# Household Inventory, Temporary Sales, and Price Indices 

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## Motivations

- Fluctuations in household inventories are one of the sources of macroeconomic fluctuations.
- COVID-19, consumption-tax increases, earthquake.
- Stockpiling occurs on a daily basis, especially during a temporary sale.
- Many important consequences.

ฝ Source of intertemporal price discrimination; financial returns; consumption inequality; price index (chain drift)

- A difficulty in observing household inventories
- Aims
- Propose a methodology to estimate changes in household inventories.
- Examine the validity of the methodology, focusing on the intertemporal substitution bias in consumer price indices.


## Contributions

- Construct a simple partial equilibrium model on stockpiling by consumers.
- Quasi-dynamic
- Infer consumption, consumption prices, and inventories
- Construct a COLI
- Apply our methodology to the issue of intertemporal substitution bias in price indices.
- Show that intertemporal substitution bias disappears for a particular type of price index if we switch from purchase-based data to consumption-based data.
- Apply our methodology to a range of other issues (skip today)
- The estimation of the elasticity of substitution, responses to consumption tax hikes, and business cycles.


## Literature

- Storable goods; household inventory
- Empiric: Boizot, Robin, and Visser (2001), Griffith et al (2009), Hendel and Nevo (2006a), Kano (2018); Models: Boizot, Robin, and Visser (2001), Hendel and Nevo (2006a, 2006b), Cashin and Unayama (2016);
- Relation to sales: Empirics: Blattberg and Neslin (1989), Neslin and Schneider Stone (1996), Hendel and Nevo (2003QME); Models of stockpiling: Salop and Stiglitz (1982), Hong, McAfee, and Nayyar (2002), Hendel, Lizzeri, and Roketskiy (2014)
- Chain drift in the chain index
- Frisch (1936), Reinsdorf (1999), Feenstra and Shapiro (2003), ILO (2004) etc; Solutions proposed by Feenstra and Shapiro (2003), Ivancic, Diewert, and Fox (2011), Haan and van der Grient (2011)
- COLI
- Static COLI: Konus (1924), Diewert (1976), Feenstra and Shapiro (2003), Chevalier and Kashyap (2018); Dynamic COLI: Reis (2009), Gowrisankaran and Rysman (2012), Wang (2013), Osborne (2018)
- We propose a simple method to calculate a static COLI while incorporating goods storability.
- A new quasi dynamic model.
- Deep investigation on the source of chain drift.


## Model

## Quasi Dynamic Model Setup

- Storable goods
- Partial equlibrium
- Price: high (regular) or low (sales); stochastic and exogenous
- Households consist of consumers and household producers.
- Household producers
- purchase goods from manufacturers, hold inventories, and sell them to consumers
- inventory cost; no depreciation; free entry
- Consumers
- do not hold inventory.
- Purchase equals consumption.
- Purchase goods from household producers and/or manufacturers.
- The problem to solve the COLI is static,
- while propositions in Hendel and Nevo (2006a) continue to hold in the quasi-dynamic model.


## Consumers

Hold no inventory. The consumer's cost minimization problem:

$$
\begin{equation*}
\min _{c_{t}^{k}}\left\{\sum_{k \in K_{t}} r_{t}^{k} c_{t}^{k}+\lambda_{t}\left\{U-\left[\sum_{k \in K_{t}} b^{k}\left(c_{t}^{k}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right\}\right\} \tag{1}
\end{equation*}
$$

where $r_{t}^{k}$ represents the consumer price of storable goods $k$ in period $t$ that the consumer purchases.

This optimization problem is static.

## Household producers

A household producer maximizes the firm value:

$$
\begin{equation*}
V_{t}^{k}=\mathrm{E}_{t}\left[\sum_{j=0}^{\infty} \beta^{j}\left\{\sum_{k \in K_{t+j}}\left(r_{t+j}^{k} y_{t+j}^{k}-p_{t+j}^{k} x_{t+j}^{k}-C\left(i_{t+j}^{k}\right)\right)\right\}\right] \tag{2}
\end{equation*}
$$

subject to

$$
\begin{equation*}
i_{t}^{k}=i_{t-1}^{k}-y_{t}^{k}+x_{t}^{k} \tag{3}
\end{equation*}
$$

The inventory as well as the purchase must be nonnegative:

$$
\begin{equation*}
x_{t}^{k}, i_{t}^{k} \geq 0 \tag{4}
\end{equation*}
$$

and $C(0)>0, C(i)^{\prime}>0$ and $C(i)^{\prime \prime} \geq 0$
The first-order conditions with respect to $x_{t}^{k}$ and $i_{t}^{k}$ are

$$
\begin{align*}
0 & =r_{t}^{k}-p_{t}^{k}+\psi_{t}^{k}  \tag{5}\\
C^{\prime}\left(i_{t}^{k}\right) & =\beta \mathrm{E}_{t}\left[r_{t+1}^{k}\right]-r_{t}^{k}+\mu_{t}^{k} \tag{6}
\end{align*}
$$

Consumption price $r_{t}$ may differ from purchase price $p_{t}$. Consumption $y_{t}\left(c_{t}\right)$ may differ from purchases $x_{t}$.

