# The Nature of Chain Drift

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# 1 Introduction

- For inflation measurement, scanner data have many advantages.
- However, they can give rise to
  - *measurement bias* caused by changes in the universe of items,
  - *chain drift* caused by
    – post sales dip
    – delayed consumer adjustment.

- The presentation proceeds in three stages:

  Stage 1: Two forces of chain drift are identified:

  1. *pendular quantities* (post sales dip),
  2. *sticky quantities* (delayed consumer adjustment).

  Stage 2: A utility framework is introduced that generates pendular and sticky quantities.

  Stage 3: A stress test for different RGEKS indices is conducted.

# 2 Synchronous Quantity Responses to Price Changes

- Let $P^{t/t\text{-}1}$ denote a direct price index (e.g. Laspeyres, Törnqvist) for the comparison period $t$ and the base period $t-1$.
- A direct price index $P^{t/t\text{-}1}$ exhibits chain drift, if
  - in period $T$ all prices and quantities reverse to their values of period 0, but
  - chaining $P^{t/t\text{-}1}$ gives

$$P^{1/0} \cdot P^{2/1} \cdot \ldots \cdot P^{T/T-1} \neq 1$$

- The Törnqvist index is a weighted average of price ratios:

$$\ln P_{\text{Tö}}^{t/t\text{-}1} = \sum \frac{1}{2}\left(\frac{p_i^{t\text{-}1}x_i^{t\text{-}1}}{\sum p_j^{t\text{-}1}x_j^{t\text{-}1}} + \frac{p_i^t x_i^t}{\sum p_j^t x_j^t}\right)\ln\frac{p_i^t}{p_i^{t\text{-}1}}$$

- The weight attached to item $i$
  - increases in both prices ($p_i^{t\text{-}1}$ and $p_i^t$) and both quantities ($x_i^{t\text{-}1}$ and $x_i^t$),
  - is symmetric in the two periods.
- Therefore, the Törnqvist index belongs to the class of SWAP indices (**S**ymmetrically **W**eighted **A**verages of **P**rice ratios).

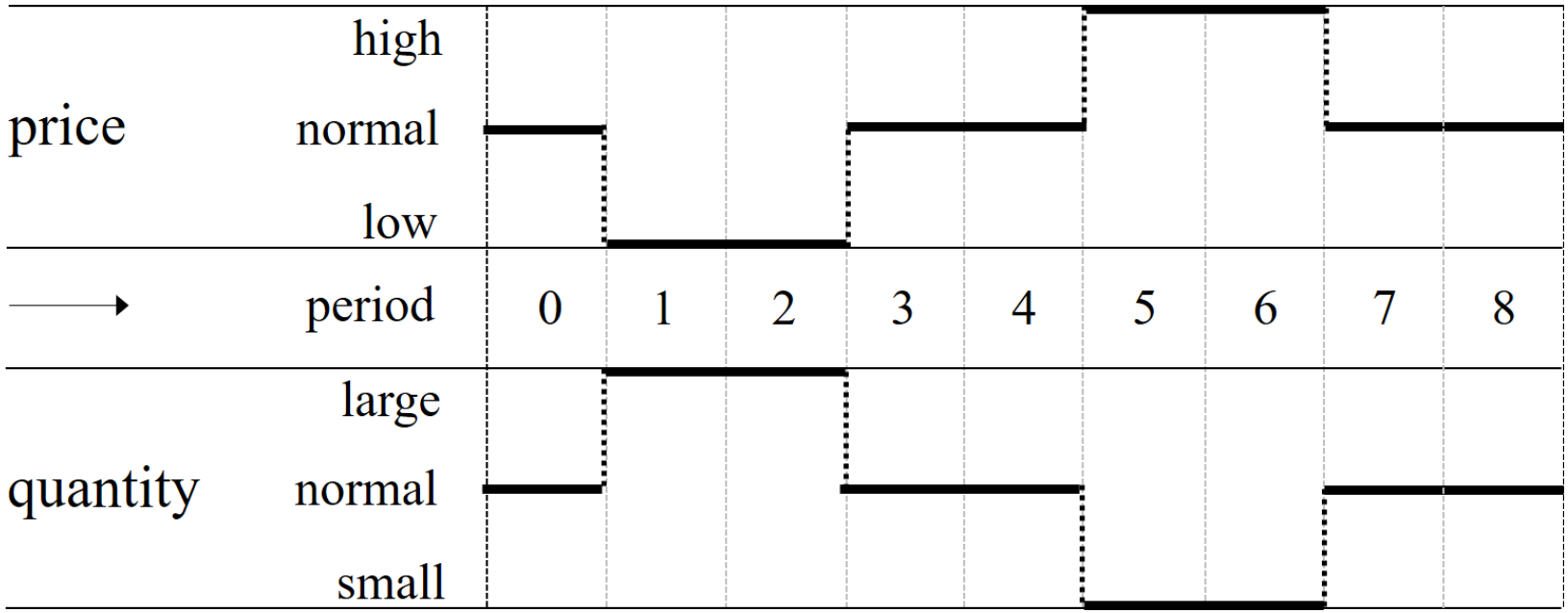
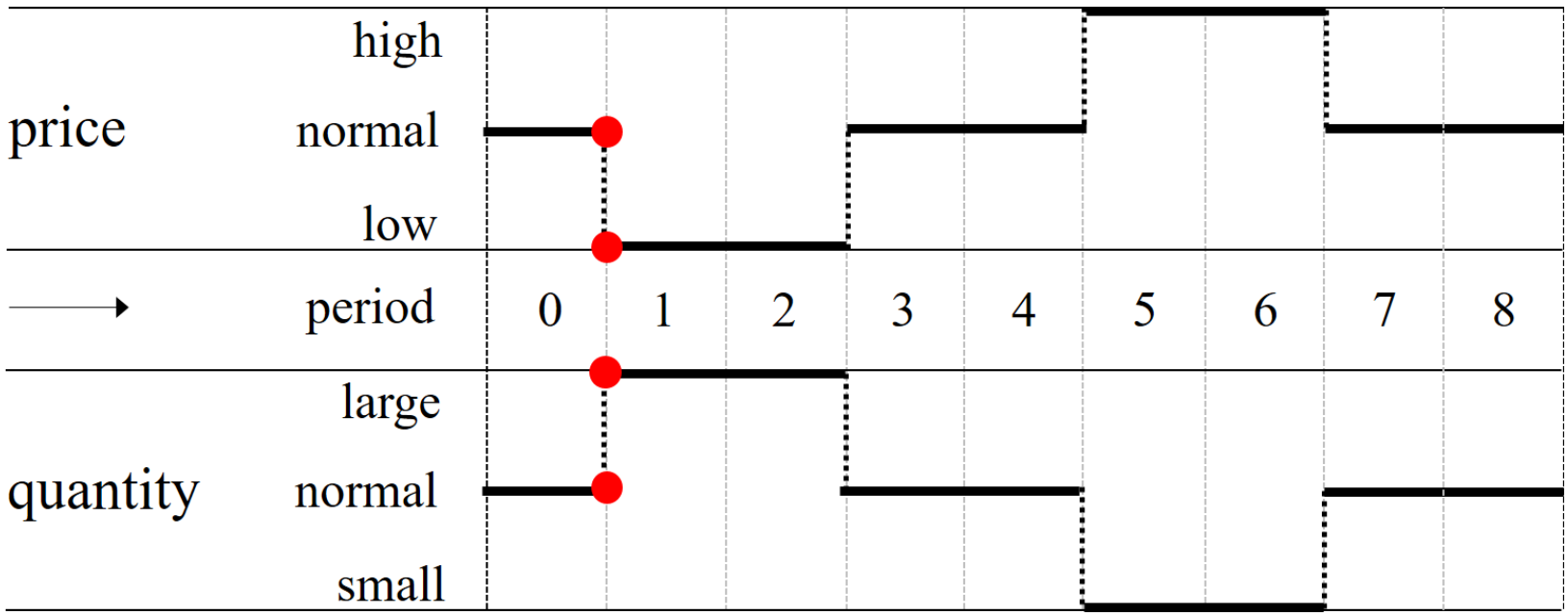



Figure 1: Synchronous Quantity Reactions to Price Changes.

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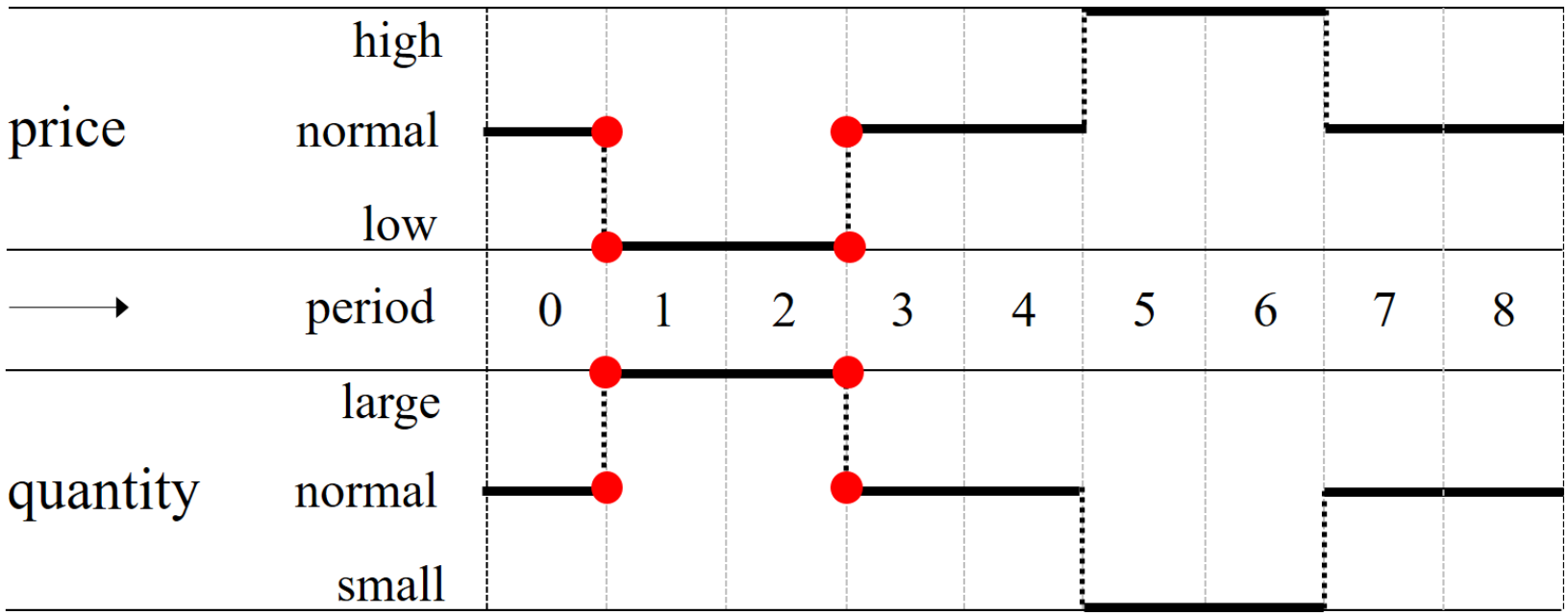


