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Comparison of Price Index Methods for the CPI Measurement Using Scanner Data

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Ottawa Group Meeting, Rio de Janeiro, 2019.

GTIN-13
(EAN / UCC-13)

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Agenda

1. **Contemporary Polish experiences and knowledge**
2. **Considered index methods for CPI calculations using scanner data**
3. **Propositions of price index modifications**
4. **Simulation study**
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1. Contemporary Polish experiences and knowledge

We have no IT for:

(a) **Combining different data sources** from different retailers (supermarkets) written in different file formats. Nevertheless some small sub-team works on this topic (machine learning, text mining methods).
(b) **Extracting the Mathematica results** to the main (present) IT system in Statistics Poland.

At this moment we use the ***Mathematica*** software for programming data filters and price index methods. We use the Wolfram's ***Link for Excel*** for cooperating with Excel files.

- Several helpful software, programming languages or potential possibilities are considered (like R, SAS, Phyton, Mathematica scripts or others….). Potentially, the IT system can be made in any programming languages (C++, Delphi, others…) – this just the task for IT team.
- We have collected a team which consists of IT experts, academics and practitioners. On the present stage we intend to create a methodological concept and some conceptual procedure for processing scanner data. To reach this aim, we are going to study more real cases and data sets and also some simulation studies, concerning properties of price indices, are planned.
- We have started to implement the big economical and statistical project (*InstatCeny*) which is financed by the National Centre of Research and Development (NCBR). The 3-year project is focused on the revision of existing CPI methodology in Poland with taking into consideration new possible data sources (scanner data, web scrapped data, others?). Statistics Poland is one of performers in that project.

2. Considered index methods for CPI calculations using scanner data

Let us denote the sets of homogeneous products belonging to the same product group in months 0 and t by G_0 and G_t respectively, and let $G_{0,t}$ denote the set of matched products in both moments 0 and t. A product may refer to a single item (GTIN) or to a sub-group of items (GTINs) having the same characteristics, and thus being in the same homogeneity group. In the next part of the paper, we consider the second scenario, i.e. **a homogeneous group of different GTINs but having identical characteristics**. We also consider **a month** as a time period over which scanner data are aggregated. In fact, one month is the longest interval among time intervals recommended by Eurostat for the scanner data aggregation (see *Practical Guide for Processing Supermarket Scanner data* (2017), page 13).

Bilateral index methods - *Unweighted formulas*

If expenditure information is not available, the European Commission recommends the **Jevons** (1865) price index (see also Diewert (2012) or Levell (2015)), which can written as follows

$$P_J^{0,t} = \prod_{i \in G_{0,t}} \left(\frac{p_i^t}{p_i^0}\right)^{\frac{1}{N_{0,t}}} ,$$

where p_i^τ denotes the price of the i-th product at the time $\tau \in \{0,t\}$ and $N_{0,t} = card\ G_{0,t}$. On the other hand, the same recommendation takes also into consideration ("in exceptional cases") the **Carli** (1804) price index, which can be written as follows

$$P_C^{0,t} = \frac{1}{N_{0,t}} \sum_{i \in G_{0,t}} \frac{p_i^t}{p_i^0} .$$

In our research, we consider only the first formula together with its monthly chained version which is denoted here by $P_{CH-J}^{0,t}$.

Bilateral index methods - *Weighted formulas*

Since scanner data contain information about the expenditure, it is possible in their case to calculate weighted bilateral indices. *Superlative* price indices, firstly proposed by Diewert (1976), are the most recommended index formulas for the scanner data case (as base formulas). In the paper the following superlative price index formulas are considered:

the Fisher price index (1922)

$$P_F^{0,t} = \sqrt{P_{La}^{0,t} \cdot P_{Pa}^{0,t}}$$

the Törnqvist price index (1936)

$$P_T^{0,t} = \prod_{i \in G_{0,t}} \left(\frac{p_i^t}{p_i^0}\right)^{\frac{s_i^0 + s_i^t}{2}}$$

Multilateral index methods

Multilateral index methods have their genesis in comparisons of price levels across countries or regions. These methods satisfy the **transitivity**, which is a desirable property for spatial comparisons due to the fact that the results are independent of the choice of base country (region). Commonly known methods are the GEKS method (also known as the EKS method – see Gini (1931), Eltetö and Köves (1964), Szulc (1964), the Geary-Khamis (GK) method (Geary (1958), Khamis (1972)), the CCDI method (Caves, Christensen and Diewert (1982), Inklaar and Diewert (2016)) or the *real time index method* (Chessa (2015)).

In the paper, the following multilateral methods are considered:

The quality adjusted unit value index (QU) and the Geary-Khamis (GK) method

$$P_{QU}^{0,t} = \frac{\sum_{i \in G_t} p_i^t q_i^t / \sum_{i \in G_0} p_i^0 q_i^0}{\sum_{i \in G_t} v_i q_i^t / \sum_{i \in G_0} v_i q_i^0}$$

Note also that the quantity weights v_i are the only unknown factors in the above formula and different choices of factors v_i lead to different prices index formulas.

In the GK method, the weights v_i are defined as follows

$$v_i = \sum_{z=0}^{T} \varphi_{i,GK}^{z} \frac{p_i^z}{P_{QU}^{0,z}} , \qquad \text{where} \qquad \varphi_{i,GK}^{z} = \frac{q_i^z}{\sum_{\tau=0}^{T} q_i^{\tau}} ,$$

where $[0,T]$ is the entire time interval of the product observations (typically $T = 12$, see Diewert & Fox (2017)).

The augmented Lehr index

$$P_{AL}^{0,t} = \frac{\sum_{i \in G_t} p_i^t q_i^t / \sum_{i \in G_0} p_i^0 q_i^0}{\sum_{i \in G_t} v_i^{AL} q_i^t / \sum_{i \in G_0} v_i^{AL} q_i^0}, \quad \text{where} \quad v_i = \frac{\sum_{\tau=0}^{T} p_i^\tau q_i^\tau}{\sum_{\tau=0}^{T} q_i^\tau}.$$

The real time index

Let us note that price imputations are not needed when prices from each month of the current year are included in weights v_i. Chessa (2015) suggests the following procedure of calculating *the real time index*: (1) For the current year, we use a time window with December of the previous year as the fixed base month and the window is enlarged each month with the current month; (2) The price index of the current month t is calculated by using the updated quantity weights according to a special algorithm. In particular, this algorithm repeats updating weights $v_i = \sum_{z=0}^{t} \varphi_{i,GK}^{z} \frac{p_i^z}{P_{QU}^{0,z}}$ and next updating values of price indices $P_{QU}^{0,\tau} : 0 \le \tau \le t$ until the difference between indices from the last two iterations is small enough.

The GEKS method

The **GEKS** price index between months 0 and t is an unweighted geometric mean of $T+1$ ratios of bilateral price indices which are based on the same price index formula. The bilateral price index formula should satisfy the **time reversal test**.

$$P_{GEKS}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_F^{\tau,t}}{P_F^{\tau,0}}\right)^{\frac{1}{T+1}}$$

In the paper of Diewert and Fox (2017), the multilateral price comparison method that uses the GEKS method based on the Törnqvist price index is called the **CCDI** method:

$$P_{CCDI}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_T^{\tau,t}}{P_T^{\tau,0}}\right)^{\frac{1}{T+1}}.$$

Alternative weighting schemes in the QU method

In the classical form, the **GK** method uses **quantity shares** as weight in the construction of v_i. In the literature, we can find at least two other weighting schemes in quantity weights for the GK price index. The first variant was proposed by Hill (2000) and it assumes that deflated prices, i.e. $p_i^z / P_{QU}^{0,z}$, are weighted by the ratio of the **turnover share** of the i-th product in the month and the sum of turnover shares of the same product over different months: $\varphi_{i,TS}^{z} = \frac{s_i^z}{\sum_{\tau=0}^{T} s_i^{\tau}}$.

The other weighting scheme assumes that deflated prices in months with sales receive **equal weight**, and thus it is denoted here by the **EW** method, i.e. $\varphi_{i,EW}^{z} = \frac{\delta_i^z}{\sum_{\tau=0}^{T} \delta_i^{\tau}}$, where $\delta_i^z = 1$ if $q_i^z > 0$ and $\delta_i^z = 0$ otherwise.

The corresponding multilateral indices will be denoted by $P_{GK}^{0,t}$, $P_{TS}^{0,t}$ and $P_{EW}^{0,t}$ respectively.

Updating problem and window updating methods

In the case of **bilateral methods**, a fixed base month (period) is used and the current period is shifted each month. In **monthly chained index methods**, the base and the current month are both moved one month.

The problem with proceeding with the next month arises in the case of **multilateral index methods**. Adding information from a new month may influence the values of quality adjustment parameters and values of the corresponding multilateral indices. In this paper, we consider four commonly used rolling-window updating methods which shift the estimation window (often 13 months) forward each period (a month as a rule) and then splice the new indices onto the existing time series. The considered methods are as follows:

The movement splice method (de Haan & van der Grient (2011)):

$$P_{MS}^{0,t} = P_{MS}^{0,t-1} \cdot P_{t-T,t}^{t-1,t}$$

*The window splice method (*Krsinich (2014)*)*

$$P_{WS}^{0,t} = P_{WS}^{0,t-1} \cdot \frac{P_{t-T,t}^{t-T,t}}{P_{t-T-1,t-1}^{t-T,t-1}}$$

The half splice method (De Hann (2015))

$$P_{HS}^{0,t} = P_{HS}^{0,t-1} \cdot \frac{P_{t-T,t}^{t-t_0,t}}{P_{t-T-1,t-1}^{t-t_0,t-1}}$$ ($t_0 = \frac{T+1}{2}$ if T is an odd integer and at $t_0 = \frac{T}{2}$ if T is an even integer)

The mean splice method (Diewert & Fox (2017))

$$P_{GMS}^{0,t} = P_{GMS}^{0,t-1} \cdot \prod_{t_0=1}^{T} \left(\frac{P_{t-T,t}^{t-t_0,t}}{P_{t-T-1,t-1}^{t-t_0,t-1}}\right)^{\frac{1}{T}}$$

3. Propositions of price index modifications

- *Modification of the Jevons formula*

$$P_{MJ}^{0,t} = \sqrt[\sum_{j\in G_{0,t}} \frac{q_j^0 + q_j^t}{2}]{\prod_{i\in G_{0,t}} \left(\frac{p_i^t}{p_i^0}\right)^{\frac{q_i^0 + q_i^2}{2}}} = \prod_{i\in G_{0,t}} \left(\frac{p_i^t}{p_i^0}\right)^{\frac{q_i^0 + q_i^t}{\sum_{j\in G_{0,t}} (q_j^0 + q_j^t)}}$$

- *Modifications of the GEKS index*

$$P_{MJ-GEKS}^{0,t} = \prod_{\tau=0}^{T} \left(\frac{P_{MJ}^{\tau,t}}{P_{MJ}^{\tau,0}}\right)^{\frac{1}{T+1}}$$

- *Modification of the Geary-Khamis index ($P_{EX}^{0,t}$)*

In the classical form, the GK method uses quantity shares as weight in the construction of v_i. Now we suggest considering a different system of weights based on observed and available **ex**penditures, namely $\varphi_{i,EX}^{z} = \frac{p_i^z q_i^z}{\sum_{\tau=0}^{T} p_i^\tau q_i^\tau}$.

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4. Simulation study

Case 1. In the first experiment, we are going to verify the chain drift effect in the case of bilateral and multilateral indices (sic!). Chain drift occurs when an index does not return to unity when prices in the current period return to their levels in the base period (ILO 2004, p. 445). For instance, Szulc (1983), (1987) demonstrated how big the chain problem could be with chained Laspeyres indices but also, as it is commonly known, chain drift can also be a problem with chained superlative indices. Some authors consider the chain drift problem more narrowly, i.e. they assume that only when both prices and quantities in the current period revert back to their levels in the base period, a corresponding price index should indicate that no price change occurred (Diewert and Fox (2017), von Auer (2019)). Potentially, multilateral methods should deal with the chain problem in this "narrow" sense. Nevertheless, **even multilateral indices may not return to unity when prices revert back to the levels in the base period but quantities do not.**

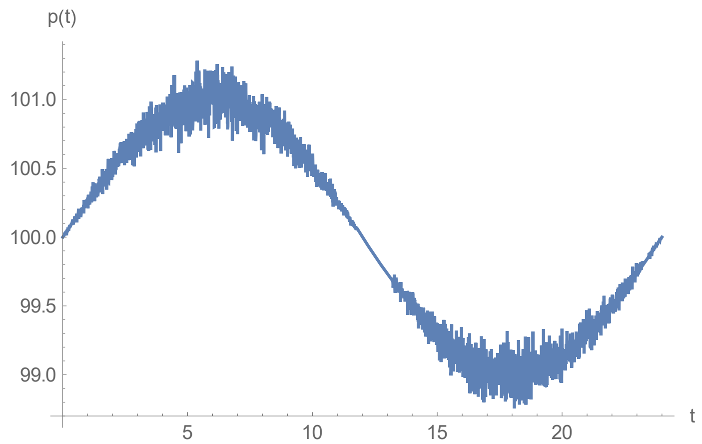
Let us consider a group of $N = 40$ matched items observed during two years, i.e. each month during the time interval [0, 24]. Let us assume that the price of k-th item can be described by the following stochastic process: $p_k^t = 100 + k \cdot \sin(x \cdot t) \cdot Y$,

where $x = \frac{2\pi}{24}$ and the random variable Y is normally distributed, i.e. $Y \sim N(1;0.1)$.

