Elementary aggregation: A not so elementary story!

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1. Introduction

The compilation of a CPI is often presented in two stages. First, prices are aggregated without weights at the elementary level. Prices are typically obtained from dedicated surveys for which price collectors visit outlets and record the observed prices. These elementary price indices are then aggregated to the higher levels using expenditure weights.

Nowadays, CPIs are becoming a multi-source statistics where prices are obtained not only from price collection in the field but also from transaction data, administrative data or from the Internet using web scraping techniques. Depending on the data source, different strategies can be adopted for constructing the elementary aggregates and for compiling elementary price indices. A CPI may be compiled in more than two stages and weights may be available even within the elementary aggregates.

With scanner data, the index compiler must make two main structural decisions which can have a significant impact on inflation measurement. First, the item which is being aggregated must be defined. Second, the level must be fixed up to which these items are first aggregated. To discuss this second issue, we distinguish two strategies for a category that can be divided into sub-categories. Either the items are directly aggregated to the category level, possibly using a multilateral method. Alternatively, the multilateral method aggregates only up to the sub-category level, and these intermediate sub-category level indices are then aggregated to the category level using for instance a Laspeyres-type index formula. The objective of this paper is to examine the impact of introducing this additional level of fixity in the CPI structure.

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The paper is organized as follows. Section 2 discusses some of the concepts involved in elementary aggregation. In section 3, we formally analyse if the introduction of an additional level of fixity has an upward effect on the index. Empirical results obtained with real transaction data are presented in section 4. Finally some conclusions are drawn in section 5.

2. Concepts

The definition of an elementary aggregate

The glossary of the CPI manual gives the following definition for an elementary aggregate: *The smallest aggregate for which expenditure data are available and used for CPI purposes. The values of the elementary aggregates are used to weight the price indices for elementary aggregates to obtain higher-level indices. The range of goods and services covered by an elementary aggregate should be relatively narrow, and may be further narrowed by confining the goods and services to those sold in particular types of outlet or in particular locations. Elementary aggregates also serve as strata for the sampling of prices.*

Additional advice is provided in Chapter 4 of the CPI Manual on how elementary aggregates should best be constructed: *Elementary aggregates should consist of groups of goods or services that are as similar as possible. They should also consist of goods or services that may be expected to have similar price movements. The objective is to minimize the dispersion of price movements within the aggregate.* The idea is to make the elementary aggregate as "homogeneous" as possible in terms of types of products covered and in terms of price changes. Diewert (2018) defines elementary price indices as indices that are *constructed at the first stage of aggregation for closely related products.*

In order to clarify this concept, one first needs to define the target universe. For the Harmonised Index of Consumer Prices (HICP)[2] for instance, the target universe consists of all transactions which fall within the scope of household final monetary consumption expenditure (HFMCE). The latter is defined as that part of final consumption expenditure incurred by households, in monetary transactions, on the economic territory of the Member State, on products that are used for the direct satisfaction of individual needs or wants, as defined in paragraph 3.101 of ESA 2010, in one or both of the time periods being compared. The target universe can be divided according to the COICOP (Classification of individual consumption by purpose), or ECOICOP, which is the European 5-digit version of the COICOP. At Member State level, an ECOICOP subclass is usually further broken down according to a national classification. The elementary aggregates

[2] For more details on the concepts and methods of the HICP, see the HICP manual (2018).

are the smallest aggregates that are included in the index aggregation structure and to which a share of the subclass weight is assigned.

The weight of an individual elementary aggregate defines how much that elementary aggregate represents in the target universe and the weights of all the elementary aggregates together represent the entire target universe. The elementary aggregates weights could be updated regularly, for instance annually, in order to capture any changes in the expenditure patterns between the different elementary aggregates.

The definition of the item

Conceptually, the transactions from the target universe have three dimensions: the product dimension, the time dimension and the outlet dimension. In general, we do not observe individual transactions of the target universe that fall within an elementary aggregate. Therefore, a decision must be made on how to define the "item" for which prices are aggregated within an elementary aggregate. The practical construction of the item depends on the data source.

