Scanner Data, Chain Drift, Superlative Price Indices and the Redding-Weinstein CES Common Varieties Price Index

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Objective

The main objective of the paper is to examine and demonstrate the suitability of the CES Common Varieties (CCV) Price Index by Redding and Weinstein (2020) for use with Scanner Data.

Summary of the Results

- We show that the recently proposed CES Common Varieties Price Index (CCV) by Redding and Weinstein (2020) is a transitive index.

- The CCV index is a COLI for CES preferences with preference shocks.

- One serious problem of the index by Redding and Weinstein (2020) is that the index number depends on the choice of the normalization condition of parameters. Data and demand theory cannot help identify the normalization condition. There can be many normalizations that can lead to many different index numbers for the same COLI.

Summary of the Results

- Using axiomatic index number theory, we offer a solution to this problem. We show that the commensurability axiom, the independence of the index from measurement units, characterizes the normalization condition to the Cobb-Douglas functional form

- In addition, if we treat all the commodities equally/symmetrically, the normalization condition becomes unique. Other normalization conditions such as the arithmetic mean or the generalized mean function of order r violate the commensurability.

- Other axiomatic properties such as monotonicity and linear homogeneity are also established.

Summary of the Results

- We present a few applications of the CCV index number using high frequency scanner data, demonstrating the importance of the transitivity.
- The Fisher and Tornqvist indexes exhibit strong chain drifts whereas the price index by Redding and Weinstein (2020) does not.
- Recent experience of the COVID-pandemic gives us an example of a massive demand shocks.
- We show that when a large-scale demand shock occurs, the CCV exhibits very different pattern from the Fisher, Sato-Vartia or other standard superlative indexes in which preferences are assumed to be fixed.

Background

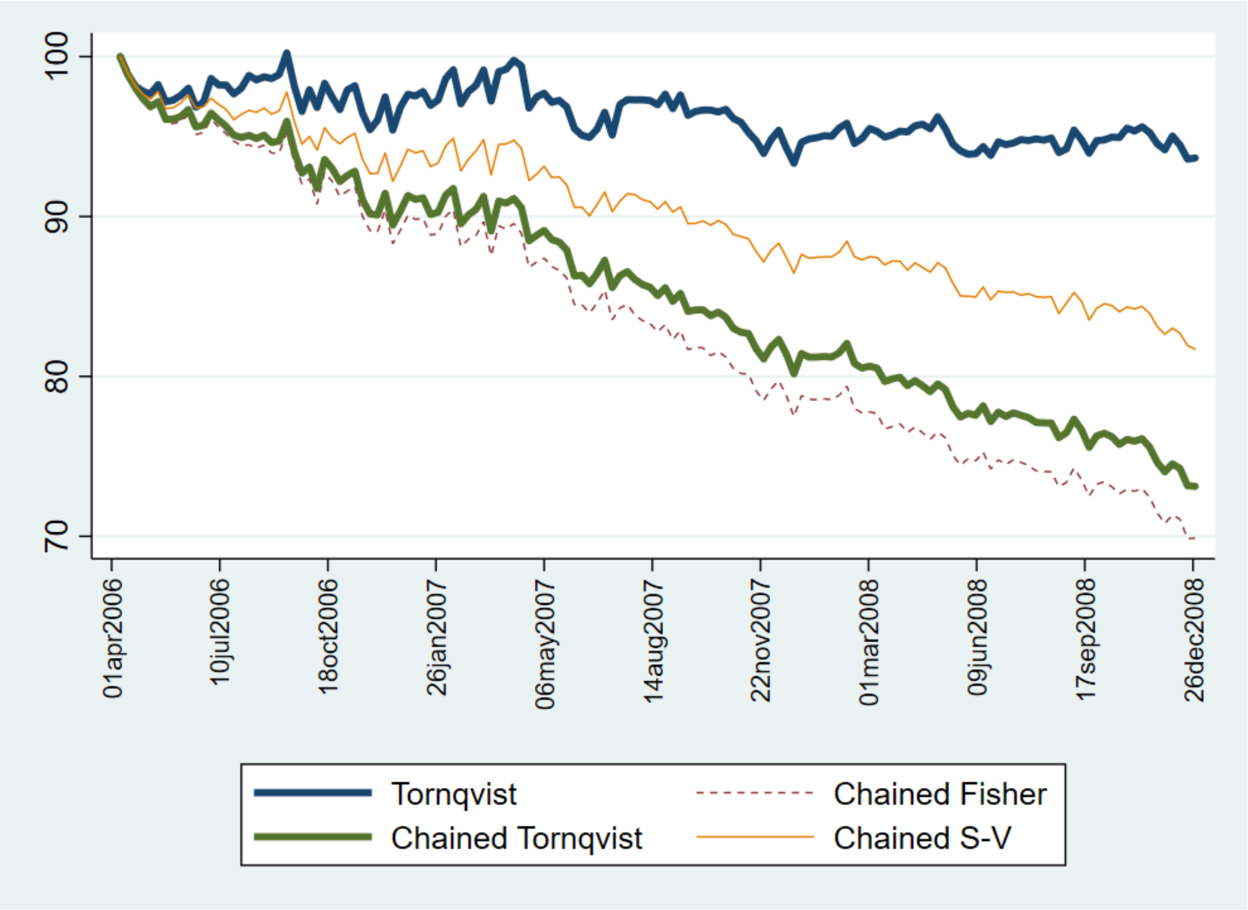
Transitivity and Index Numbers

- The Cost of Living Index (COLI), defined as the ratio of two expenditure functions, is transitive.

- Superlative index which is a nice approximation of the COLI is not generally transitive. In this case, chain drift occurs.

- When using high frequency scanner data, chain drift can be a serious problem for superlative indexes.

An Example: Scanner ata of cereal prices
Fixed-base and Chained Tornqvist, Fisher and S-V Indices



Two types of Chain Drift

- There are two types of chain drifts.