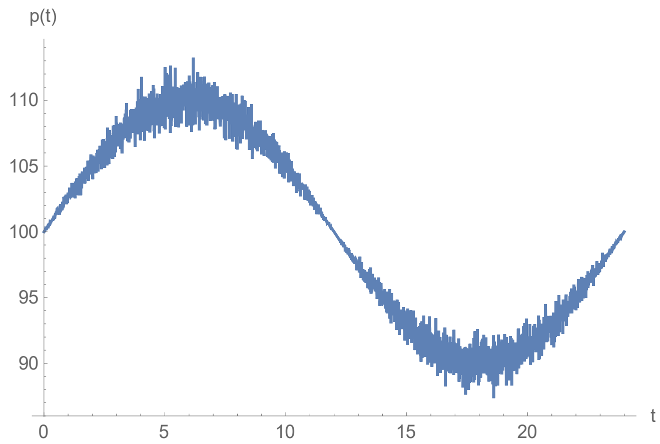
Thus, we have: $E(p_k^t) = 100 + k\sin(x \cdot t)$, $D(p_k^t) = k\sin(x \cdot t)$ and $p_k^0 = p_k^{12} = p_{24}^{24}$.

Fig. 1. Sample realisations of price processes for $t \in [0,24]$

a) $k = 1$



b) $k = 10$



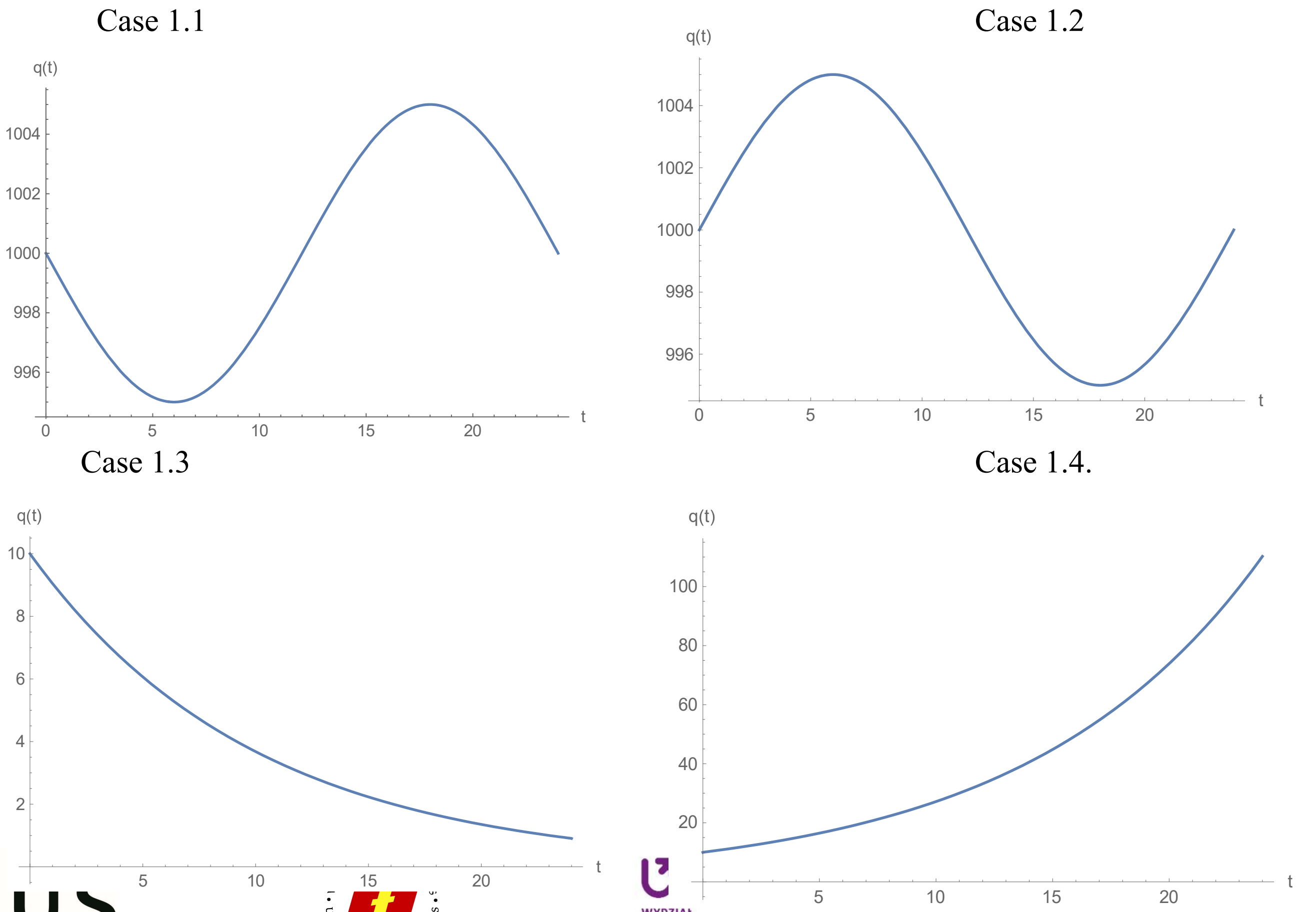
Case 1.1. (periodic quantities negatively correlated with prices): $q_k^t = 1000 - k \cdot \sin(x \cdot t)$

Case 1.2. (periodic quantities positively correlated with prices): $q_k^t = 1000 + k \cdot \sin(x \cdot t)$

Case 1.3. (strongly decreasing quantities uncorrelated with prices): $q_k^t = 10 \cdot \exp(-\frac{k}{50} \cdot t)$

Case 1.4. (strongly increasing quantities uncorrelated with prices): $q_k^t = 10 \cdot \exp(\frac{k}{50} \cdot t)$.

Fig. 2. Sample realisations of quantity processes for $t \in [0,24]$ and for $k = 5$.



In **Cases 1.1 and 1.2**, when quantity processes are not strongly fluctuated and are correlated with price movements, all indices (unweighted, weighted, including multilateral ones) equal 1 for $t \in \{12,24\}$. In these cases, quantities revert to the starting level after one and two years. The differences between indices are negligible.

Fig. 3. Values of selected indices (Case 1.1)

(in the case of multilateral indices, a 13-month window is considered, $T = 12$, $t \in [0,12]$)

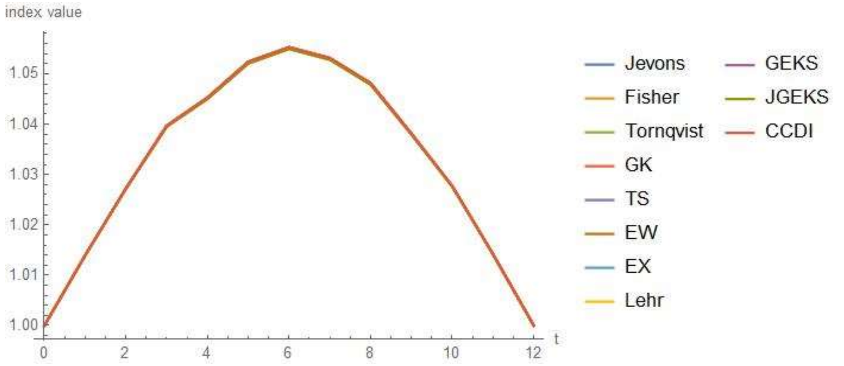


Fig. 4. Values of selected multilateral indices (Case 1.1)

(the whole time window is available, $T = 24$, $t \in [0,24]$)

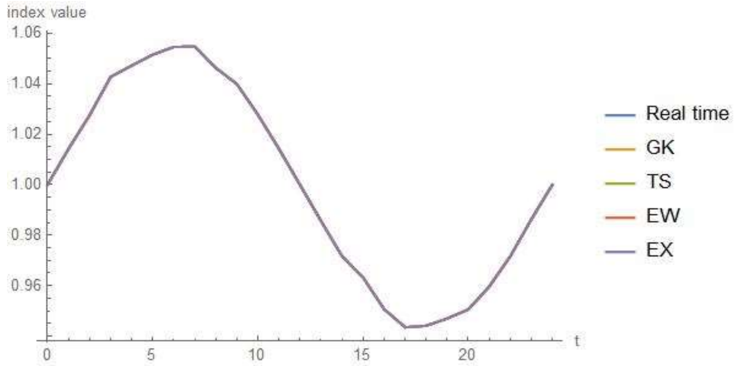
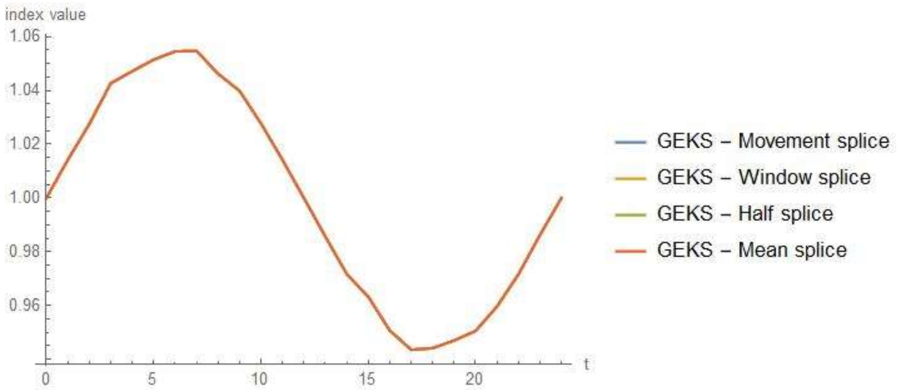


Fig. 5. Values of the GEKS index (Case 1.1)

(a 13-month window is considered, $T = 12$, $t \in [0,24]$)



The situation in Cases 1.3 and 1.4 is different. For instance, when quantities strongly decrease (Case 1.3), chained superlative indices and multilateral indices seem to slightly overestimate the real price change for time intervals [0,12] and [0,24]:

Fig. 6. Case 1.3, $t \in [0,12]$

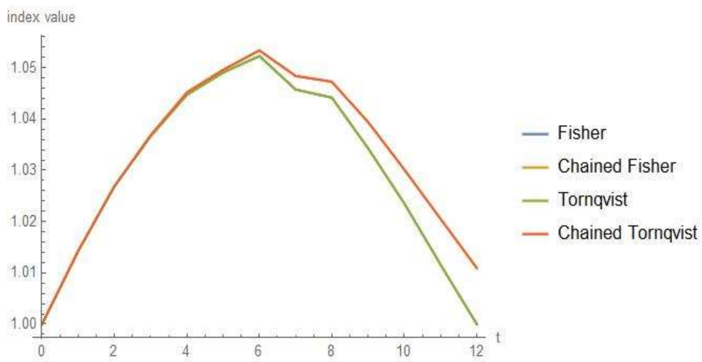


Fig. 7. Case 1.3, $T = 12, t \in [0,12]$

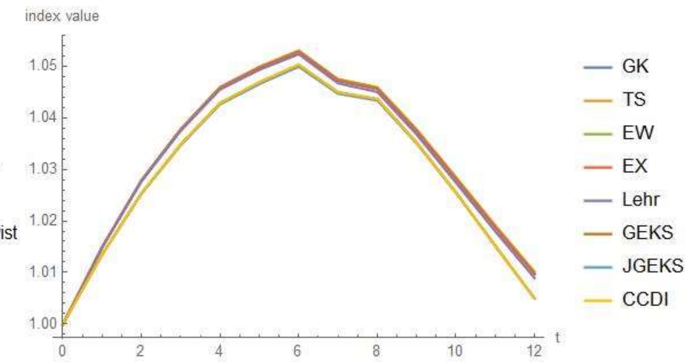


Fig. 8. Case 1.3, $T = 24, t \in [0,24]$)

