



STATISTICS

**« *Le substitution bias*  
est mort, vive le  
*substitution bias* ! »**

**JUNE 7–10, 2022**

Jens Mehrhoff, Senior Economist  
Real Sector Division

# Outline

Substitution bias is dead...

Long live substitution bias!

A “consumption” price index

**Substitution bias is dead...**

# The economic approach

Using the assumption of homothetic preferences, the **traditional economic approach** allows to compile price indexes using only observable price and quantity data that are free of substitution bias. To be precise, the thus derived index numbers are free of ***cross-product substitution bias***, but the fairly restrictive assumption leads to **income bias**.

- It implies that **all income elasticities of demand are unity**; empirical evidence suggests however that consumers do not buy a second Volkswagen Beetle but rather one Porsche 911 if their income allows.
- It is the only known approach to-date which allows that the index can be calculated **using observable price and quantity data**.
- Further restricting the preference function to a so-called flexible functional form, the Fisher and Tornqvist indexes emerge as “**superlative**”.

## The economic approach – continued

At the **elementary-index** level, cross-product substitution bias is essentially extinct thanks to most statistical offices now using the Jevons index. At the **aggregate-index** level, many statistical offices have made progress with more frequent weight updates using a variant of the Laspeyres index, thus reducing cross-production substitution bias.

- The updated CPI Manual (IMF et al., 2020) gives guidance on directly using the new weights **assuming the expenditure shares have remained the same** (Young index).

The more fundamental problem with the economic approach lies in **equaling purchases and consumption in any given period**. However, it is a well-established fact that consumer stockpile a product when it goes on sale. For storable products, the economic approach thus suffers from ***intertemporal* substitution** bias.

# Consumer inventory models

On the other hand, there is a large economics and marketing literature on what is known as **consumer inventory models**.

- The typical setup includes a utility function  $u$  based on the quantity consumed in period  $t$ ,  $c^t$ , and the consumer decides at each period – facing prices  $p^t$  – how much to buy, denoted by  $q^t$ , and how much to consume. Since the **good is storable**, quantity not consumed is kept as inventory  $x^t$  for **future consumption**:

$$x^t = x^{t-1} + q^t - c^t.$$

The major difference between these models and the economic approach lies in the **intertemporal utility maximization**. This means that the utility and hence the economic index do **not only depend on current prices** but also on current inventories (and future expected prices).

# Consumer inventory models – continued

Since the consumer's utility is defined through the quantity consumed, a reasonable price index might be based on the **principles of valuation of inventory and the consumption basket in accounting standards**. The main hurdle in documenting demand patterns of storable goods is that **inventories are generally unobserved**.

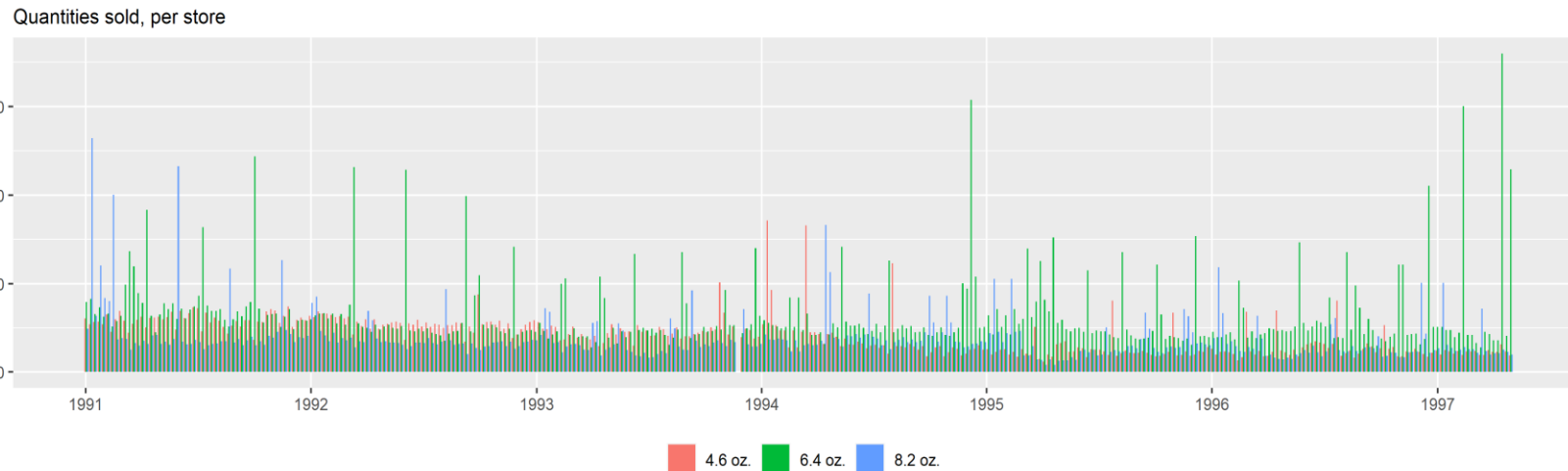
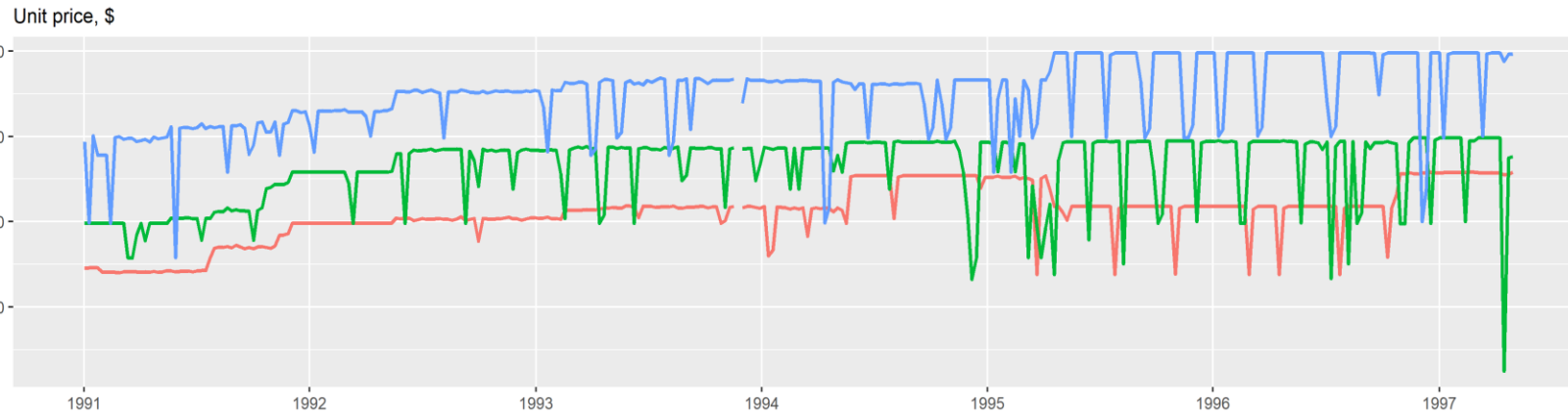
- This new paradigm is exemplified using scanner data from **Dominick's Finer Foods**. See Mehrhoff (2018), <https://github.com/eurostat/dff>

Notably, cross-production substitution is dwarfed by intertemporal substitution; a stylized fact from scanner data is that **most storable goods are bought almost exclusively when they are on discount** and only in negligible amounts during a non-sale period.