- The first drift occurs because of the correlation between lagged quantities and prices. Jevons index is free from this first type of chain drift.

- The second chain drift occurs because of product turnover, which makes the product sets for the direct and chained index different. Jevons index is not free from this second chain drift.

- In this paper, we consider the first type of chain drift only. To deal with the second chain drift, we need a model of product turnover. (Note that Feenstra's variety effects can cause chain drift as well.)

Utility Functions with Taste or Demand Shocks

Utility and Expenditure Functions

Utility Function:

$$U_\tau(q_\tau,\varphi_\tau,\sigma)=\left(\sum_{i=1}^{N}\left(\varphi_{i\tau}q_{i\tau}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad N>1;\ \sigma>1;\quad \tau=s,t$$

Expenditure Function:

$$E\left(p_t,U_t\right)=C\left(p_t;\varphi_t,\sigma\right)\times U_t=\left(\sum_{i=1}^{N}\left(\frac{p_{it}}{\varphi_{it}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\times U_t.$$

Taste parameters

φ_{it} : Taste Parameters that vary over time

The Role of Taste Parameters

$$U_\tau(q_\tau,\varphi_\tau,\sigma)=\left(\sum_{i=1}^{N}\left(\varphi_{i\tau}q_{i\tau}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad N>1;\ \sigma>1;\ \ \tau=s,t$$

- If φ_{it} changes over time, the marginal utility from the goods change which in turn shifts the demand functions.

- Traditional COLI assumes that the demand function are fixed. All the changes in quantities and prices on the fixed demand functions.

- If there are demand shocks, the observed quantities and prices can move on the supply curve.

Example of a Demand Shock

- During COVID-Pandemic in 2020, there was a large-scale increase in demand for face masks in Japan.

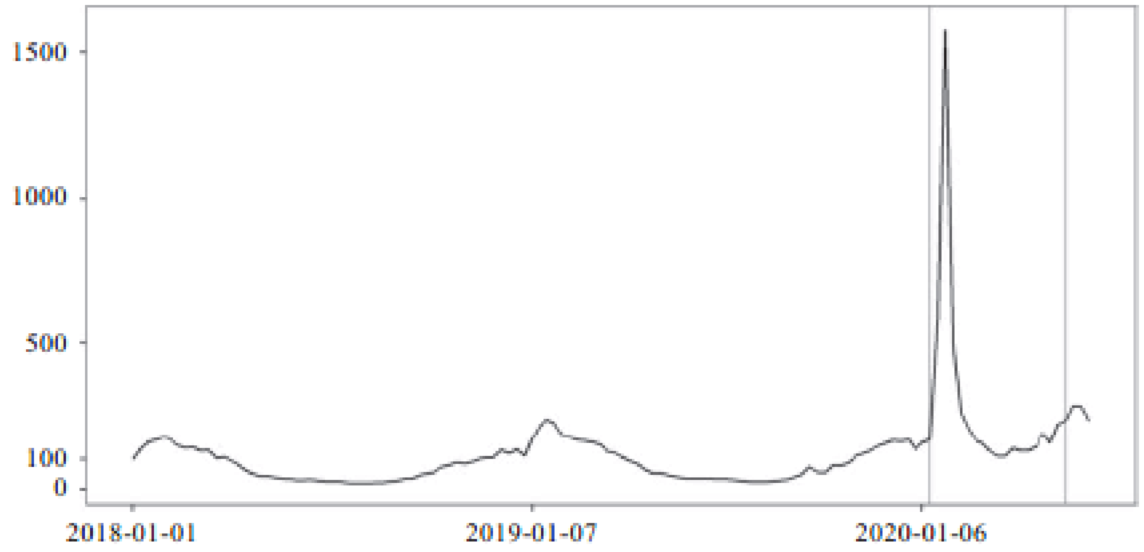
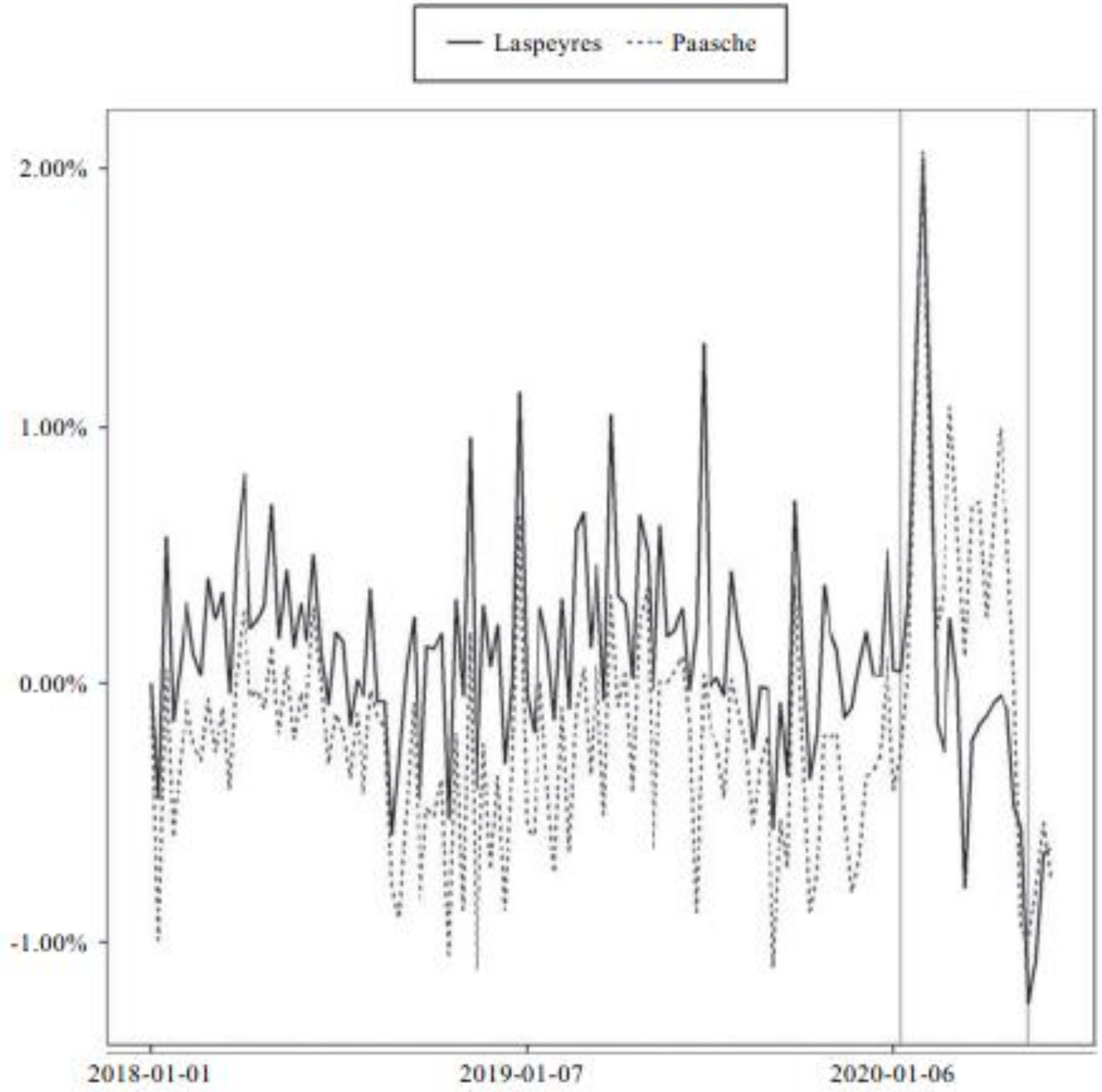