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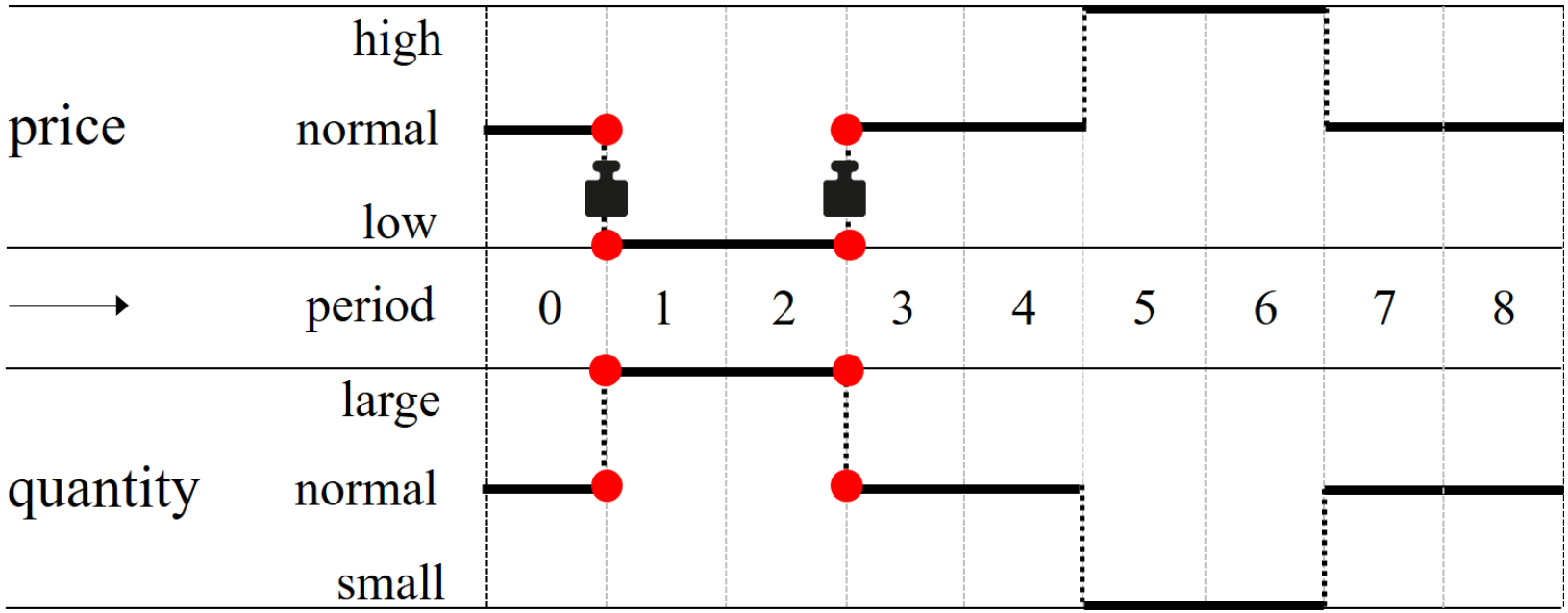


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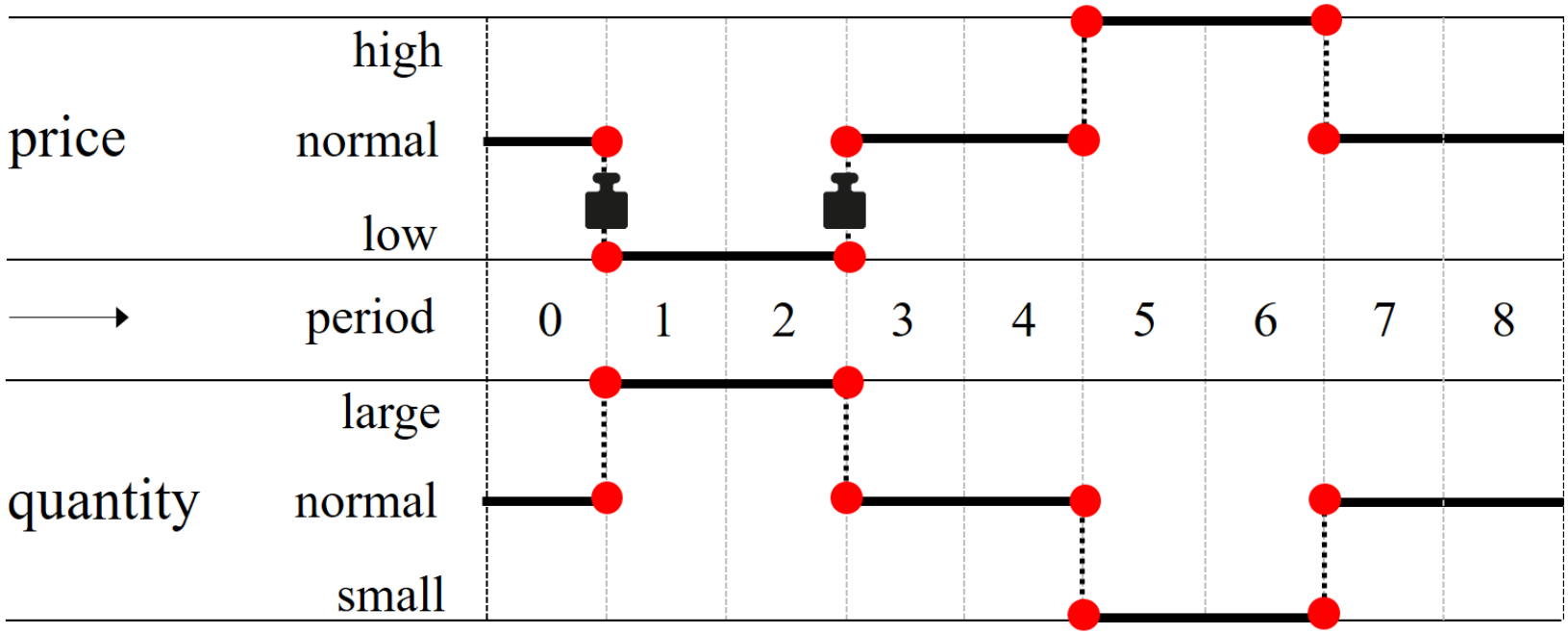


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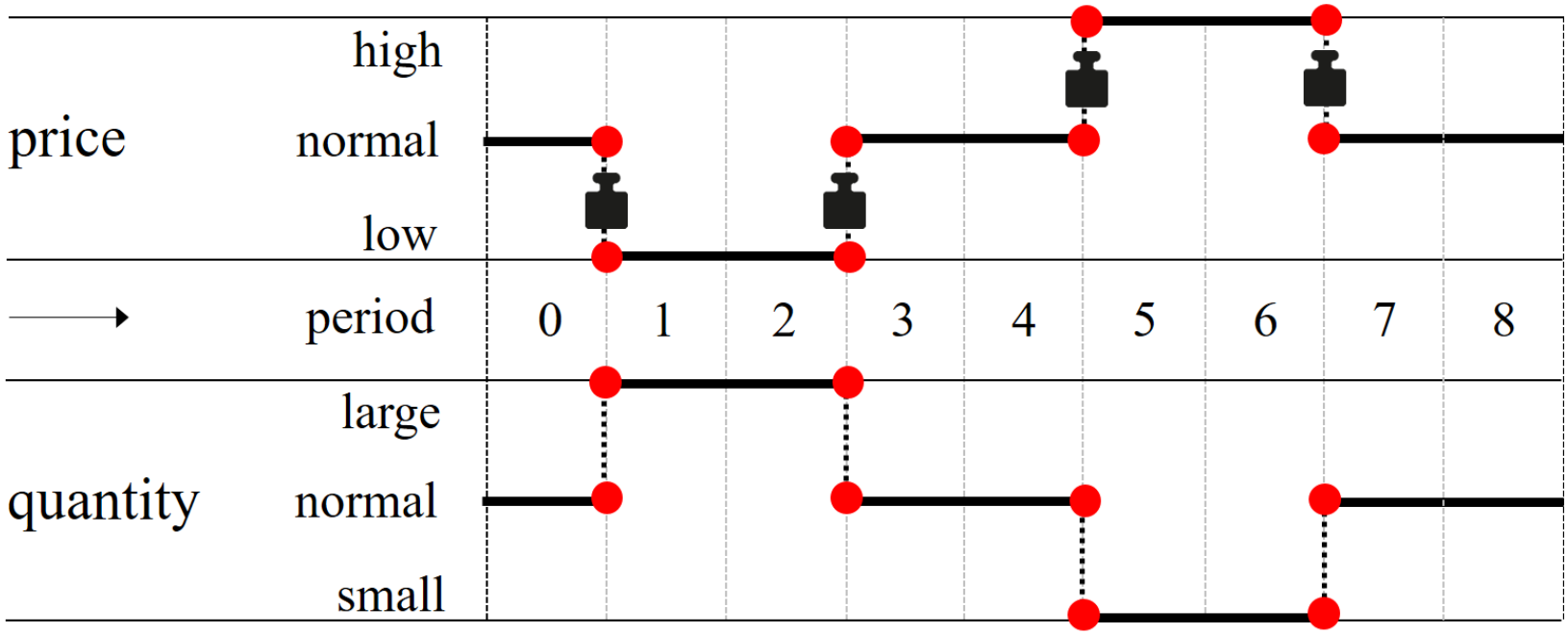


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# 3 Pendular Quantities

- Synchronous price and quantity changes:
  SWAP indices are immune to chain drift, while the Laspeyres index and Paasche index are not (if elasticities of demand differ from 1).
- Asynchronous price and quantity changes:
  SWAP indices generate chain drift.
- Usually this is attributed to sales in conjunction with inventory behaviour of consumers.
- Such behaviour leads to *pendular quantities*.
- Also price spikes can trigger pendular quantities.
- Pendular quantities generate *downward* chain drift.

