = \frac{\frac{1}{K}\sum_{k=1}^{K} p_k^{mt}}{\frac{1}{K}\sum_{k=1}^{K} p_k^{0t}} = P_D^{0t,mt}. \end{aligned} \tag{8.13}$$

$CP_D^{0t,mt}$ becomes the simple ratio of arithmetic means $P_D^{0t,mt}$ (or similarly with the geometric formula described above). Hence, the index for an elementary aggregate may be calculated as a chain-linked month-on-month index using one of the above two preferred formulae. However, if the set of products available for sampling varies from month to month, using monthly chain-linking and replenishment is *not* a preferred method in many situations.[17]

The chain-linked Carli index would have provided the following elementary aggregate index:

$$\begin{aligned} CP_C^{0t,mt} &= P_C^{0t,1t} \cdot P_C^{1t,2t} \cdot \ldots \cdot P_C^{(m-1)t,mt} \\ &= \left(\frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{1t}}{p_k^{0t}}\right) \cdot \left(\frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{2t}}{p_k^{1t}}\right) \cdot \ldots \cdot \left(\frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{mt}}{p_k^{(m-1)t}}\right) \neq \frac{1}{K}\sum_{k=1}^{K} \frac{p_k^{mt}}{p_k^{0t}} = P_C^{0t,mt}. \end{aligned} \tag{8.14}$$

$CP_C^{0t,mt}$ does *not* reduce to a direct Carli index $P_C^{0t,mt}$ if the samples do not change.

8.3.3 Compilation issues

There has been much research effort spent on finding the best index for an elementary aggregate, and quite a lot is known about the behaviour of the various proposals under various assumptions, but only a small number of hard facts have been established. Expanding $P_J^{0t,mt}$ by a second-order Taylor series approximation around the arithmetic mean prices $p_k^{0t} = \bar{p}^{0t}$ and $p_k^{mt} = \bar{p}^{mt}$ for all $k = 1, \ldots, K$ (K being sufficiently large), it

[16] Where misunderstanding is possible, here and in the sequel, months and years, such as $m-1$ or $t-1$, are put within brackets.

[17] Indeed, Chapter 6 gives details how appropriate replacements and quality adjustments keep the samples basically unchanged throughout the year. This is particularly important when products disappear on discount since chain-linking only matched products then introduces a severe downward drift.

can be verified that the difference between the Dutot and the Jevons index depends on the change over time of the squared coefficient of variation of individual prices:

$$P_J^{0t,mt} \approx P_D^{0t,mt}\left(1+\frac{1}{2}\frac{\mathrm{Var}\left[p_k^{0t}\right]}{(\mathrm{E}[p_k^{0t}])^2}-\frac{1}{2}\frac{\mathrm{Var}[p_k^{mt}]}{(\mathrm{E}[p_k^{mt}])^2}\right). \tag{8.15}$$

Likewise, expanding $P_D^{0t,mt}$ around the geometric mean prices $\ln p_k^{0t} = \ln \bar{p}^{0t}$ and $\ln p_k^{mt} = \ln \bar{p}^{mt}$, the following second-order approximate relationship is obtained:

$$P_D^{0t,mt} \approx P_J^{0t,mt}\left(1-\frac{1}{2}\mathrm{Var}\left[\ln p_k^{0t}\right]+\frac{1}{2}\mathrm{Var}[\ln p_k^{mt}]\right). \tag{8.16}$$

However, whether the difference between the Dutot and Jevons index is positive or negative, large or small, is an empirical matter. Still, Silver and Heravi (2007, J. Econometrics) show that this difference depends on the change over time in price dispersion. Some of the price dispersion will be due to product heterogeneity.

The Dutot index has the drawback of tending to primarily reflect the price development of products at relatively high prices. This can be seen as follows. Consider again Equation (8.10),

$$P_D^{0t,mt} = \frac{\frac{1}{K}\sum_{k=1}^{K} p_k^{mt}}{\frac{1}{K}\sum_{k=1}^{K} p_k^{0t}} = \sum_{k=1}^{K}\frac{p_k^{mt}}{p_k^{0t}}\cdot\frac{p_k^{0t}}{\sum_{l=1}^{K} p_l^{0t}}. \tag{8.10a}$$

It appears that the Dutot index can be written as a weighted arithmetic mean of individual price relatives, its weights being relative prices in the price reference period. Products with higher relative prices get a higher weight, and products with lower relative prices get a lower weight in the Dutot index. Thus, it is advised to use the Dutot index only for elementary aggregates in which the relative prices exhibit small variance, i.e. the price levels are similar.