- The examples presented in the present paper are not the final word on all this. Eventually, the intention of this contribution is to start the **search for a new yardstick for economic indexes**.

# Not only increases demand during sales, it also accumulates between sales

Weekly prices and quantities of Crest Tartar Protection Regular Paste by size\*



4.6 oz. 6.4 oz. 8.2 oz.

\* No data for week ending November 24, 1993. Sources: Dominick's Finer Foods data set; and IMF staff calculations.

There are **three items** (4.6 oz., 6.4 oz., and 8.2 oz.) and **331 weeks** (January 2, 1991, through April 30, 1997) distinguished. The “regular” prices—at the end—are \$2.09, \$2.49, and \$2.99, respectively.

The **prices are temporarily reduced** to \$1.69, \$1.99, and \$2.49, respectively.



# The elasticity of substitution parameter $\sigma$ takes on the value 5 – that means substitution, right?

Using **constant elasticity of substitution (CES)** preferences and assuming optimizing behavior on the part of the consumers, the period  $t$  expenditure shares will be

$$s_i^t = \frac{\alpha_i (p_i^t)^{1-\sigma}}{\sum_{j=1}^n \alpha_j (p_j^t)^{1-\sigma}},$$

where  $\sigma$  is the elasticity of substitution, which is constant for all pairs of commodities, and the  $\alpha_i$  are positive parameters, which sum to unity.

Estimating the system of equations by fitting an iterated **seemingly unrelated regression** returns:

	<b>4.6 oz.</b>	<b>6.4 oz.</b>	<b>8.2 oz.</b>
$\alpha_i$	0.1303	0.3632	0.5064
<b>(s.e.)</b>	(0.0043)	(0.0060)	(0.0091)
$\sigma$	4.988		
<b>(s.e.)</b>	(0.1488)		
$R^2$	0.4163	0.5954	0.5189
<b>McElroy-<math>R^2</math></b>	0.5228		

However, asynchronous sales yield dynamic intertemporal substitution but **no *static* CES effects!**

**Long live substitution bias!**

# Get ready for the \$10 tube of toothpaste

Source: [Reuters, March 1, 2022](#)

- *Colgate-Palmolive Co CEO Noel Wallace said last week at an industry conference that the household goods maker sees its new Optic White Pro Series toothpaste as the type of **premium product** “vital” to its ability to raise prices, which will help drive profit growth this year.*
- *Colgate expects its **margins to widen** this year, due in part to higher prices.*
- ***Raising prices** is a “key capability” for Colgate that will help **drive profit growth**, Wallace said last week.*
- *A Colgate spokesperson said in a statement that the company has a wide portfolio of products at different price points, and touted its **new \$10 toothpaste** as the first with 5% hydrogen peroxide, with “demonstrated efficacy to whiten teeth.”*

Simon-Kucher (the pricing consultancy): “An improvement in **pricing can boost profits easily**. Increasing volumes and cutting costs can only take you so far.”

# For storable products, the economic approach suffers from intertemporal substitution bias

Weekly prices and quantities of Crest Tartar Protection Regular Paste by size\*—zoomed



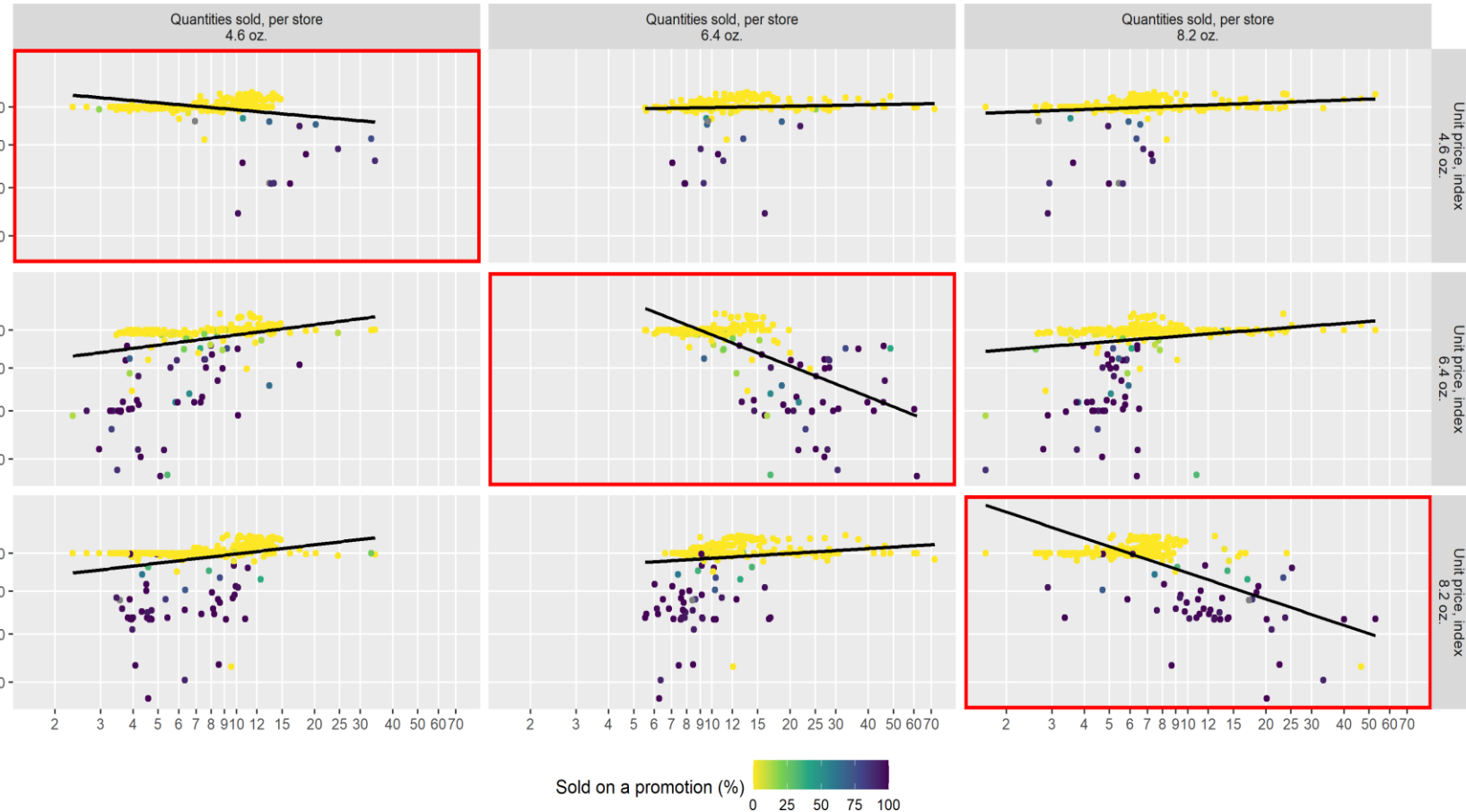
\* No data for week ending November 24, 1993. Sources: Dominick's Finer Foods data set; and IMF staff calculations.

The fundamental problem with the economic approach lies in **equaling purchases and consumption in any given period**. However, it is a well-established fact that **consumers stockpile a product when it goes on sale.**

A stylized fact from scanner data is that **most storable goods are bought almost exclusively when they are on discount** and only in negligible amounts during a non-sale period.