Usually, the item can be understood as an individual product, specified by its characteristics, and for which a price is collected in an outlet at a given time. In the context of the HICP, the term "product-offer" is used to designate *a specified good or service that is offered for purchase at a stated price, in a specific outlet or by a specific provider, under specific terms of supply, and thus defines a unique entity at any one time*. In most cases, there is only one price observation for an item. In some circumstances, there can be several price observations relating to the same item. For instance, for manual field collection, the prices for a specific good or service may be recorded four times per month. In that case, the average price of these four price quotes could enter the index calculations. In bulk web scraping, prices are collected at a very high frequency for sometimes very detailed products. These individual price observations are first combined together to obtain an overall price for an item. As no data are available on the quantities sold, the sampled prices of the sub-items belonging to the same item are averaged together using for instance a geometric mean or an arithmetic mean. However, such unweighted means may be biased compared to an average price based on actual transactions.

In scanner data, an item is a composed of transactions which refer to one or more individual goods or services, from one or more places of purchase, over a period of time, and for which an average price can be computed. The price of an item is obtained as the average price (unit value aggregation) of its underlying transactions. The item either corresponds to the level for which the transaction data are supplied (e.g. by Global Trade Item Number (GTIN), by outlet, by week), or it may be constructed by further aggregating the supplied transaction data. Diewert and Von der Lippe (2012) concluded "that some use of unit value aggregation is inevitable”[3]. The operational meaning of items (or “homogenous products”) in the context of scanner data has been analysed by Dalèn (2016). Chessa (2018) argues that, when constructing items, trade-offs must be made between homogeneity and stability over time. If items are defined too broadly, there is a risk of a unit value bias. If they are defined too tightly, there is a risk that relaunches are not captured. Von Auer (2017) discusses this trade-off by referring to “assignment bias” and “assortment bias”. Another rationale for defining the items is to allow for full substitution between the individual products that belong to the same item[4]. Items for services can also be defined in various ways based on the different dimensions or characteristics according to which transaction data for services may be supplied (see Stahl (2019) with an example for dental services).

The construction of the elementary aggregates

Price collection in the field is typically organised according to a top-down perspective. First elementary aggregates are defined based on available expenditure data. In addition to the product dimension, it is common practice to stratify according to the outlet or the regional dimensions. In practice, cut-off sampling techniques can be used to select and define the elementary aggregates to be included in the index structure. Within an elementary aggregate, items are selected in a sample of outlets. The prices of the sampled items are typically aggregated to the level of an elementary aggregate, using either the ratio of arithmetic means (Dutot price index) or the ratio of geometric means (Jevons price index). Still, some additional information, such as market shares, can be used to explicitly or implicitly weight the sampled prices during this step of aggregation.

Expenditure data permitting, it is generally thought that elementary aggregates should be defined relatively tightly. It can be easier to sample items for more tightly defined aggregates. The price levels and changes within such elementary aggregates are likely to be more similar. Possible biases stemming from the use of unweighted index formulas are reduced. There are however limitations to fully apply this principle. Reliable and detailed weights are not always available. Tightly defined elementary aggregates are probably more

[3] *It should be noted that some use of unit value aggregation is inevitable; i.e., it will always be necessary to aggregate household or establishment purchases or sales of individual products over time in order to obtain total purchases or sales of the unit under consideration and then these (within the period time aggregated) prices and quantities are used as inputs into a bilateral index number formula.* (Diewert and Von der Lippe (2012)).

[4] This issue is discussed in Silver (2009) who provided the following example: *“Say, for example, the price of good A was 10 in both the reference and current period and the price of good B was 12 in both periods, but there was a shift in quantities from say 6, for both A and B in the reference period, to 8 for A and 4 for B in the current period. A superlative, or any other index number formula for heterogeneous goods, would give an answer of unity, no overall price change. However, the correct answer for homogeneous goods would be a unit value fall of 3 per cent appropriately reflecting the shift in the quantity basket in the current period from the higher price level of 12 for A to the lower price level of 10 for B.”*

numerous and more efforts have to be put in defining the elementary aggregates and estimating the corresponding weights. Detailed elementary aggregates are also less stable so there is a need to continuously review and update the elementary aggregate structure. Finally, sufficient prices must be collected to ensure the compilation of reliable price indices for each elementary aggregate.