A quick glance at the formula of the Jevons index, Equation (8.11), reveals that it is not a linear index. Despite this, a linear approximation to the Jevons index yields the BMW index and "weights" square root of the inverse price relatives:[18]

$$P_J^{0t,mt} = \left(\prod_{k=1}^{K}\frac{p_k^{mt}}{p_k^{0t}}\right)^{\frac{1}{K}} \approx \sum_{k=1}^{K}\frac{p_k^{mt}}{p_k^{0t}}\cdot\frac{\sqrt{p_k^{0t}/p_k^{mt}}}{\sum_{l=1}^{K}\sqrt{p_l^{0t}/p_l^{mt}}}. \tag{8.11a}$$

Thus, this index is more robust with respect to the variance of relative prices within an elementary aggregate.

There are two final important tests that should be added to the circularity (transitivity) test above. The first one is the commensurability test, i.e. if the units of measurement for each product are changed, then the elementary aggregate index remains unchanged. The Dutot index $P_D^{0t,mt}$ fails the commensurability test since the price levels are affected by the measurement unit. If there are heterogeneous products in the elementary aggregate,

[18] See Mehrhoff, in: v.d. Lippe (2007); Balk (2008) independently described this index as the equally weighted Walsh index. Because it is a linear index, it can directly be compared to other linear indices such as Dutot or Laspeyres(-type) using a theorem of v. Bortkiewicz. It should be noted that the approximation is exact when the number of products is not greater than two.

this is a rather serious failure and, hence, price statisticians should be careful in using this index under these conditions. The other one is the test of determinateness as to prices, i.e. if any single price tends to zero, then the index should not tend to zero or plus infinity. It can be verified that the Jevons index does not satisfy this test. Thus, when using the Jevons index $P_J^{0t,mt}$, care must be taken to bound the prices away from zero in order to avoid a meaningless index number value (see Section 7.5).

8.4 Annual chain-linking

8.4.1 December as the linking month

The Laspeyres-type index defined by Equation (8.2) compares prices of month m of year t to December of the preceding year, $t-1$. When t moves through time, there is for each year a series of 13 index numbers, running from December of year $t-1$ (its index number being equal to 100) to December of year t.[19]

Now these separate 13-month series can be chain-linked together into a single long-term series, which compares month m of year t to some earlier period. The HICP uses thereby December as the linking month (see Chart). The annually chain-linked Laspeyres-type index

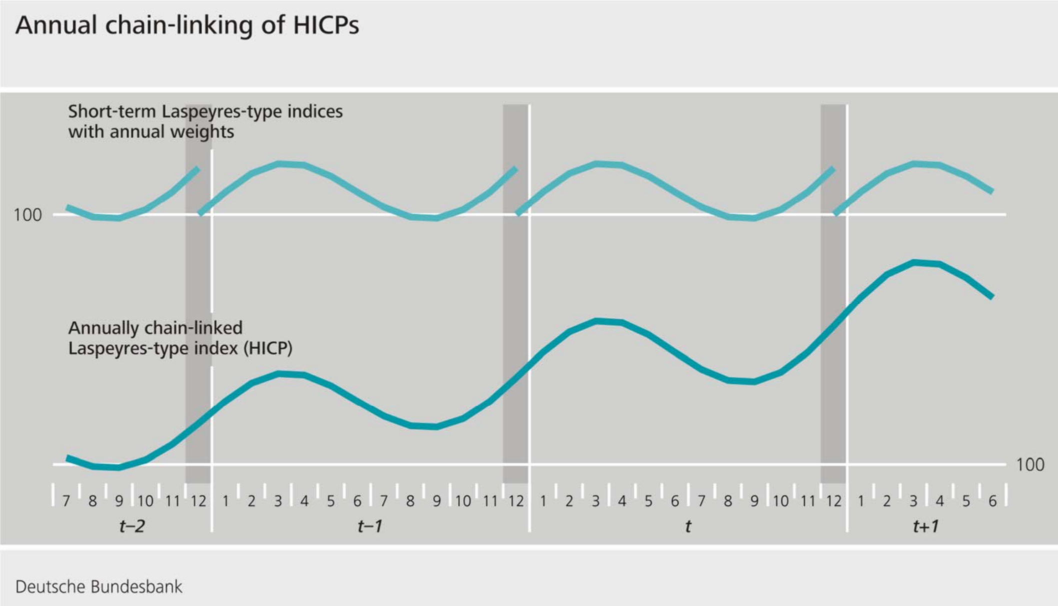
$$\begin{aligned} CP^{b,mt} &= \left(P^{b,12(0)} \cdot P^{0(1),12(1)} \cdot \ldots \cdot P^{0(t-2),12(t-2)} \cdot P^{0(t-1),12(t-1)}\right) \cdot P^{0t,mt} \\ &= CP^{b,12(t-1)} \cdot P^{0t,mt} \end{aligned} \qquad (8.17)$$

compares month m of year t with a certain year b.[20] Recall that month 0 of any year τ is the same as month 12 of year $\tau-1$. Notice that, in principle, each short-term index of this chain-linked series employs a different weight reference period and the set of products may vary through time as some disappear from the market and others enter the market. In this case year b, used in the initial link of the long-term series, is the *index reference period*.[21]

[19] Changes in the production methods could be incorporated each December. One should be certain, however, that such changes do not affect significantly the structural characteristics of the entire series of index numbers (see Chapter 10).