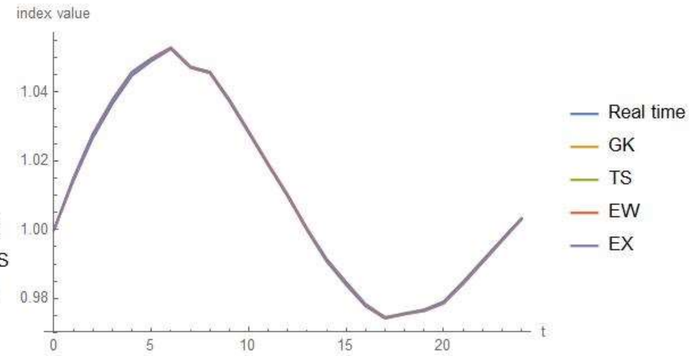


Fig. 9. Case 1.4, $t \in [0,12]$

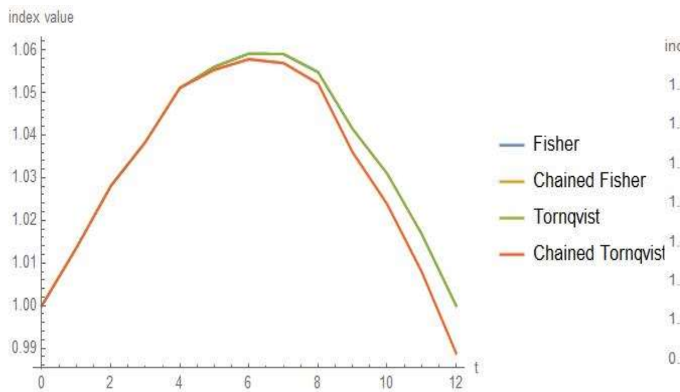


Fig. 10. Case 1.4, $T = 12, t \in [0,12]$

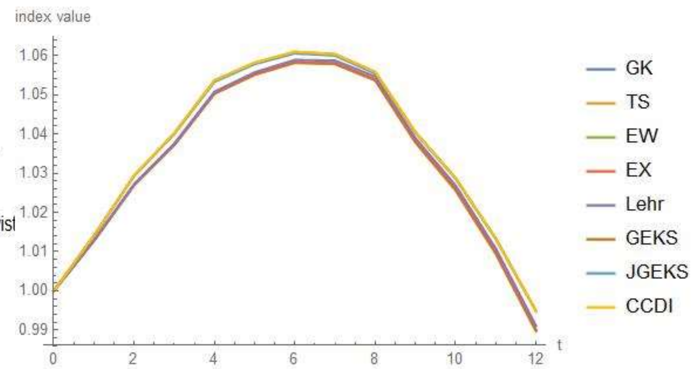


Fig. 11. Case 1.4, $T = 24, t \in [0,24]$)

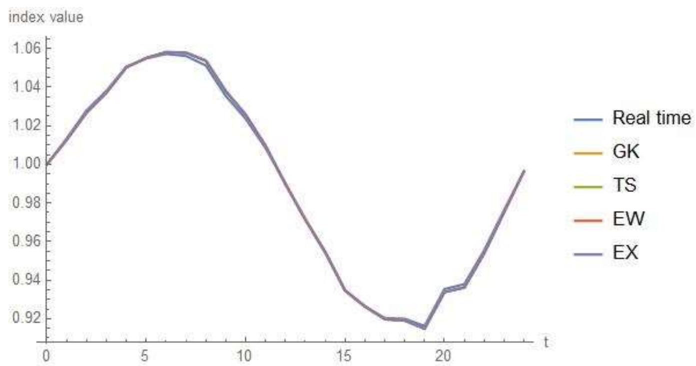


Fig. 12. Values of selected multilateral indices for different window updating methods

(**Case 1.3**, a 13-month window is considered, $T = 12$, $t \in [0,24]$)

a) Lehr index b) GEKS index c) JGEKS index d) CCDI index

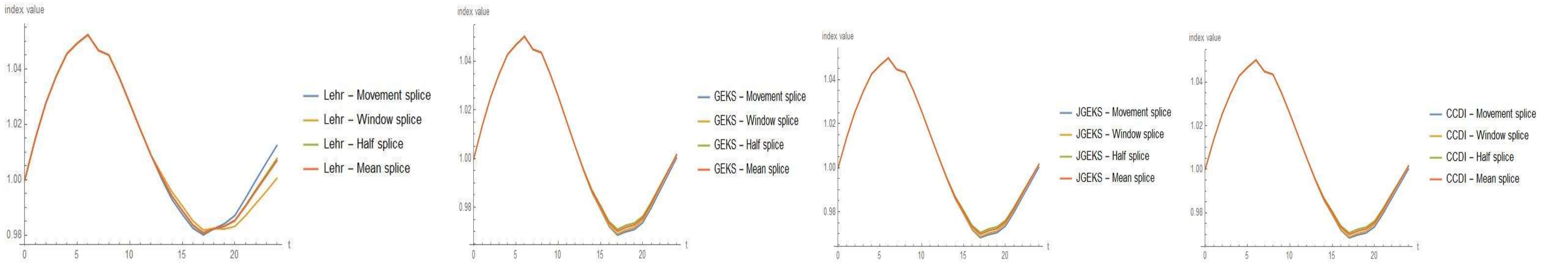
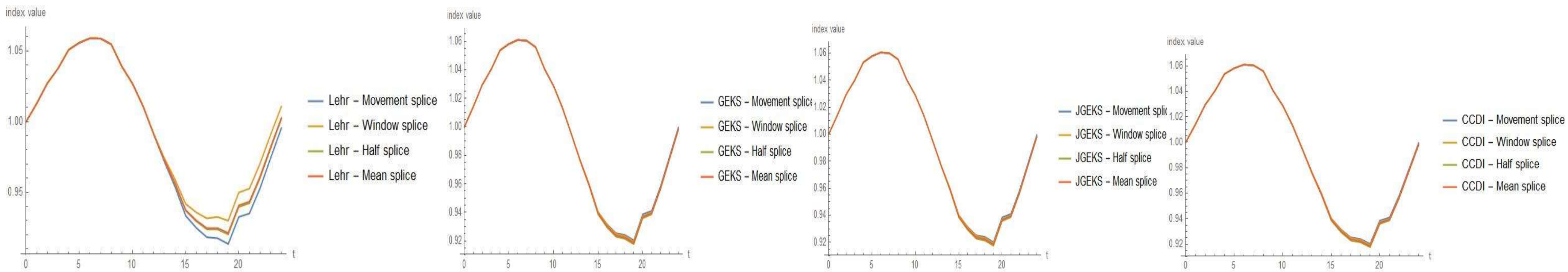


Fig. 13. Values of selected multilateral indices for different window updating methods

(**Case 1.4**, a 13-month window is considered, $T = 12$, $t \in [0,24]$)

a) Lehr index b) GEKS index c) JGEKS index d) CCDI index



Tab. 1. Values of considered indices

(**Case 1.3**, a 13-month window is considered, $T=12$, $t \in \{12,24\}$)

Index formula	Time interval	
	[0,12]	[0,24]*
Classical indices		
Jevons	1.00000	1.00000
Chained Jevons	1.00000	1.00000
Fisher	1.00000	1.00000
Chained Fisher	1.01096	1.00351
Törnqvist	1.00000	1.00000
Chained Törnqvist	1.01096	1.00350
Multilateral indices		
GK	1.00965	1.00894
TS	1.01017	1.00359
EW	1.01003	0.99933
EX	1.00977	1.00939
Real Time	1.00965	1.00894
GEKS	1.00500	1.00153
JGEKS	1.00498	1.00143
CCDI	1.00498	1.00140
Lehr	1.00891	1.00680

(*) the mean splice method is used

Tab. 2. Values of considered indices

(Case 1.4, a 13-month window is considered, $T = 12$, $t \in \{12,24\}$)

Index formula	**Time interval**	
	[0,12]	[0,24]*
Classical indices		
Jevons	1.00000	1.00000
Chained Jevons	1.00000	1.00000
Fisher	1.00000	1.00000
Chained Fisher	0.98885	0.99609
Törnqvist	1.00000	1.00000
Chained Törnqvist	0.98885	0.99608
Multilateral indices		
GK	0.99012	1.00882
TS	0.98967	0.99648
EW	0.98975	1.00021
EX	0.99000	1.00840
Real Time	0.99012	1.00882
GEKS	0.99486	0.99815
JGEKS	0.99485	0.99806
CCDI	0.99482	0.99803
Lehr	0.99112	1.00257

(*) the mean splice method is used

Case 2

A geometric Brownian motion (GBM) (also known as exponential Brownian motion) is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion (also called a Wiener process) with drift (see Oksendal, 2002; Privault, 2012). The main arguments for using the GBM price model are as follows: (a) the expected returns (relative price changes) are independent of the value of the process (price), which is consistent with what we would expect in reality; (b) the GBM process only assumes positive values, just like real commodity prices; (c) the GBM process shows the same kind of 'roughness' in its paths as we see in real prices; (d) estimations of its parameters are relatively easy. In our simulation study, we use the GBM model for generating price processes and, having known the expected value of obtained price shares, we compare values of calculated multilateral indices with these theoretical ones.

We assume that the given $i-$th price process satisfies the following stochastic differential equation

$$dp_i^t = \alpha p_i^t dt + \beta p_i^t dW_i^t ,$$

where the percentage drift α and the percentage volatility β are constant, and $\{W_i^t : 0 \le t < \infty, i = 1,2,\ldots,N\}$ are independent Wiener processes.

The solution for the above stochastic differential is as follows (Oksendal, 2002, Jakubowski et al., 2003):

$$p_i^t = p_i^0 \exp((\alpha - \frac{\beta^2}{2})t + \beta W_i^t)$$

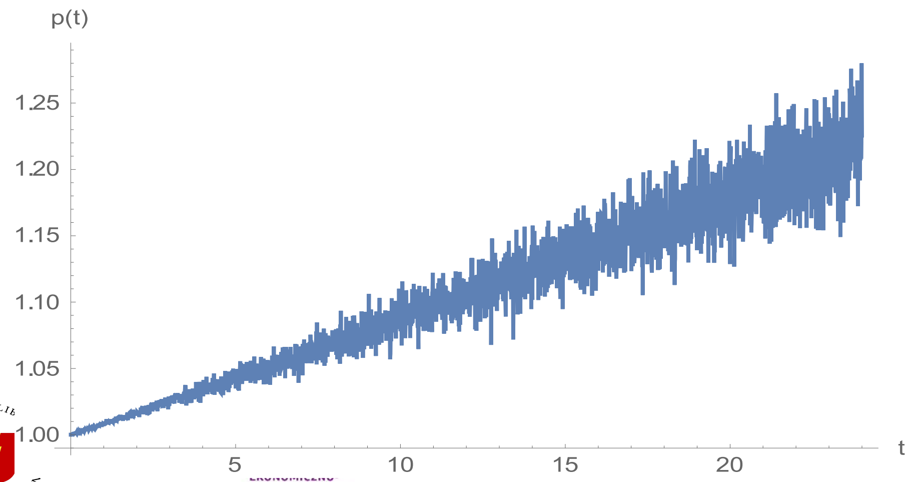
and we assume that all initial prices p_i^0 are deterministic. As a consequence, we obtain

$$E(P^t) = E(P_i^t) = \exp(\alpha t),$$

$$Var(P^t) = Var(P_i^t) = \exp(2\alpha t)[\exp(\beta^2 t) - 1]$$

where P_i^t is the $i-$th price relative and $P^t = \frac{p^t}{p^0}$ denotes the (unknown) population price index that we want to estimate.

Fig.14. Sample realisation of price process (for $\alpha = 0.1$ and $\beta = 0.02$)



We consider the following cases of quantity processes:

Case 2.1 (periodic quantities)

$$q_k^t = 1000 + k \cdot \sin(x \cdot t)$$

Case 2.2. (strongly decreasing quantities)

$$q_k^t = 10 \cdot \exp(-\frac{k}{50} \cdot t)$$

Case 2.3. (strongly increasing quantities)

$$q_k^t = 10 \cdot \exp(\frac{k}{50} \cdot t).$$

We consider the following values of parameters: $\alpha = 0.1$ and $\beta \in \{0.02,\ 0.05,\ 0.1\}$.