# Cross-production substitution is dwarfed by intertemporal substitution

Own and cross-price elasticity of demand for Crest Tartar Protection Regular Paste by size\*  
Log scale



\* No data for week ending November 24, 1993. Sources: Dominick's Finer Foods data set; and IMF staff calculations.

**Static (CES or index) estimation may provide misleading results** since the consumer's utility is defined through the quantity consumed, and hence does not only depend on current prices but also on current inventories (and future expected prices).

The main hurdle in documenting demand patterns of storable goods is that **inventories are generally unobserved.**

## Some stylized facts about sales

	4.6 oz.	6.4 oz.	8.2 oz
<b>Sales occur every ... weeks</b>	20.6	5.5	6.2
<b>Sale prices are reduced by ...%</b>	11.6	17.8	16.0
<b>Sale quantities increase ...-fold</b>	2.2	2.5	2.5
<b>...% of units are sold during sales</b>	10.0	35.9	32.1

Except for the 4.6 oz. size (which was not sold on a promotion frequently in the early years), sales occur **every five to seven weeks** on average.

Prices are reduced by **about one sixth** during sales.

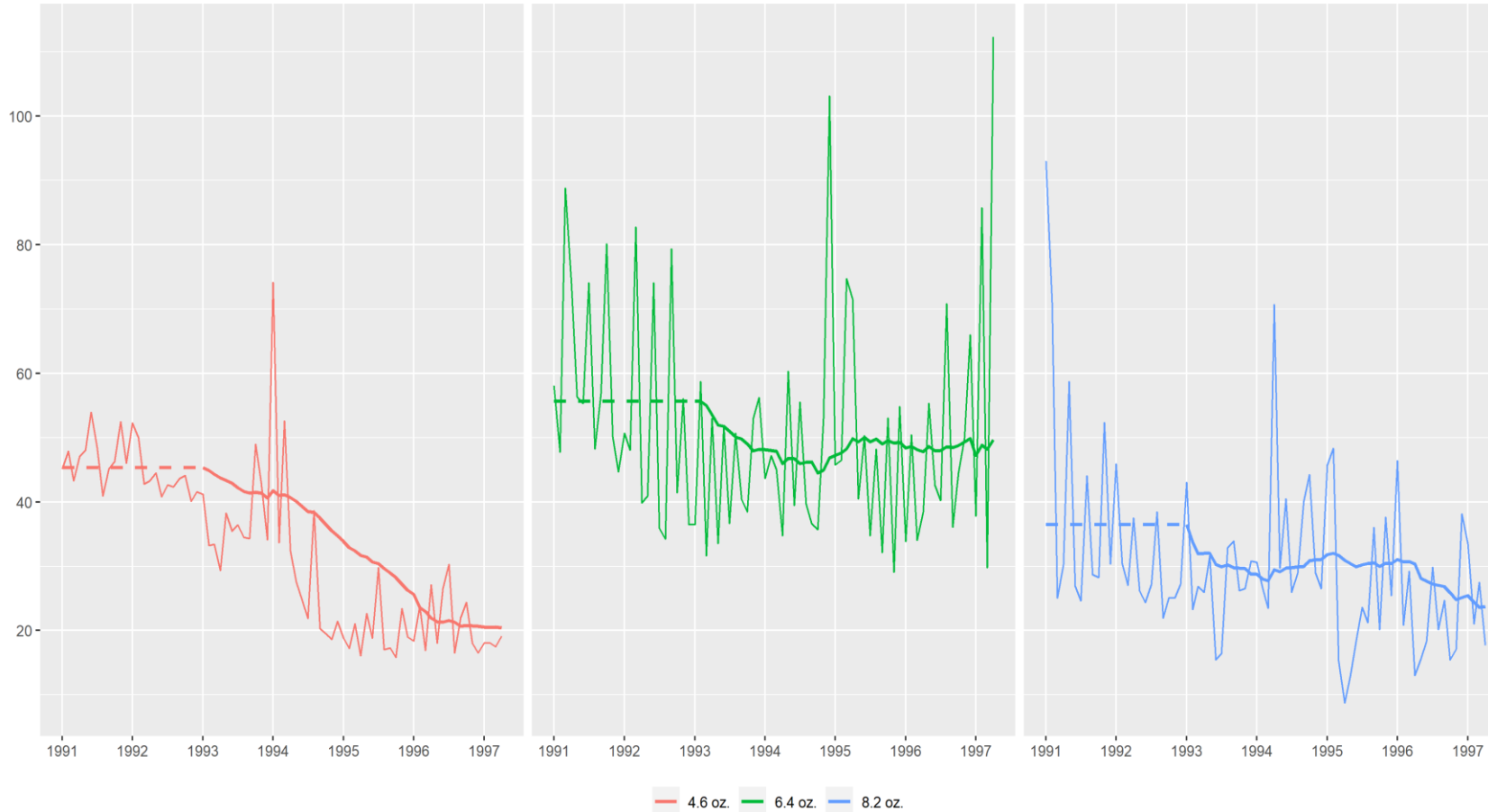
Quantities more than **double to almost triple** during a typical sale.

Since consumers stockpile during sales, **around a third** of total purchases are made on promotions alone.

# A “consumption” price index

# Consumption patterns

25-months rolling window average quantities of Crest Tartar Protection Regular Paste by size  
Quantities sold, per store



Sources: Dominick's Finer Foods data set; and IMF staff calculations.

**Consumption**  $c^t$  is smoothed, and likely lagged, compared to purchases  $q^t$ .

- Smoothed because **stockpiling** on sale does not increase contemporaneous consumption.
- Lagged because **inventories** are used rather than tossed. (4.6 oz. size is declining in demand over time.)

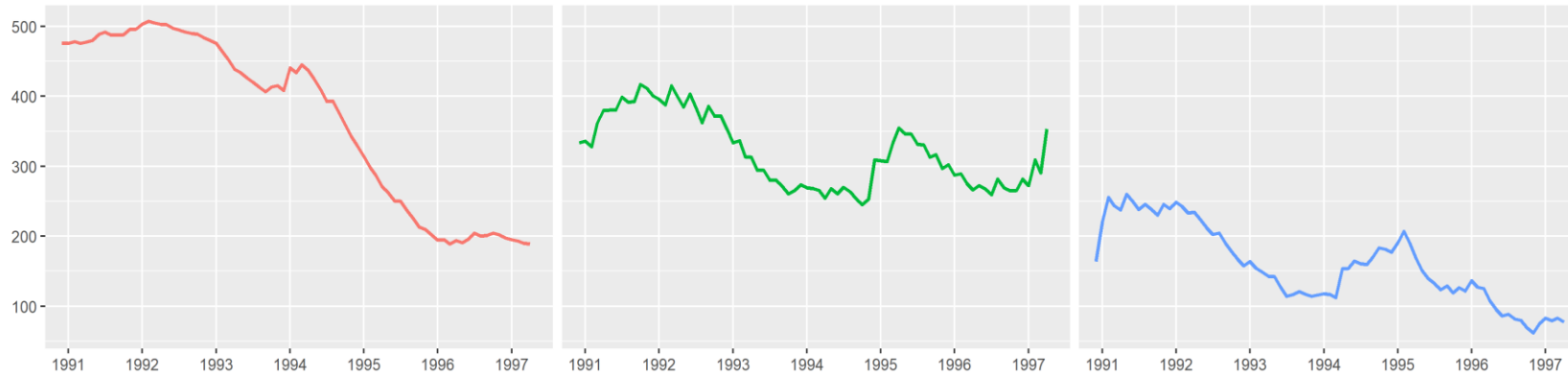
Consumption patterns are estimated as the **average of purchases** over a 25-months rolling window.