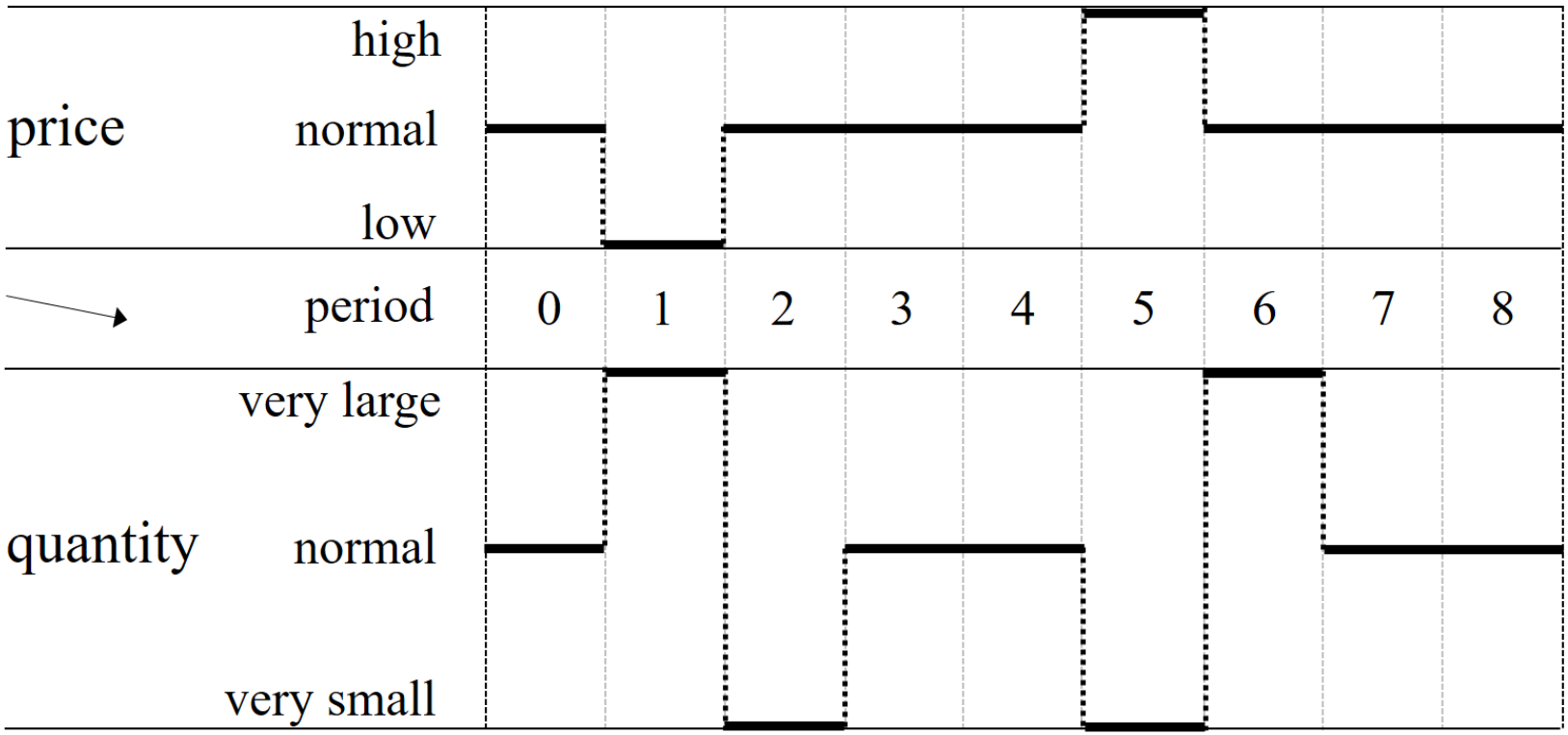
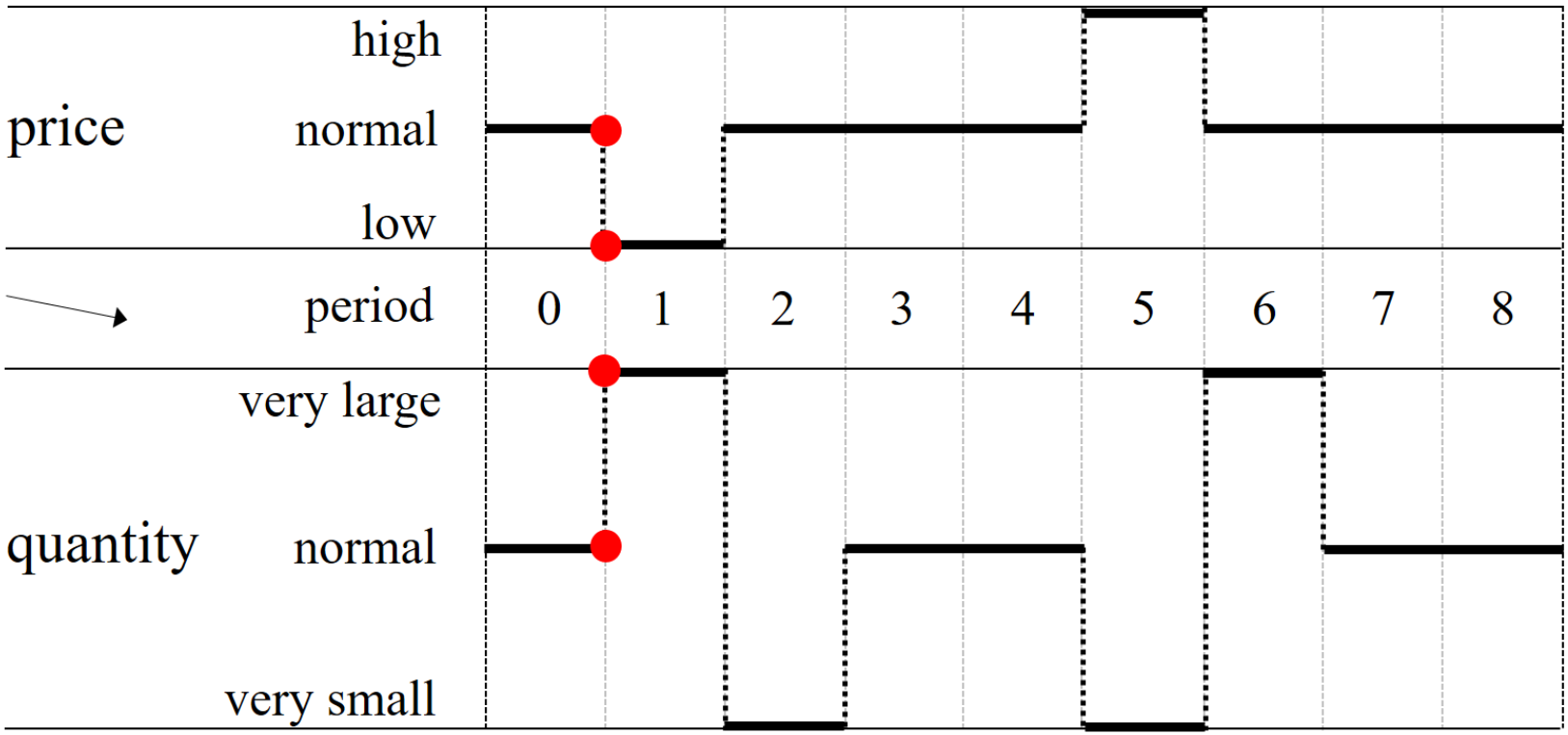



Figure 2: Pendular Quantity Reactions Caused by Sales Or Price Spikes.

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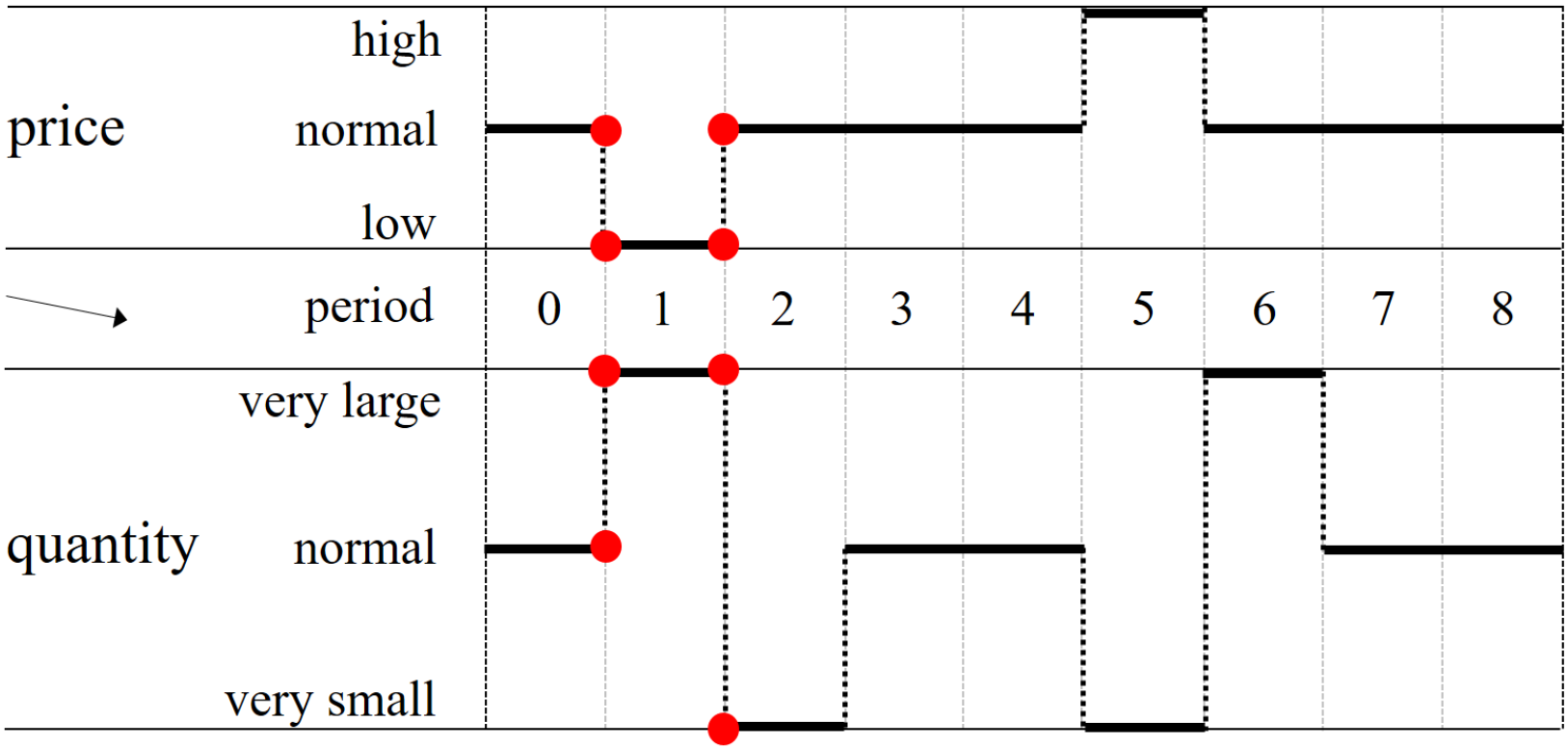