Our results are as follows:

Tab.3. Values of selected multilateral indices for considered time intervals – **Case 2.1**

Index value	Time interval [0,12] (theoretical index value = 1.10517)			Time interval [0,24] (theoretical index value = 1.2214)		
	$\beta = 0.02$	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.02$	$\beta = 0.05$	$\beta = 0.1$
GK	1.10492	1.10064	1.13713	1.22484	1.21391	1.26426
TS	1.10492	1.10064	1.13713	1.22484	1.21391	1.26426
EW	1.10492	1.10064	1.13713	1.22484	1.21391	1.26426
EX	1.10492	1.10064	1.13713	1.22484	1.21391	1.26426
Real Time	1.10492	1.10064	1.13713	1.22484	1.21391	1.26426
GEKS	1.10495	1.10072	1.13721	1.22497	1.21390	1.26400
JGEKS	1.10471	1.09907	1.13096	1.22426	1.20550	1.23890
CCDI	1.10495	1.10073	1.13726	1.22497	1.21392	1.26387
Lehr	1.10492	1.10064	1.13713	1.22487	1.21389	1.26405

Tab.4. Values of selected multilateral indices for considered time intervals – **Case 2.2**

Index value	Time interval [0,12] (theoretical index value = 1.10517)			Time interval [0,24] (theoretical index value = 1.2214)		
	$\beta = 0.02$	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.02$	$\beta = 0.05$	$\beta = 0.1$
GK	1.09738	1.09406	1.09444	1.21575	1.27987	1.32131
TS	1.09738	1.09447	1.09450	1.21527	1.27571	1.30300
EW	1.09782	1.09469	1.09556	1.21516	1.27580	1.31908
EX	1.09803	1.09387	1.09345	1.21566	1.27758	1.30853
Real Time	1.09837	1.09406	1.09444	1.21575	1.27987	1.32131
GEKS	1.09786	1.09475	1.09825	1.21402	1.27322	1.30844
JGEKS	1.09766	1.09358	1.09539	1.21404	1.26845	1.28782
CCDI	1.09786	1.09467	1.09810	1.21403	1.2739	1.30823
Lehr	1.08685	1.08296	1.08447	1.20766	1.25881	1.30362

Tab.5. Values of selected multilateral indices for considered time intervals – **Case 2.3**

Index value	Time interval [0,12] (theoretical index value = 1.10517)			Time interval [0,24] (theoretical index value = 1.2214)		
	$\beta = 0.02$	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.02$	$\beta = 0.05$	$\beta = 0.1$
GK	1.10087	1.10428	1.09444	1.22130	1.23098	1.32131
TS	1.10173	1.10570	1.09450	1.22237	1.25137	1.30300
EW	1.10194	1.10632	1.09556	1.22226	1.25209	1.31908
EX	1.10091	1.10428	1.09345	1.22148	1.23256	1.30853
Real Time	1.10087	1.10428	1.09444	1.22130	1.23098	1.32131
GEKS	1.10232	1.10570	1.09825	1.22118	1.24958	1.30844
JGEKS	1.10205	1.10432	1.09539	1.22059	1.24529	1.28782
CCDI	1.10229	1.10561	1.09810	1.22107	1.24885	1.30823
Lehr	1.09109	1.09422	1.08447	1.22743	1.23190	1.30362

GUS

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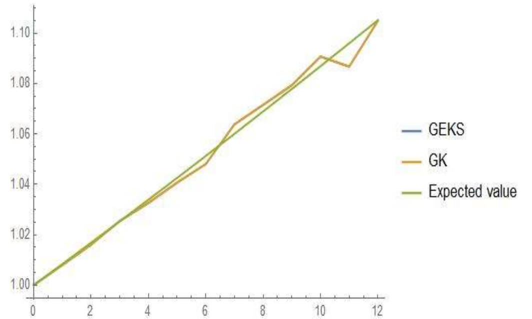
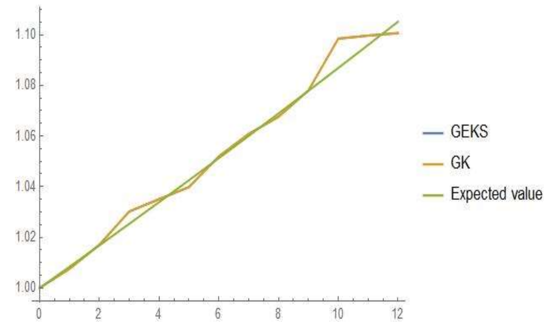
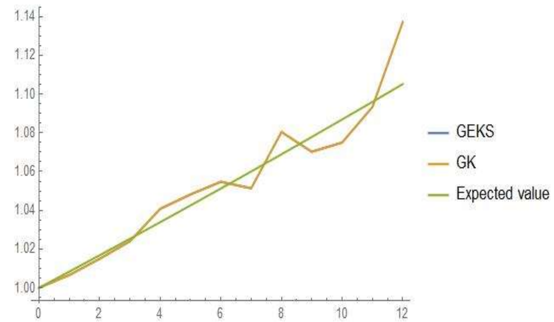
Fig. 15. Comparison of values of the **Geary-Khamis and GEKS** indices with the theoretical price dynamics ($t \in [0,12]$)

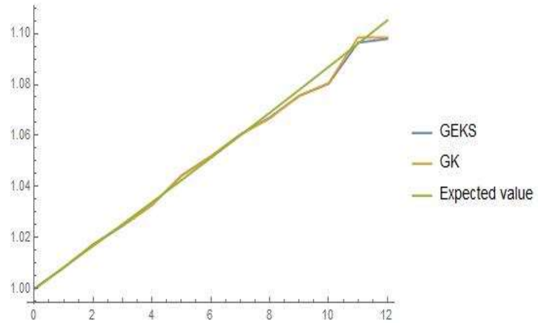
Case 2.1. $\beta = 0.02$ $\beta = 0.05$ $\beta = 0.1$

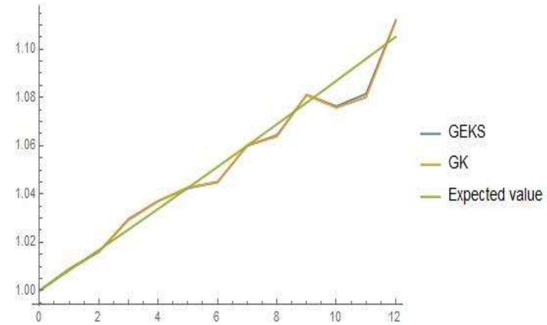


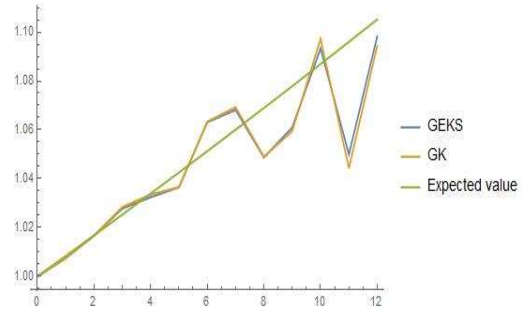




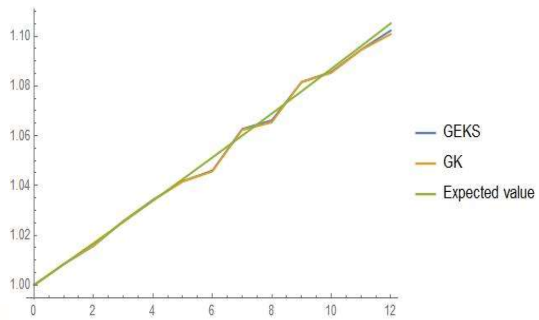
Case 2.2. $\beta = 0.02$ $\beta = 0.05$ $\beta = 0.1$

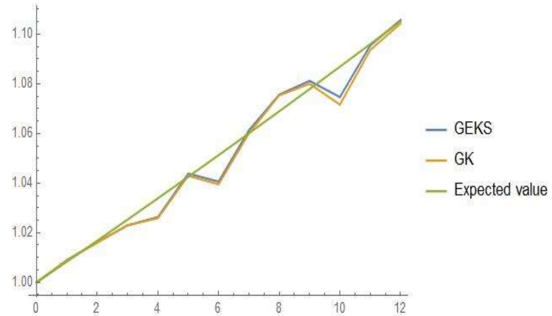


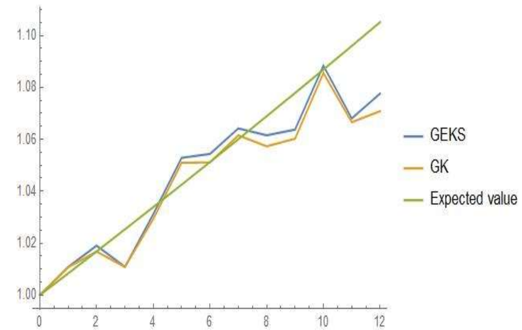




Case 2.3. $\beta = 0.02$ $\beta = 0.05$ $\beta = 0.1$







Tab. 6. Values of *RMSDs* calculated for considered multilateral indices in Cases 2.1 – 2.3 and for $\beta \in \{0.02;\ 0.1\}$

Index formula	Case 2.1		Case 2.2		Case 2.3	
	$\beta = 0.02$	$\beta = 0.1$	$\beta = 0.02$	$\beta = 0.1$	$\beta = 0.02$	$\beta = 0.1$
GEKS MS	0.00334	0.02024	0.00821	0.05531	0.00675	0.04122
GEKS WS	0.00334	0.02023	0.00808	0.05468	0.00664	0.04045
GEKS HS	0.00334	0.02025	0.00816	0.05575	0.00671	0.04093
GEKS GMS	0.00334	0.02025	0.00815	0.05534	0.00673	0.04086
JGEKS MS	0.00343	0.02213	0.00825	0.04564	0.00669	0.04321
JGEKS WS	0.00343	0.02212	0.00812	0.04512	0.00658	0.04240
JGEKS HS	0.00343	0.02215	0.00820	0.04599	0.00665	0.04291
JGEKS GMS	0.00343	0.02215	0.00819	0.04564	0.00667	0.04283
CCDI MS	0.00334	0.02020	0.00822	0.05581	0.00675	0.04132
CCDI WS	0.00334	0.02019	0.00809	0.05527	0.00664	0.04057
CCDI HS	0.00334	0.02021	0.00817	0.05633	0.00671	0.04102
CCDI GMS	0.00334	0.02021	0.00816	0.05591	0.00673	0.04095
Lehr MS	0.00335	0.02024	0.01132	0.05747	0.00799	0.04685
Lehr WS	0.00335	0.02021	0.01123	0.05635	0.00795	0.04013
Lehr HS	0.00335	0.02025	0.01125	0.05725	0.00799	0.04397
Lehr GMS	0.00335	0.02024	0.01123	0.05709	0.00796	0.04356
GK MS	0.00334	0.02021	0.00832	0.05583	0.00665	0.04142
GK WS	0.00334	0.02017	0.00808	0.05426	0.00624	0.04057
GK HS	0.00334	0.02022	0.00827	0.05630	0.00661	0.04133
GK GMS	0.00334	0.02021	0.00826	0.05631	0.00663	0.04125
Real Time	0.00334	0.02023	0.00897	0.05712	0.00643	0.03807

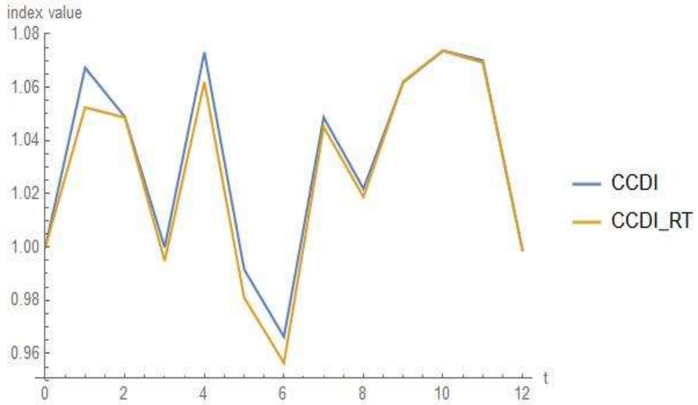
5. Empirical study

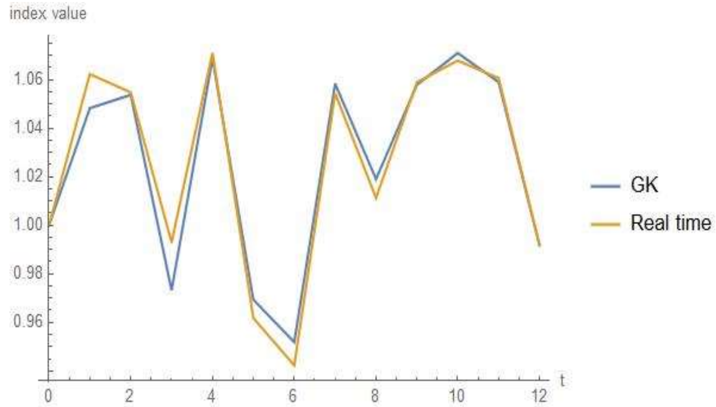
In the following empirical study, we consider two scanner data sources: (a) the first is "classical", i.e. data sets come from one supermarket and they concern the following group of products: plain flour (COICOP group: 011121), milk 3.2% (COICOP group: 011411) and rice (COICOP group: 011111). In this case, we have only a 13-month time series (Dec. 2014 – Dec. 2015), so our analysis is limited here; (b) the other scanner data source is *allegro.pl,* which is one of the biggest online e-commerce platform in Poland. We use transaction data on mountain bikes, touring bicycles and children's bicycles from the group "bicycles" (COICOP group: 071301). This time, the length of the considered time interval is 25 months (Dec. 2016 – Dec. 2018), and thus window updating methods (for a 13-month window) can be used here. In both cases (a) and (b), we use data aggregated to one month. Matching products to the proper group is supported by using some text mining methods and also some manual verification is made to avoid the "re-launch problem". To be included in the calculations, a product has to have a turnover above a minimum threshold. Products that show extreme pricing changes from one month to another are also excluded.