Level	**Weights**	**Aggregation method**
CPI category	Fixed	Laspeyres-type
CPI category by outlet-type or by region = Elementary aggregate	Fixed	Laspeyres-type
Item	No weights	Jevons (or Dutot)
Sub-item		Average price

Table 1. Typical aggregation structure in a CPI

For scanner data, a bottom-up perspective is often applied. First, items are constructed from the transaction data. The items are classified according to possibly retailer specific classification which form the elementary aggregates. These retailer-specific categories are linked to the retailer, or an outlet of the retailer. The different retailers or outlets are then linked to a common category which is part of the CPI structure. The prices of the items are aggregated to obtain a price index for the elementary aggregate. With scanner data, there is the opportunity to use weights from each period at the level of the item. Multilateral methods (see Diewert (2018)) have emerged as a solution to compile price indices from scanner data. They provide in principle chain-drift free results, cope with the dynamic aspect of scanner data sets and make explicit use of the available weights data. Above the level of the elementary aggregate, a Laspeyres-type aggregation is often applied to obtain the indices for the higher-level aggregates.

Level	**Weights**	**Aggregation method**
CPI category	Fixed	Laspeyres-type
Retailer/Outlet	Fixed	Laspeyres-type
Product category = Elementary aggregate	Fixed	Laspeyres-type
Item	Variable	Multilateral method
Individual transaction		Unit value

Table 2. Aggregation structure for scanner data

Moving down the levels in table 2, there is a decreasing degree of "stability". At the top, the CPI category may be based on classifications which are kept unchanged over longer periods. Elementary aggregates are considered to be rather stable objects in the sense that they are included in the index over a longer period, at least for a full calendar year. Items are not stable objects in the sense that in every period, they may appear or disappear as a consequence of a dynamic target universe. However, items can still be matched over time more easily than individual transactions.

In practice, the elementary aggregate could correspond to the level where we switch from the use of variable weights for the aggregation of the items to fixed weights. The elementary aggregates can be defined rather tightly, for instance "organic tomatoes in outlet A", or rather broadly, for instance "vegetables in retail chain B". In other words, we need to decide on the level up to which the prices of the items are first aggregated.

3. Has the introduction of additional levels an upward effect on the index?

We are going to distinguish two compilation approaches. Under the "1-step approach", the items are directly aggregated up to the level of a broad category. Under the "2-step approach", the prices of the items are first aggregated up to the level of a sub-category. The resulting sub-category indices are then further aggregated into a price index for the whole category.

We use some simplifying assumptions in order to study this question from an index formula point of view. First we assume that the set of items is constant over time. Second, we use as an underlying price index a direct Törnqvist index. In the empirical study in the next section we will use a GEKS-Törnqvist (CCDI) index[5]. Third, we assume that the sub-category indices are combined using a geometric Laspeyres-type index[6].

We assume that the items of the product category can be partitioned into K sub-categories, each of which consists of N_k items. The 1-step index is obtained as a Törnqvist index compiled over all items of all sub-categories:

[5] Diewert (2018) notes that typically, the Törnqvist index and the CCDI index will approximate each other fairly closely.

[6] The relationships between the geometric and the arithmetic versions of Laspeyres-type indices are examined in Silver and Armknecht (2016).

$$\ln\left(I_{1-step}^{0,t}\right) = \sum_{k=1}^{K} \sum_{i=1}^{N_k} \left(\ln\left(p_i^t\right) - \ln\left(p_i^0\right)\right) * 0.5 * \left(s_i^t + s_i^0\right) \tag{1}$$

where s_i^t is the share of item *i* in period *t* with respect to all the items in all sub-categories. The 2-step index is obtained as follows. First the sub-category index is calculated as a Törnqvist index over the items that fall within this sub-category. The sub-category indices are then aggregated to the category level using fixed expenditure weights w_k from some past period.

$$\ln\left(I_{2-step}^{0,t}\right) = \sum_{k=1}^{K} w_k \sum_{i=1}^{N_k} \left(\ln\left(p_i^t\right) - \ln\left(p_i^0\right)\right) * 0.5 * \left(s_{i,k}^t + s_{i,k}^0\right) \tag{2}$$

where $s_{i,k}^t$ is the share of item *i* in period *t* with respect to the products of the sub-category *k*. We also define the sub-category weights for the current and base periods as follows:

$$w_k^0 = \sum_{i=1}^{N_k} s_i^0 \text{ and } w_k^t = \sum_{i=1}^{N_k} s_i^{\mathrm{t}} \tag{3}$$

We then have :

$$s_i^0 = w_k^0 * s_{i,k}^0 \text{ and } s_i^t = w_k^t * s_{i,k}^t \tag{4}$$

With this notation, we have:

$$\ln\left(I_{2-step}^{0,t}\right) - \ln\left(I_{1-step}^{0,t}\right)$$

$$= \sum_{k=1}^{K} \sum_{i=1}^{N_k} \left(\left(ln\left(p_i^t\right) - ln\left(p_i^0\right)\right) 0.5\left(s_{i,k}^t(w_k - w_k^t) + s_{i,k}^0(w_k - w_k^0)\right)\right) \tag{5}$$