- Results **robust** to choice of window length.

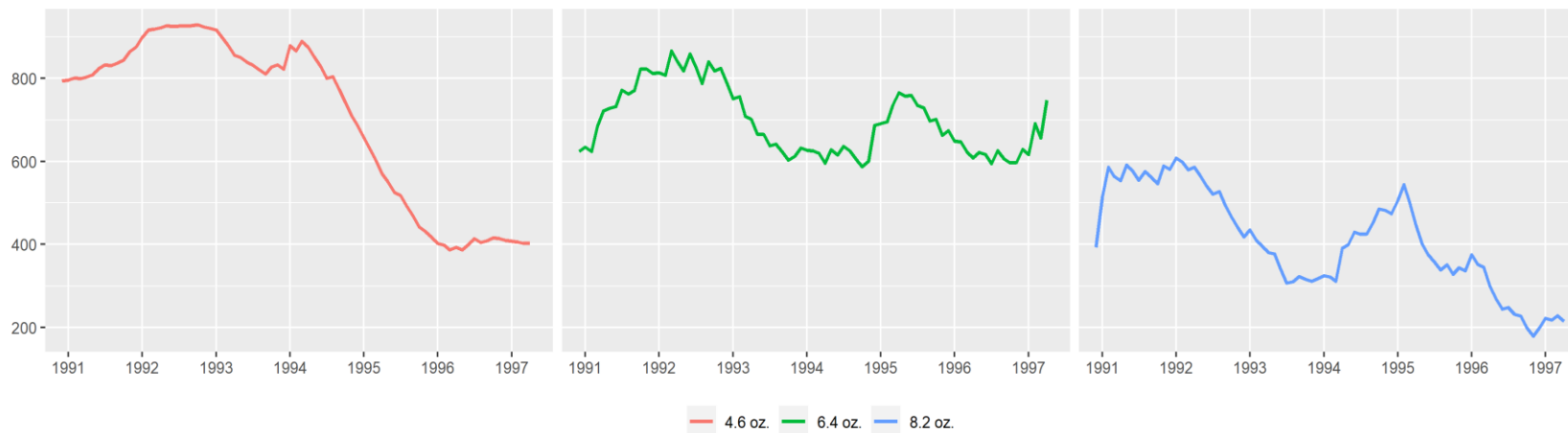


# Inventories

Inventories of Crest Tartar Protection Regular Paste by size  
Quantities, per store



Values, \$



Sources: Dominick's Finer Foods data set; and IMF staff calculations.

Once consumption patterns are established, and since purchases are known from the scanner data, **inventories**  $x^t$  can be derived:

$$x^t = x^{t-1} + q^t - c^t.$$

**Inventories before the first period**, i.e., end-December 1990, are inferred from the – available – data for 1990.

- **4.6 oz.:** Above average inventories, recurring promotions during 1990.
- **6.4 oz.:** High inventories, last promotion in December 1990.
- **8.2 oz:** Below average inventories, first promotion in January 1991.

# Price of consumption

Prices of consumption of Crest Tartar Protection Regular Paste by size  
Unit price, \$



Sources: Dominick's Finer Foods data set; and IMF staff calculations.

With inventories estimated, the **price of consumption**  $p_c^t$  can be derived:

$$p_c^t = p_x^{t-1} \frac{x^{t-1}}{x^{t-1} + q^t} + p^t \frac{q^t}{x^{t-1} + q^t}$$

The **initial inventory** is valued:

- **4.6 oz:** Below the current non-sale price, inventories from promotions earlier in 1990 are still in stock.
- **6.4 oz:** At the last sale price, inventories were restocked just in December 1990.
- **8.2 oz:** At around the current non-sale price, inventories are restocked immediately in January 1991.

# Price of consumption – continued

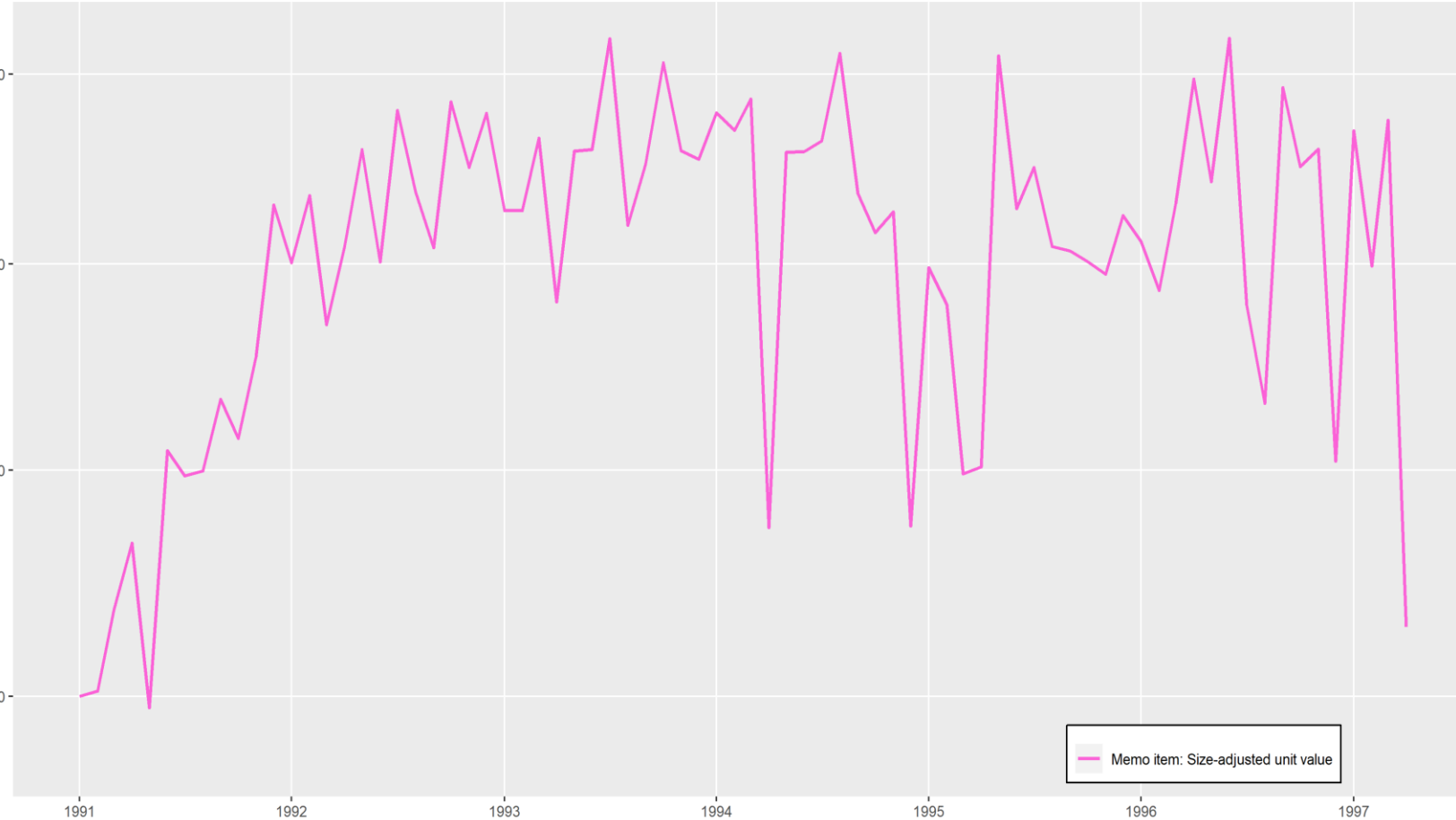
The price of consumption does not correspond to replacement cost or the last transaction price. Instead, we have the **average acquisition cost** of the items in inventory. There are two main sources of discrepancy between replacement cost and the price of consumption.