The price of storable goods follows a Markov process. It takes either of the two, non-sale $P_{H}$ or sale $P_{L}\left(P_{H}>P_{L}\right)$ and

$$
\begin{align*}
\operatorname{Prob}\left(P_{L} \mid P_{H}\right) & =\bar{q} \\
\operatorname{Prob}\left(P_{L} \mid P_{L}\right) & =\underline{q} . \tag{7}
\end{align*}
$$

Free entry: the zero value for the entrant firm with zero inventory,

$$
\begin{equation*}
V^{k}\left(i_{t-1}^{k}=0, p_{t}^{k}\right) \leq 0 \tag{8}
\end{equation*}
$$

## Equilibrium

- Typical pattern

- Asymmetry in the quantity purchased
- $X_{H}^{1}$ (the quantity purchased just before a sale) $>X_{H}^{2}$ (the quantity purchased just after the sale)
- $X_{L}^{1}$ (the average quantity purchased during the first half of the sale) $>X_{L}^{2}$ (the average quantity purchased during the second half of the sale)


## Intertemporal Substitution Bias for Chained Price Indices

- It is known that the Törnqvist index is a good approximation of the cost-of-living index (COLI) (Diewert 1976).
- Changes from $t-d t$ to $t$ based on Laspayres, Paasche, and Törnquist:

$$
\begin{align*}
\pi_{t}^{L} & =\sum_{k \in K_{t-d t} \cap K_{t}} W_{t-d t}^{k}\left(K_{t-d t} \cap K_{t}\right) \log \left(\frac{p_{t}^{k}}{p_{t-d t}^{k}}\right)  \tag{9}\\
\pi_{t}^{P} & =\sum_{k \in K_{t-d t} \cap K_{t}} W_{t}^{k}\left(K_{t-d t} \cap K_{t}\right) \log \left(\frac{p_{t}^{k}}{p_{t-d t}^{k}}\right)  \tag{10}\\
\pi_{t}^{T} & =\sum_{k \in K_{t-d t} \cap K_{t}} \frac{W_{t-d t}^{k}\left(K_{t-d t} \cap K_{t}\right)+W_{t}^{k}\left(K_{t-d t} \cap K_{t}\right)}{2} \log \left(\frac{p_{t}^{k}}{p_{t-d t}^{k}}\right), \tag{11}
\end{align*}
$$

where the purchase-based weight share $W_{t}^{k}\left(K_{t-d t} \cap K_{t}\right) \equiv p_{t}^{k} x_{t}^{k} / \sum_{k^{\prime} \in K_{t-d t} \cap K_{t}} p_{t}^{k^{\prime}} x_{t}^{k^{\prime}}$.

- The model suggets $\pi^{P}<\pi^{T}<0<\pi^{L}$.
- Because of asymmetry in the quantity purchased


## Stylized Facts Based on Japanese Scanner Data

## Data

- Retailer-side data, namely, the point-of-sale (POS) scanner data
- collected by Nikkei Inc
- The number of units sold and the sales amount (price times the number of units sold) for each product and retailer on a daily basis.
- From March 1, 1988 to December 31, 2019.
- Processed food and daily necessities, covering 170 of the 588 categories in the CPI and making up about 20 percent of households' expenditure

Fact 1. When weights are based on purchases, the relationship between the different price indices is given by the following inequality: $\pi^{P}<\pi^{T}<0<\pi^{L}$.