Fig. 3. Movements of the Total Sales of Face Masks.

Data: weekly retail scanner data, SRI by INTAGE

The Laspeyres-Paasche gap for face masks during COVID-Pandemic actually becomes negative!

If we ignore the demand shocks, the negative LP gap indicates that the face mask is a Giffen good during the pandemic, which is unlikely.



The Redding-Weinstein (2020) COLI and the Constant Elasticity of Substitution Unified Price Index (CUPI)

$$R-W\,COLI_{st} = \frac{E_t(p_t,U)}{E_s(p_s,U)} = \frac{C(p_t,\varphi_t,\sigma)}{C(p_s,\varphi_s,\sigma)} = \frac{\left(\sum_{i=1}^{N}\left(\frac{p_{it}}{\varphi_{it}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}{\left(\sum_{i=1}^{N}\left(\frac{p_{is}}{\varphi_{is}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}$$

RW (2020) shows that for ALL the quantities and prices,

$$\ln CUPI_{st} = \ln SV_{st} - \left[\sum_{i=1}^{N}\omega_{ist}^{*}\ln\left(\frac{\varphi_{it}}{\varphi_{is}}\right)\right] = \ln\left(R-WCOLI\right)$$

The Sato-Vartia Index and COLI

- COLI for CES preferences: $COLI = \left(\sum_{i=1}^{N} \left(\frac{p_{i1}}{\varphi_i} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} / \left(\sum_{i=1}^{N} \left(\frac{p_{i1}}{\varphi_i} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$

- The Sato-Vartia Index: $SV = \prod_{i=1}^{N} \left(\frac{p_{i1}}{p_{i0}} \right)^{\omega_{ist}^{*}} \qquad \omega_{ist}^{*} = \frac{w_{it} - w_{is}}{\ln(w_{it}) - \ln(w_{is})} / \sum_{i=1}^{N} \left(\frac{w_{it} - w_{is}}{\ln(w_{it}) - \ln(w_{is})} \right).$

- These two indexes take the same value only when quantities are on the demand functions.

Indeterminacy of the CUPI

- The unit cost function is homogenous of degree -1 with respect to taste parameters. (Note that the standard COLI is homogeneous of degree zero w.r.t. taste parameters).

$$C\left(p_t,\varphi_t;\sigma\right)=\left(\sum_{i=1}^{N}\left(\frac{p_{it}}{\varphi_{it}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

- From the data, we can identify up to only N-1 of φ_{it}'s One parameter remains unidentified. R-W uses the following normalization condition,

$$\prod_{i=1}^{N}\left(\varphi_{it}\right)=\varphi$$

The Role of Normalization Condition

- Comparisons of levels of two different utility functions requires cardinal utilities.

- Standard demand theory is based on ordinal utility. For example, any monotonic transformation of utility functions will give us the same demand system.

- To compare two utility levels, we need to normalize the utility function at some point.

- For example, u(1, 1, 1,⋯.,1)=1 is an example of a normalization condition. The problem is that our economic theory does not help us select one of the normalization conditions.

- Kurtzon (2020) compares the R-W indexes under different normalization conditions, finding significant differences induced by the choice of different normalization conditions.

Commensurability

- An index number is said to be commensurable if it is free from the measurement units of commodities.

- Commensurability is regarded as one of the most important axioms for index numbers. Almost all the index numbers (except for the Dutot index) are commensurable.