Because the item weights within a sub-category sum to unity, we must have that:

$$\sum_{i=1}^{N_k} 0.5\left(s_{i,k}^t(w_k - w_k^t) + s_{i,k}^0(w_k - w_k^0)\right) = w_k - 0.5\left(w_k^t + w_k^0\right) \tag{6}$$

Given (6), the equation (5) can be expressed using a covariance[7] operator as follows:

$$\sum_{k=1}^{K} N_k Cov_{N_k}\left(ln\left(p_i^t\right) - ln\left(p_i^0\right), 0.5\left(s_{i,k}^t(w_k - w_k^t) + \left(s_{i,k}^0(w_k - w_k^0)\right)\right) + \frac{1}{N_k}\left(\sum_{i=1}^{N_k} ln\left(p_i^t\right) - ln\left(p_i^t\right)\right)\left(w_k - 0.5\left(w_k^t + w_k^0\right)\right) \tag{7}$$

[7] Let us suppose that there are two vectors x_i and y_i of length *N*. Let $Cov_N(x_i, y_i)$ denote the usual covariance between these two vectors. It is then well known that the following holds: $\sum_{i=}^{N} x_i y_i = N Cov_N(x_i, y_i) + \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right)$.

Note that the covariance is defined between vectors of length N_k, which is the number of items within the sub-category *k*. After dividing the first covariance term of (7) into two additive terms, the following expression is obtained which now consists of three parts:

$$\sum_{k=1}^{K} N_k Cov_{N_k}\left(ln\left(p_i^t\right) - ln\left(p_i^0\right), s_{i,k}^t\right)0.5(w_k - w_k^t)$$
$$+\sum_{k=1}^{K} N_k Cov_{N_k}\left(ln\left(p_i^t\right) - ln\left(p_i^0\right), s_{i,k}^0\right)0.5\left(w_k - w_k^0\right)$$
$$+\sum_{k=1}^{K} \frac{1}{N_k}\left(\sum_{i=1}^{N_k} ln\left(p_i^t\right) - ln\left(p_i^0\right)\right)\left(w_k - 0.5\left(w_k^t + w_k^0\right)\right) \quad (8)$$

As the weights of the *K* sub-categories sum to unity, we have the following:

$$\sum_{k=1}^{K} 0.5(w_k - w_k^t) = 0 \quad (9)$$
$$\sum_{k=1}^{K} 0.5\left(w_k - w_k^0\right) = 0 \quad (10)$$
$$\sum_{k=1}^{K} \left(w_k - 0.5\left(w_k^t + w_k^0\right)\right) = 0 \quad (11)$$

Given (9), (10) and (11), equation (8) can be expressed as follows, using a covariance operator:

$$K\ Cov_K\left(N_k Cov_{N_k}\left(ln\left(p_i^t\right) - ln\left(p_i^0\right), s_{i,k}^t\right), 0.5(w_k - w_k^t)\ \right) \quad (12a)$$
$$+\ K\ Cov_K\left(N_k Cov_{N_k}\left(ln\left(p_i^t\right) - ln\left(p_i^0\right), s_{i,k}^0\right), 0.5\left(w_k - w_k^0\right)\ \right) \quad (12b)$$
$$+K\ Cov_K\left(\frac{1}{N_k}\left(\sum_{i=1}^{N_k} ln\left(p_i^t\right) - ln\left(p_i^0\right)\right), w_k - 0.5\left(w_k^t + w_k^0\right)\ \right) \quad (12c)$$

Note that these covariance terms are defined between vectors of length K, which is the number of sub-categories. The difference between the 1-step index and the 2-step index can thus be explained by the three terms (12a), (12b), and (12c)[8]. The term (12a) includes the covariance between the prices within a sub-category and its current weight. This within covariance is then put in relation to the gap between the fixed sub-category weight and the current sub-category weight. We refer to this term as "within covariance – current expenditures". The term (12b) includes the covariance between the prices within a sub-category and its base period weights. This within covariance is then put in relation to the gap between the base period sub-category weight and its fixed weight. Hence we refer to this term as "within covariance – base period

[8] Parniczky (1974) also examined the compilation of an index in two stages where the first stage is based on unit value aggregation and the second stage is based on usual index number theory.

expenditures". Finally, the term (12c) deals with the covariance between the sub-category average price and the gap between the fixed sub-category weight and the average of the sub-category current and base period weights. We refer to this term as "in-between covariance".