## Chain Drift Test

Large inflation or deflation may not be necessarily a bias.
Define chain drift $d_{0, \tau, d t}^{X}$ as

$$
\begin{equation*}
d_{0, \tau, d t}^{X}=\sum_{s=1}^{(\tau-1) / d t} \pi_{(s-1) d t, s d t}^{X}-\pi_{0, \tau-1}^{X} \tag{12}
\end{equation*}
$$

where $\tau$ represents a time span to measure chain drift and set to 365 days.

|  | Price indices |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $d t$ | $\begin{array}{c}\text { Törnqvist } \\ \text { (purchase-weighted) }\end{array}$ | $\begin{array}{c}\text { Törnqvist } \\ \text { (consumption-weighted) }\end{array}$ | $\begin{array}{c}\text { Order r superlative } \\ \text { (consumption-weighted) }\end{array}$ |  |
|  | $-40.44^{* * *}$ | Annual chain drift |  |  |$]$

Note: Chain drift is the average from 1989 to 2019. We apply the sign test for the null hypothesis that the probability to take positive/negative chain drift is equal to 0.5 . The annualized inflation rate is calculated as $\exp \left(365 \times \overline{x_{d t=1}}\right)-1$, where $\overline{x_{d t=1}}$ is the mean of daily log inflation from 1990 to 2018.

## Fact 2. There is asymmetry in the quantity purchased when prices increase and when they decrease.

$X_{H}^{1}$ : the quantity purchased just before a sale
$X_{L}^{1}$ : the average quantity purchased during the first half of the sale
$X_{L}^{2}$ : the average quantity purchased during the second half of the sale
$X_{H}^{2}$ : the quantity purchased just after the sale
$Q_{H}=\log \left(X_{H}^{2}+\sqrt{1+\left(X_{H}^{2}\right)^{2}}\right)-\log \left(X_{H}^{1}+\sqrt{1+\left(X_{H}^{1}\right)^{2}}\right)$ : negative $\rightarrow Q$ smaller after a sale than before it
$Q_{L}=\log \left(X_{L}^{2}+\sqrt{1+\left(X_{L}^{2}\right)^{2}}\right)-\log \left(X_{L}^{1}+\sqrt{1+\left(X_{L}^{1}\right)^{2}}\right)$ : negative $\rightarrow Q$ smaller during the second half of a sale than the first half of it


## Fact 3. Chain drift is associated with household stockpiling during temporary sales.

|  | Dependent variable: |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Chain drift for purchase-weighted Törnqvist |  |  |  |
|  | (1) | (2) | (3) | (4) |
| $Q_{H}$ | $\begin{aligned} & 0.698^{* *} \\ & (0.315) \end{aligned}$ |  |  |  |
| $Q_{L}$ | $\begin{gathered} 1.781^{* * *} \\ (0.463) \end{gathered}$ |  |  |  |
| $q$ |  | $\begin{gathered} -1.916^{* * *} \\ (0.483) \end{gathered}$ |  |  |
| $\bar{q}$ |  |  | $\begin{aligned} & -0.807 \\ & (1.039) \end{aligned}$ | $\begin{gathered} -2.190^{* *} \\ (1.089) \end{gathered}$ |
| $\underline{q}$ |  |  | $\begin{gathered} -0.708^{* * *} \\ (0.154) \end{gathered}$ | $\begin{gathered} 0.014 \\ (0.259) \end{gathered}$ |
| $\log \left(P_{L} / P_{H}\right)$ |  | $\begin{gathered} 3.979 * * * \\ (0.644) \end{gathered}$ | $\begin{gathered} 3.929^{* * *} \\ (0.618) \end{gathered}$ | $\begin{gathered} 2.899^{* * *} \\ (0.673) \end{gathered}$ |
| $\log m$ |  |  |  | $\begin{gathered} -0.344^{* * *} \\ (0.101) \end{gathered}$ |
| Constant | $\begin{gathered} -0.320^{* * *} \\ (0.039) \end{gathered}$ | $\begin{gathered} 0.178 * * * \\ (0.062) \end{gathered}$ | $\begin{gathered} 0.418^{* * *} \\ (0.075) \end{gathered}$ | $\begin{aligned} & 0.223^{* *} \\ & (0.093) \end{aligned}$ |
| Adjusted $R^{2}$ | 0.102 | 0.400 | 0.440 | 0.470 |
| Observations | 189 | 195 | 195 | 195 |

Note: $Q_{H}$ and $Q_{L}$ represent a degree of an asymmetry for the quantity purchased when price increases and decreases, $q$ is the probability that a product is on sale, $\bar{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $q$ is the probability that a product will continue to be on sale on the following day given that it is currently on


## Inference of Consumption and Consumption Prices and Calculation of the COLI

## Motivation

- We want to know
- consumption $\left(c_{t}\right)$, the consumption price $\left(r_{t}\right)$ and the degree of stockpiling ( $m$ )
$\star$ necessary for the COLI
* however, usually unobservable, unlike purchase and the posted price