- The first is the familiar one of **sluggish adjustment**. A retail price cut today only **gradually** works itself into the price of consumption as old, higher priced inventory is consumed.
- The second arises from the occasional practice of manufacturers to inform the buyer in advance of an **impending temporary price reduction**. This permits the buyer to completely deplete inventory and then “overstock” at the lower price. In this case the price of consumption declines **precipitously** to the lower price and stays there until the large inventory acquired at that price runs off. Thus, the accounting cost shows the low price for some time after the replacement cost has gone back up.

Source: Peltzman, S. (2000), “Prices Rise Faster than They Fall,” *Journal of Political Economy*, 108(3), 466–502.

# Index number 1: Size-adjusted unit value

GEKS indexes for Crest Tartar Protection Regular Paste  
January 1991 = 100, log scale

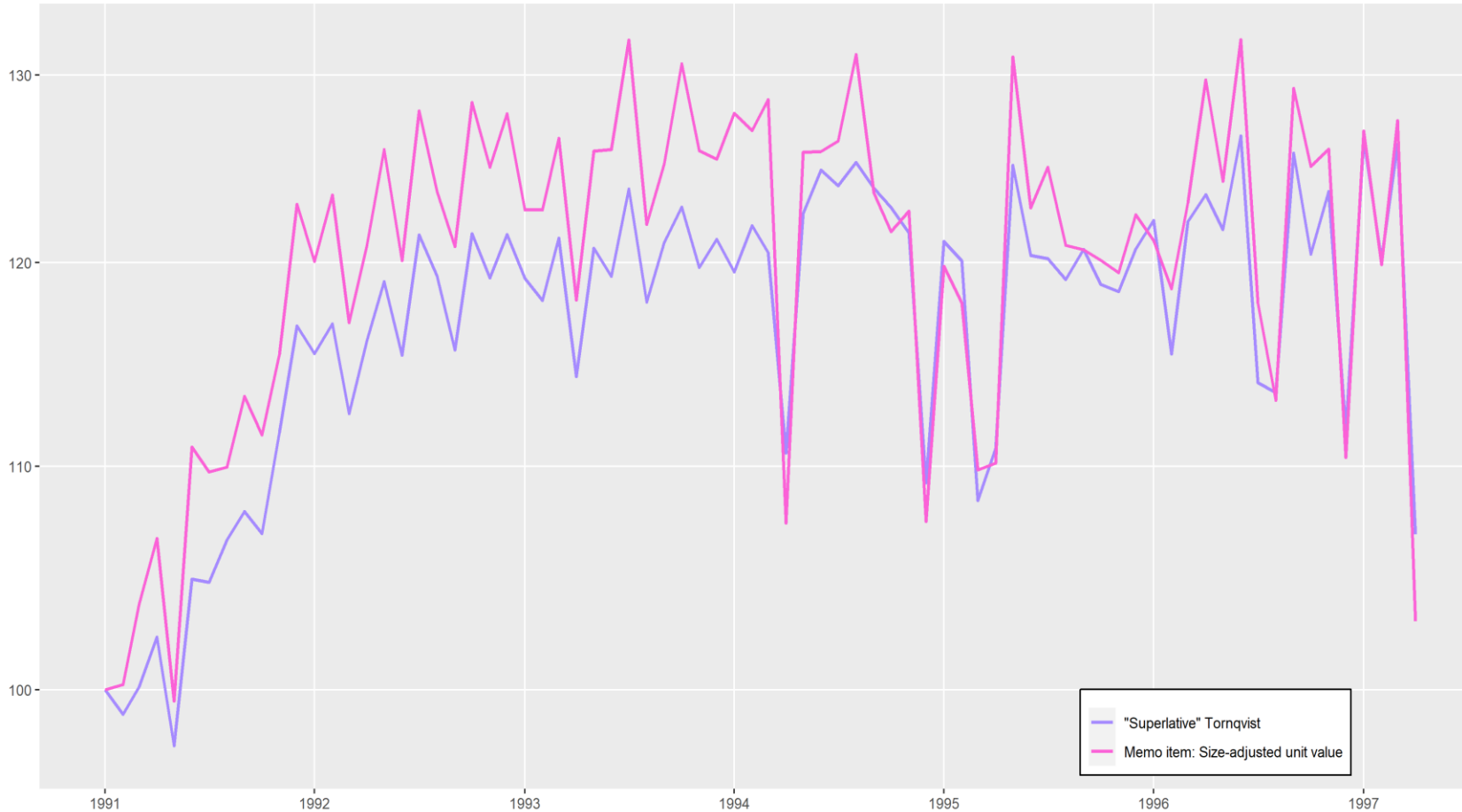


The first index, a **size-adjusted unit value using *purchase prices and quantities***, is quite **volatile**.

Sources: Dominick's Finer Foods data set; and IMF staff calculations.

# Index number 2: “Superlative” Tornqvist

GEKS indexes for Crest Tartar Protection Regular Paste  
January 1991 = 100, log scale