- If we adopt the following arithmetic mean as the normalization condition, the CUPI fails the commensurability test.

$$\sum_{i=1}^{N} \varphi_{it} \Big/ N = \varphi$$

Our Results – Proposition 1

The CCV index passes commensurability test *if and only if* the normalization condition can be written in the form :

$$\varphi_1 = A \times \left(\prod_{i=2}^{N} x_{ti}^{c_i} \right) \qquad c_i \in \mathbf{R},\ A > 0$$

Corollary

φ_i

If all the taste parameters are considered equally important and treated symmetrically, the necessary and sufficient condition can be written in the form of a simple geometric mean, which is identical to the normalization condition by R-W (2020):

$$\prod_{i=1}^{N}\left(\varphi_i\right) = \varphi$$

Our Results – Proposition 2

The R-W CCV with geometric normalization of taste parameters is independent of units of measurement.

$$\prod_{i=1}^{N}\left(\varphi_{it}\right)=\varphi$$

Other Properties

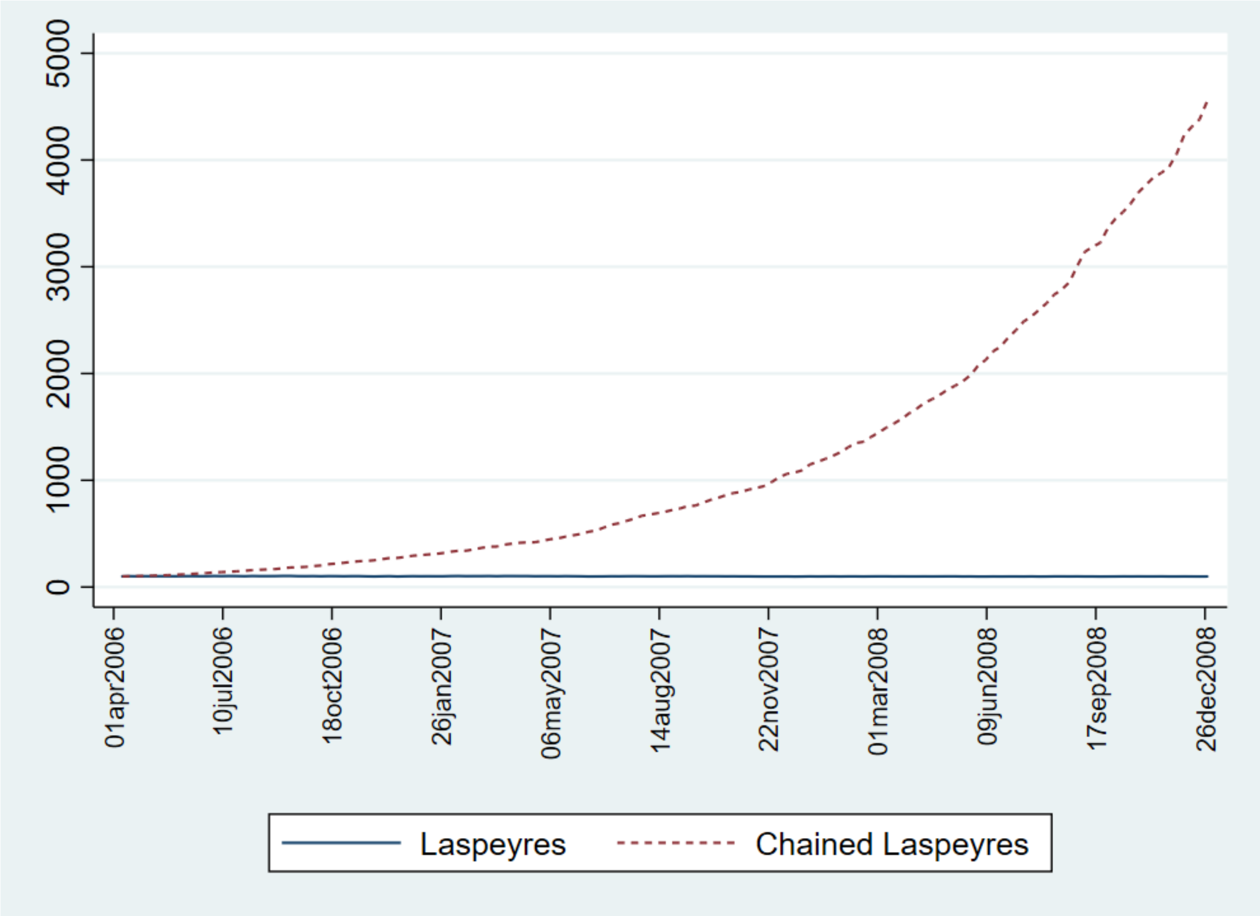
- Suppose we use the following normalization condition,