When $p_{t}=P_{H}$, household producers' optimization problem is given by

$$
\begin{equation*}
C^{\prime}\left(i_{H} ; I_{t-1}\right)=\beta\left\{(1-\bar{q}) r_{H}\left(I_{t}\right)+\bar{q} P_{L}\right\}-r_{H}\left(I_{t-1}\right)+\mu_{t} . \tag{13}
\end{equation*}
$$

costs of holding inventories $=$ benefits of holding inventories (price increase)

- When inventory cost function $C(\cdot)$ is written in a certain form, $r_{H}\left(I_{t}\right)-r_{H}\left(I_{t-1}\right)$ is a positive constant.
- The expectation of a linear consumption-price increase.
- Consumers' stockpiling behavior can be conveniently summarized by a single variable: the degree of stockpiling during a sale ( $m$ )
- which expresses how long an inventory lasts after sales end.
- Case of $m=5$.



## Degree of stockpiling $m$

Calculated $m$ is mostly a few days.


## Chain Drift at a Product Category Level

Simulation using the frequency of sales, the size of sale discounts, the degree of stockpiling for each product category.


Note: Each circle indicates a 3-digit product category. The inflation rates are daily average and based on the purchase-weighted Törnqvist index. The red dashed line represents the 45 degree line.

## Price Indices (Consumption Weighted)

|  | Price indices |  |  |
| :---: | :---: | :---: | :---: |
| $d t$ | TÖrnqvist <br> (purchase-weighted) | Törnqvist <br> (consumption-weighted) | Order r superlative <br> (consumption-weighted) |
|  | $-40.44^{* * *}$ | Annual chain drift |  |
| 1 | $-7.24^{* * *}$ | $-5.69^{* * *}$ | -1.20 |
| 7 | $-2.43^{* * *}$ | $-2.28^{* * *}$ | $-1.23^{* * *}$ |
| 14 | $-0.97^{* * *}$ | $-1.10^{* * *}$ | $-0.59^{* * *}$ |
| 28 | $-0.66^{* * *}$ | $-0.46^{* * *}$ | -0.04 |
| 52 | $-0.61^{* * *}$ | $-0.20^{* *}$ | 0.23 |
| 91 | $-0.49^{* * *}$ | 0.06 | 0.17 |
| 182 |  | 0.01 | 0.14 |
|  | $\underline{\text { Annualized inflation rate }}$ |  |  |
| 1 | -46.34 | -13.06 |  |

Note: We apply the sign test for the null hypothesis that the probability to take positive/negative chain drift is equal to 0.5 . The annualized inflation rate is calculated as $\exp \left(365 \times \overline{x_{d t=1}}\right)-1$, where $\overline{x_{d t=1}}$ is the mean of daily log inflation from 1990 to 2018 .

- Even if we use a consumption-weighted Törnqvist index, chain drift is not completely disappeared.
- Consumption path at the start and after a sale is asymmetric
- Because of nonnegative constraints on both inventory and purchases
- Causes a systematic higher-order approximation errors in price indices,
- although it is well known that the Törnqvist price index provides a good approximation to the COLI up to the second order (Diewert 1976).
- The order $r$ superlative index, where we define $P_{r}$ as

$$
P_{r}\left(r_{0}, r_{1}, c_{0}, c_{1}\right)=\left\{\sum_{k \in K} s_{0}^{k}\left(\frac{r_{1}^{k}}{r_{0}^{k}}\right)^{\frac{1-\sigma}{2}}\right\}^{1 /(1-\sigma)} /\left\{\sum_{k \in K} s_{1}^{k}\left(\frac{r_{0}^{k}}{r_{1}^{k}}\right)^{\frac{1-\sigma}{2}}\right\}^{1 /(1-\sigma)} .
$$

## Annualized Inflation Rates



## Comparison with Rolling Window GEKS (RWGEKS)

- Multilateral index so that the circularity or transitivity requirement is satisfied
- Ivancic, Diewert, and Fox (2011), de Haan and van der Grient (2011) and Diewert (2021)
- The change in the RWGEKS between $t-1$ and $t$ is defined as

$$
\begin{equation*}
\pi_{t}^{R W G E K S} \equiv \log \left[\prod_{I=t-d+1}^{t}\left(P_{t-1, I} / P_{t, I}\right)^{1 / d}\right] \tag{14}
\end{equation*}
$$

where $d$ represents the length of a window in days and $P_{t, l}$ is the Törnqvist index.