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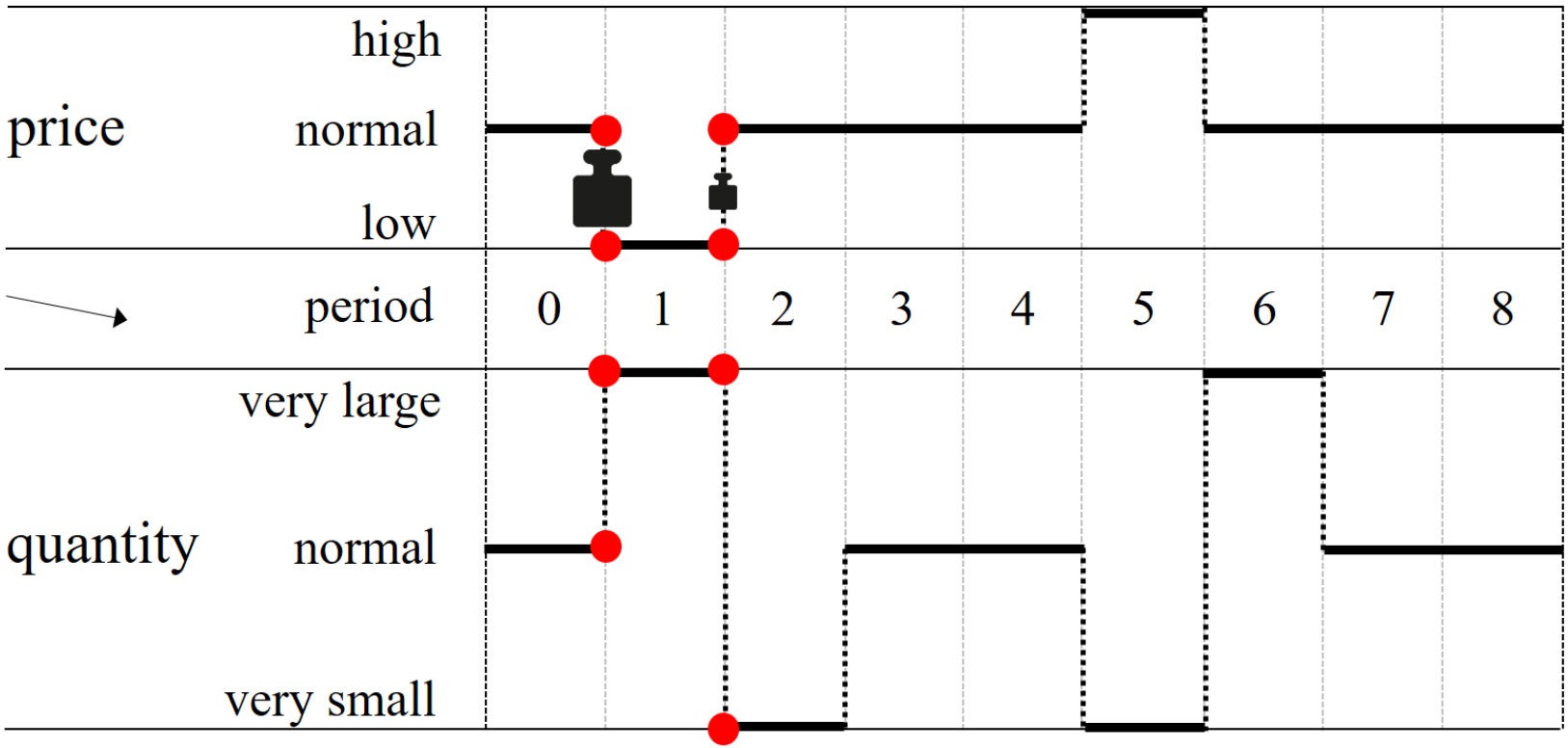


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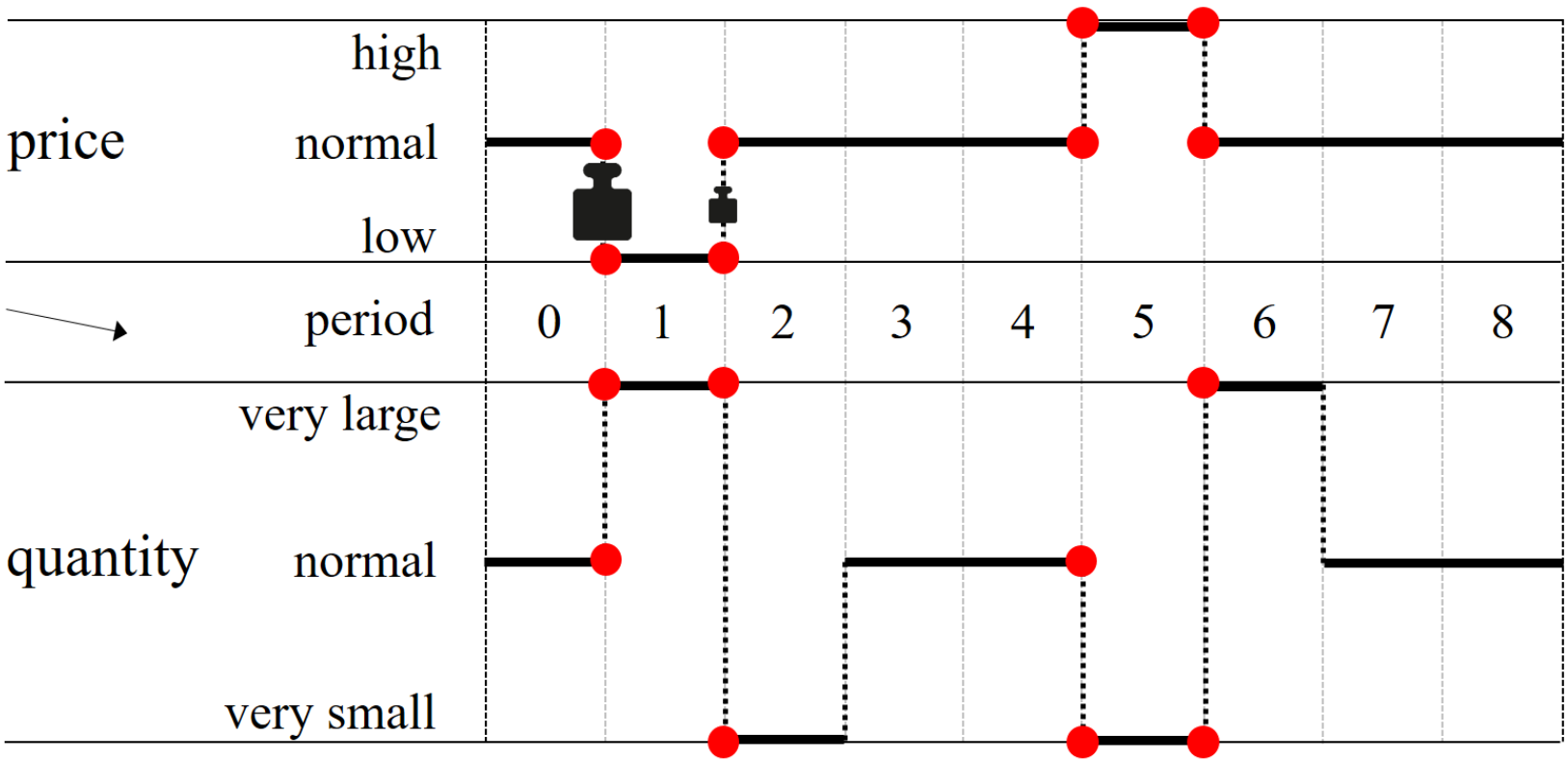


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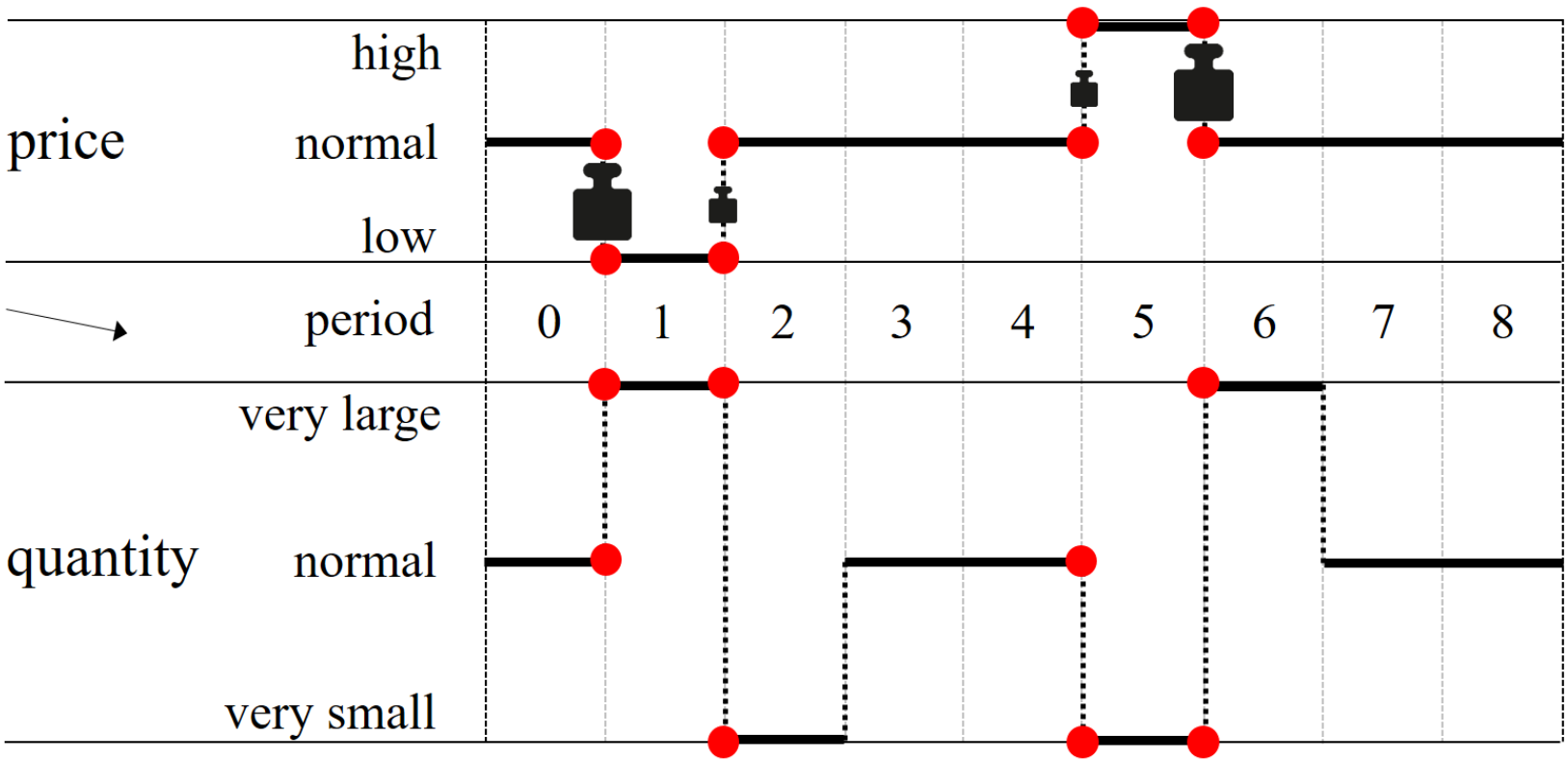


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# 4 Sticky Quantities

- Another form of asynchronous quantity responses are the consumers' delayed quantity responses to price changes (e.g., due to adjustment cost or search cost).
- This case is denoted here as *sticky quantities*.
- They lead to *upward* chain drift.