Case A

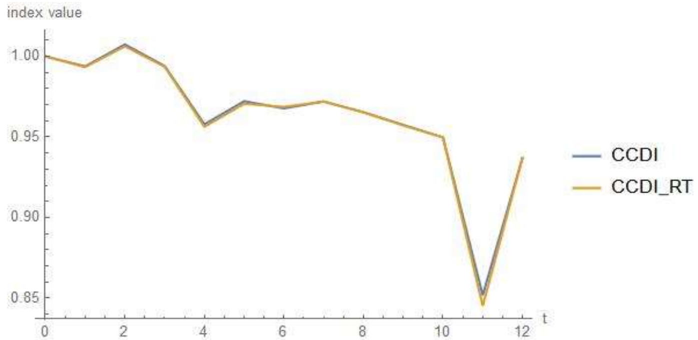
Fig. 16. Comparison of selected multilateral indices (**CCDI, GK**) for fully and "currently" available time windows (calculated for plain flour, milk and rice).

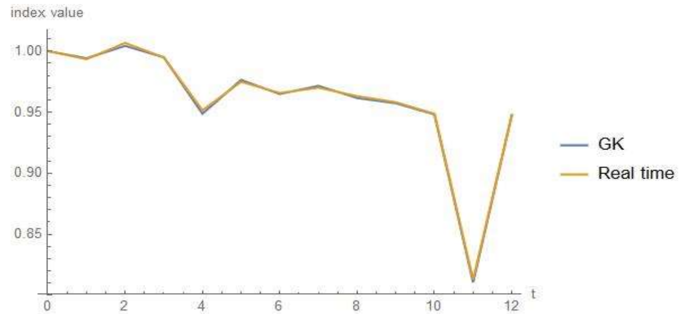
a) plain flour



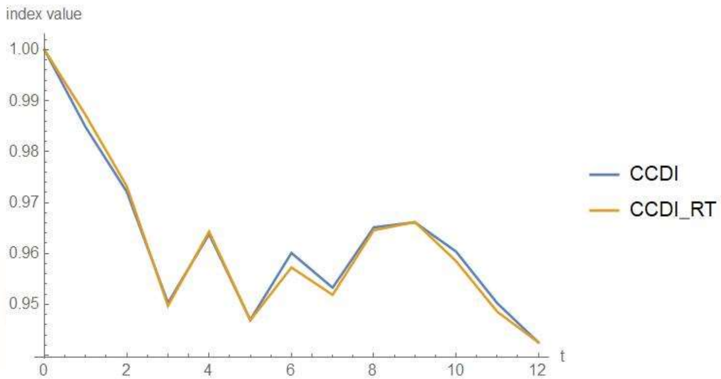


b) milk





c) rice



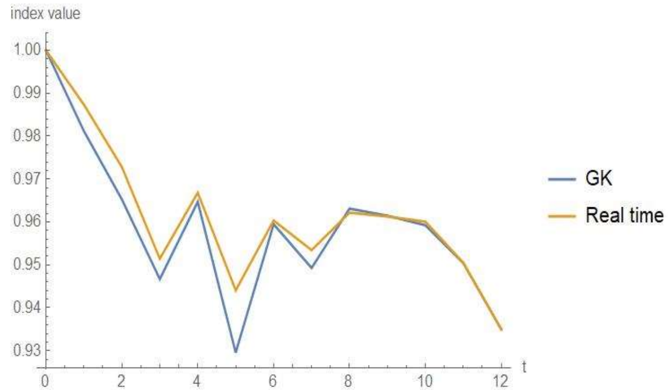
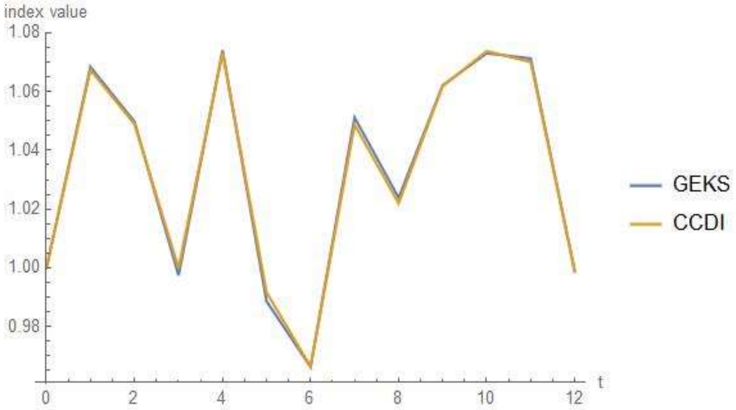
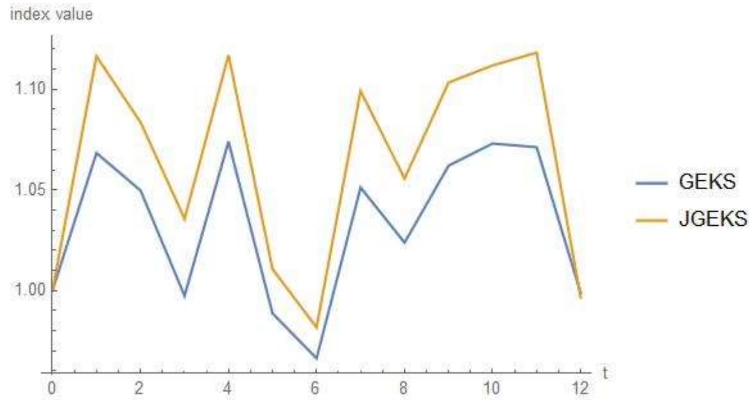


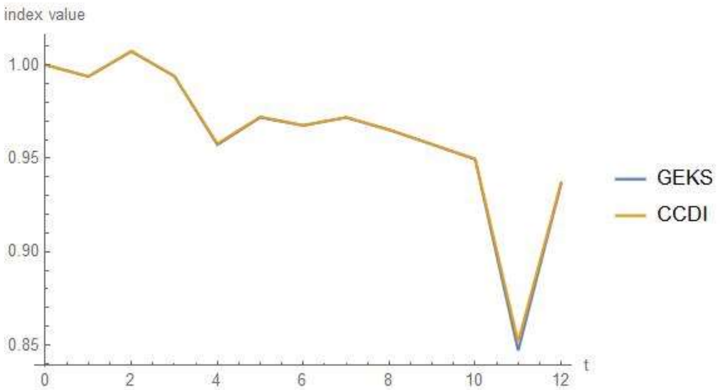
Fig. 17. Comparison of the **GEKS** index with the **CCDI and JGEKS** indices calculated over the whole period of 13 months for plain flour, milk and rice.

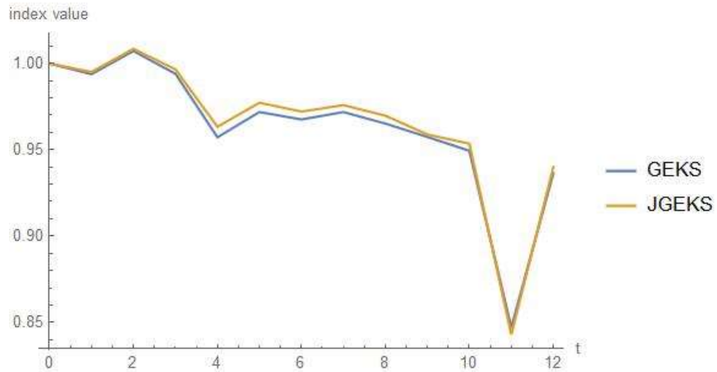
a) plain flour



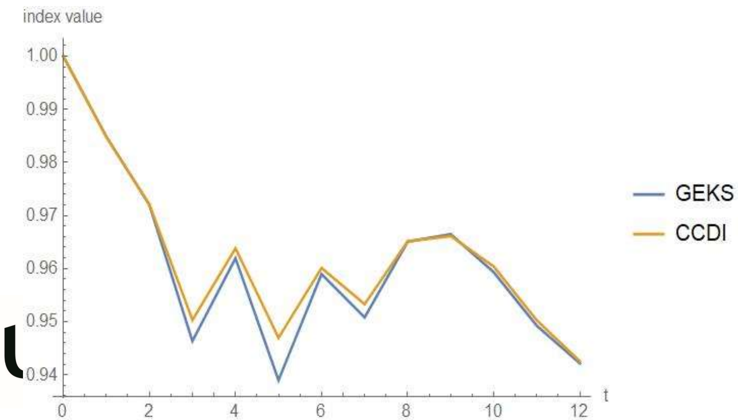


b) milk





c) rice



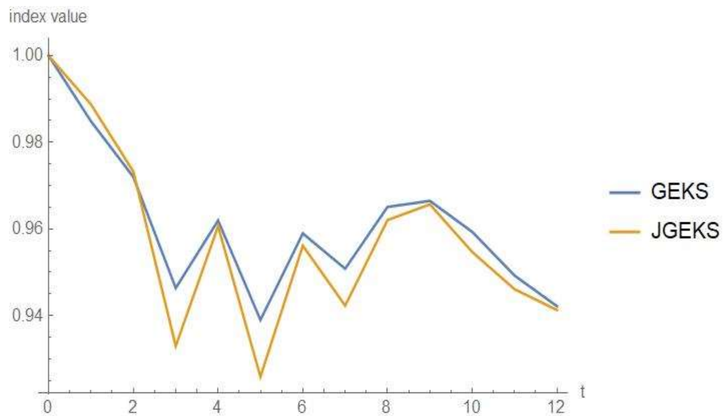
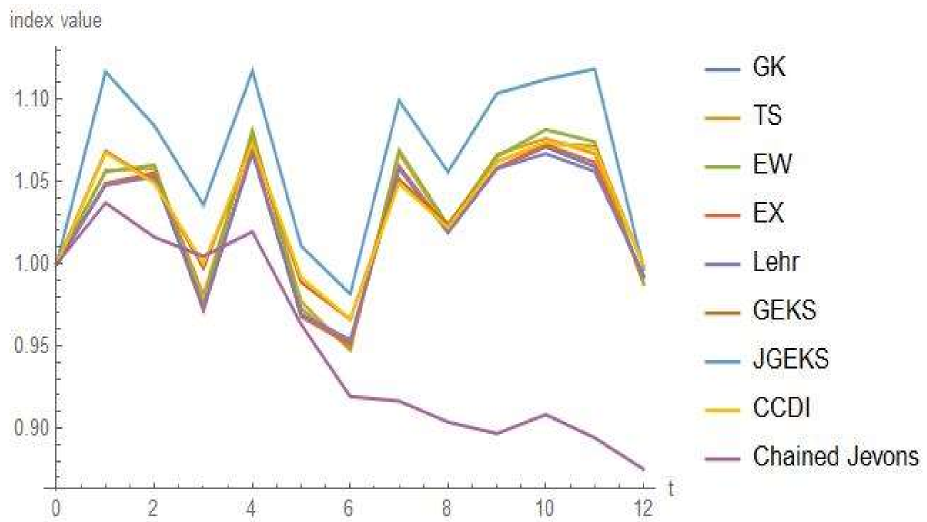
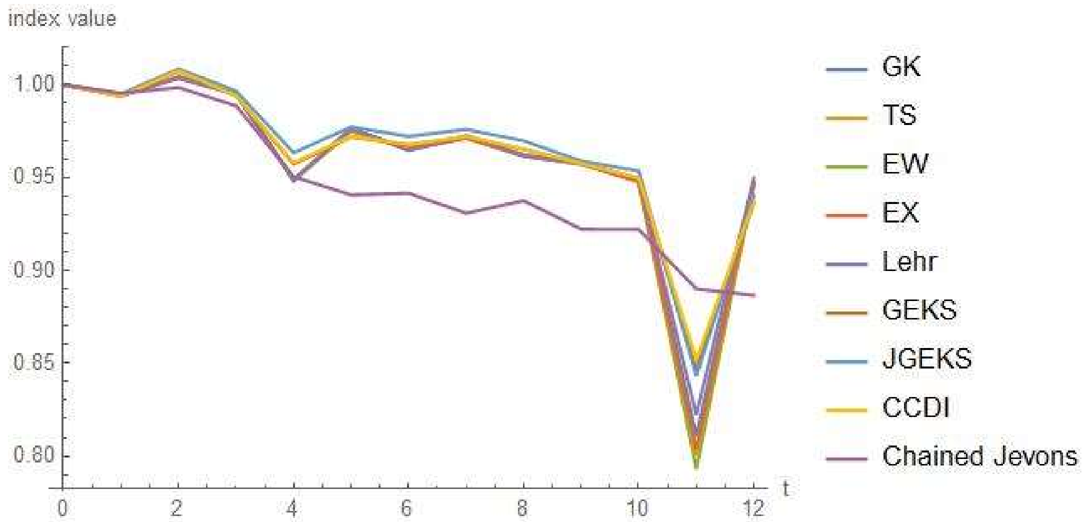


Fig. 18. All considered multilateral indices together with the chained Jevons index calculated over the whole period of 13 months for plain flour, milk and rice.