$$\prod_{i=1}^{N}(\varphi_i) = \varphi$$

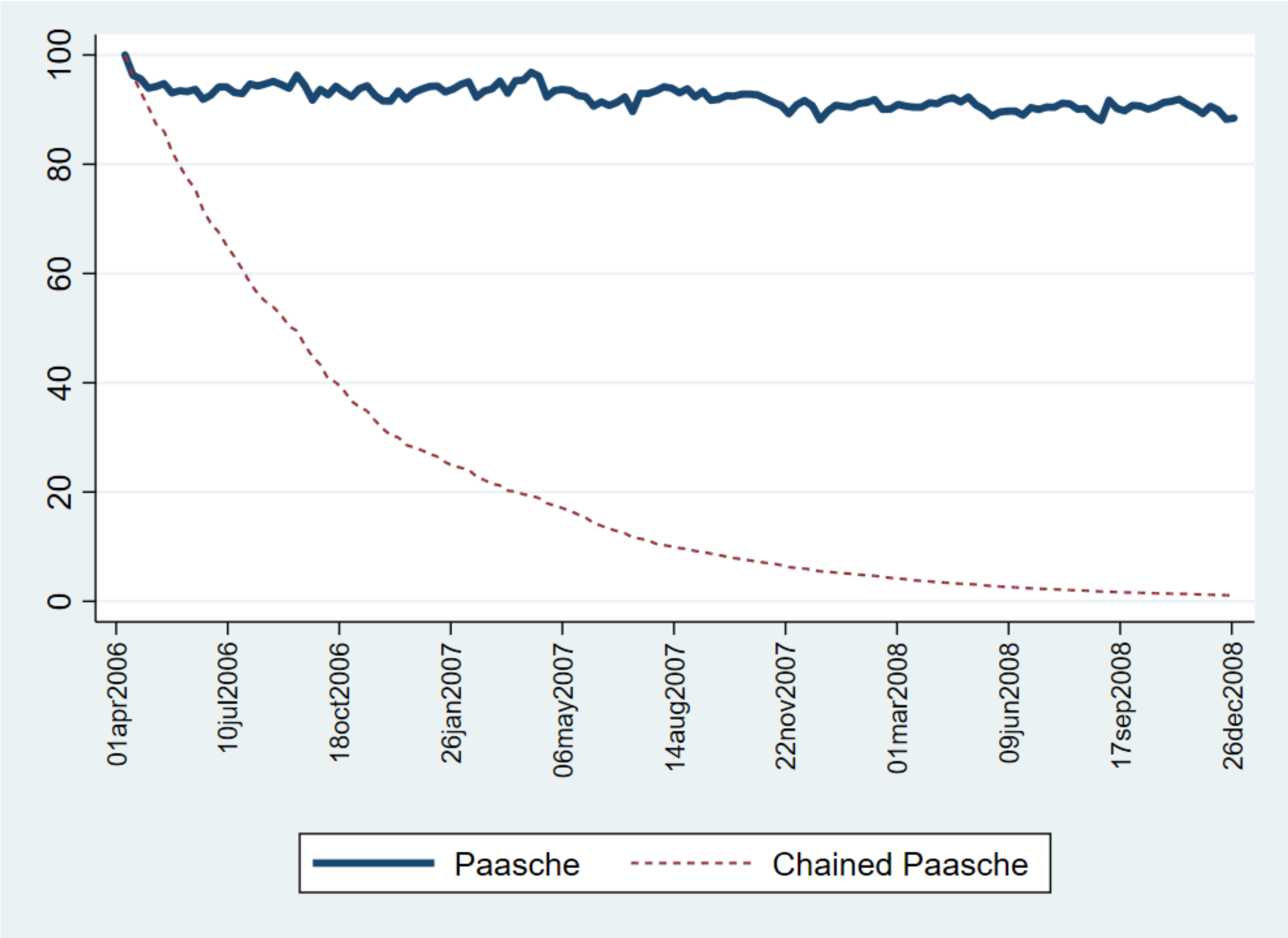
Then, the CCV is: (1) an increasing function of current prices; (2) transitive; and (3) linear homogenous with respect current prices.

- Unfortunately, the CCV fails the identity test, that is, even if all the prices are the same between the two time periods, the index number is not unity. But, this is a natural consequence of introducing the taste shocks in the CCV.

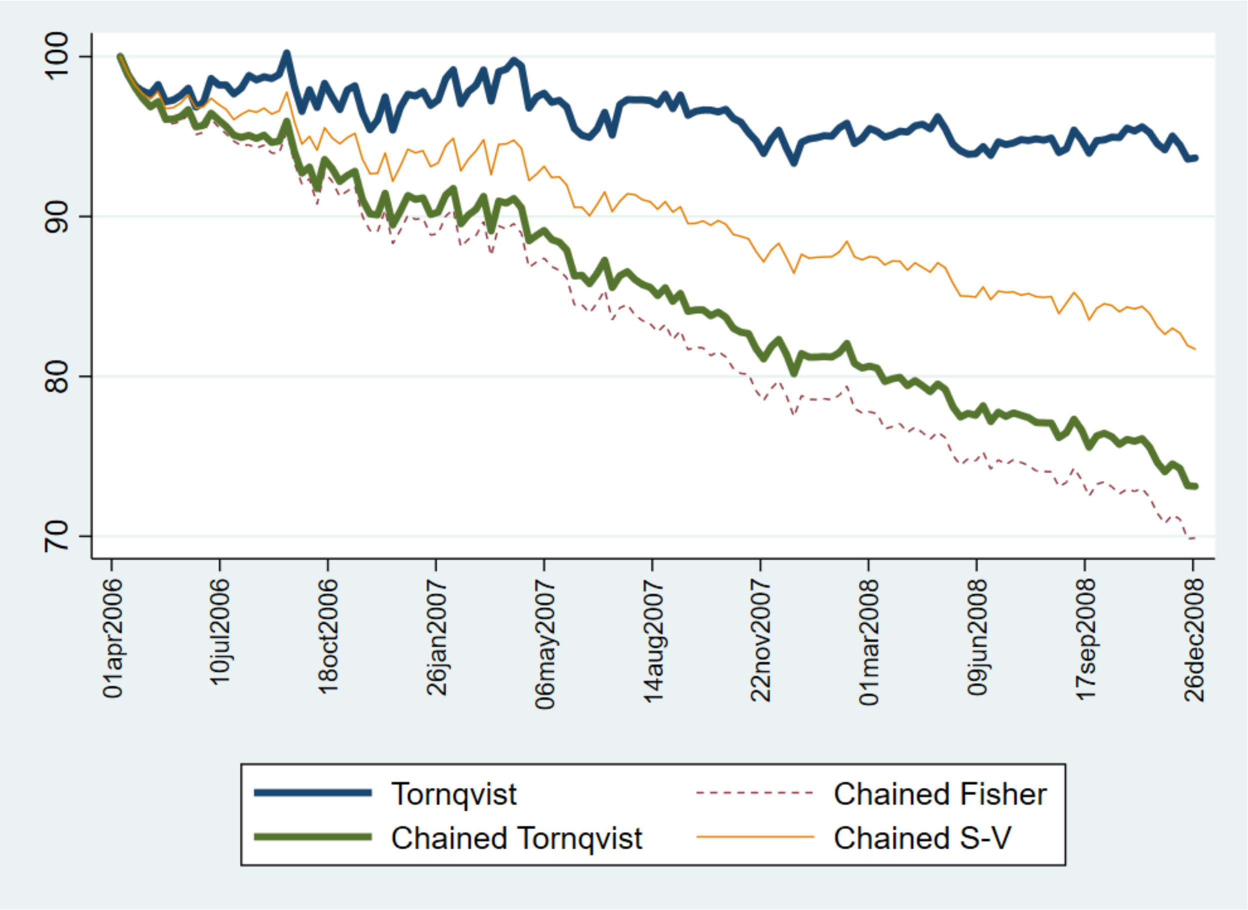
An Example with Scanner Data of Cereals: Laspeyres