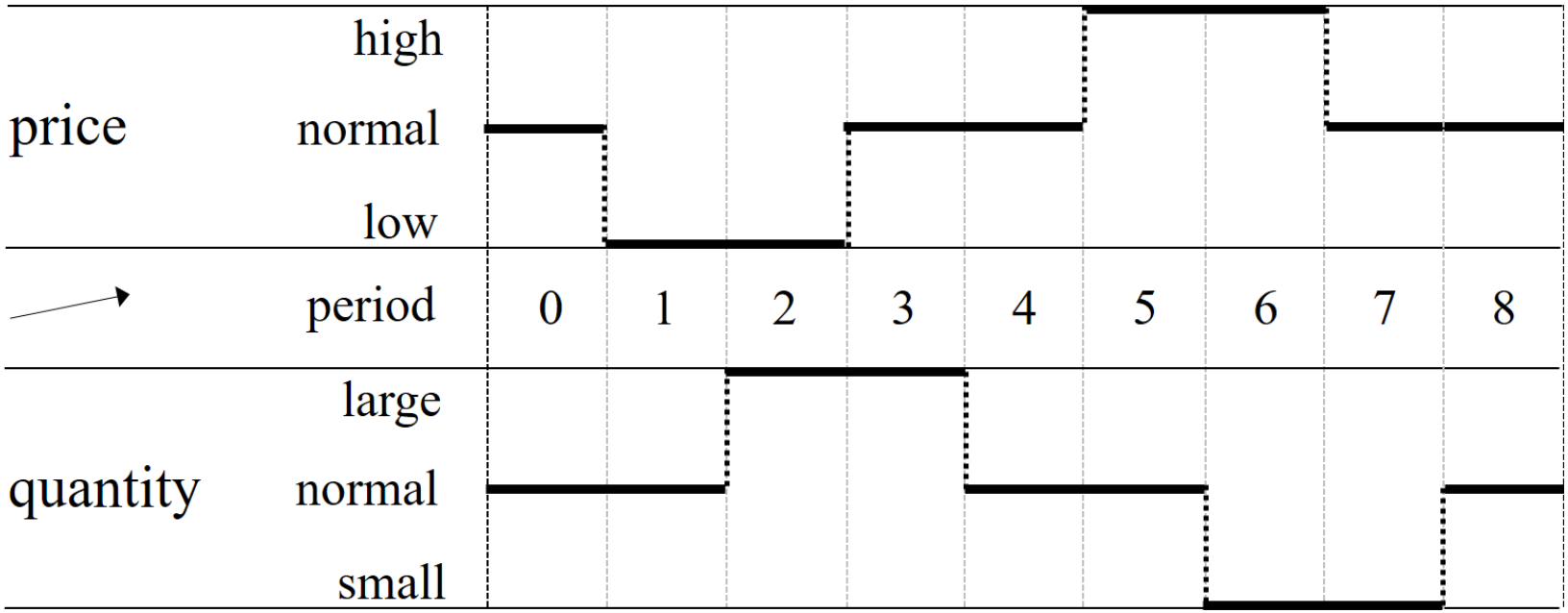
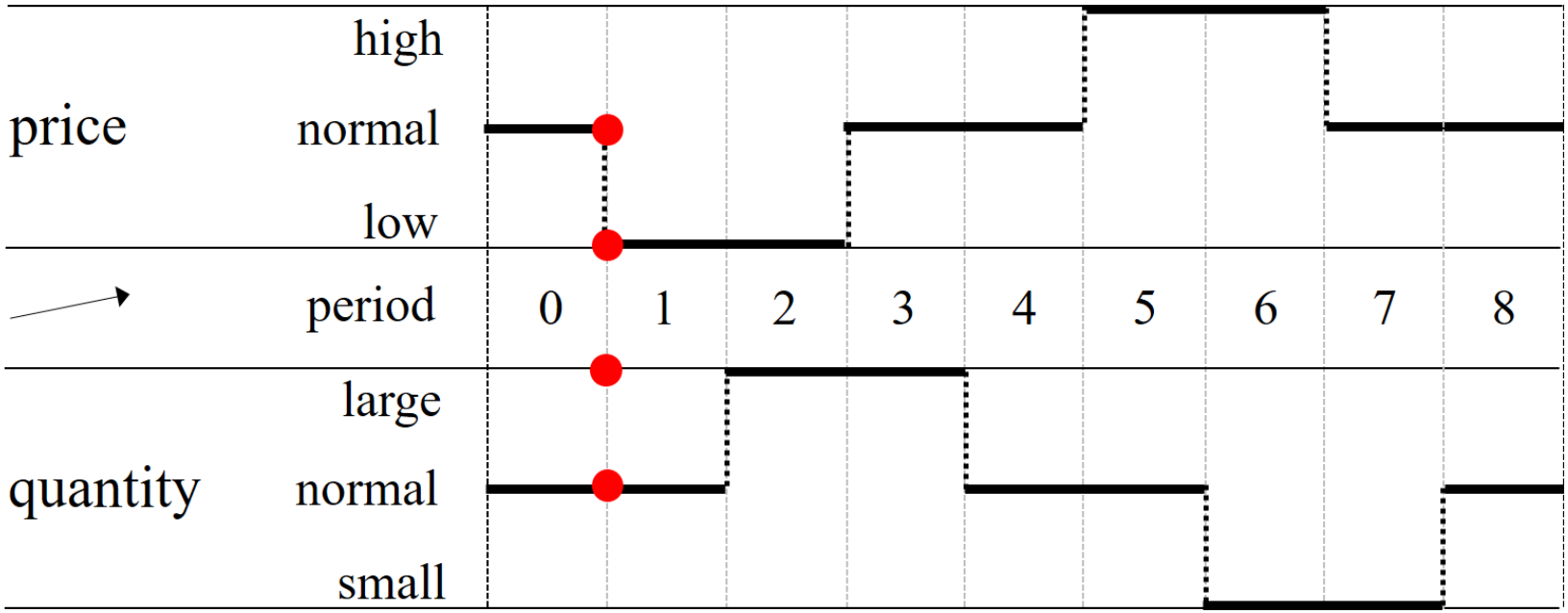



Figure 3: Sticky Quantity Reactions to Price Changes.

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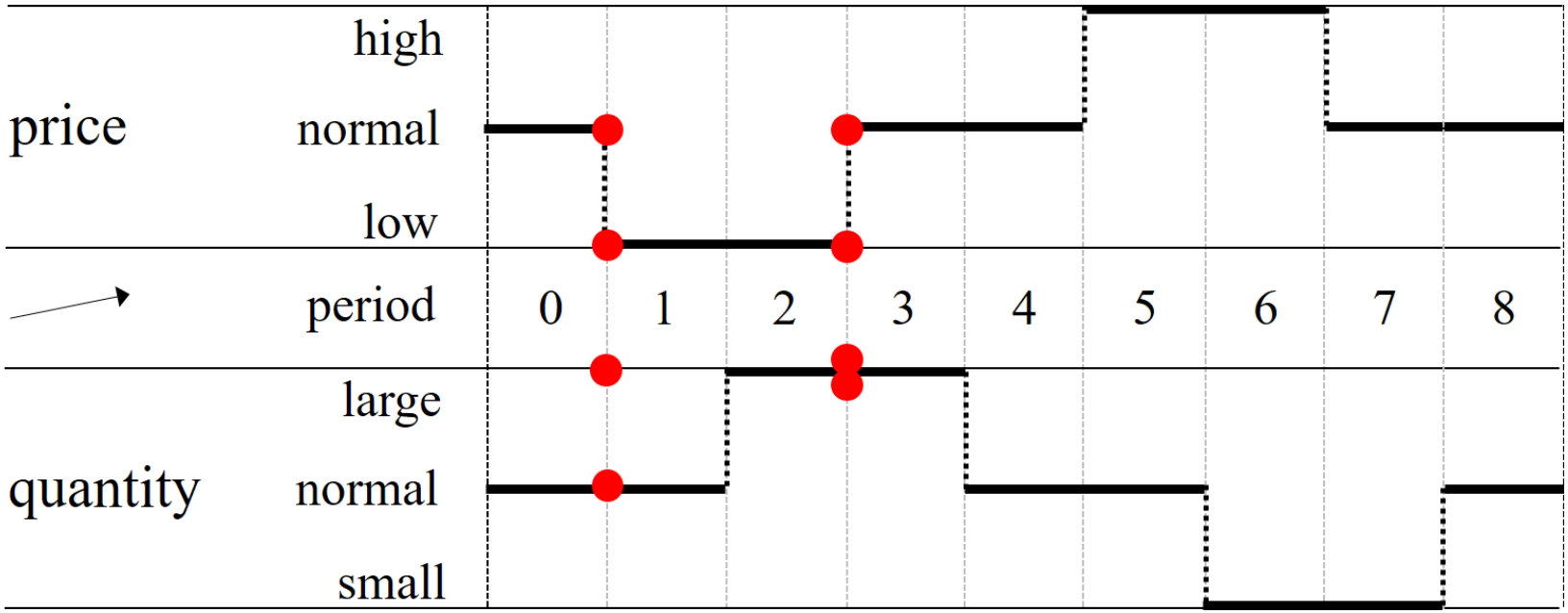


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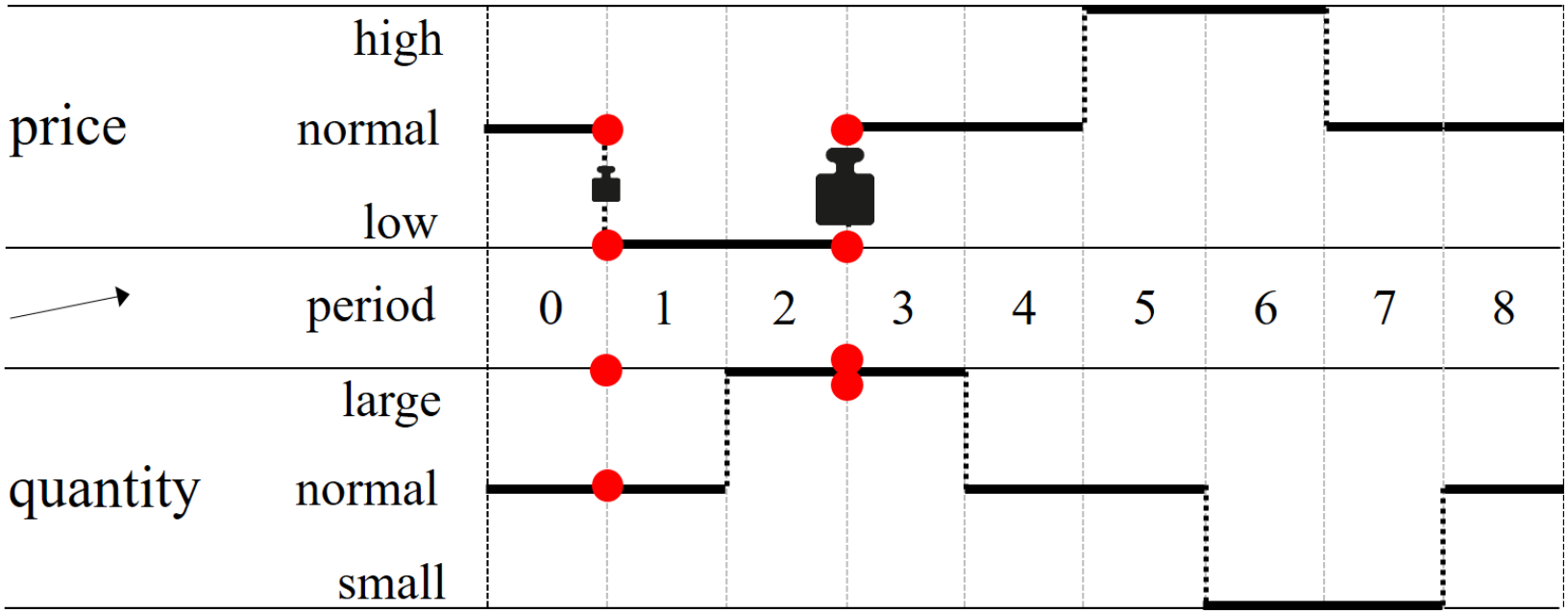