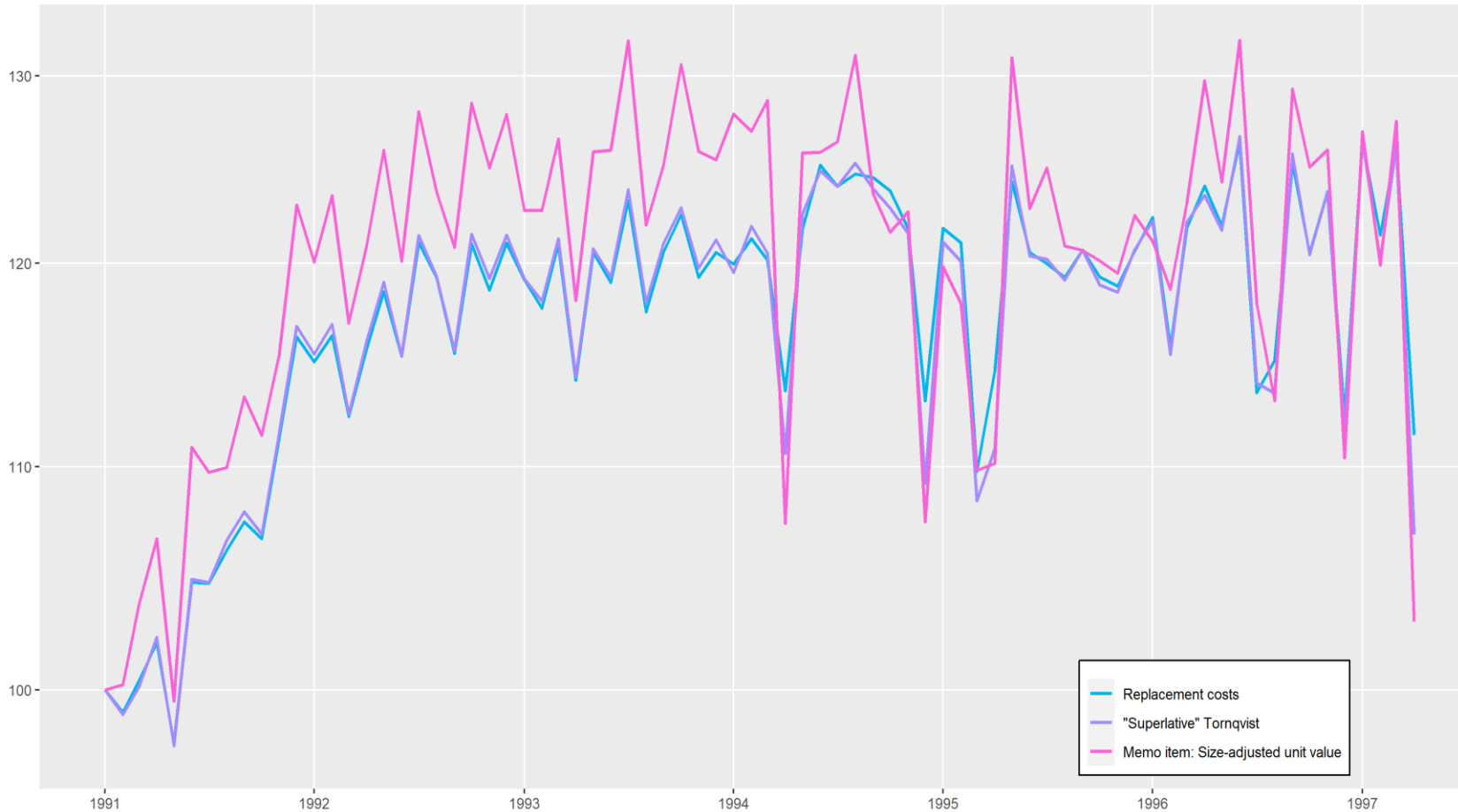
Sources: Dominick's Finer Foods data set; and IMF staff calculations.

The first index, a **size-adjusted unit value using *purchase prices and quantities***, is quite **volatile**.

The second index, a **Tornqvist using *purchase prices and quantities***, is considered to be **“superlative”**.

# Index number 3: Replacement costs

GEKS indexes for Crest Tartar Protection Regular Paste  
January 1991 = 100, log scale



Sources: Dominick's Finer Foods data set; and IMF staff calculations.

The first index, a **size-adjusted unit value using *purchase prices and quantities***, is quite **volatile**.

The second index, a **Tornqvist using *purchase prices and quantities***, is considered to be **"superlative"**.

The third index, using ***purchase prices but consumption quantities***, reflects **replacement costs**.

# Index number 4: Purchase prices Jevons

GEKS indexes for Crest Tartar Protection Regular Paste  
January 1991 = 100, log scale



Sources: Dominick's Finer Foods data set; and IMF staff calculations.

The first index, a **size-adjusted unit value using *purchase prices and quantities***, is quite **volatile**.

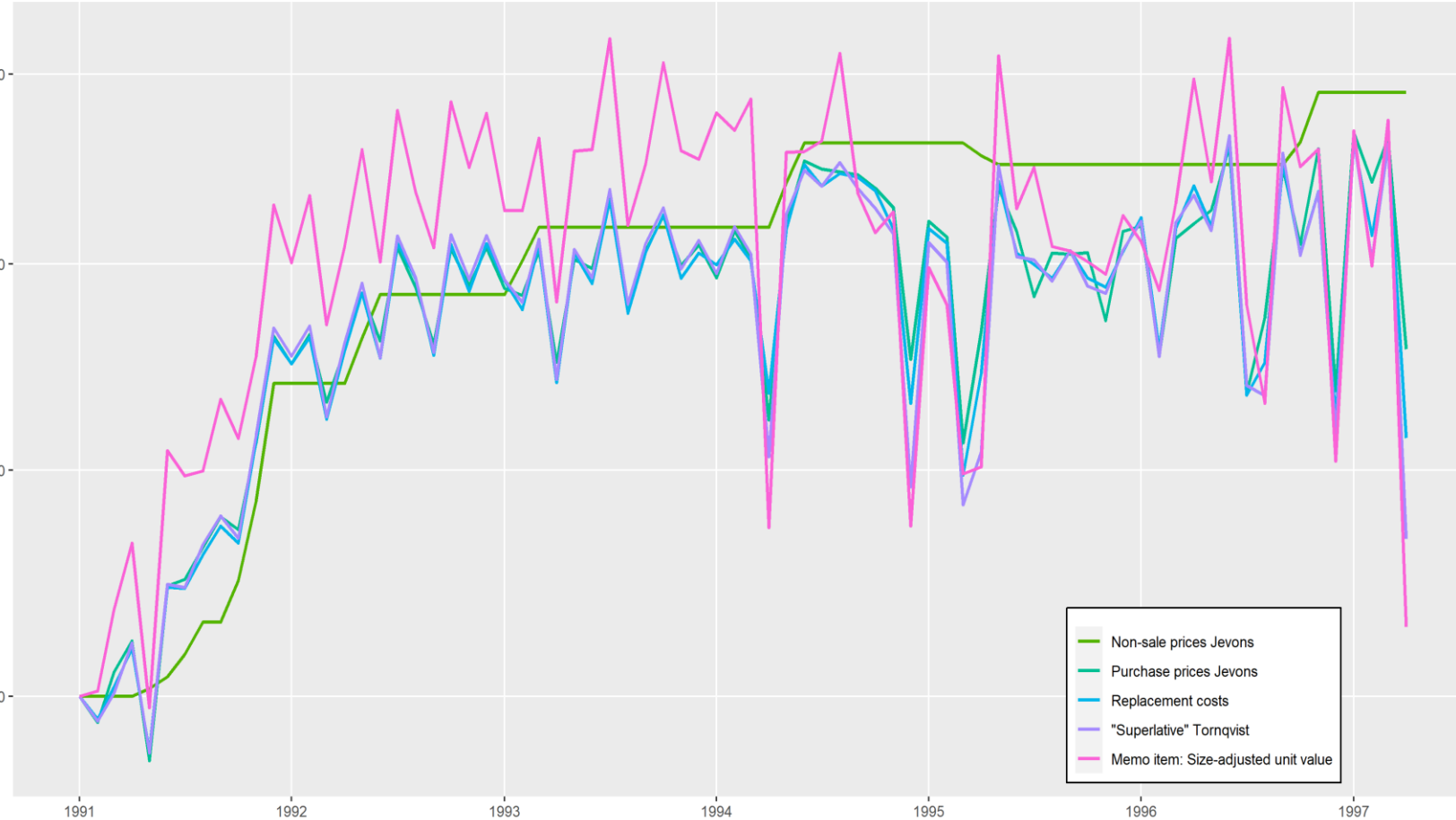
The second index, a **Tornqvist using *purchase prices and quantities***, is considered to be **"superlative"**.