- Simulation based on our model
- Given that our model is correct, how do the RWGEKS perform?
- Data
- Comparison between our index and RWGEKS

Simulate the paths of price indices assuming $m=5, \bar{q}=0.03, \underline{q}=0.50$, and $P_{L} / P_{H}=0.9$. The price level after 365 days, where the initial price level is set to one. If there is no bias, the price level after 365 days should reach one.

|  | COLI | Order $r$ <br> superlative | Tornqvist <br> $(\mathrm{C})$ | Tornqvist <br> $(\mathrm{Q})$ | Laspeyres <br> $(\mathrm{Q})$ | Paasche <br> $(\mathrm{Q})$ | RWGEKS <br> $(d=30)$ | RWGEKS <br> $(d=7)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) Benchmark | 1.000 | 1.000 | 0.991 | 0.163 | $2.71 \mathrm{e}+01$ | $1.01 \mathrm{e}-03$ | 1.001 | 0.943 |
|  | $(4.60 \mathrm{e}-03)$ | $(4.60 \mathrm{e}-03)$ | $(4.58 \mathrm{e}-03)$ | $(1.09 \mathrm{e}-02)$ | $(3.80 \mathrm{e}+00)$ | $(2.10 \mathrm{e}-04)$ | $(9.11 \mathrm{e}-03)$ | $(7.83 \mathrm{e}-03)$ |
| (2) Low $m$ | 1.000 | 1.000 | 1.000 | 0.448 | $2.73 \mathrm{e}+00$ | $7.36 \mathrm{e}-02$ | 1.001 | 0.998 |
| $(m=1)$ | $(3.88 \mathrm{e}-03)$ | $(3.88 \mathrm{e}-03)$ | $(3.91 \mathrm{e}-03)$ | $(1.15 \mathrm{e}-02)$ | $(1.02 \mathrm{e}-01)$ | $(5.82 \mathrm{e}-03)$ | $(5.34 \mathrm{e}-03)$ | $(4.20 \mathrm{e}-03)$ |
| (3) High $m$ | 0.999 | 0.999 | 0.992 | 0.054 | $3.42 \mathrm{e}+02$ | $9.13 \mathrm{e}-06$ | 1.001 | 0.897 |
| $(m=10)$ | $(5.08 \mathrm{e}-03)$ | $(5.08 \mathrm{e}-03)$ | $(5.06 \mathrm{e}-03)$ | $(6.20 \mathrm{e}-03)$ | $(8.82 \mathrm{e}+01)$ | $(3.15 \mathrm{e}-06)$ | $(1.22 \mathrm{e}-02)$ | $(1.02 \mathrm{e}-02)$ |
| (4) Low $\sigma$ | 1.000 | 1.000 | 0.999 | 0.177 | $1.71 \mathrm{e}+01$ | $1.87 \mathrm{e}-03$ | 1.001 | 0.941 |
| $(\sigma=2)$ | $(4.29 \mathrm{e}-03)$ | $(4.29 \mathrm{e}-03)$ | $(4.29 \mathrm{e}-03)$ | $(1.14 \mathrm{e}-02)$ | $(2.16 \mathrm{e}+00)$ | $(3.63 \mathrm{e}-04)$ | $(8.61 \mathrm{e}-03)$ | $(7.51 \mathrm{e}-03)$ |
| (5) Low $P_{L} / P_{H}$ | 0.999 | 0.999 | 0.808 | 0.005 | $2.50 \mathrm{e}+05$ | $1.44 \mathrm{e}-10$ | 1.004 | 0.860 |
| $\left(P_{L} / P_{H}=0.75\right)$ | $(1.48 \mathrm{e}-02)$ | $(1.48 \mathrm{e}-02)$ | $(1.33 \mathrm{e}-02)$ | $(1.01 \mathrm{e}-03)$ | $(1.25 \mathrm{e}+05)$ | $(9.40 \mathrm{e}-11)$ | $(2.77 \mathrm{e}-02)$ | $(2.11 \mathrm{e}-02)$ |
| (6) High $\bar{a}$ | 1.001 | 1.001 | 0.980 | 0.005 | $1.87 \mathrm{e}+05$ | $1.46 \mathrm{e}-10$ | 1.002 | 0.937 |
| $(\bar{q}=1 / 7)$ | $(6.06 \mathrm{e}-03)$ | $(6.06 \mathrm{e}-03)$ | $(5.94 \mathrm{e}-03)$ | $(3.50 \mathrm{e}-04)$ | $(4.24 \mathrm{e}+04)$ | $(2.59 \mathrm{e}-11)$ | $(7.30 \mathrm{e}-03)$ | $(1.23 \mathrm{e}-02)$ |
| $(7)$ High $\boldsymbol{q}$ | 1.000 | 1.000 | 0.991 | 0.163 | $2.71 \mathrm{e}+01$ | $1.01 \mathrm{e}-03$ | 1.001 | 0.943 |
| $(\underline{q}=0.5)$ | $(4.60 \mathrm{e}-03)$ | $(4.60 \mathrm{e}-03)$ | $(4.58 \mathrm{e}-03)$ | $(1.09 \mathrm{e}-02)$ | $(3.80 \mathrm{e}+00)$ | $(2.10 \mathrm{e}-04)$ | $(9.11 \mathrm{e}-03)$ | $(7.83 \mathrm{e}-03)$ |