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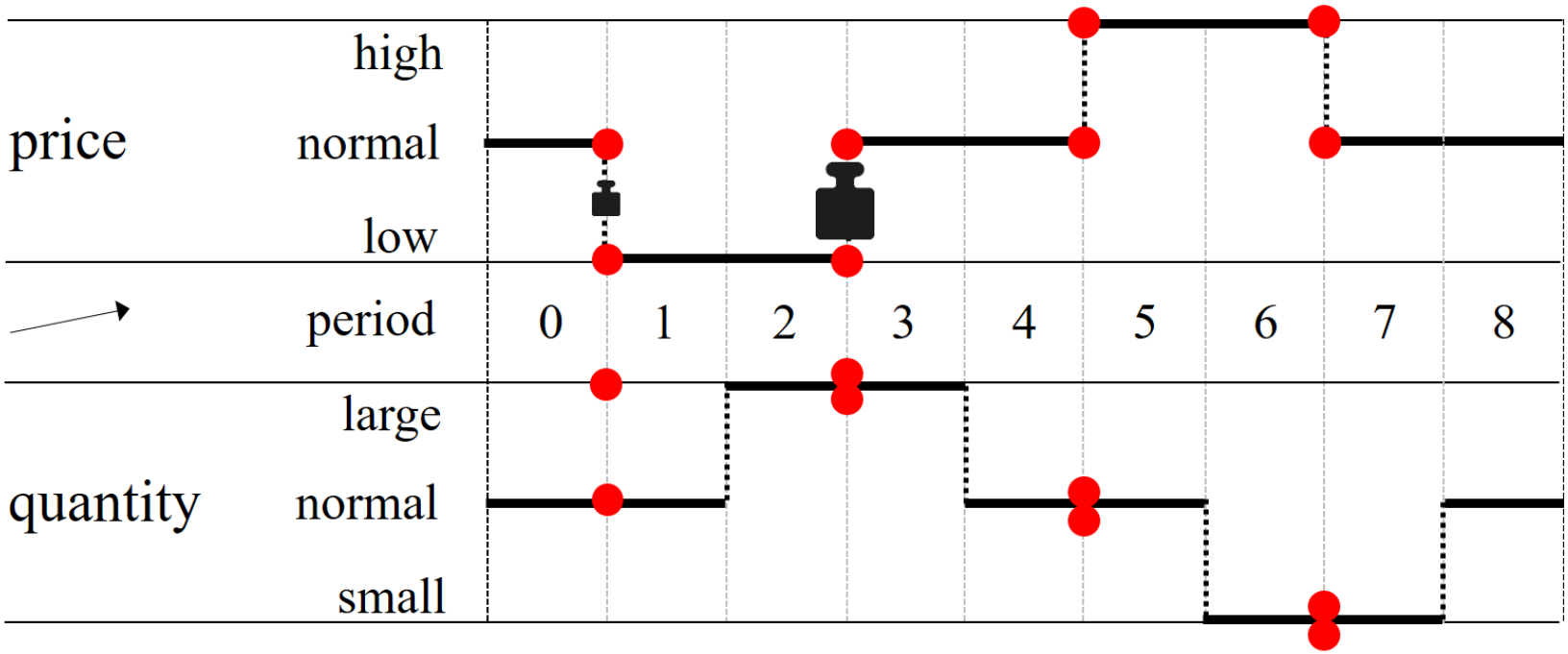


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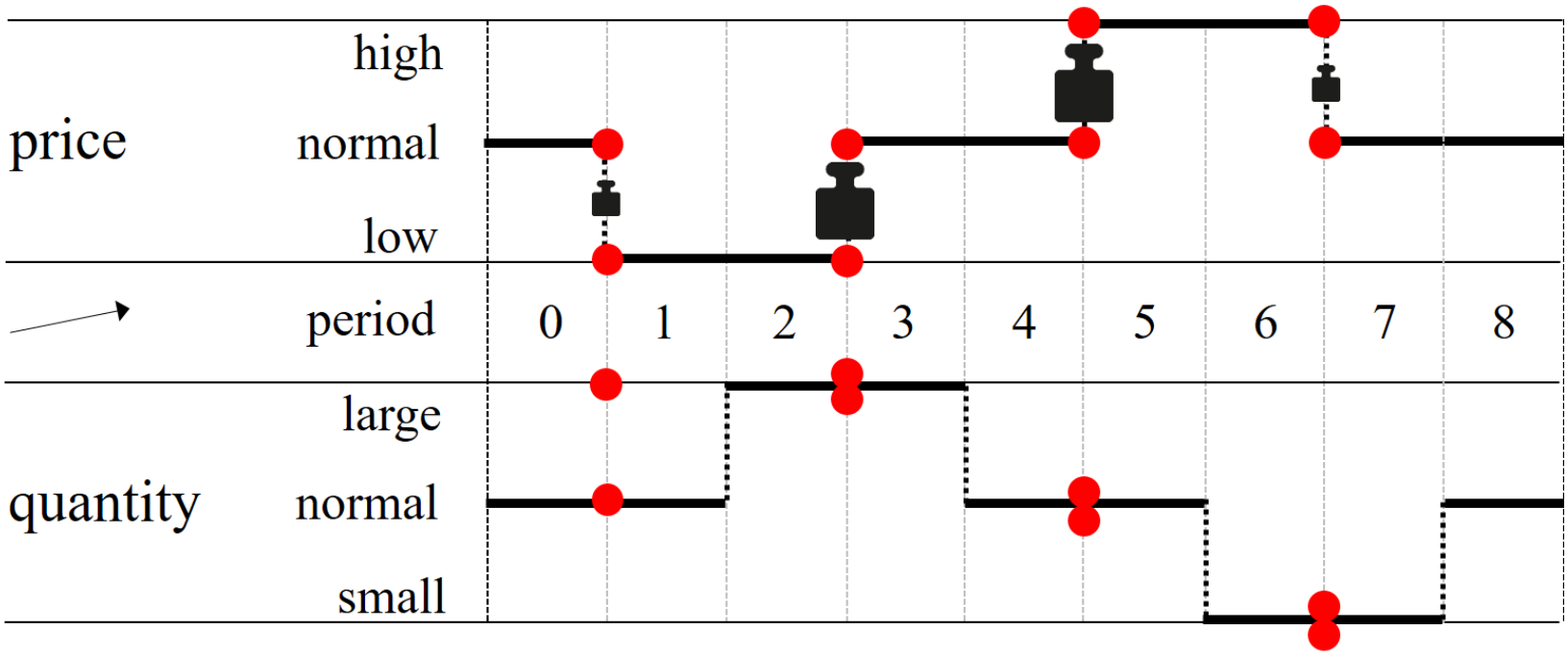


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# 5 RGEKS Indices

- Observed chain drift is the net effect of two counteracting forces:
  1. downward chain drift from pendular quantities (sales in conjunction with inventory behaviour),
  2. upward chain drift from sticky quantities (delayed consumer adjustment due to search and adjustment costs).
- Multilateral price indices have been proposed as a remedy.
- One such option are RGEKS indices.

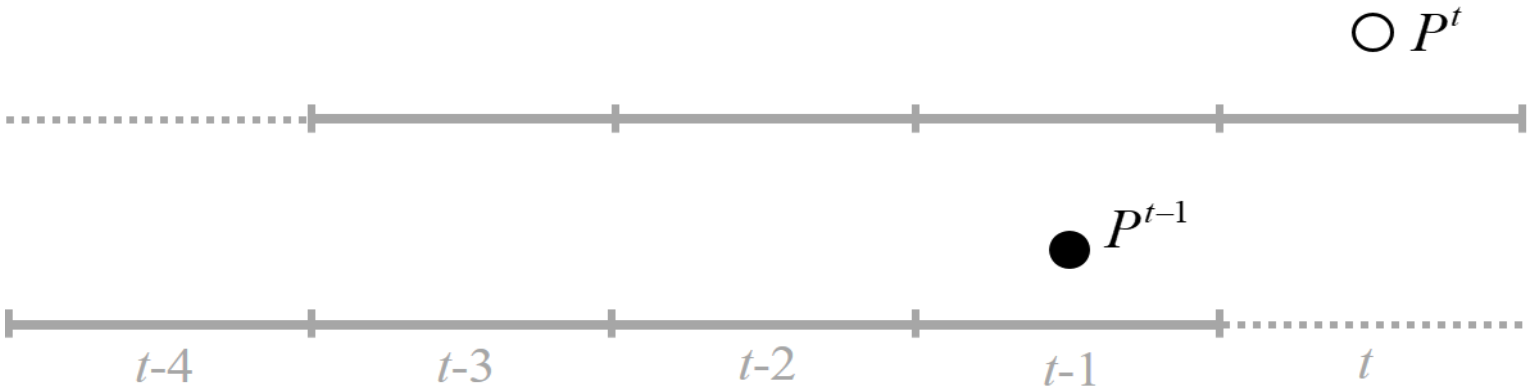
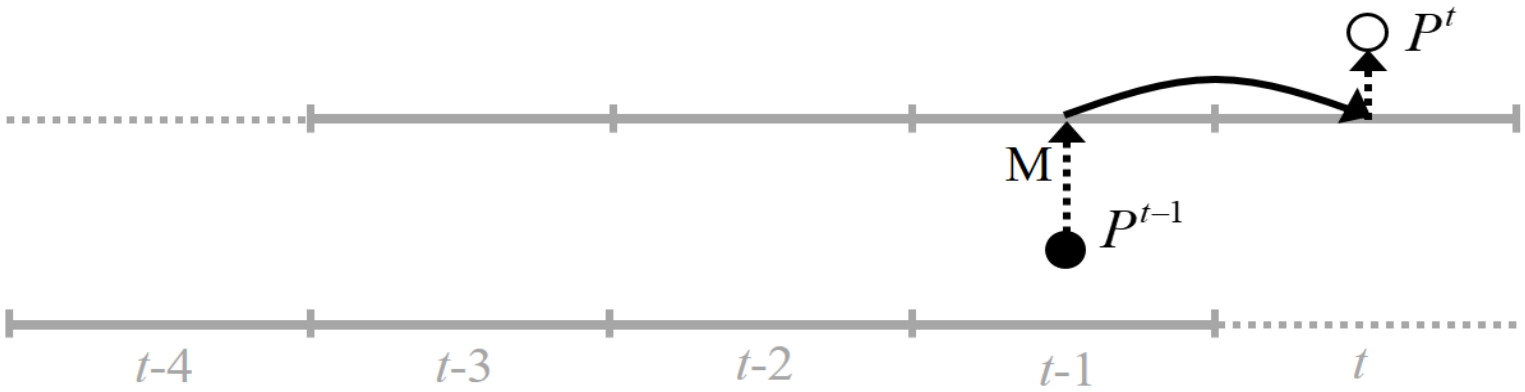



Figure 4: Different Variants of RGEKS Indices.