[20] For the HICP this has initially been the year 1996, where the annual weights of 1996 have *not* been price-updated to December of that year. Strictly speaking, the HICP started with the index for January 1997, where 1996 weights were price-updated from the annual average to December.

[21] Article 2 (15) of the Framework Regulation specifies that 'index reference period' means the period for which the index is set to 100 index points.



8.4.2 Loss of consistency in aggregation

The technique of chain-linking indices implies losing consistency in aggregation also at higher levels. To see this, return to Equation (8.5) and recall that the set of elementary aggregates[22] N, now without loss of generality assumed to be constant over time, is divided into mutually disjoint subsets N_h $(h = 1, \dots, H)$. Then, as was explained above, the consistency in aggregation of the short-term Laspeyres-type index (now at higher levels) implies that

$$P^{0t,mt} = \sum_{h=1}^{H} P_h^{0t,mt} \cdot w_h^{0t,t-1}. \tag{8.18}$$

This holds for any month $m = 1, \dots, 12$, any year t and any subset of elementary aggregates.

Once chain-linked, however, there does not exist such a relation; that is, there does *not* exist a set of weights (adding up to unity) such that

$$CP^{b,mt} = \sum_{h=1}^{H} CP_h^{b,mt} \cdot w_h \,, \tag{8.19}$$

where $CP_h^{b,mt}$ is the chain-linked index for subset $h = 1, \dots, H$. Even should the weights be constant over the entire time span this would not imply consistency in aggregation of the chain-linked index, because then the chain-linked index still does not reduce to a direct Laspeyres-type index.[23]

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[22] See Section 8.3.2 for the change in notation as regards the elementary aggregates.
[23] This is due to the failure of the transitivity in prices for fixed weights test, a (weak) variant of the circularity test, where the weights are held constant while making all price comparison. The argument found in the literature that the Lowe index is (strongly) transitive would hold true only if the weights, starting from some out-of-date base period, were derived by continual price-updating rather than being genuinely newly observed every year. The only index satisfying the circularity test is a weighted geometric mean of all the individual price ratios, the weights being

8.4.3 Aggregation of chain-linked sub-indices

Aggregation is a *hierarchical process*, i.e. elementary aggregates are first aggregated to elementary product groups, which are then aggregated to ECOICOP 5-digit sub-class indices, which are in turn aggregated to ECOICOP 4-digit class indices, etc. For maximum precision, aggregation should be performed on unrounded indices.

It is important to note that only unchained indices should be aggregated. This applies to all levels of index aggregation. Once chain-linked, index number series are no longer consistent in aggregation.

As noted above, the HICP is calculated as a series of 13 month (December to December) aggregate index numbers where the December of each year is chosen as the linking month. These index numbers are chain-linked by multiplying the chain-linked index for December of the previous year series by the index number for every month of the short-term series (divided by 100). The chain-linked index number for month m of year t is calculated as in the second part of Equation (8.17):

$$CP^{b,mt} = CP^{b,12(t-1)} \cdot P^{0t,mt}. \tag{8.20}$$

For the HICP, chain-linking is required each year, and December is the linking month.

From time to time, it may be necessary to produce bespoke aggregates which are often requested by users. To achieve this index compilers must use unchained indices. To obtain unchained index numbers from their chain-linked counterparts, one must divide the chain-linked index number of each month of each year by the chain-linked December index of the previous year (and multiply by 100). The unchained index for month m of year t is calculated by solving Equation (8.20) for the short-term series:

$$P^{0t,mt} = \frac{CP^{b,mt}}{CP^{b,12(t-1)}}. \tag{8.21}$$

In order to produce bespoke aggregates from published chain-linked index numbers, the first step is always to unchain the relevant components; the starting point is *unchained index number series*. Starting from this point the unchained aggregate index numbers are aggregated together using their relevant weights to produce unchained index number series for the new bespoke aggregate. These index numbers are then chain-linked.

8.4.4 Re-referencing

Every ten years, it is required to re-reference, or rescale, the HICP to a more recent index reference period.[24] This can be achieved by rescaling the chain-linked index (8.17); that is, dividing by the arithmetic mean of the index numbers for the months of the index reference year. Thus, the chain-linked index for month m of year t relative to the current index reference period 2015[25] is defined by

constant through time. After all, the whole point of chain-linking has been to enable the weights to be continually updated to take account of the changing consumption patterns.

[24] Article 5 (6) of the Framework Regulation. This article also requires rescaling in the case of a major methodological change.