It follows from this that the two indices $I_{1-step}^{0,t}$ and $I_{2-step}^{0,t}$ are identical in the following situations:

- The prices in the current period are proportional to the prices in the base period.
- The weights for all sub-categories are constant across time: $w_k^0 = w_k^t = w_k$.
- The three terms "within covariance – current expenditures", "within covariance – base period expenditures" and "in-between covariance" cancel out each other.

In practice, it is highly unlikely that any of these three situations occurs. If the sub-category weight in the base period is close to the fixed weight, the "within covariance – base period expenditures" will be close to zero. If, over time, there is some substitution taking place between the sub-categories, the "in-between covariance" is likely to be positive. The sign and magnitude of the term "within covariance – current expenditures" is more difficult to assess. If the positive term "in-between covariance" is larger than (in absolute terms) a possible negative term "within covariance – current expenditures", the index $I_{2-step}^{0,t}$ will lie above the index $I_{1-step}^{0,t}$. This is the expected scenario as we suppose that introducing fixed weights from a past period at a certain level will introduce some substitution bias compared to an index without fixed weights.

Apart from substitution, differences can also arise because of seasonality. The weights w_k^0, w_k^t and w_k can very well differ over time following a more or less regular pattern. This is line with the observation made by H. van der Grient and J. de Haan (2011). They compiled multilateral indices at different levels of aggregation and attributed the obtained differences to seasonal effects[9].

In order to illustrate the effect of weak seasonality, we apply the decomposition on the "Israeli vegetable data set" available in Diewert (2014). This data set contains prices and quantities for seven vegetables (from January 1997 to December 2002. The data set is characterized by significant fluctuations in prices and quantities for these items. We split the data set into two sub-categories. The first sub-category contains

[9] In their paper, they state that *"Expenditures on these seasonal goods are much greater in summer than in winter, and the prices of fresh produce are lower in summer, which has a downward effect on the "true" RYGEKS index".*

three vegetables (cabbages, cauliflower and cucumber). The second sub-category contains the four remaining vegetables (potatoes, carrots, lettuce and eggplants). Based on the data from the calendar year 1997, the first sub-category accounts for 38.8% and the second sub-category for 61.2% of the expenditures. We then calculate a 1-step index over all 7 vegetables using the formula (1), and a 2-step index based on the two sub-categories using the formula (2), using the year 1997 as the base period. Because the weight and price reference periods are both set to 1997, the term "within covariance – base period expenditures" vanishes.

The difference between the 2-step and the 1-step index can be decomposed into the terms "within covariance – current expenditures" and "in-between covariance as shown in figure 1. In this example the difference between the two indices never exceeds roughly minus 0.3% or plus 0.1%. The two terms often compensate each other, but not always. This example also shows that in some months, the 1-step index is larger than the 2-step index, which means that the introduction of a lower level of fixity does not necessarily lead to a lower index, especially in the context of seasonal patterns. In this example, the first sub-category is characterized by high prices and expenditures in the summer. Therefore, the 1-step index is in those months higher than the 2-step index which relies on weights that are kept fixed at a lower amount.