The third index, using ***purchase prices but consumption quantities***, reflects **replacement costs**.

The fourth index is the geometric average of ***purchase prices***.

# Index number 5: Non-sale prices Jevons

GEKS indexes for Crest Tartar Protection Regular Paste  
January 1991 = 100, log scale



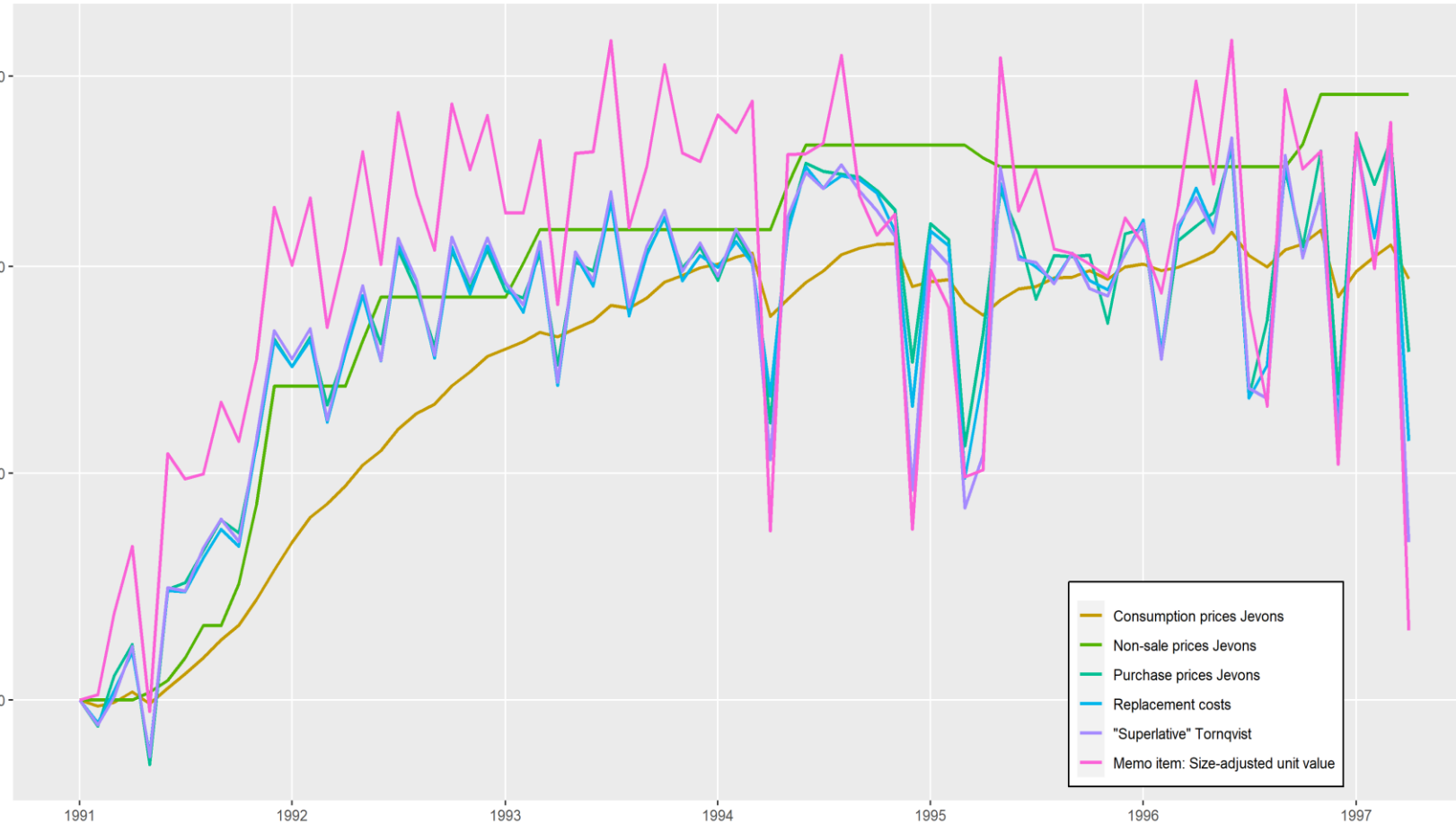
The fifth index takes the geometric average of *non-sale prices*.

Sources: Dominick's Finer Foods data set; and IMF staff calculations.



# Index number 6: Consumption prices Jevons

GEKS indexes for Crest Tartar Protection Regular Paste  
January 1991 = 100, log scale



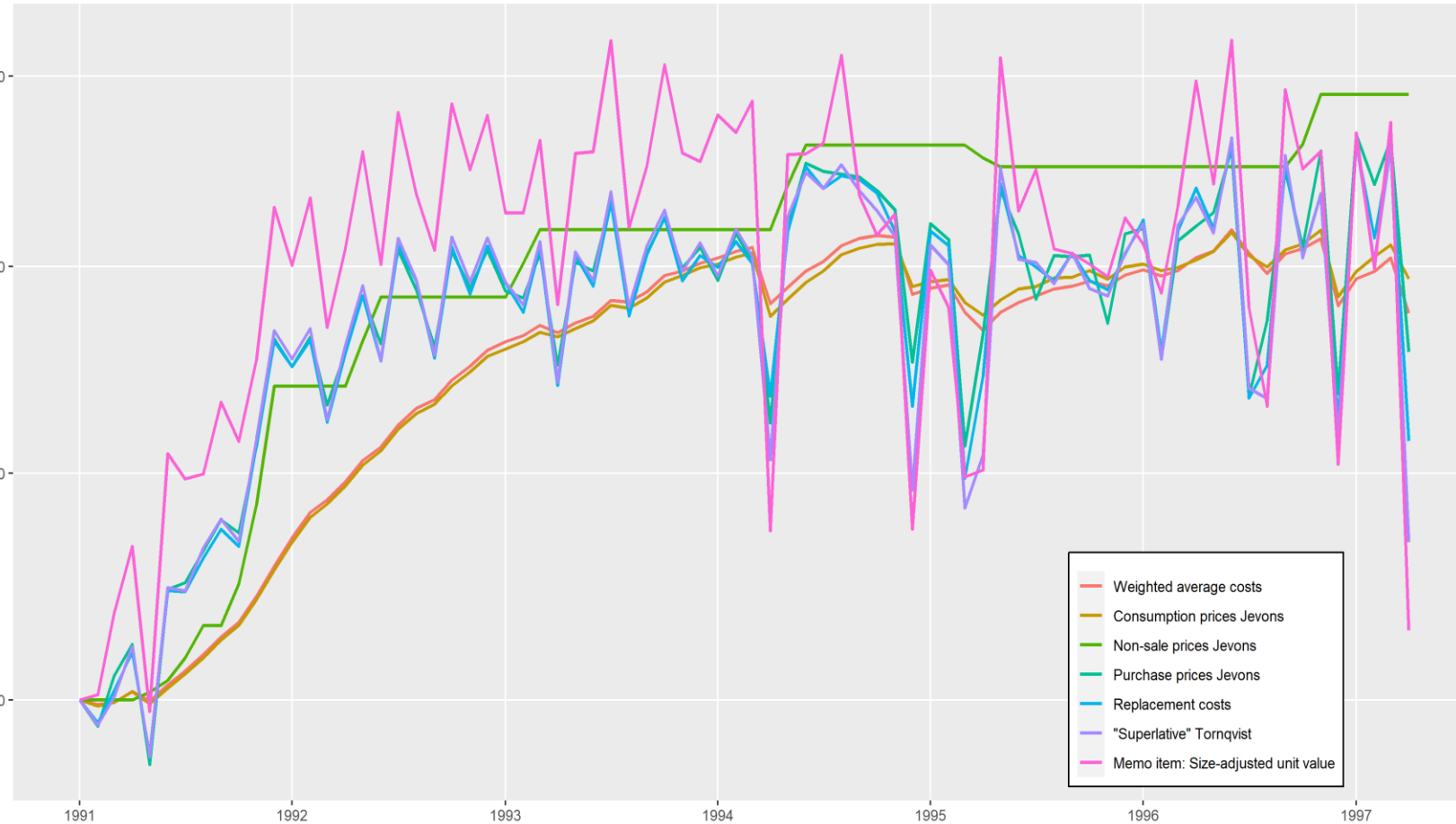
Sources: Dominick's Finer Foods data set; and IMF staff calculations.