[25] Article 5 (5) of the Framework Regulation.

$$CP_{2015}^{b,mt} = \frac{CP^{b,mt}}{\frac{1}{12}\sum_{m=1}^{12} CP^{b,m(2015)}}. \tag{8.22}$$

It is instructive to see what happens in the months of the index reference year. It turns out that

$$\begin{aligned} CP_{2015}^{b,m(2015)} &= \frac{P^{0(2015),m(2015)}}{\frac{1}{12}\sum_{m=1}^{12} P^{0(2015),m(2015)}} = \frac{P^{0(2015),m(2015)}}{P^{0(2015),2015}} \\ &= \sum_{i=1}^{N} \left(\frac{P_i^{0(2015),m(2015)}}{P_i^{0(2015),2015}} \right) \cdot \left(w_i^{0(2015),2014} \cdot \frac{P_i^{0(2015),2015}}{P^{0(2015),2015}} \right) \\ &= \sum_{i=1}^{N} P_i^{2015,m(2015)} \cdot w_i^{2015,2014} = P^{2015,m(2015)}, \end{aligned} \tag{8.23}$$

where $P^{0(2015),2015}$ is the arithmetic mean of the short-term index in this period. Thus, in this situation, the chain-linked index is again a Laspeyres-type index, where the price reference period and index reference period coincide, i.e. the price-updated weights are:

$$w_i^{2015,2014} = w_i^{0(2015),2014} \cdot \frac{P_i^{0(2015),2015}}{P^{0(2015),2015}}. \tag{8.4a}$$

Apart from rounding errors, re-referencing of a series of index numbers has no impact on monthly or annual rates of change.

8.5 Rates of change

8.5.1 Monthly rate

The rate of price change between month $m-1$ and month m ($m = 1, \dots, 12$), both of year t, is calculated as the relative change of the chain-linked Laspeyres-type indices, and usually presented as a percentage; that is, times 100%:

$$\frac{CP^{b,mt}}{CP^{b,(m-1)t}} - 1 = \frac{P^{0t,mt}}{P^{0t,(m-1)t}} - 1. \tag{8.24}$$

Recall that month 0 of year t is the same as month 12 of year $t-1$. Also, the computations are dependent on the short-term indices, rather than the chain-linked index.

Using Equation (8.2), Equation (8.24) can be rewritten as

$$\frac{P^{0t,mt}}{P^{0t,(m-1)t}} - 1 = \frac{P^{0t,mt} - P^{0t,(m-1)t}}{P^{0t,(m-1)t}} = \frac{\sum_{i=1}^{N}\left(P_i^{0t,mt} - P_i^{0t,(m-1)t}\right)\cdot w_i^{0t,t-1}}{P^{0t,(m-1)t}}$$
$$= \sum_{i=1}^{N}\left(\frac{P_i^{0t,mt}}{P_i^{0t,(m-1)t}} - 1\right)\cdot\left(w_i^{0t,t-1}\cdot\frac{P_i^{0t,(m-1)t}}{P^{0t,(m-1)t}}\right) \quad (8.25)$$
$$= \sum_{i=1}^{N}\left(P_i^{(m-1)t,mt} - 1\right)\cdot w_i^{(m-1)t,t-1} = P^{(m-1)t,mt} - 1,$$

where $P^{(m-1)t,mt}$ is again a Laspeyres-type index, but now with month $m-1$ as price reference period; that is, the weights are price-updated to month $m-1$ of year t:

$$w_i^{(m-1)t,t-1} = w_i^{0t,t-1}\cdot\frac{P_i^{0t,(m-1)t}}{P^{0t,(m-1)t}}. \quad (8.4b)$$

The *contribution* of sub-index N, for example, to the overall monthly price change is

$$\Delta_N^{(m-1)t,mt} = \left(P_N^{(m-1)t,mt} - 1\right)\cdot w_N^{(m-1)t,t-1}$$
$$= \left(\frac{P_N^{0t,mt}}{P_N^{0t,(m-1)t}} - 1\right)\cdot\left(w_N^{0t,t-1}\cdot\frac{P_N^{0t,(m-1)t}}{P^{0t,(m-1)t}}\right), \quad (8.25a)$$

i.e. rate of change times price-updated weight.[26] The contribution of a higher aggregate is the sum of contributions of constituent sub-indices.

8.5.2 Annual rate

Very much like the monthly rate, the annual rate of price change between month m of year t and the same month of year $t-1$ is calculated as the relative change of the chain-linked Laspeyres-type indices. The result when the chain-linked index for month m of year t is divided by the chain-linked index for month m of year $t-1$ is a chain-linked index consisting of two parts:

1. a ratio of two Laspeyres-type indices for December of year $t-1$ and month m of year $t-1$, respectively, both relative to December of year $t-2$, multiplied by
2. a Laspeyres-type index for month m of year t relative to December of year $t-1$.