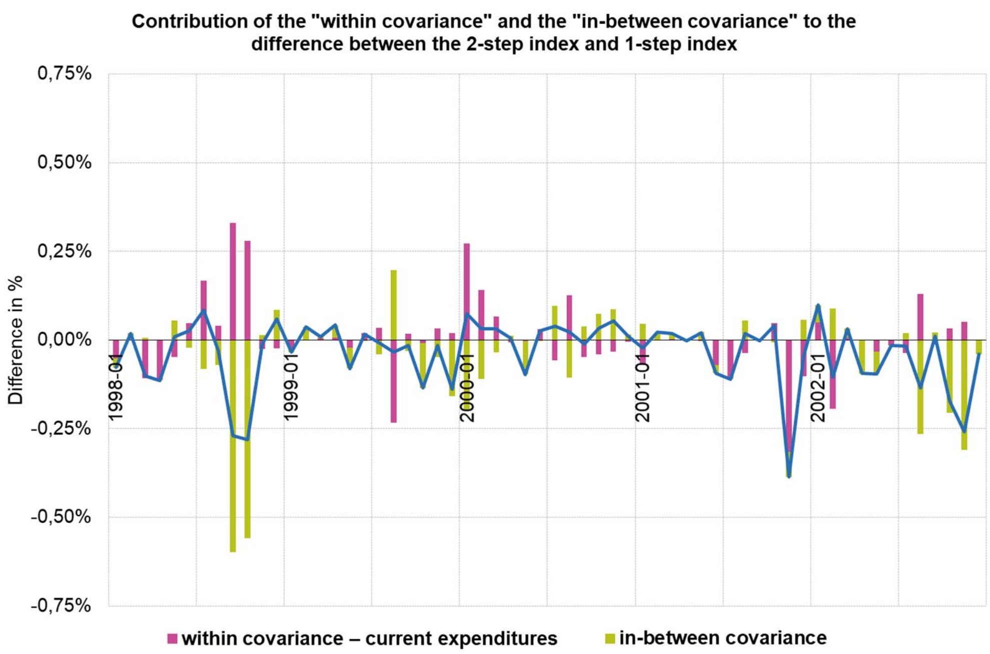

Figure 1. Contribution of the "within-covariance – current expenditures" and the "in between-covariance" to the difference between the 2-step index and the 1-step index

3. An empirical application to scanner data

In order to further explore the question from an empirical point of view, we compile different price indices from the publicly available Dominick's scanner data set. This data set is presented in detail in Mehrhoff (2019). The "item code"[10], and not the Universal Product Code (UPC), has been used to define the product. The data has been aggregated across all stores up to the chain level. The weekly data has been transformed into monthly data by assigning each week to the month in which the week starts.

Dominick's data set is classified according to main products. We concentrate on six categories (soft drinks, dish detergents, crackers, cookies, grooming products, cheese) which can be conveniently subdivided into sub-categories using the pre-defined Dominick's Commodity Code. The weights for the sub-categories are defined using the total expenditure within these groupings in the 12 months prior to April 1995 (April 1994 - March 1995). Some of the commodity codes were grouped together in order to obtain more balanced sub-categories (see table 3).

The CCDI (Caves, Christensen and Diewert) index corresponds to a GEKS index, where the bilateral comparisons are made using a Törnqvist index. For a given category, the 1-step CDDI index[11] is defined as follows over a time window T which covers 25 months (April 1995 – April 1997):

$$I^{0,t}_{1-step}(\text{CCDI}) = \prod_{k\in T}\left(I^{0,k} * I^{k,t}\right)^{\frac{1}{|T|}} \quad , \tag{13}$$

where the underlying bilateral indices are matched Törnqvist indices. In practice, the multilateral methods are compiled over a rolling time window. When a new period becomes available, the time window is moved forward so that the new period is included and the first period of the previous time window is removed. Splicing techniques can be used to combine the series from these moving time windows. In our simulations however, no splicing techniques were used.

According to the 2-step approach, the category is partitioned according to K sub-categories using the commodity codes. We denote by $I^{0,t}_{k}(CCDI)$ the CCDI index compiled for sub-category *k (k=1,...,K)*. The

[10] Dominick's Data Manual claims the following: *[The item code] is [our] attempt at tracking products across multiple upcs. If two different upcs have the same item, then one is a newer version of the other. This scheme is not foolproof. There are many instances where two upcs have different nitem codes although they are the same products*.

[11] We prefer here the CCDI index over other multilateral methods. Whereas the Geary-Khamis method or the Time Product Dummy method assumes rather detailed elementary aggregates because their (approximate) additivity is not supported by economic theory, this not necessarily required for the CCDI (see paragraph 10.119 of the draft Chapter 10 (version February 2019) of the update of the CPI manual).

index in the 2-step approach is then obtained using a Laspeyres-type index formula. Such an approach simply pushes the elementary aggregate level one level down in the index hierarchy.

$$I_{2-step}^{0,t}(\mathrm{CCDI} + \mathrm{Lasp}) = \sum_{k=1}^{K} w_k I_k^{0,t}(CCDI) \tag{14}$$

where w_k is the normalized expenditure weight. Alternatively, one could also adopt a geometric formulation. Such an approach would still rely on fixed weights and it would be consistent with a Jevons index typically used at the elementary level.

$$I_{2-step}^{0,t}(\mathrm{CCDI} + \mathrm{GeoLasp}) = \prod_{k=1}^{K} \left(I_k^{0,t}(CCDI)\right)^{w_k} \tag{15}$$

Finally, one could also consider in this second stage a Törnqvist index, using the actual expenditure shares in the current and price reference periods.

$$I_{2-step}^{0,t}(\mathrm{CCDI} + \mathrm{Tornqvist}) = \prod_{k=1}^{K} \left(I_k^{0,t}(CCDI)\right)^{0.5*(w_k^0 + w_k^t)} \tag{16}$$

The CCDI indices were compiled with the help of the *R* package *indexNumR*.