The fifth index takes the geometric average of ***non-sale prices***.

The sixth index uses the geometric average of ***consumption prices***.

# Index number 7: Weighted average costs

GEKS indexes for Crest Tartar Protection Regular Paste  
January 1991 = 100, log scale



Sources: Dominick's Finer Foods data set; and IMF staff calculations.

The fifth index takes the geometric average of ***non-sale prices***.

The sixth index uses the geometric average of ***consumption prices***.

The seventh and final index, the ***weighted average costs***, is based on ***consumption prices and quantities***.

# Summary and outlook

The general pattern of the **new, preferred weighted average costs index** is that it is

- **increasing at a systematically lower rate** compared to e.g., the Tornqvist index – due to dampening effect of stockpiling; and
- **likewise, not as reactive to sale prices** – due to the sluggish adjustment of the average acquisition cost in the inventory.

Moreover, the “consumption” price index should **not be mistaken as just being smoothed, or lagged**; it still takes into account **sale prices and price increases**.

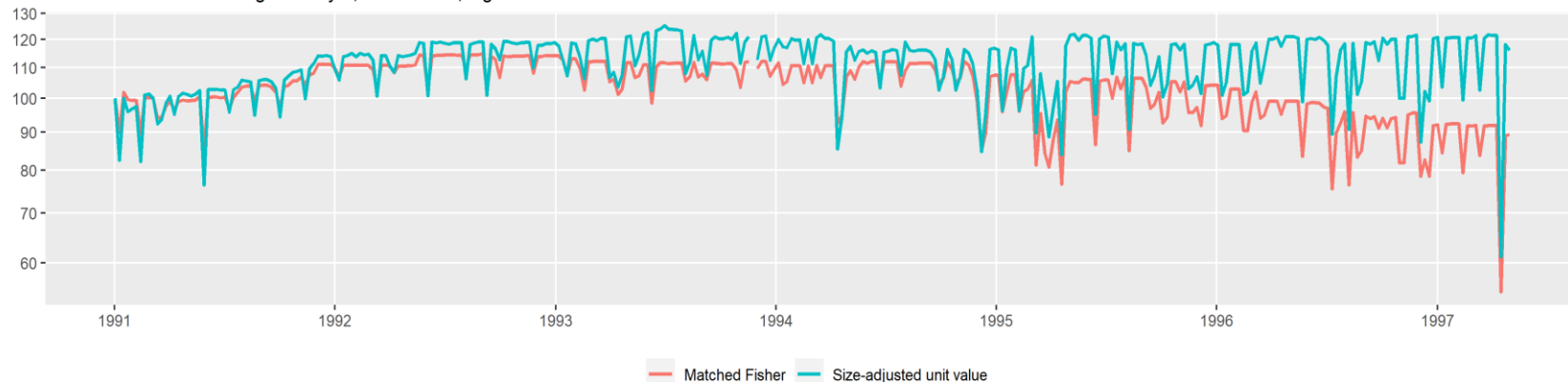
To some extent, the quantitative (but not the qualitative!) results are influenced by **assumptions about consumption, inventories, and (initial) prices**.

This contribution: Start the **search for a new yardstick for economic indexes**. → Is such an **index feasible without these assumptions** (like for “superlative” indexes)?

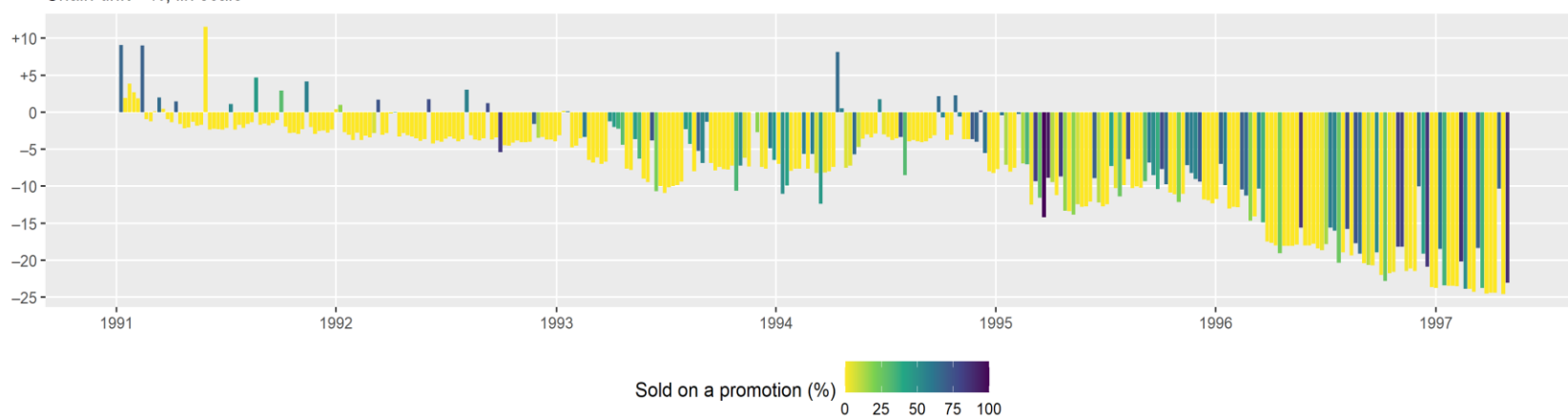
# Annex: A numerical example of chain drift in the matched Fisher index

Weekly Fisher and unit value indexes for Crest Tartar Protection Regular Paste\*

Price index—Week ending January 2, 1991 = 100, log scale



Chain drift—%, lin scale



\* No data for week ending November 24, 1993. Sources: Dominick's Finer Foods data set; and IMF staff calculations.

In case of promotional sales with reduced prices, the quantities purchased often increase substantially. But when the prices return to their original level, the **quantities purchased of storable goods may not return to their “normal” level.**

The **price change from the normal price to the reduced price has a bigger weight than vice versa.**

This type of asymmetric behavior can cause **chain drift in superlative price indices**, which is typically downward.