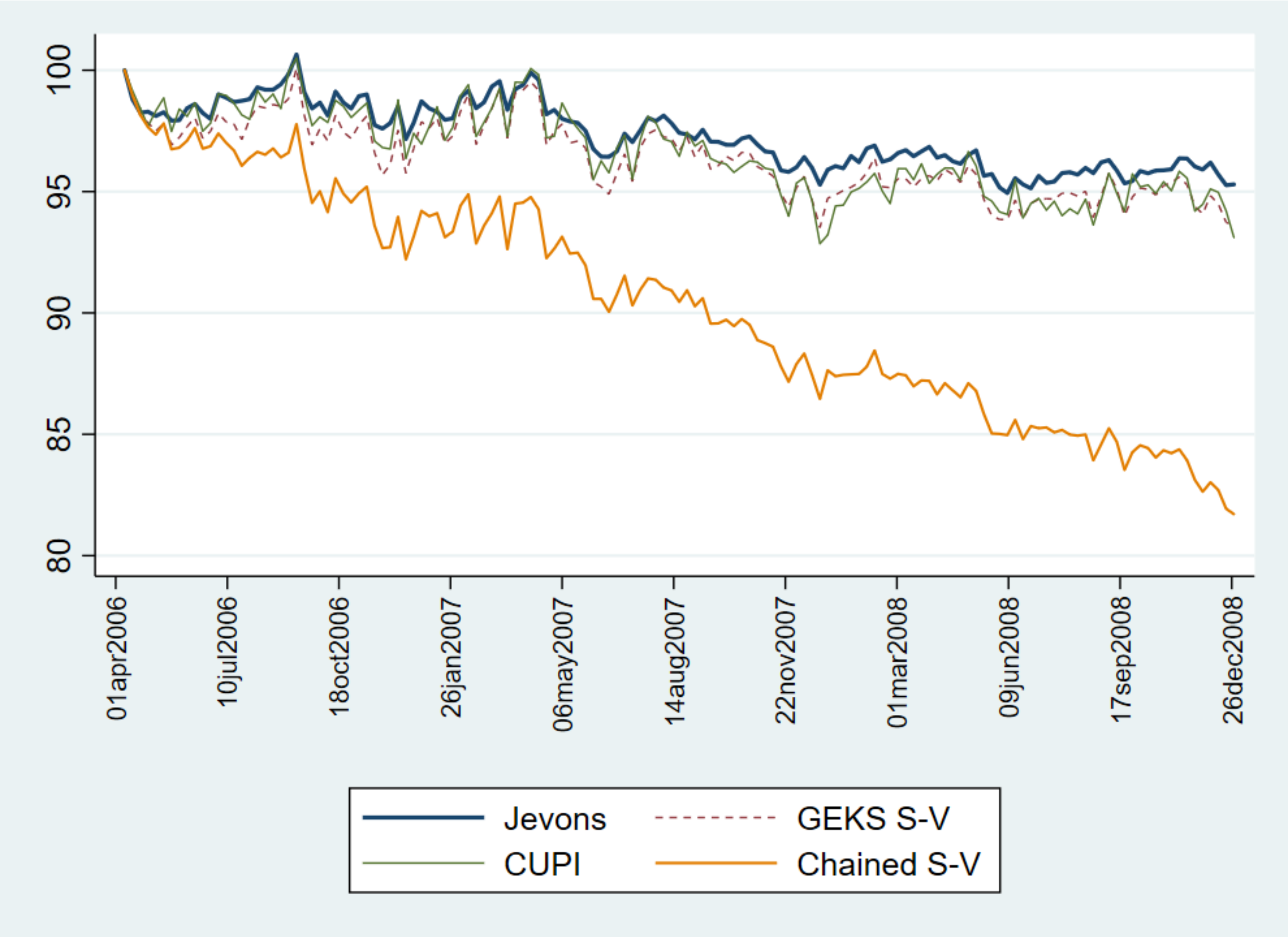
An Example with Scanner Data of Cereals: Paasche