Soft drinks (SDR)	
Commodity codes	**Weight**
227 & 225	5.5%
233 & 234	7.9%
235	33.5%
236	13.0%
237	14.8%
238	19.0%
239 & 240 & 241	6.4%

Dish detergents (DID)	
Commodity codes	**Weight**
653	50.5%
654	22.0%
655	27.5%

Crackers (CRA)	
Commodity codes	**Weight**
477 & 478 & 479	22.8%
480	55.7%
485	21.5%

Cookies (COO)	
Commodity codes	**Weight**
477	68.2%
478	17.0%
479	14.8%

Grooming products (GRO)	
Commodity codes	**Weight**
967	32.0%
968	5.2%
969	7.5%
972	13.0%
973	30.8%
974	11.5%

Cheese (CHE)	
Commodity codes	**Weight**
152	6.7%
153	32.6%
154	28.2%
155	32.4%

Table 3. Fixed weights by commodity code groupings (April 1994 - March 1995)

Figure 2 compares a 1-step CCDI index with a 2-step CCDI-Laspeyres index for soft drinks. This is an example where the differences are small at the beginning of the period but they become quite pronounced at the end of the period. This results from significant increases in the expenditure share of commodity code 235 (April 97 expenditure share stands at 57%, compared to a fixed weight of 33.5%) combined with price decreases (April 97 index is down by 10% compared to April 95).

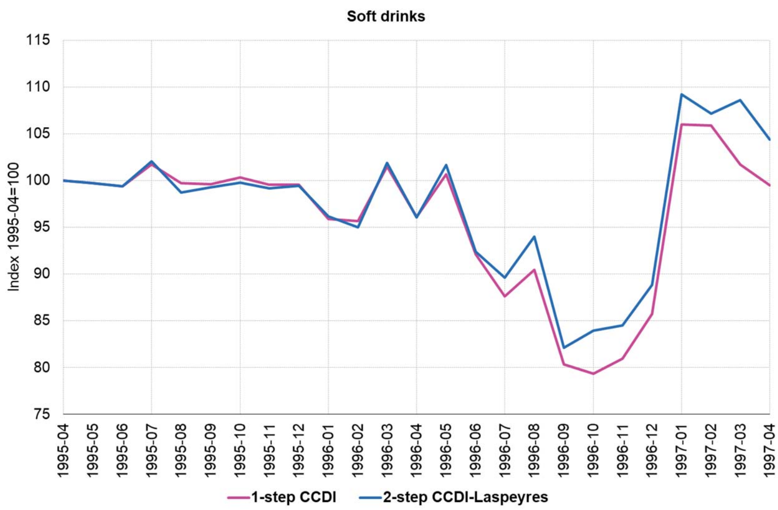

Figure 2. The 1-step CCDI index the 2-step CCDI & Laspeyres index for soft drinks

Figure 4 summarizes the results by looking at the difference (in %) between the various 2-step CCDI indices and the 1-step CCDI index. In addition to soft drinks, differences can also be observed for dish detergents, albeit to a lesser extent. For the other products, the different approaches all lead to fairly similar results, suggesting that the introduction of an additional lower level very often does not change the results compared to an index without this additional level. For the 2-step approaches, the Laspeyres-type index lies of course above the geometric Laspeyres-type index, which itself usually lies above the Törnqvist index. In general, the CCDI & Törnqvist index approximates best a 1-step CCDI index.

The compilation of elementary price indices from prices manually collected in the field differs from the scanner data framework in a number of ways. The price collector samples only a few items, often focusing on those which are well sold and which are likely to be available in the subsequent periods whereas scanner data sets contain all items purchased in a given outlet or group of outlets. Moreover, unweighted index formulas are used at the first step of aggregation.