M = movement splice (Ivancic et al., 2011)

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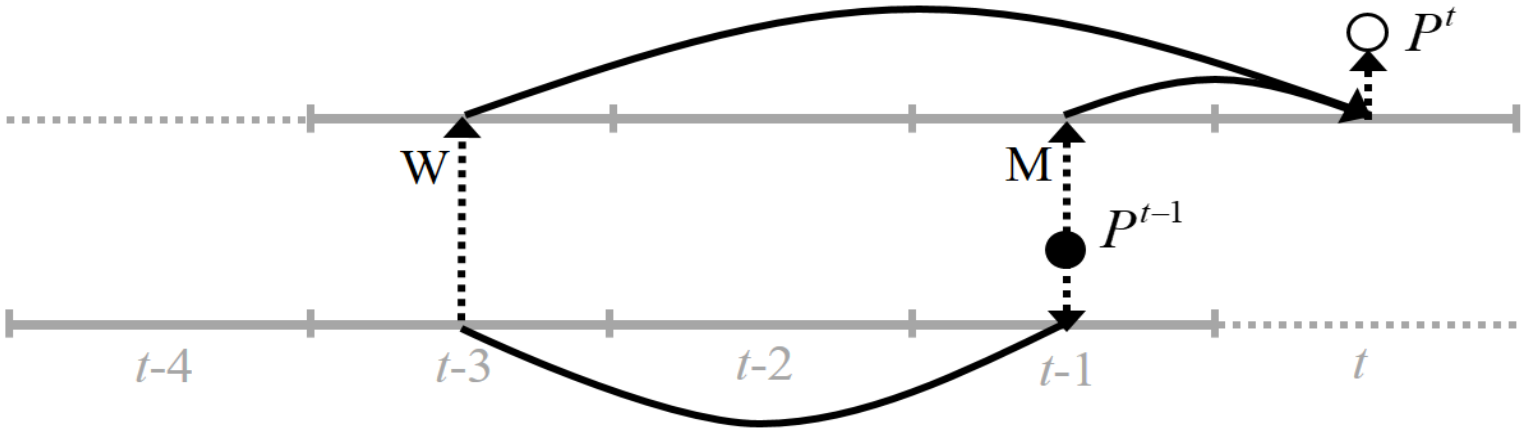


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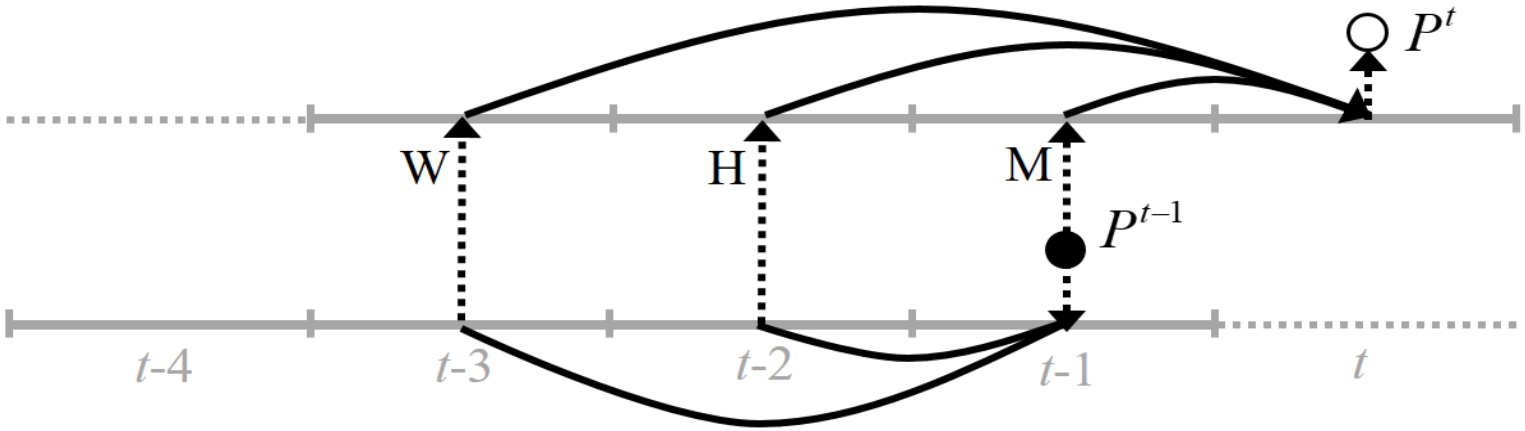


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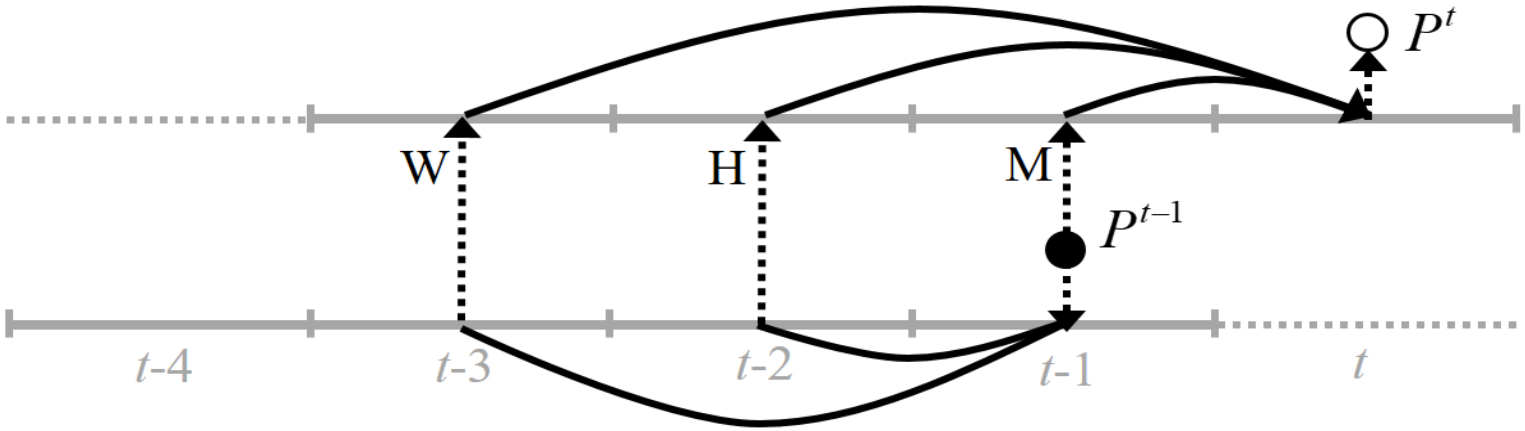


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geometric average gives mean splice (Diewert and Fox, 2017)

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- The mean movement splice RGEKS (Melser, 2018) also uses the averaging principle.
- It is easier to compute than the mean splice.

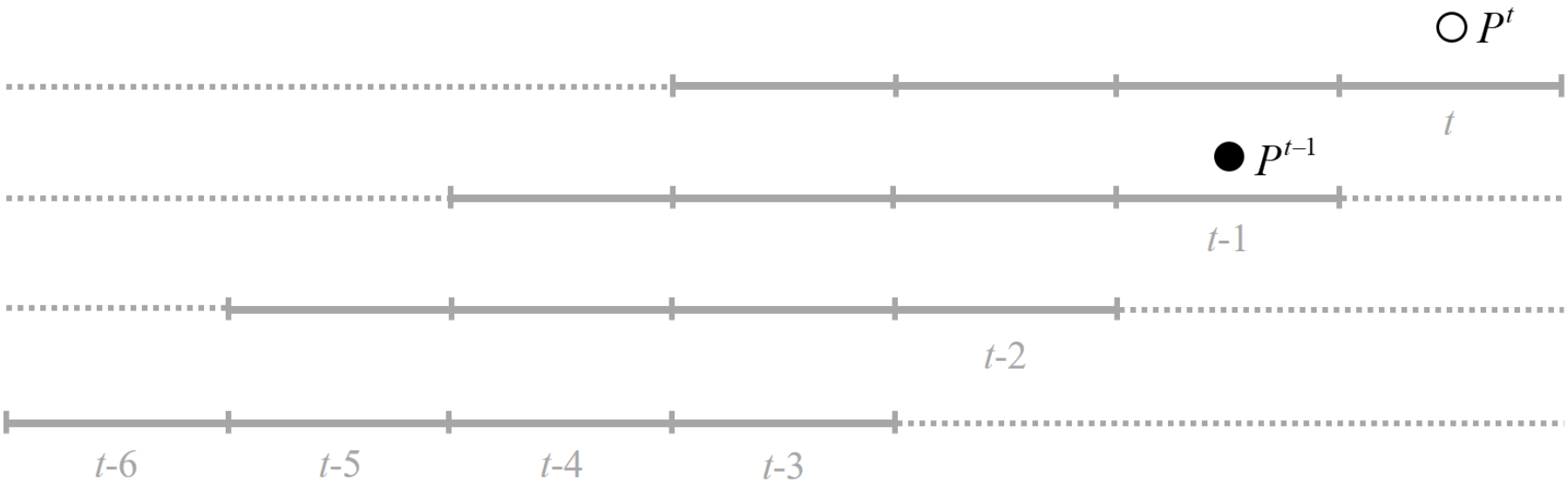
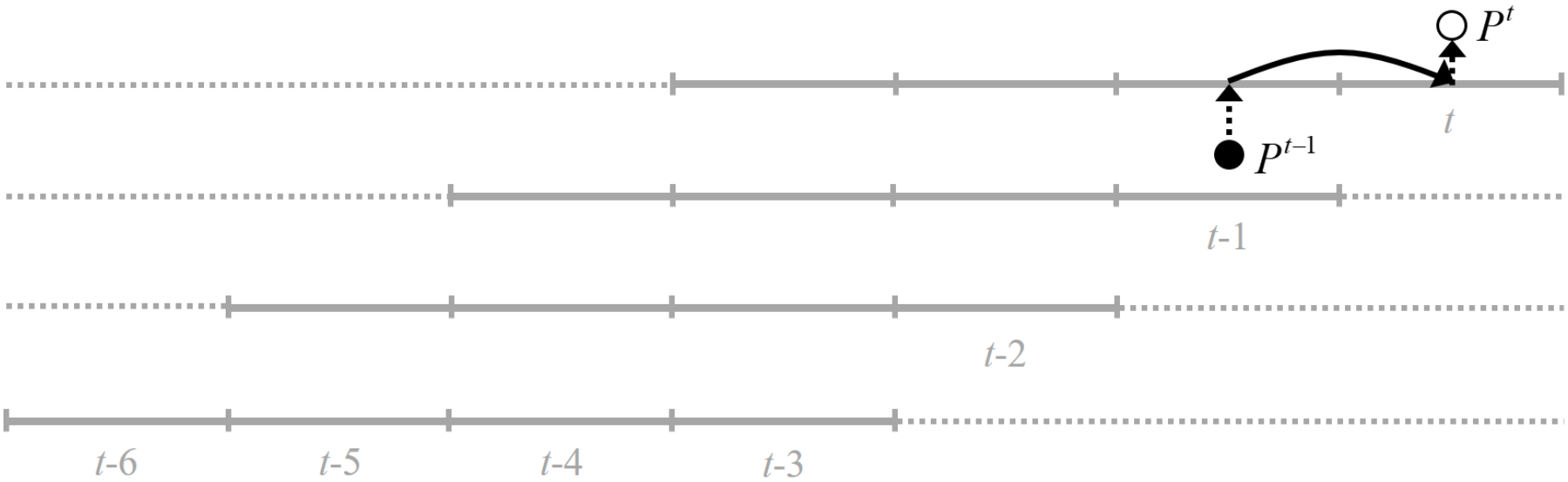