Thus,

$$\frac{CP^{b,mt}}{CP^{b,m(t-1)}} = \frac{P^{0(t-1),12(t-1)}}{P^{0(t-1),m(t-1)}}\cdot P^{0t,mt} = P^{m(t-1),12(t-1)}\cdot P^{0t,mt}. \quad (8.26)$$

The right-hand side of Equation (8.26) shows that the ratio of two Laspeyres-type indices can be written as a Laspeyres-type index for December of year $t-1$ relative to month m of the same year with price-updated weights:

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[26] It should be noted that the contribution in January becomes just $\left(P_N^{0t,1t} - 1\right)\cdot w_N^{0t,t-1}$, which means that neither the short-term index is rescaled nor the weight price-updated.

$$w_i^{m(t-1),t-2} = w_i^{0(t-1),t-2} \cdot \frac{P_i^{0(t-1),m(t-1)}}{P^{0(t-1),m(t-1)}}. \tag{8.4c}$$

Since Equation (8.26) is a chain-linked index, the rate of change between corresponding months m of adjacent years $t-1$ and t cannot be written as a weighted mean of sub-indices. It is, therefore, not possible to express the *contribution* of a sub-index to the overall annual price change as a *simple* formula.

This is because chain-linked time series for the HICP may contain statistically-related breaks from one year to another.[27] To see this, resort to the basket interpretation of the Laspeyres index in Equation (8.1) and rewrite Equation (8.26) as follows:[28]

$$\begin{aligned} P_L^{m(t-1),12(t-1)} \cdot P_L^{0t,mt} &= \frac{\sum_{i=1}^N p_i^{12(t-1)} \cdot q_i^{t-2}}{\sum_{i=1}^N p_i^{m(t-1)} \cdot q_i^{t-2}} \cdot \frac{\sum_{i=1}^N p_i^{mt} \cdot q_i^{t-1}}{\sum_{i=1}^N p_i^{0t} \cdot q_i^{t-1}} \cdot \frac{P_L^{m(t-1),mt}}{P_L^{m(t-1),mt}} \\ &= \frac{\sum_{i=1}^N p_i^{mt} \cdot q_i^{t-1}}{\sum_{i=1}^N p_i^{m(t-1)} \cdot q_i^{t-1}} \cdot \left(\frac{\sum_{i=1}^N p_i^{m(t-1)} \cdot q_i^{t-1}}{\sum_{i=1}^N p_i^{0t} \cdot q_i^{t-1}} \Big/ \frac{\sum_{i=1}^N p_i^{m(t-1)} \cdot q_i^{t-2}}{\sum_{i=1}^N p_i^{0t} \cdot q_i^{t-2}} \right). \end{aligned} \tag{8.27}$$

The first factor of the second part of Equation (8.27) measures pure price change. The second factor in brackets is a technical distortion that will generally differ from unity. More specifically, the less the price structure in month m of the previous year deviates from that of December of the same year and the smaller the relative quantity change from year to year, the smaller such breaks arising from the change of the weight basis are.

Using the theorem of v. Bortkiewicz, one arrives at the following expression for the annual rate of a fixed basket in an annually chain-linked Laspeyres index:[29]

$$P_L^{m(t-1),12(t-1)} \cdot P_L^{0t,mt} = P_L^{m(t-1),mt} \cdot \left(1 + \frac{\operatorname{Cov}\left[\frac{p_i^{m(t-1)}}{p_i^{0t}}, \frac{q_i^{t-1}}{q_i^{t-2}}\right]}{\operatorname{E}\left[\frac{p_i^{m(t-1)}}{p_i^{0t}}\right] \cdot \operatorname{E}\left[\frac{q_i^{t-1}}{q_i^{t-2}}\right]} \right). \tag{8.27a}$$

Thus, the technical distortion vanishes if and only if there is no (weighted) correlation between the price change from December of the previous year to month m of year $t-1$ and quantity changes from year $t-2$ to year $t-1$.[30] It is very unlikely that this criterion holds in reality. Worst of all, not even the sign is determined *a priori*.

The measure of inflation that is given prominence is the annual rate in the HICP. When the change in this rate between two consecutive months, i.e.

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[27] See Section 8.4.2 for why this holds true even should the weights be constant over the entire time span (reference to the failure of the transitivity in prices for fixed weights test).

[28] The extension of this decomposition to the Young and Lowe indices is straightforward using the expressions involving quantities.

[29] In principle, this distortion of the annual rate could be avoided by chain-linking over the same month of the previous year (which would then also be the price reference period) rather than December throughout. However, the results from this so-called over-the-year technique would allow only for the meaningful interpretation of annual rates, while the infra-annual pattern of the chain-linked series could be spurious and distorted. Due to the disturbing time series properties, the over-the-year technique should be avoided.