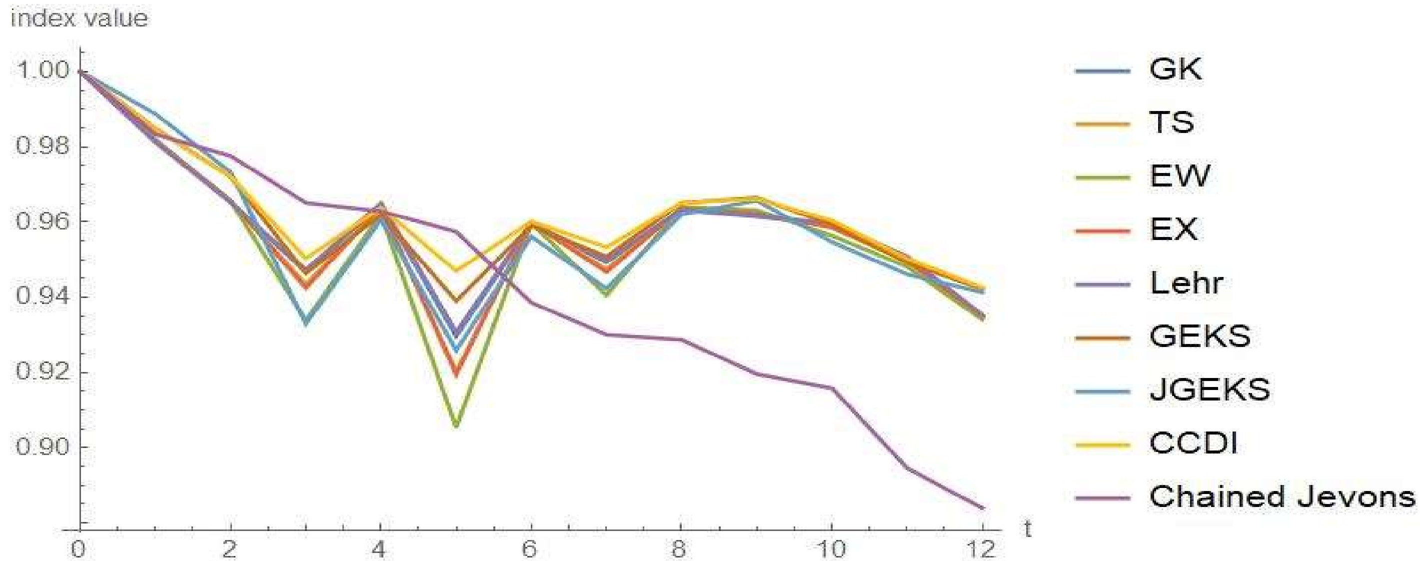
a) plain flour



c) milk



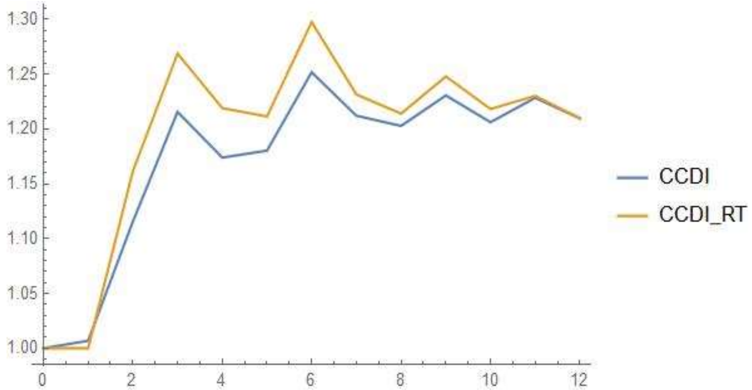
b) rice

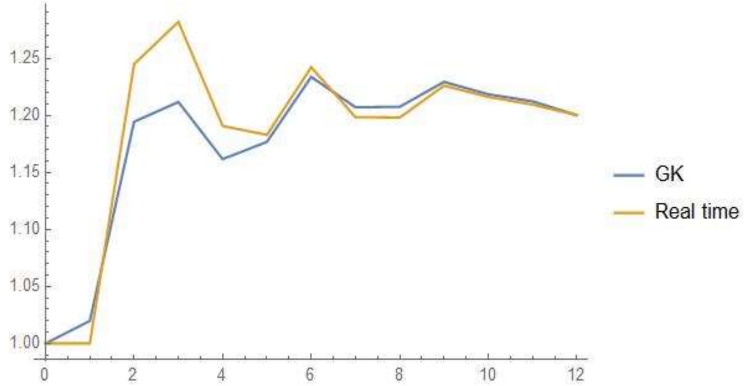


Case B (data from allegro.pl)

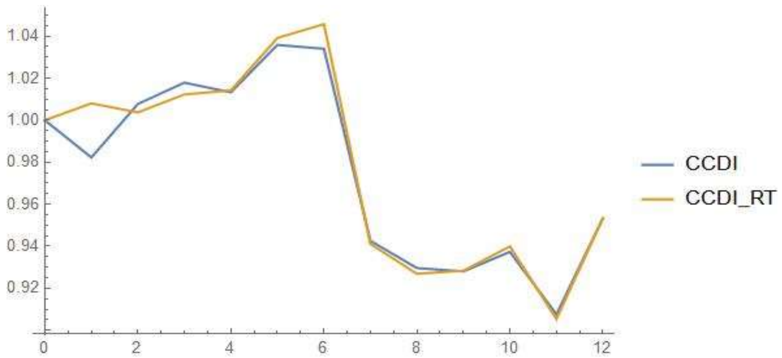
Fig. 19. Comparison of selected multilateral indices (**CCDI, GK**) for fully and “currently” available time windows (for mountain bikes, touring bicycles and children’s bicycles sold in 2018).

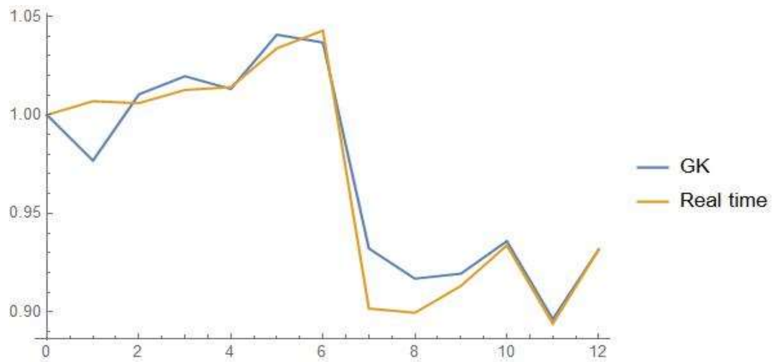
a) mountain bikes



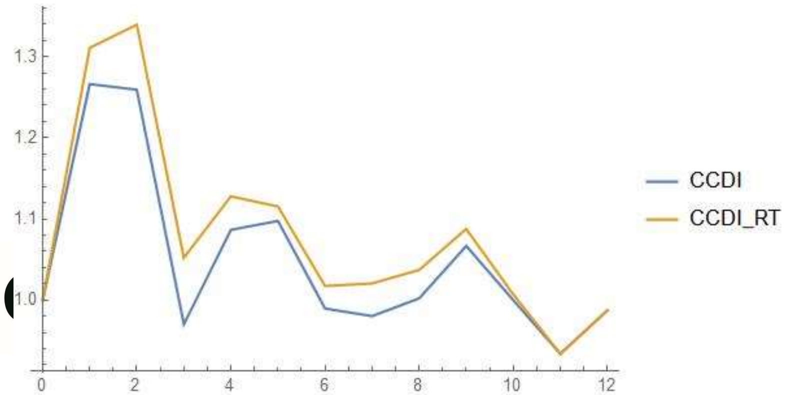


b) touring bicycles





c) children’s bicycles



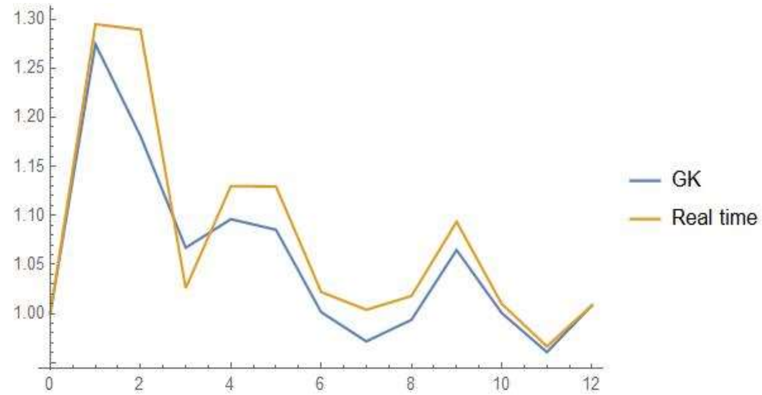
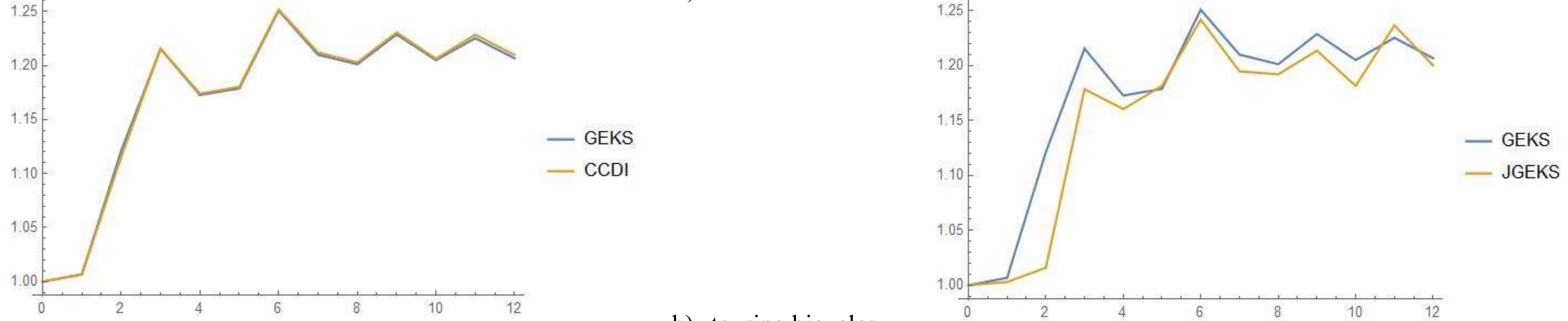
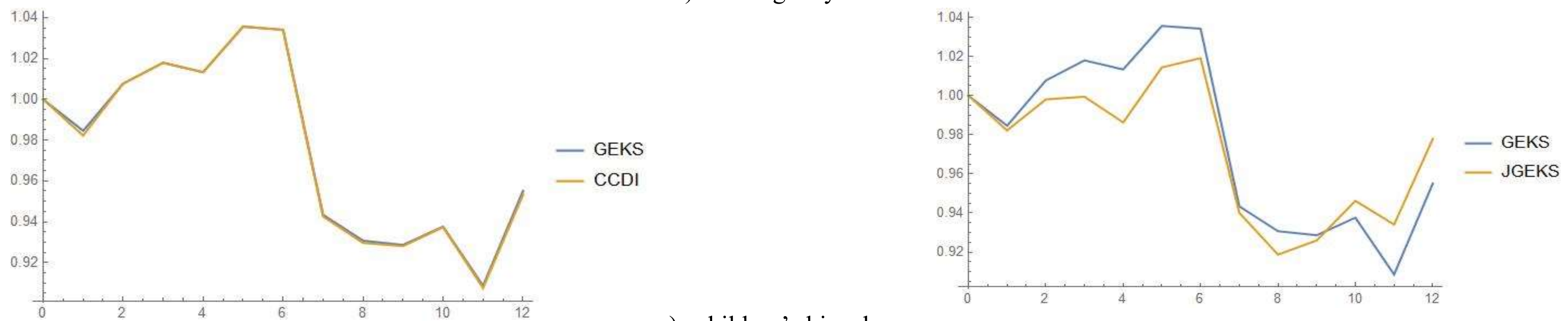