Note: The table shows the means of the price levels after 365 days, where the initial price level is set to one (so that a value of one indicates no change). RWGEKS represents the rolling window Gini, Elteto, Koves, and Szulc (GEKS) index. Standard deviations in parentheses.

## Concluding Remarks

- Summary
- Provided some evidence of household inventory, and they are consistent with theory
- Inferred consumption/stockpiling
$\star$ Bias is mitigated by constructing consumption-based indices.
- Stockpiling should not be ignored in business-cycle studies (not presented today).
- Use of interesting household scanner data (not presented today).
- Remaining tasks
- Heterogeneity wrt products and households
- Other product categories (gasoline, car, PC, travel)


## Some Other Results

## On Use of Lower Frequency Data

- The use of lower frequency data alone does not completely eliminate the bias.
- Using both lower frequency data and fixed base indices ignores some products that are short lived or new.
- Some of the high frequency (e.g., daily) fluctuations in prices and quantities may be closely related to business cycle fluctuations.


## Household-side Shoku-map data (food map)

- Respondents: mainly housewives, 400 households each month
- Daily from Sep 1998 to present
- Food only
- Records
- who, what, when and where purchased, when consumed, when consumption ends
- no price information
- not on how much (i.e. weights) consumed
- Example: a salt product for a household



## A beer product for a household



## Fact 4. Consumption tends to decrease as household inventories decrease.

- We estimate the following equation for the sample of $\Lambda_{i j t}>0$ :

$$
\begin{equation*}
y_{i j t}=c_{i}+d_{j}+\alpha \Lambda_{i j t}+\varepsilon_{i j t}, \tag{15}
\end{equation*}
$$

for household $i$ in product category $j$ at date $t$.

- $y_{i j t}$ is consumption. It takes one if goods in product category $j$ are used. Alternatively, we define $y_{i j t}$ as the sum of uses of goods in product category $j$.
- $\Lambda_{i j t}$ is the sum of inventory. Suppose that household $i$ uses goods $k$ in date $t_{l} \in t_{1}, t_{2}, \cdots, t_{n_{i k t}}$, where $t$ represents the purchase date. Thus, $n_{i k t}$ represents the number of uses. We define the inventory in date $t^{\prime}$ $\left(t \leq t^{\prime} \leq t_{n_{i k t}}\right)$ as $\lambda_{i k t^{\prime}}=n_{i k t}-\sum_{l=1}^{n_{i k t}} 1$. Then, aggregating it in product category $j$ leads to

$$
\begin{equation*}
\Lambda_{i j t}=\sum_{k \in j} \lambda_{i k t} \tag{16}
\end{equation*}
$$

- Is $\alpha$ positive?


## Table: State-dependent Consumption

Dependent variable 1 if household uses product $\quad$ Number of times a product is used

| Inventory | $0.0006895^{* * *}$ <br> $(0.0000171)$ | $0.00410^{* * *}$ <br> $(0.000901)$ |
| :---: | :---: | :---: |
| Observations | $90,545,020$ | $90,545,020$ |
| No. of HH | 3,602 | 3,602 |
| $R^{2}$ | 0.12 | $\mathrm{HH} /$ category |

Note: Figures in parentheses represent robust standard errors. ${ }^{* * *}$, **, and * indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively.

## Elasticity of Substitution

- We obtain $\sigma$ for each 3-digit product category.
- V shape filter to identify sales.
- for each sales event $s$ of product $k$ and shop $s$, obtain $P_{H}, X_{H}, P_{L}, X_{L}$ (the lower of the quantity purchased in the first- and second-half spell of sales).
- calculate $\sigma$ for each 3-digit product category.

$$
\star-\log \left(X_{L} / X_{H}\right) / \log \left(P_{L} / P_{H}\right) .
$$

- For each sales event $s$ of product $k$ and shop $s$, we obtain
- Consumption at sales: $X_{L}^{*}=\left(P_{L} / P_{H}\right)^{-\sigma} X_{H}$
- Inventory at the end of sales: $I_{L}=\Sigma_{j=1}^{T} X_{t+j}-T X_{L}^{*}$.
- $r_{H}\left(I_{t-1}\right)$ and $m$