[30] Of course, should all prices or quantities change at the same rate, the variance and also the covariance would be zero.

$$\pi^{mt} - \pi^{(m-1)t} = \left(\frac{CP^{b,mt}}{CP^{b,m(t-1)}} - 1 \right) - \left(\frac{CP^{b,(m-1)t}}{CP^{b,(m-1)(t-1)}} - 1 \right), \tag{8.28}$$

is described, "base effects"[31] are often mentioned. In a purely technical sense, the contribution of the monthly rate between month $m-1$ and month m, both of year $t-1$, to the change in the annual rate could be referred to as a base, or denominator, effect.

To see this, approximate Equation (8.28) by:

$$\pi^{mt} - \pi^{(m-1)t} \approx \left(\frac{CP^{b,mt}}{CP^{b,(m-1)t}} - 1 \right) - \left(\frac{CP^{b,m(t-1)}}{CP^{b,(m-1)(t-1)}} - 1 \right), \tag{8.28a}$$

where the approximation $\pi^{mt} \approx \ln CP^{b,mt} - \ln CP^{b,m(t-1)}$ (and similarly with $\pi^{(m-1)t}$) is used.[32]

Thus, the difference between the annual rates in two subsequent months is approximately the same as the difference between the monthly rate in the current month and the monthly rate one year earlier. This illustrates the fact that the change in the annual rate from one month to the next reflects both recent price changes and price movements twelve months earlier. For example, if the index declines in the period from November to December of year $t-1$, this will increase the change in the annual rate between November to December of year t.[33]

8.5.3 Contributions to the annual rate

In general, the concept of contributions *per se* is no longer well defined in chain-linked indices and different approaches give different results with different properties. Two competing lines will be presented here: one that assures additivity ("Ribe contributions") and one that has a meaningful interpretation ("statistical contributions").

As one of many possible conventions circumventing the problems associated with the abovementioned statistical break, Ribe (1999, *mimeo*) showed how the rate of change of a chain-linked index can be decomposed into the sum of the contributions of the sub-indices covered by the higher aggregate.

The annual rate can be decomposed additively into a "this-year term" (TYT) and a "last-year term" (LYT), according to[34]

$$\begin{aligned} P^{m(t-1),12(t-1)} \cdot P^{0t,mt} - 1 &= \left[P^{m(t-1),12(t-1)} \cdot \left(P^{0t,mt} - 1 \right) \right] \\ &\quad + \left[\left(P^{m(t-1),12(t-1)} - 1 \right) \right] \\ &= TYT^{m(t-1),mt} + LYT^{m(t-1),mt}, \end{aligned} \tag{8.29}$$

[31] See *Monthly Bulletin*, ECB, January 2005.
[32] Since the total HICP base effect is the monthly rate one year earlier, the contribution of a sub-index to the total HICP base effect is its contribution to the previous year monthly rate.
[33] However, the monthly rate and, hence, the base effect, are affected by seasonally fluctuating prices, which is not desirable from an economic perspective.
[34] Note that the short-term indices are rescaled, see the third part of Equation (8.26), and the weights are price-updated, see Equation (8.4c).

where the first bracketed term is the this-year term from December of year $t-1$ to month m of year t, adjusted "to take account of the differences in the overall price levels involved in the comparisons", and the second one is the last-year term from month m of year $t-1$ to December of the same year.

Again, the interesting point is to decompose the overall annual price change into components by sub-indices. The this-year term and the last-year term can now each be decomposed, analogously to the monthly contribution for sub-index N, according to

$$\begin{aligned} TYT_N^{m(t-1),mt} &= P^{m(t-1),12(t-1)} \cdot \left(P_N^{0t,mt} - 1\right) \cdot w_N^{0t,t-1}; \\ LYT_N^{m(t-1),mt} &= \left(P_N^{m(t-1),12(t-1)} - 1\right) \cdot w_N^{m(t-1),t-2}. \end{aligned} \tag{8.29a}$$

It should be noted that the factor $P^{m(t-1),12(t-1)}$ in TYT pertains to the overall index. It is worth mentioning that this choice is arbitrary, since the factor is also part of the price change to be decomposed (see LYT). Had $P^{0t,mt}$ been held constant in LYT instead, which is equally justified, the resulting contributions would be different, though still additive.