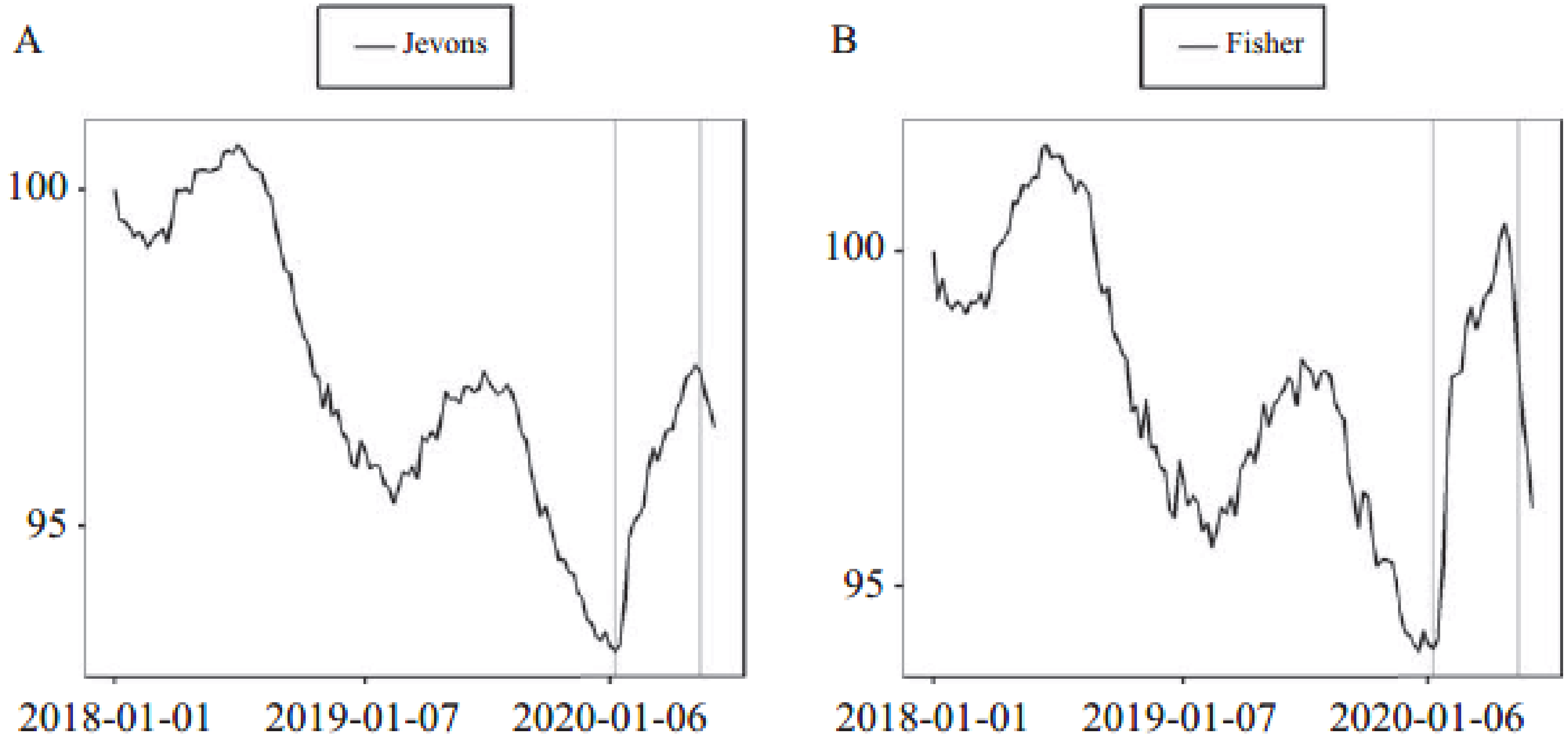
An Example with Scanner Data of Cereals : The Tornqvist and Chained Tornqvist, Fisher, S-V



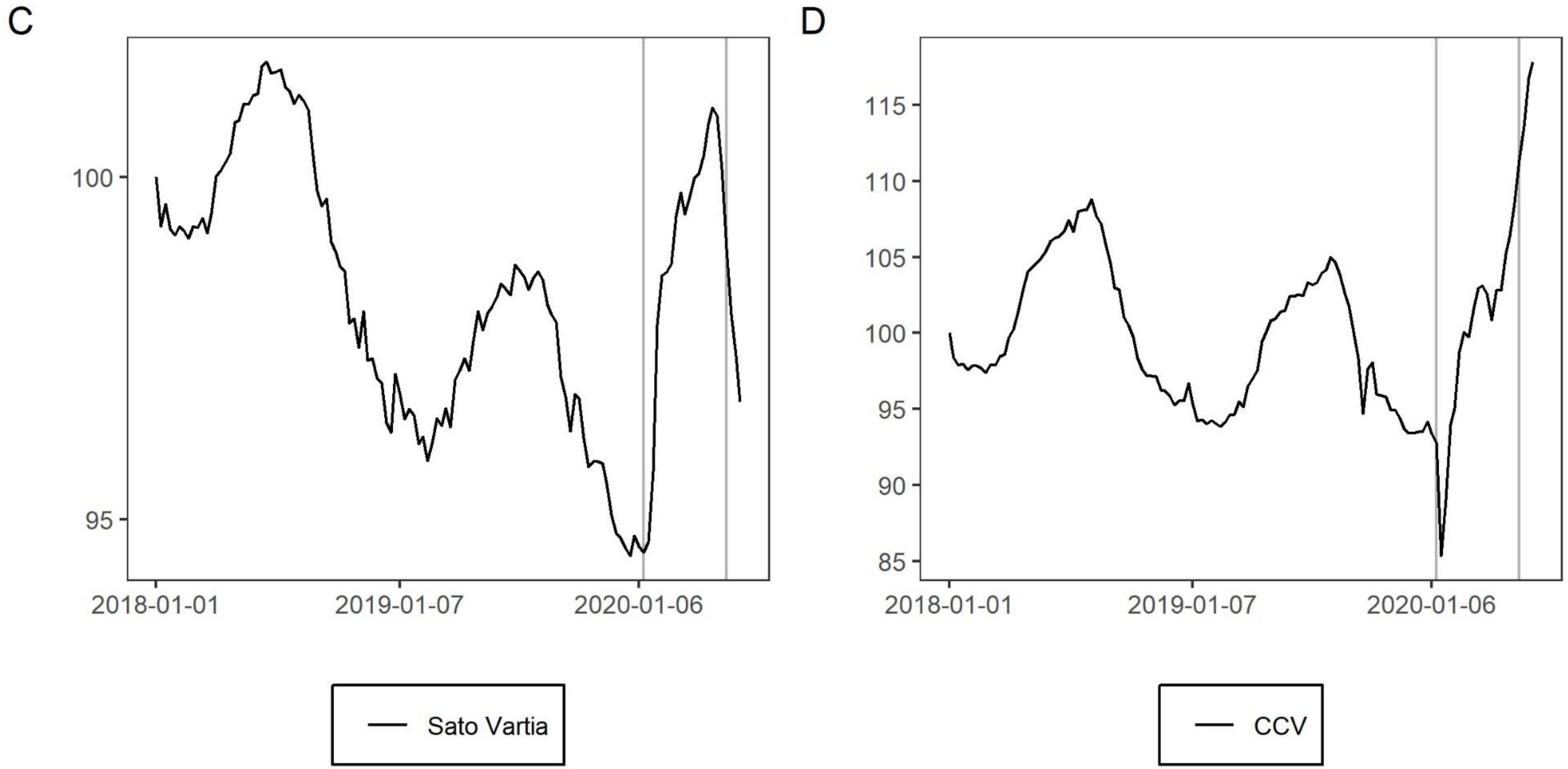
An Example with Scanner Data of Cereals: The Jevons, GEKS S-V, CUPI, and the Chained SV