In that context, we construct an additional experimental price index as follows. We assume that the sub-categories act as strata for sampling. Within each sub-category we consider all items that are continuously available in the 25 periods. For each item, total sales for the 25 periods is computed. We select n=10 items with the largest sales. A Jevons price index (unweighted geometric mean) is compiled over these 10 items.

$$I^{0,t}(Jevons) = \prod_{i \in S} \left(\frac{p_i^t}{p_i^0}\right)^{\frac{1}{n}} \tag{17}$$

These Jevons cut-off price indices are then aggregated using the fixed expenditure shares of each sub-category:

$$I_{2-step}^{0,t}(\text{Jevons} + \text{Lasp}) = \sum_{k=1}^{K} w_k I_k^{0,t}(Jevons) \tag{18}$$

The 2-step Jevons-Laspeyres price index ("tight" elementary aggregate and no weights within the elementary aggregate) can now be compared to a 2-step CCDI & Laspeyres index ("tight" elementary aggregate and variable weights within the elementary aggregate). The short-term movements of the two indices can differ because of the selection of stable and well-sold products and the lack of explicit weights in the Jevons price index. This is illustrated in figure 3 for dish detergents.

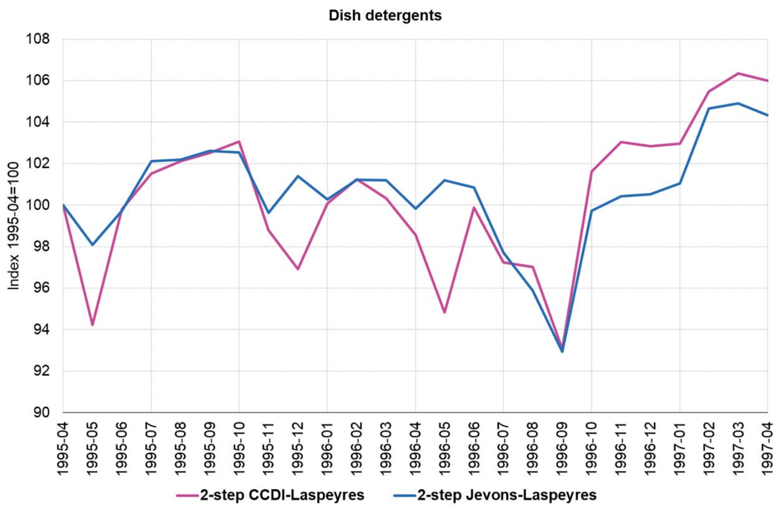

Figure 3. The 2-step CCDI-Laspeyres index the 2-step Jevons-Laspeyres index for dish detergents

Figure 5 summarizes to what extent these two indices deviate from each other for all product categories. On average, the 2-step Jevons-Laspeyres indices slightly lie above the 2-step CCDI-Laspeyres indices, except for Cookies.

Figure 4. Difference in % between the 2-step CCDI indices and the 1-step CCDI index

Figure 5. Difference in % between the 2-step Jevons-Laspeyres index and the 2-step CCDI-Laspeyres index

4. Conclusion

In this paper we analysed the impact of dividing a category into sub-categories. We compiled various hybrid 2-step indices and compared them to a 1-step index which does not take into account this intermediate level. In theory, the impact can go either way. Empirically, the impact can often be negligible but there can also be situations where it matters. We mainly focused on the product dimension. A similar discussion could be held with respect to the outlet or retailer dimensions.

This leads to the following tentative conclusions:

- In the standard CPI context, the construction of the elementary aggregates depends to a large extent on the availability of expenditure data. The elementary aggregate should be defined relatively tightly and some kind of fixed weights, if available, could still be used within the elementary aggregate. From a harmonisation point of view, the same approach remains fully valid also for scanner data, which means that the elementary aggregates could be defined in a relatively tight manner. At the same time variable weights can now be used within the elementary aggregates.

- Alternatively, for scanner data, the elementary aggregates could be defined slightly broader. Under certain circumstances, this would approximately be the same as sub-dividing the elementary aggregate into sub-categories and assigning fixed weights to them. In particular, this holds if the expenditures for the sub-categories remain fairly stable over time or follow smooth trends which would be reasonably well captured by an annual update of the elementary aggregate weights.

- A decision must be made if an elementary aggregate is composed of sub-categories whose expenditures are changing in the short-term. On the one hand, fixing the weights at the sub-category level means that such fluctuations would not be captured. On the other hand, aggregating sub-category level indices with variable weights comes close to skipping the sub-category level all together and aggregating directly to the level above. Eventually, such choices also need to be made by taking into account the conceptual framework of the CPI and the main users' opinions on the measurement objectives of the CPI.

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