The special case of December ($m = 12$) gives

$$TYT_N^{m(t-1),mt} = \left(P_N^{0t,12t} - 1\right) \cdot w_N^{0t,t-1}; LYT_N^{m(t-1),mt} = 0. \tag{8.29b}$$

In order to ascertain the importance of a sub-index for price dynamics, a *statistical contribution* is a mechanical measure showing the difference between the actual annual rate and that which one would obtain if, under otherwise equal conditions, this sub-index had remained constant vis-à-vis the comparison period. Unlike the Ribe contributions, thus, these contributions have a meaningful interpretation. For example, the statistical contribution of sub-index N is

$$\begin{aligned} \Delta_N^{m(t-1),mt} &= P^{m(t-1),12(t-1)} \cdot P^{0t,mt} \\ &\quad - \left(P^{m(t-1),12(t-1)} - \left(P_N^{m(t-1),12(t-1)} - 1\right) \cdot w_N^{m(t-1),t-2}\right) \\ &\quad\quad \cdot \left(P^{0t,mt} - \left(P_N^{0t,mt} - 1\right) \cdot w_N^{0t,t-1}\right) \\ &= P^{m(t-1),12(t-1)} \cdot \left(P_N^{0t,mt} - 1\right) \cdot w_N^{0t,t-1} \\ &\quad + P^{0t,mt} \cdot \left(P_N^{m(t-1),12(t-1)} - 1\right) \cdot w_N^{m(t-1),t-2} \\ &\quad - \left(P_N^{0t,mt} - 1\right) \cdot \left(P_N^{m(t-1),12(t-1)} - 1\right) \cdot w_N^{0t,t-1} \cdot w_N^{m(t-1),t-2}. \end{aligned} \tag{8.30}$$

The expression has three terms. The first two terms relate to the price change of the sub-index before and after the chain-linking in December, the third term refers to the whole 12-month period. The latter term will normally be dominated by the former two terms because the weight part is approximately square. Owing to the statistical break from one year to another, the sum of the statistical contributions of the sub-indices for this period is not necessarily equal to the annual rate.[35]

The special case of December ($m = 12$) gives

[35] To reiterate, the additivity of the Ribe contributions is imposed and the contributions are derived accordingly. The difference of the statistical contributions presented here to the Ribe contributions is:
$\left(P_N^{m(t-1),12(t-1)} - 1\right) \cdot w_N^{m(t-1),t-2} \cdot \left(\left(P^{0t,mt} - 1\right) - \left(P_N^{0t,mt} - 1\right) \cdot w_N^{0t,t-1}\right)$, which is expected to be close to zero.

$$\Delta_N^{m(t-1),mt} = \left(P_N^{0t,12t} - 1\right) \cdot w_N^{0t,t-1}, \tag{8.30a}$$

which is again additive.

8.5.4 Annual average rate

Aggregation of the Laspeyres-type index from months to the year is performed by means of *arithmetic* averaging the twelve months of year t as the weights remain constant within a *calendar* year:[36]

$$CP^{b,t} = \frac{1}{12}\sum_{m=1}^{12} CP^{b,mt} = CP^{b,12(t-1)} \cdot \left(\frac{1}{12}\sum_{m=1}^{12} P^{0t,mt}\right) = CP^{b,12(t-1)} \cdot P^{0t,t}. \tag{8.31}$$

The annual average rate is obtained by dividing the arithmetic mean of the chain-linked indices for year t by a mean of the same indices for year $t-1$; that is,

$$\frac{CP^{b,t}}{CP^{b,t-1}} = \frac{P^{0(t-1),12(t-1)}}{P^{0(t-1),t-1}} \cdot P^{0t,t} = P^{t-1,12(t-1)} \cdot P^{0t,t}. \tag{8.31a}$$

Very much like the annual rate, the result is a chain-linked index consisting of two parts:

1. a ratio of two Laspeyres-type indices for December of year $t-1$ and the entire previous year, respectively, both relative to December of year $t-2$, multiplied by
2. a Laspeyres-type index for the entire year t relative to December of year $t-1$.

The first part can be written as a Laspeyres-type index for December of year $t-1$ relative to the entire previous year. The corresponding price-updated weights are obtained as follows:

$$w_i^{t-1,t-2} = w_i^{0(t-1),t-2} \cdot \frac{P_i^{0(t-1),t-1}}{P^{0(t-1),t-1}}. \tag{8.4d}$$

It is straightforward to see that the relative importance of a price change for the annual average rate also depends on the month in which it occurs. A permanent upward shock to the price level, say, in January has a twelvefold impact than the same shock in December. A first-order Taylor series approximation of Equation (8.31a) around $P^{\tau-1,\tau} = 1$ $(\tau = 2(t-1), \ldots, 12t)$ yields:

$$\begin{aligned} & P^{0(t-1),12(t-1)} \cdot \frac{\frac{1}{12}\sum_{m=1}^{12} P^{0t,mt}}{\frac{1}{12}\sum_{m=1}^{12} P^{0(t-1),m(t-1)}} - 1 \\ & \approx \frac{1}{12}P^{1(t-1),2(t-1)} + \frac{2}{12}P^{2(t-1),3(t-1)} + \cdots + \frac{11}{12}P^{11(t-1),12(t-1)} \\ & \quad + \frac{12}{12}P^{0t,1t} + \frac{11}{12}P^{1t,2t} + \cdots + \frac{2}{12}P^{10t,11t} + \frac{1}{12}P^{11t,12t}. \end{aligned} \tag{8.31b}$$

[36] Should the 12-month average not correspond to a calendar year, nothing can be said about its properties.