## Elasticity of Substitution



Note: "Our estimate" represents our calculation of the elasticity of substitution $\sigma$ from $\Gamma \equiv-\log \left(c_{L} / c_{H}\right) / \log \left(r_{L} / r_{H}\right)$ using the inferred series of consumption $c$ and consumption price $r$. "Simple estimate" represents the calculation of $\sigma$ from $-\Delta \log X_{t} / \Delta \log p_{t}$, where $X$ and $p$ represent the quantity purchased and posted price, respectively. The left-hand panel shows the histogram of the values of $\sigma$ for 3-digit product categories, while the right-hand panel shows a scatter plot where each dot represents a 3-digit product category.

Table: Top and Bottom Five Categories for the Degree of Stockpiling

|  | Product category | m |  | Product category | m |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Top 5 | Instant cup noodles | 4.98 | Bottom 5 | Razors | 1.15 |
|  | Diluted beverages | 3.74 |  | Prepared bread meals | 1.19 |
|  | Frozen meals | 3.58 |  | Cosmetic accessories | 1.26 |
|  | Packaged instant noodles | 3.23 |  | Home medical supplies | 1.27 |
|  | Coffee beverages | 3.08 |  | Batteries | 1.30 |

Note: The degree of stockpiling $m$ is inferred using the POS data.

## External Validations

(1) Comparison of the inflation rates: data and simulation
(2) Comparison with the household-side scanner inventory data

## Comparison of the Inflation Rates: Data and Simulation

- Data
- The daily average of the inflation rate from January 1989 to December 2011 based on the purchase-weighted Törnqvist index for each 3-digit product category $j$ (denote $\pi_{j}$ ).
- At the same time, we record the mean of the following variables for each 3-digit product category: $m_{j}, \overline{q_{j}}, \underline{q_{j}}$, and $\left(P_{L} / P_{H}\right)_{j}$.
- Simulation
- Using the above values, we simulate the model and calculate the average size of the chain drift, where $\sigma$ is set at 5 .
- We generate randomized price paths for the period of $T=405$ days and discard the first and last 20 days to calculate changes in the price indexes in 365 days. We repeat this for $N=100$ times.


## Relation between Shoku-map data and the estimate of $m$

No correlation between $\log$ (inventory period) and $\log m$ over the entire observation periods
$\rightarrow m$ does not necessarily represent storability.
But a significant correlation just before the consumption tax increase



## Relation between Shoku-map data and the estimate of $m$

Positive correlation between $\log$ (the quantity of purchase) and $\log m$ over the entire observation periods
$\rightarrow m$ captures stockpiling when price decreases (or is expected to increase).



## Developments in Consumers' Stockpiling Behavior



Time fixed effect $d_{t}$ for the regression of $\log \left(m_{j t}\right)=c_{j}+d_{t}+A X_{j t}+\varepsilon_{j t}$, where $X_{j t}=\left\{\log \left(1-\overline{q_{j t}}\right), \log \left(1-\underline{q_{j t}}\right), \log \left(\left(P_{H}-P_{L}\right) / P_{H}\right)_{j t}\right\}$.

## Determinants of $m$

- Prices $X_{j t}=\left\{\log \left(1-\overline{q_{j t}}\right), \log \left(1-\underline{q_{j t}}\right), \log \left(\left(P_{H}-P_{L}\right) / P_{H}\right)_{j t}\right\}$
- Labor market conditions: hours worked, unemployment rate
- Longer hours worked may increase $m$, if an increase in income alleviates financial constraints (positive sign)
- Longer hours worked may decrease $m$, if a decrease in shopping time prevents search for sales (negative sign).
- Interest rate (negative sign)
- A higher interest rate increases the opportunity cost of stockpiling, decreasing $m$.