Fig. 20. Comparison of the **GEKS** index with the **CCDI and JGEKS** indices (a full window of 13 months is available) for mountain bikes, touring bicycles and children's bicycles sold in 2018.

a) mountain bikes



b) touring bicycles



c) children's bicycles

Fig. 21. **Window updating methods** in the case of the **CCDI, GEKS, JGEKS and Lehr** indices (a 13-month time window is considered)

For instance: **mountain bikes**

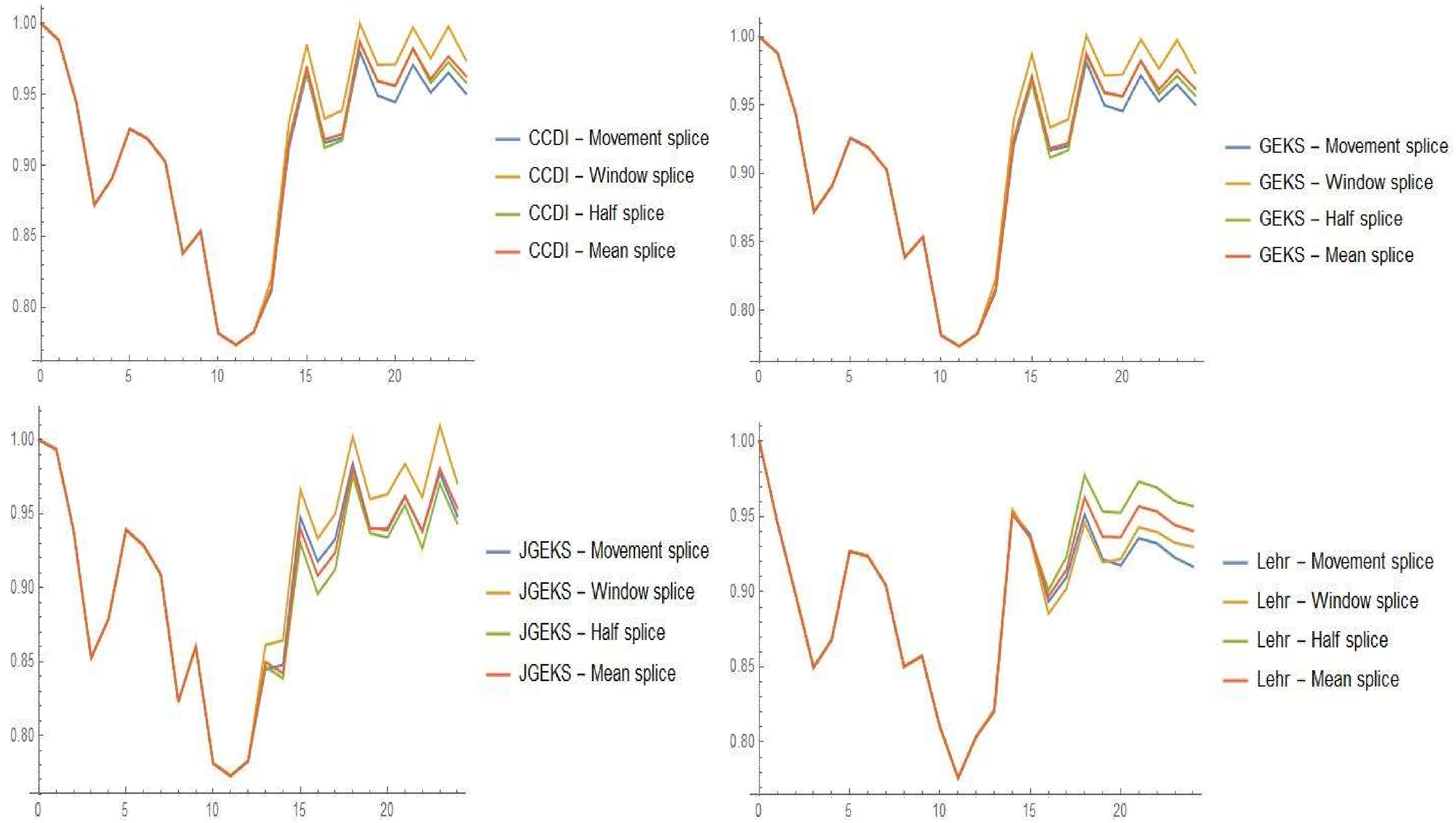
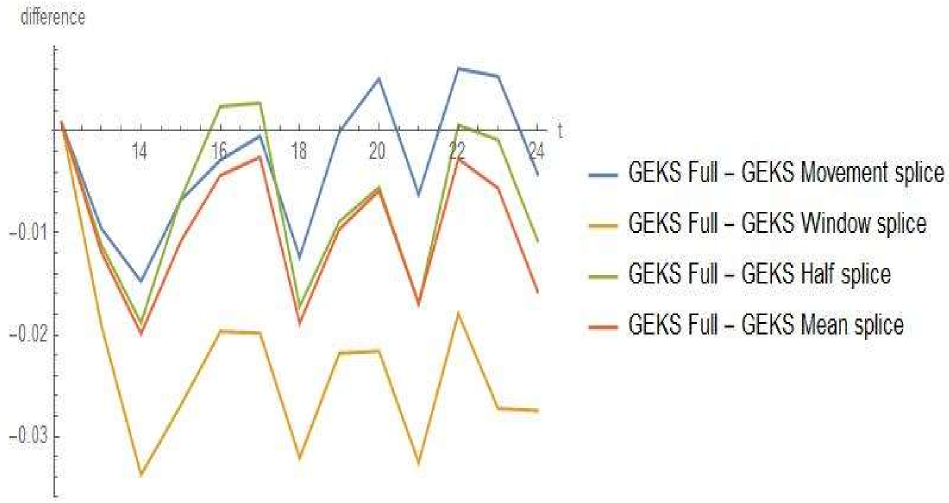
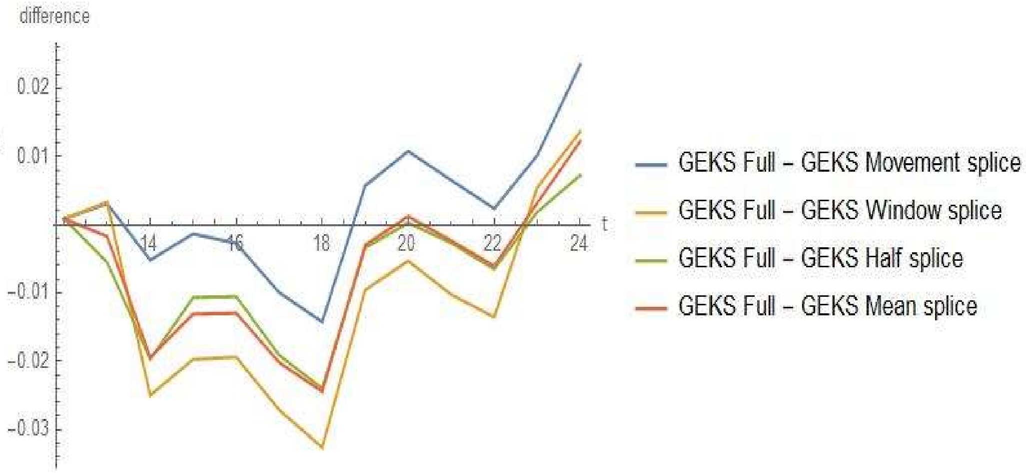


Fig. 22. Differences between the **GEKS** index and the corresponding **splice indices** (year: 2018)

a) mountain bikes



c) touring bikes



b) children's bicycles

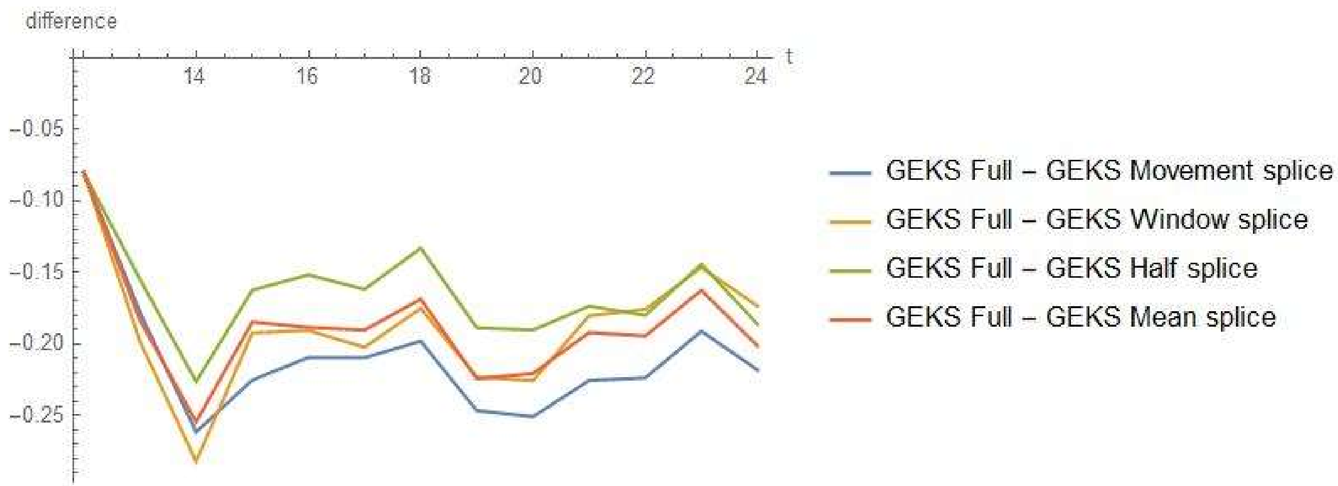
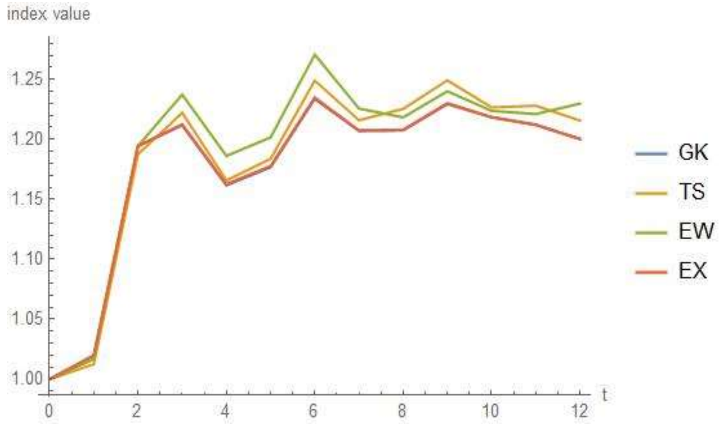
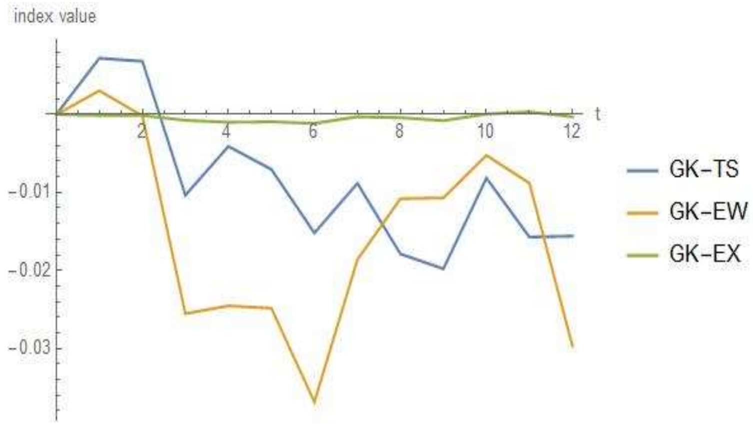


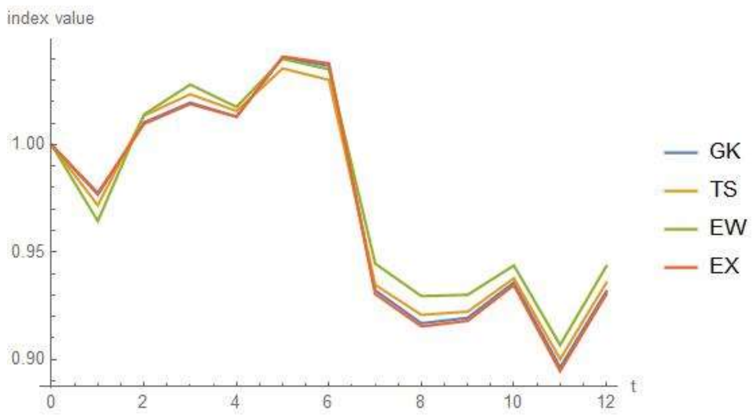
Fig. 23. Comparison of **weighting schemes** in the **QU method** (year: 2018, a 13-month time window is used)