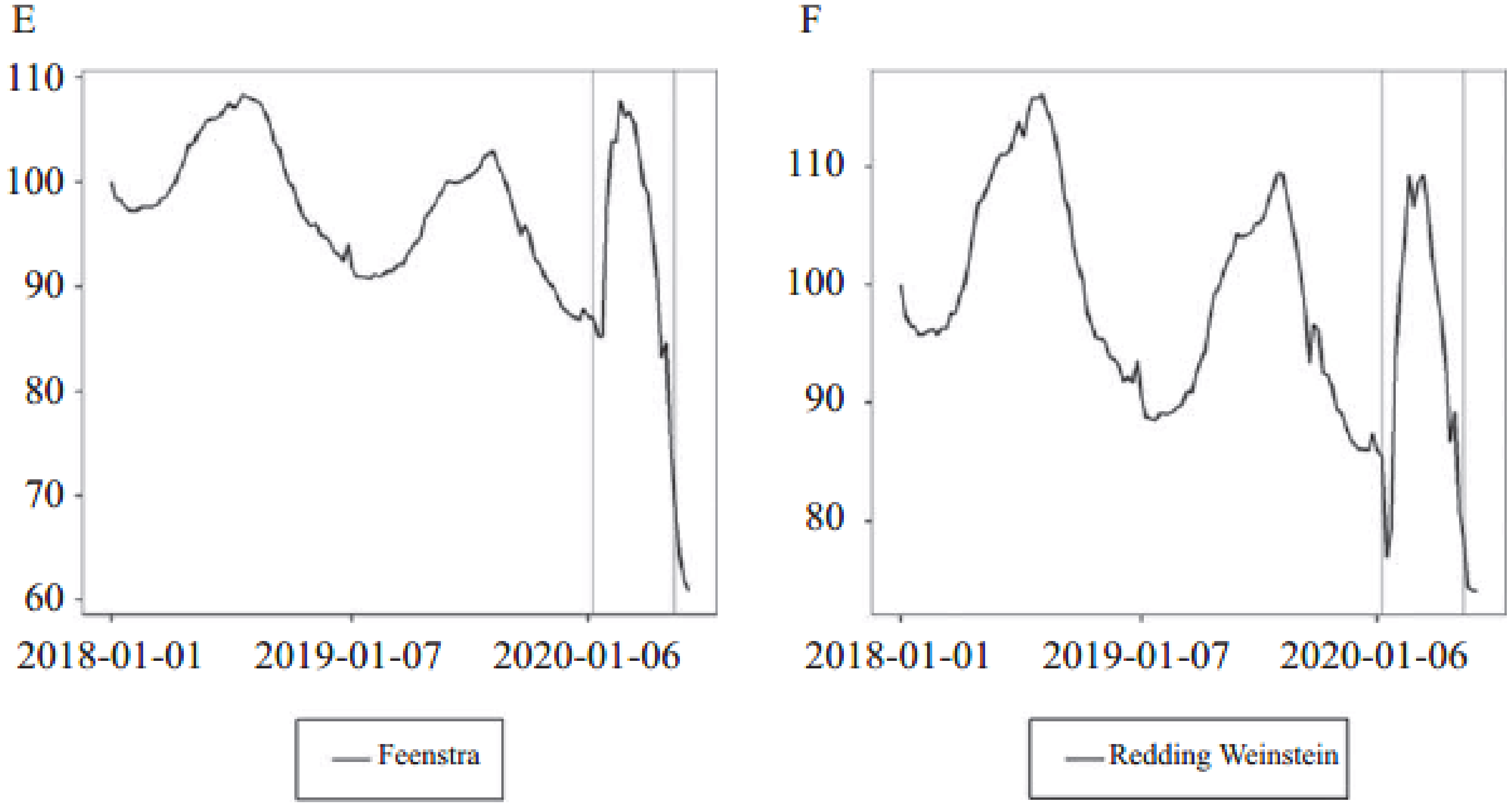
An Example with Face Masks



An Example with Face Masks



An Example with Face Masks: With Variety Effects



Conclusion

- The CCV by Redding and Weinstein (2020), has several important characteristics that solves the longstanding problem in the aggregation of consumer products, lack of transitivity.
- The serious problem of indeterminacy can be greatly mitigated by appealing to the commensurability property of index numbers.
- The CCV index numbers give us quantitatively very different values from the standard superlative and GEKS index numbers.
- The differences become very noticeable when large scale demand shock occurred such as face masks during COVID-Pandemic.

Remaining Tasks

- Although our result on the commensurability significantly reduces the indeterminacy problem, still, the utility function is assumed to be cardinal.

- Another problem of the CCV is that it cannot be defined when the elasticity of substitution is unity.

- The second type of chain drift caused by product turnover, that is, the different product sets for direct and chain indexes, needs to be minimized.

- Balk (1989) proposed the ordinal COLI that also provides us with transitive index. Balk's index can be defined at unit elasticity, free from the normalization condition, and free from the second type of chain drifts. But, we need to choose a reference quantity vector to compare two different ordinal utilities. The characterization of the reference vector is in progress.