Figure 5: Another RGEKS Variant: Mean Movement Splice.

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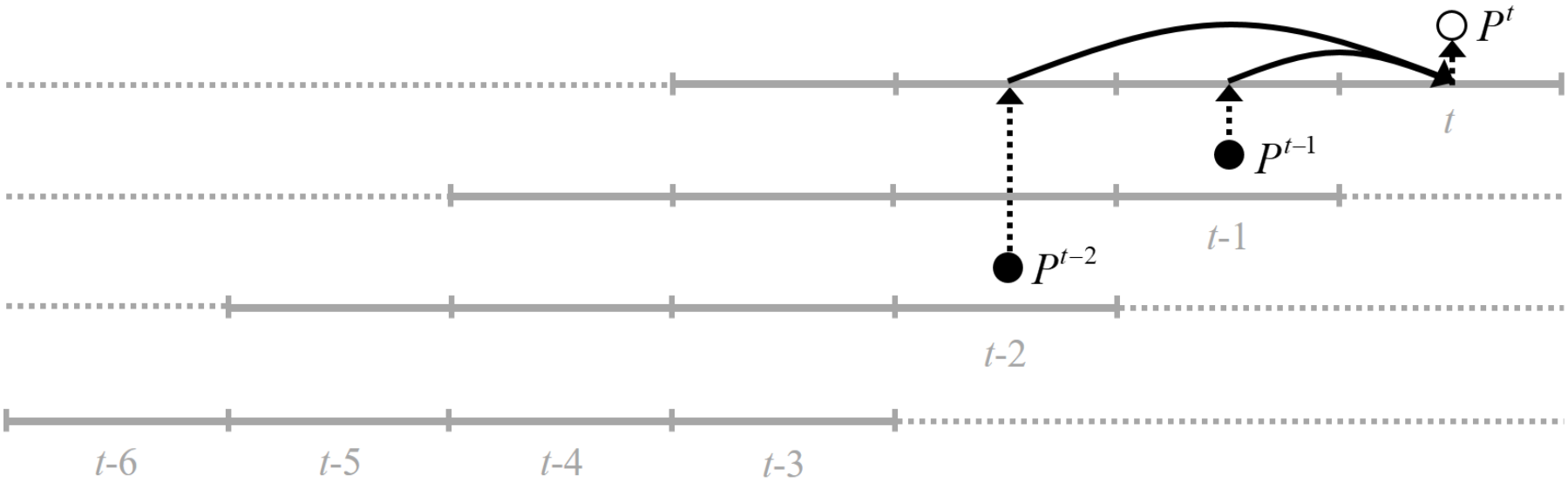


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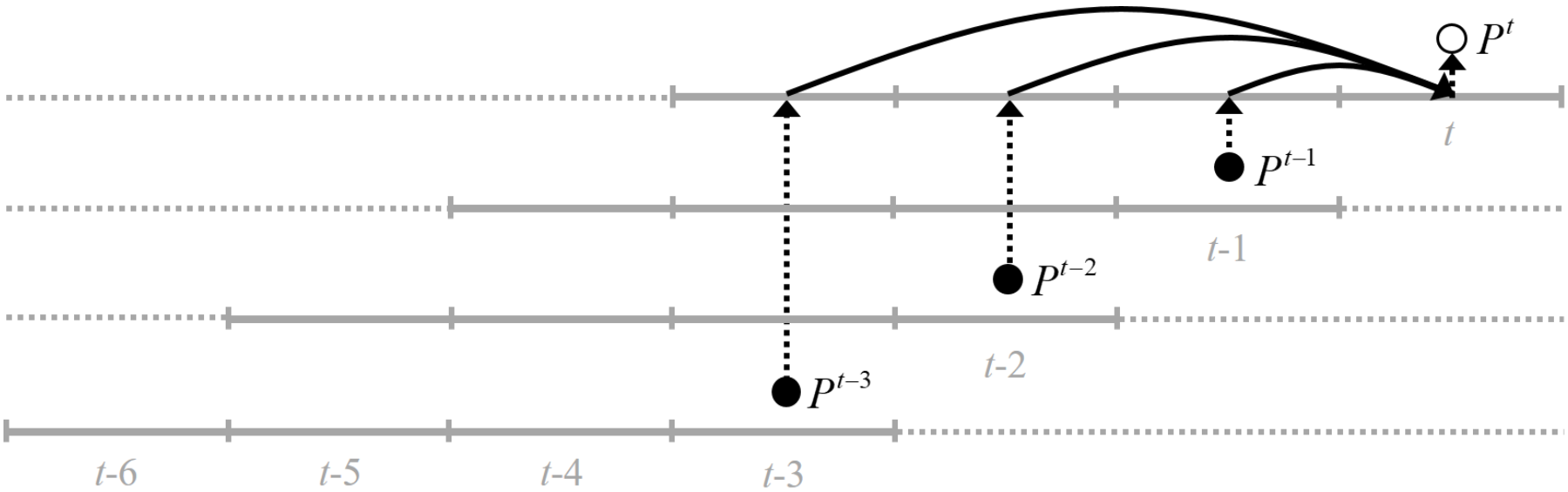


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# 6 A Simple Utility Framework Generating Pendular and Sticky Quantities

- In real world data, price and quantities do not return to their original levels.
- Therefore, no unassailable reference exists for assessing the extent of chain drift.
- The various chained direct price indices (e.g. chained Törnqvist) and RGEKS indices are exposed to a stress test with the following features:
  - 40 items (no churn),
  - *phase-in interval:* 10 periods in which all items are sold at the "base price",
  - *core-interval:* 100 periods with longer lasting price changes (10 items) or/and short sales (10 items),
  - *phase-out interval:* 10 periods in which all items are sold at their "base price".

- Quantities are the result of the consumers' utility maximizing behaviour.
- This requires a utility function that allows for
  - stocking behaviour (leading to pendular quantities) and
  - delayed adjustment (leading to sticky quantities).
- Since scanner data items represent differentiated goods, we use a myopic Dixit-Stiglitz CES utility function and amend it to allow for sticky and pendular quantities.
- In this utility function, stocking directly contributes to current utility.
- Strong deviations from former purchasing behaviour directly cause "disutility" (e.g., search or adjustment costs); this is a feature borrowed from habit formation models.

# 7 Results of the Stress Test

- Three scenarios are considered:
  1. pendular quantities
     (stocking, but no deferred adjustment)
  2. sticky quantities
     (deferred adjustment, but no stocking)
  3. pendular + sticky quantities
     (stocking and deferred adjustment).

Table 1: Chain Drift of SWAP Indices (in %).

| | Pendular | Sticky | Hybrid |
|---|---|---|---|
| Törnqvist | -25.15 | 7.41 | -1.62 |
| Walsh-2 | -37.69 | 7.52 | -2.29 |
| Walsh-Vartia | -37.78 | 7.53 | -2.31 |
| Theil | -34.19 | 7.49 | -2.07 |
| Vartia | -33.93 | 7.49 | -2.08 |

Table 2: Chain Drift of Some Other Direct Price Indices (in %).

| | Pendular | Sticky | Hybrid |
|---|---|---|---|
| Laspeyres | 468.94 | 29.76 | 337.07 |
| Paasche | -88.94 | -11.53 | -77.68 |
| Fisher | -20.66 | 7.15 | -1.24 |
| Drobisch | -18.48 | 7.19 | 0.22 |
| Walsh | -37.87 | 7.53 | -2.33 |
| Marshall-Edgeworth | -20.70 | 7.15 | -1.28 |
| Banerjee | -20.70 | 7.15 | -1.28 |
| Davies | -22.80 | 7.28 | -1.40 |
| Lehr | -44.79 | 7.88 | -3.10 |

Table 3: Chain Drift of RGEKS Indices (in %) for Different Window Lengths (4, 8, 12, and 24 Periods).

| | Pendular | | | | Sticky | | | | Hybrid | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4 | 8 | 12 | 24 | 4 | 8 | 12 | 24 | 4 | 8 | 12 | 24 |
| Mean Move. | -2.10 | 0.32 | 0.02 | 0.12 | 3.93 | 1.77 | 1.12 | 0.49 | 2.52 | 1.29 | 0.95 | 0.54 |
| Mean | -2.10 | 0.32 | 0.03 | 0.11 | 3.93 | 1.77 | 1.12 | 0.49 | 2.52 | 1.29 | 0.94 | 0.53 |
| Movement | -5.05 | -1.57 | -2.09 | -0.49 | 3.79 | 1.57 | 1.11 | 0.42 | 0.89 | 0.50 | 0.46 | 0.26 |
| Half | 0.04 | 0.31 | 0.67 | 0 | 4.08 | 1.83 | 0.87 | 0.43 | 3.35 | 0.75 | 0.32 | 0.34 |
| Window | -1.22 | 2.50 | 1.69 | 3.03 | 3.91 | 2.16 | 2.20 | 2.65 | 3.33 | 4.32 | 5.05 | 6.68 |

# 8 Conclusions

- Pendular quantities cause downward chain drift.
- Search and adjustment costs lead to sticky quantities
- Sticky quantities cause upward chain drift.
- Observed chain drift is the net effect of these two counteracting forces.
- RGEKS indices curb the chain drift problem.
- The mean movement RGEKS index shows the same results as the mean RGEKS.