## Regression of Stockpiling $m$

We estimate the following equation:

$$
\begin{equation*}
\Delta \log \left(m_{j t}\right)=c_{j}+B \Delta Z_{t}+A \Delta X_{j t}+\mu_{j t} \tag{17}
\end{equation*}
$$

or

$$
\begin{equation*}
\Delta \log \left(m_{j t}\right)=c_{j}+B \Delta Z_{t}+A \widetilde{v_{j t}}+\mu_{j t} \tag{18}
\end{equation*}
$$

where $j$ and $t$ represent a 3-digit product category and a month,

$$
\begin{align*}
\Delta X_{j t} & =e_{j}+D \Delta Z_{t}+v_{j t}  \tag{19}\\
\widetilde{v_{j t}} & \equiv \Delta X_{j t}-\left(\hat{e}_{j}+\hat{D} \Delta Z_{t}\right) \tag{20}
\end{align*}
$$

$X_{j t}=\left\{\log \left(1-\overline{q_{j t}}\right), \log \left(1-\underline{q_{j t}}\right), \log \left(\left(P_{H}-P_{L}\right) / P_{H}\right)_{j t}\right\}$ and exogenous variables associated with business cycles $Z_{t}$ consist of the unemployment rate, $\log$ hours worked, and the real interest rate.
Using $\widetilde{v_{j t}}$, we evaluate the overall effect of $\Delta Z_{t}$ on $\Delta \log \left(m_{j t}\right)$, which incorporates the indirect effect through $\Delta X_{j t}$.

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta \log (m)$ | $\Delta \log (1-\bar{q})$ | $\Delta \log (1-q)$ | $\Delta \log \left(1-P_{L} / P_{H}\right)$ | $\Delta \log (m)$ |
| $\Delta($ unemp rate $)$ | 0.0026 | $-0.0017^{* * *}$ | $-0.0338^{* * *}$ | $0.0277^{* * *}$ | $0.0312^{* * *}$ |
|  | $(0.004)$ | $(0.000)$ | $(0.003)$ | $(0.003)$ | $(0.003)$ |
| $\Delta($ unemp rate -1$))$ | $-0.0106^{* * *}$ | $0.0007^{* * *}$ | $0.0244^{* * *}$ | $-0.0300^{* * *}$ | $-0.0327^{* * *}$ |
|  | $(0.004)$ | $(0.000)$ | $(0.003)$ | $(0.003)$ | $(0.004)$ |
| $\Delta \log ($ hours worked $)$ | $-0.2940^{* * *}$ | $0.0210^{* * *}$ | $0.1341^{* * *}$ | $0.1989^{* * *}$ | $-0.3612^{* * *}$ |
|  | $(0.043)$ | $(0.002)$ | $(0.030)$ | $(0.033)$ | $(0.043)$ |
| $\Delta \log ($ hours worked $(-1))$ | $-0.6677^{* * *}$ | $0.0077^{* * *}$ | $-0.6106^{* * *}$ | $0.4365^{* * *}$ | $-0.2602^{* * *}$ |
|  | $(0.046)$ | $(0.001)$ | $(0.044)$ | $(0.040)$ | $(0.044)$ |
| $\Delta($ real r) | $-0.0133^{* * *}$ | $0.0009^{* * *}$ | $0.0147^{* * *}$ | $-0.0052^{* * *}$ | $-0.0243^{* * *}$ |
|  | $(0.003)$ | $(0.000)$ | $(0.002)$ | $(0.002)$ | $(0.002)$ |
| $\Delta($ real r(-1)) | $-0.0036^{*}$ | $-0.0002^{* * *}$ | $-0.0086^{* * *}$ | $0.0037^{* *}$ | 0.0024 |
|  | $(0.002)$ | 0.000 | $(0.002)$ | $(0.002)$ | $(0.002)$ |
| $\Delta \log (1-\bar{q})$ | $-2.4013^{* * *}$ |  |  |  | $-2.4013^{* * *}$ |
|  | $(0.615)$ |  |  | $(0.615)$ |  |
| $\Delta \log (1-\underline{q})$ | $-0.5117^{* * *}$ |  |  |  | $-0.5117^{* * *}$ |
|  | $(0.036)$ |  |  | $(0.036)$ |  |
| $\Delta \log \left(1-P_{L} / P_{H}\right)$ | $0.2604^{* * *}$ |  |  | $0.2604^{* * *}$ |  |
|  | $(0.032)$ |  |  | $(0.032)$ |  |
| Fixed effects | category | category | category | category | 53,700 |
| Observations | 53,700 | 53,700 | 53,700 | 0.005 | 150 |

Note: Variable $m$ is the degree of stockpiling, $\bar{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $q$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, $P_{L} / P_{H}$ is the ratio of the sale price to the regular price, and real $r$ is the real interest rate. In column (5), the explanatory variables corresponding to $\Delta \log (1-\bar{q}), \Delta \log (1-q)$, and $\Delta \log \left(1-P_{L} / P_{H}\right)$ are the residuals of the estimation for columns (2), (3), and (4), respectively. ${ }^{* * *},{ }^{* *}$, and ${ }^{*}$ indicate significance at the $1 \%, 5 \%$, and $10 \%$ levels, respectively.