References

Balk, B.M. (2008), *Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference*, Cambridge University Press.

ILO, IMF, OECD, UNECE, Eurostat, and The World Bank (2004), *Consumer Price Index Manual: Theory and Practice*, International Labour Office.

v.d. Lippe, P.M. (2001), *Chain Indices: A Study in Price Index Theory*, Metzler-Poeschel.

v.d. Lippe, P.M. (2007), *Index Theory and Price Statistics*, Lang.

Ralph, J., O'Neill, R., and Winton, J. (2015), *A Practical Introduction to Index Numbers*, Wiley.

Appendix: Numerical example

The Microsoft Excel file contains three groups of tabs. In the blue tabs (*Index*, and *Item weights*) the data referring to the changing composition of the euro area are presented as published by Eurostat on 22 February 2017. The indices and item weights are both rounded to two decimals, which will affect the calculation results. They are, nonetheless, exact except for rounding differences. The grey tabs (*Aggregation*, *Non-consistency*, *Re-referencing (1) and (2)*, and *Disaggregation (1) and (2)*) contain the examples connected to Section 8.4, while the yellow tabs (*Monthly rate*, *Annual rate*, *Base effect*, and *Annual average rate*) contain those connected to Section 8.5.

Aggregation

The five special aggregates – processed food, unprocessed food, non-energy industrial goods, energy, and services – are aggregated to the total HICP. The procedure involves three steps:

1. Unchain the chain-linked indices to obtain the short-term series using Equation (8.21).
2. Aggregate the unchained sub-indices to the total short-term index using the item weights provided and Equation (8.18).
3. Chain-link the short-term series together into a single long-term series using December as the linking month and Equation (8.20).

Non-consistency

Here it is shown that aggregating the chain-linked indices directly using Equation (8.19), rather than their unchained counterparts, does not give consistency in aggregation. The zero check is different from zero not only because of rounding but particularly because of the failure of the circularity test of the Laspeyres-type index.

Re-referencing

Re-referencing of the index with reference period 2005=100 to an index with reference period 2015=100 is nothing more but the Rule of Three using Equation (8.22) as shown on the first tab. The second tab derives the chain-linked index in the new index reference period from rescaled short-term indices and price-updated weights using Equations (8.23) and (8.4a).

Disaggregation

From the total HICP an exclusion measure without food and energy is derived. The procedure is the same as for the aggregation with the difference that now sub-indices are deducted from the total on the first tab rather than these are added to form the total. On the second tab the same calculation but now aggregating the constituent sub-indices is performed.

Monthly rate

The monthly rate has been defined in Equation (8.24). The calculation of contributions, here for the sub-index "energy", involves the short-term indices, which is why the chain-linked indices are first unchained using again Equation (8.21). As shown in Equation (8.25) and (8.4b) the indices need to be rescaled and the item weights price-updated, respectively. On this basis the contribution of energy to the monthly rate of the total HICP is derived using Equation (8.25a). In December 2016, for example, the contribution of energy to the monthly rate of the total HICP of 0.5% was 0.2 percentage point.

Annual rate

The calculation for the annual rate is repeated analogously to the monthly rate using Equation (8.26). Again, the contributions involve the short-term indices and after unchaining the chain-linked indices using Equation (8.21), the thus derived indices are rescaled using Equation (8.29) and the item weights price-updated using Equation (8.4c). The Ribe contribution of energy to the annual rate of the total HICP follows from Equation (8.29a) as the sum of the "this-year term" and the "last-year term". The statistical contribution, on the other hand, involves three terms as given in Equation (8.30). In December 2016, for example, the contribution of energy to the annual rate of the total HICP of 1.1% was 0.2 percentage point. As it was to be expected, the two contributions are virtually identical.

Base effect

The base, or denominator, effect to the change of the annual inflation rate in the HICP, Equation (8.28), is the monthly rate observed one year earlier, the subtrahend of Equation (8.28a). In December 2016, for example, the month-on-month change of the annual rate of energy was 3.6 percentage points; approximately half of these 3.6 percentage points were due to the monthly rate of –1.8% in December 2015. Since the base effects use information from twelve months ago, the monthly rates of the year 2016 will be the base effects for the year 2017; that is the base effect for energy in January 2017 will be 2.7 percentage points upwards (which will also translate into the total HICP base effect with 0.3 percentage point upward contribution in January 2017).[37]

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[37] See *Economic Bulletin*, Issue 1, ECB, 2017 (energy prices are assumed to show no seasonal influences).

Annual average rate

The annual average index is calculated using Equation (8.31) and the annual average rate using Equation (8.31a). The approximation of the average annual rate involving the monthly rates uses Equation (8.31b).