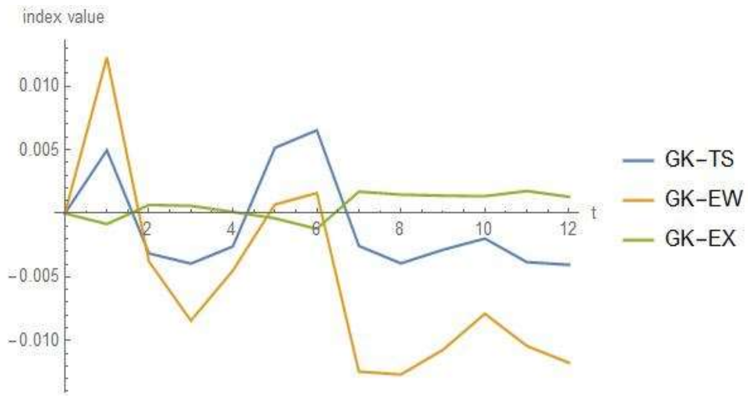
a) mountain bikes



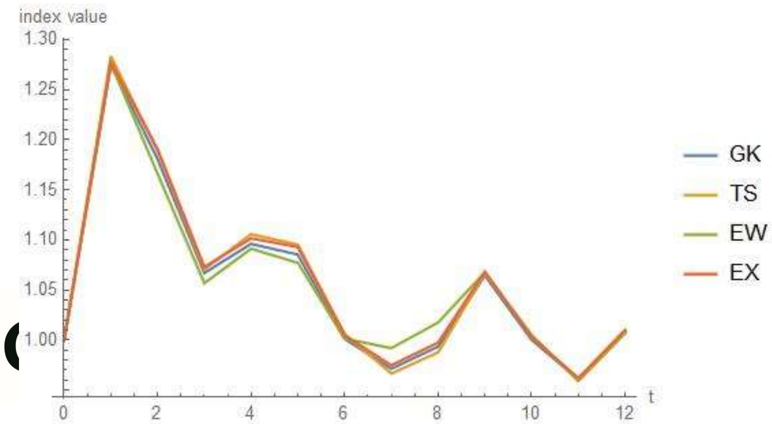


b) touring bicycles





c) children's bicycles



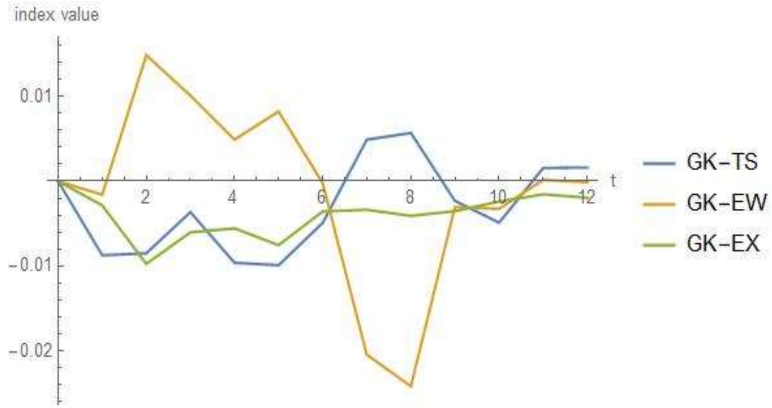
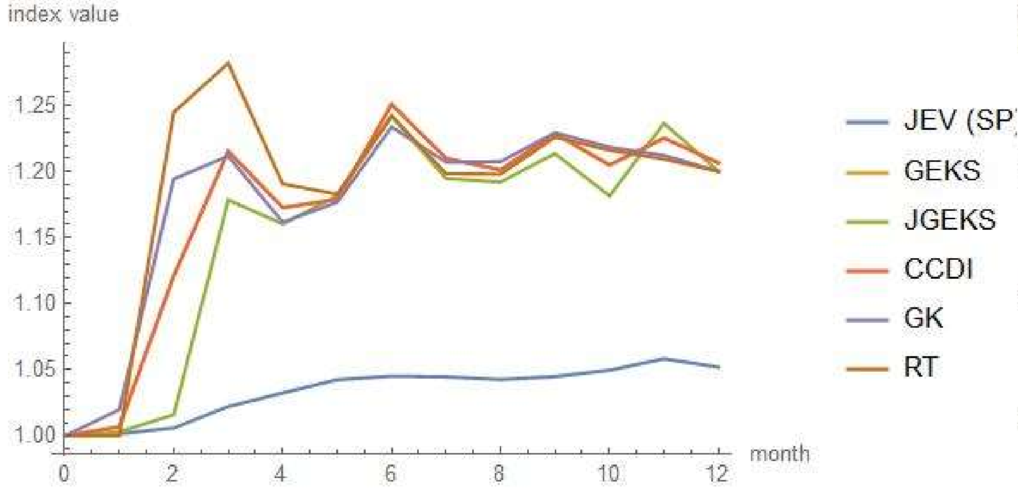
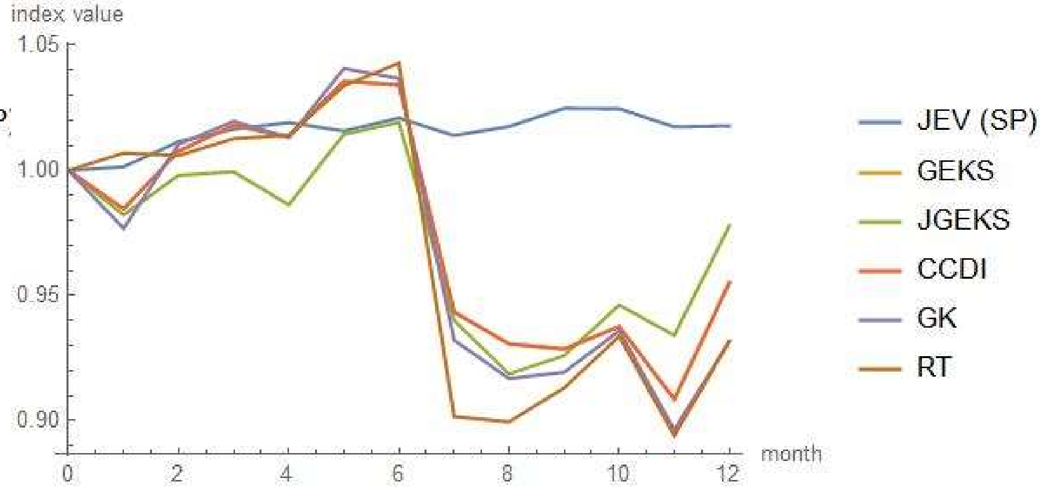


Fig. 24. Comparison of **all discussed multilateral indices** with the **chained Jevons** based on data collected in a **traditional way** by Statistics Poland (a full window of 13 months, 2018 year).

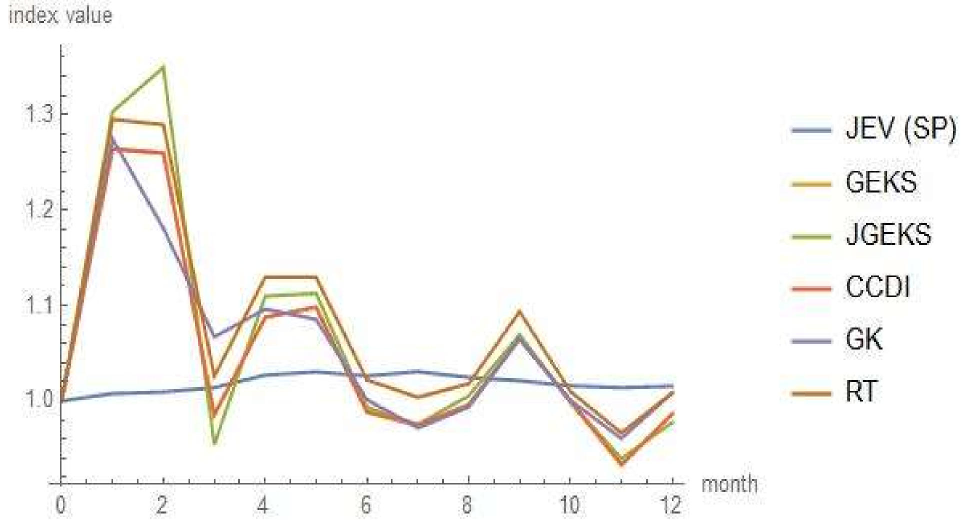
a) mountain bikes



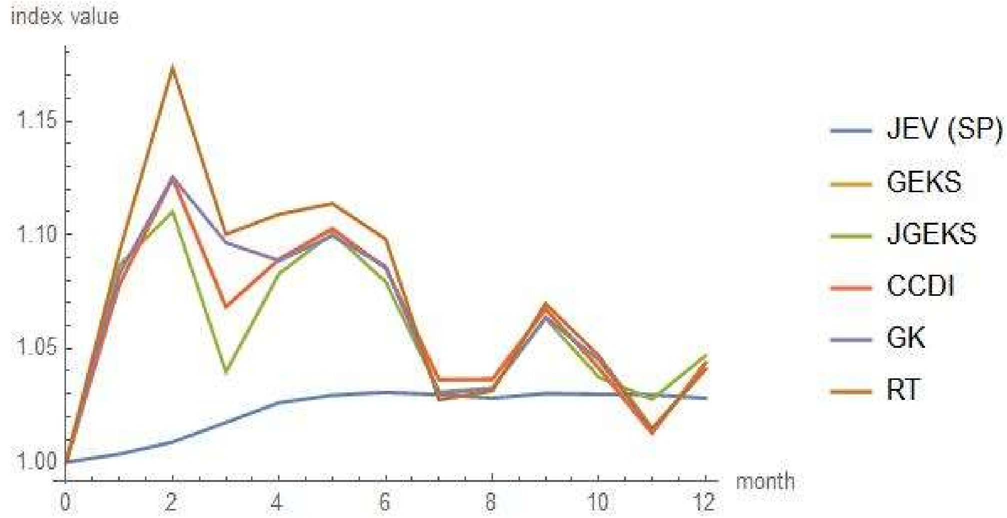
b) touring bicycles



c) children's bicycles



d) all groups of bicycles



6. Conclusions

The general conclusions from the **Simulation Study** are:

(a) Even multilateral indices may differ from unity if only prices revert back to their levels in the base period. In our study, chained superlative indices and multilateral indices seem to slightly overestimate the real price change when quantities strongly decrease. When quantities strongly decrease, as a rule chained superlative indices and multilateral indices seem to be slightly below the real price change. In our experiments (which are not presented here), we observe that the monotonicity of quantities (in particular those connected with new and disappearing goods) has a much more bigger impact on differences among multilateral indices than the level of price volatilities.

(b) When prices can be described by a geometric Brownian motion (they follow a trend instead of displaying periodical changes), differences between the theoretical (known) value of the price change and any multilateral price index (for a given time moment) are the biggest in the case of strongly decreasing quantities. In general, these differences will rise if the volatility of prices increases. In particular, for any considered cases of quantity changes, the measured root mean square error seems to be comparable for considered multilateral indices (GEKS, JGEKS, CCDI, Lehr, Real time) but it seems to be the smallest in the case of window splice method.

Our **Empirical Study** provides the following conclusions: (a) When we have no historical data from supermarkets and we start using scanner data sets, then the application of multilateral indices for the "currently" available time window is justified since differences between selected indices (CCDI, GK) for the fully and "currently" available time window are not too big, i.e. these differences are decreasing functions of time and, as a rule, after 6 – 8 months they are negligible.

(b) In practice, there are no substantial differences between the GEKS and CCDI indices and it is not surprising since superlative indices (Fisher, Törnqvist) approximate each other (Diewert (1976)). Nevertheless, the differences between the GEKS and JGEKS indices are crucial and, in our opinion, it confirms that the movements of quantities may not be (rationally) correlated with price movements;

(c) Differences between multilateral indices and the chained Jevons index may be very big (see Fig. 18 for plain flour or rice), and as a rule they are. Thus, switching the chained Jevons index to one of multilateral indices does matter in the CPI measurement;

(d) The chain drift bias may be substantial when using splice indices. In our study, the best result as a rule is obtained by using the movement splice method and the worst result (the biggest chain drift bias) is obtained by using the window splice method.

(e) The differences between the GEKS index and splice indices as a rule are negative;

(f) The choice of the weighting schemes in the QU method does matter – differences in results may be crucial (in our study time moments for which the differences between the TS, EW and EX indices exceeded 3 percentage points were observed). The EW index differs the most in relation to the Geary-Khamis index;

(g) The results of price dynamics obtained by using alternative data sources (e.g.: allegro.pl) may be completely different in comparison to those obtained by using traditionally collected data sets

(h) The Lehr price index seems to be the most sensitive in the case of the choice of window updating method (see also our Simulation Study).

Thank you for your attention!